S&P500 Case Study

In this case study, we construct the efficient frontier of the top 40 stocks by weight in the S&P500 index and compared its efficient market return to the S&P500 index.

Introduction

The S&P500 Index is one of the mostly commonly quoted indices along with Dow Jones Industrial Average and Dow 30. Not only is the index an indicator of the economy, but it is also commonly used as benchmarks for investment strategies.

The general rule of diversification in investing says an investor should trade in stocks of many companies to minimize risk inherent to a single company and reap return from the general market growth. The question naturally arises as to which stocks an investor should choose and how much of each stock should an investor invest out of the hundreds of publicly traded stocks. This is the classical problem of portfolio optimization and has been studied as early as the 1940s.

While the problem has a closed analytical solution, the assumptions required for the solution often meant a difficult execution. In real world, constraints such as no short-selling or limited leverage makes adaptation of the analytical solution difficult. Hence, we will use numerical optimization techniques.



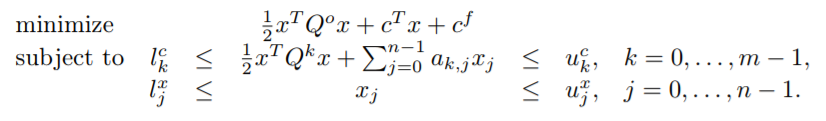
Method

Historical stock data is taken for the top 40 stocks by weight in the S&P500 index over a 10 year period from 05/01/2009 to 05/01/2019. End of Month closing prices are obtained from Yahoo Finance as CSV files and imported into MATLAB. It is noted that out of the 40 stocks, Facebook (FB), Berkshire Hathaway (BRKB) and PayPal (PYPL) have insufficient data either due to an intermittent IPO or corrupted data in the selected time horizon. The updated list is now 37 stocks. Also note that dividends, commission fees are not included in this analysis.

Monthly return rates are calculated and a mean monthly return over the 10 year horizon is obtained for each stock. Then the variances are calculated to obtain the 37x37 covariance matrix. For the mean-variance portfolio analysis, we have all the data needed to solve the problem at this stage.

The stochastic nature of the return can be visualized in Figure 2. While the variance distribution for the first 4 stocks are visualized in Figure 3. The density distribution shows an approximate form of a normal distribution.

From the MOSEK documentation for Quadratic Optimization, we need to cast our portfolio optimization problem into the standard form



Our portfolio optimization problem is defined as:

We need to cast the 2 constraints into the standard form for MOSEK:

as our 37x1 portfolio weight vector

as our 37x37 covariance matrix

as 37x37 zero matrix

as constraint matrix

as the 37x37 zero matrix

as zero

, the matrix of lower bound of constraints

, the matrix of upper bound of constraints

, the empty vector

, the empty vector

The above form is defined in MATLAB and mskqpopt()is executed to return 37x1 vector if a solution exist and return null vector otherwise.

Efficient Frontier

The efficient frontier is solved for both cases of any leverage and no leverage using the MOSEK solver. We use the method of fixed return and minimize variance to solve for the minimum possible risk at each return.

These are then plotted to yield the efficient frontier.



The dotted blue line represents the efficient frontier for any leverage (short-selling allowed for any quantity) and solid red curve is no leverage (no short-selling allowed). Any point on the curves represent a solution for the portfolio weights. The investor can then choose the target return based on the acceptable risk and obtain the portfolio weights as a 37 element vector.

Firstly, note that investing in any solution on the curves will yield a higher return than if one were to invest in an individual stock for the same risk.

Secondly, note that when no leverage is allowed, the solution set start and end at the stock with the maximum and minimum return.



Return Performance

Given the efficient frontier is constructed from EOM data from 05/01/2009 to 05/01/2019, and that we now have a new monthly close price for the month of May, we may compare the two investment strategies where one investor would invest in the S&P500 benchmark and another investor would invest based on the portfolio weights.

|  |  |
| --- | --- |
| Stock | % Change |
| AAPL | - 12.78 % |
| AMZN | - 7.86 % |
| GOOG | - 7.14 % |
| UNH | + 3.75 % |
| MA | - 1.08 % |
| BA | - 9.55 % |
| NFLX | - 7.36 % |
| MCD | + 0.35 % |
| UNP | - 5.8% |

Assuming the investor would accept a 5% risk and 2.5% return, the investor would choose a portfolio with the following stocks and weights (see Table 1). Monthly close for the 9 selected stocks during the month of May 2019 is also given (Table 2)

|  |  |
| --- | --- |
| Stock | Weight |
| AAPL | 0.1361 |
| AMZN | 0.0419 |
| GOOG | 0.0137 |
| UNH | 0.2816 |
| MA | 0.0905 |
| BA | 0.0352 |
| NFLX | 0.0449 |
| MCD | 0.1710 |
| UNP | 0.1850 |

This yields a gross return of -2.89 %, while the S&P500 benchmark has a return of -6.58%. This is a positive indicator of the optimized mean-variance portfolio strategy.