"A surrogate-assisted evolutionary algorithm using special points for expensive multi-objective optimization"supplementary materials

This is the supplementary materials of "A surrogate-assisted evolutionary algorithm using special points for expensive multi-objective optimization"

I. RVEA

For MOPs, scholars have proposed a lot of evolutionary algorithms [1]. RVEA has been widely used for multi-objective optimization problems, since it can balance the convergence and diversity of population very well [2], [3]. RVEA uses the simplex lattice method to get the set of uniformly distributed reference vectors, based on which the whole objective space is divided into multiple subspaces. Then, according to the angle between individual and reference vector, all individuals are assigned to corresponding reference vectors.

Figure S1 shows an example with two reference vectors(v_i and v_{i+1}) and three individuals ($\bar{f}^1$,$\bar{f}^2$ and $\bar{f}^3$). Here, the angle between $\bar{f}^1$ and v_i is θ_i^1, and the angle between $\bar{f}^1$ and v_{i+1} is θ_{i+1}^1. Since$\theta_i^1 < \theta_{i+1}^1$, the individual $\bar{f}^1$ is assigned to the reference vector v_i. Similarly, $\bar{f}^2$ and $\bar{f}^3$ are assigned to the reference vector v_{i+1}.

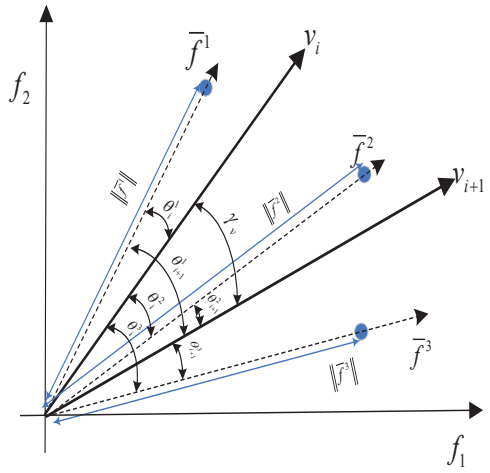


Fig. S1: Example for individuals assigned to reference vectors

Specifically, RVEA uses the angle penalized distance (APD) to select appropriate individuals for each reference vector. For the current reference vector, the smaller the APD value of an individual, the higher the probability that it is selected. The specific formula of APD is as follows:

$$d^j = \left(1 + \varphi\left(\theta^j\right)\right) \cdot \left\|\bar{f}^j\right\| \tag{1}$$

where, $\left\|\bar{f}^j\right\|$ is the distance between the j-th individual and the origin in the objective space. θ^j represents the angle between v_j and the objective vector $\bar{f}^j$; $\varphi\left(\theta^j\right)$ is a penalty function, whose value is $\mathrm{M} \cdot \left(t/t_{\max}\right)^{\alpha} \cdot \theta^j / \gamma_{v_j}$, where, α is the user-specified parameter, and γ_{v_j} is the angle of other reference vectors that form the smallest angle with v_j.

In order to find a group of non dominated individuals with good convergence and diversity, it is necessary to adaptively adjust the reference vectors in iteration process. Aimed at the issue, the reference [2] provides an adaptively updating method of reference vector, as follows:

$$v_{t+1,i} = \frac{v_{0,i} \circ \left(z_t^{\max} - z_t^{\min}\right)}{\left\|v_{0,i} \circ \left(z_t^{\max} - z_t^{\min}\right)\right\|} \tag{2}$$

where, " $\circ$" represents the Hadamard product; $v_{0,i}$ is the initial reference vector; $z_t^{\max}$ and $z_t^{\min}$ are the maximum and minimum of each objective function obtained by the population respectively.

II. Part of the figures and tables

In order to ensure the simple and beautiful layout of the main text, we display Fig.S1-Fig.S8 and Table S1-Table S12 in this section.

TABLE S1: Characteristics of each test function

Name	Number of objectives	Characteristic
DTLZ2	2,3	normal
DTLZ5	2,3	degenerated
DTLZ7	2,3	discontinuous
IMOP1	2	strongly convex
IMOP2	2	strongly concave
IMOP3	2	discontinuous
IMOP4	3	concave-convex
IMOP5	3	discontinuous
IMOP6	3	discontinuous
IMOP7	3	Mixed
IMOP8	3	discontinuous
WFG1	2,3	concave-convex
WFG2	2,3	discontinuous
WFG3	2,3	degenerated
WFG4	2,3	Deceptive
WFG5	2,3	normal
WFG6	2,3	normal
WFG7	2,3	normal
WFG8	2,3	normal
WFG7	2,3	Deceptive

III. Experiments

TABLE S2: HV values obtained by DK-SAMOEA and 7 comparison algorithms on DTLZs when D=10

Problem	Nf	ABSAEA	CSEA	EDN-ARMOEA	He-EMOEA	MCEAD	ParEGO	REMO	DK-SAMOEA
DTLZ2	2	3.3047e-1	2.5167e-1	2.4188e-1	2.0119e-1	2.7580e-1	3.0908e-1	2.6778e-1	**3.3286e-1**
		1.40e-3≈	2.44e-2+	2.03e-2+	2.77e-2+	1.61e-2+	6.26e-3+	3.07e-2+	1.22e-3
	3	2.4667e-1	1.4782e-1	3.6075e-4	1.9124e-1	2.0423e-1	1.0723e-2	1.9377e-1	**5.0616e-1**
		6.97e-2+	5.82e-2+	5.37e-4+	4.43e-2+	4.18e-2+	1.36e-2+	5.64e-2+	9.82e-3
DTLZ5	2	3.3263e-1	2.4426e-1	2.1340e-1	2.0764e-1	2.9800e-1	3.0105e-1	2.5565e-1	**3.4336e-1**
		7.08e-3+	2.27e-2+	4.32e-2+	1.48e-2+	8.49e-3+	4.17e-3+	3.21e-2+	1.40e-3
	3	**2.6804e-1**	1.2164e-1	1.3520e-2	1.0134e-1	2.1504e-1	9.3121e-2	1.8757e-1	2.4310e-1
		1.18e-2-	5.03e-2+	8.06e-3+	4.36e-2+	2.72e-2+	5.20e-2+	1.22e-2+	4.24e-3
DTLZ7	2	2.3800e-1	1.5520e-1	7.7451e-2	1.9234e-4	7.6971e-2	2.1359e-1	1.6023e-1	**8.0990e-1**
		1.75e-3+	4.31e-2+	2.60e-2+	3.57e-4+	6.47e-2+	4.40e-3+	2.52e-2+	1.48e-4
	3	1.9459e-1	4.7493e-2	1.6158e-2	/	/	8.9808e-2	1.3412e-1	**8.6586e-1**
		3.79e-2+	4.26e-2+	1.03e-2+	/	/	3.89e-2+	2.32e-2+	1.33e-3
win/tie/lose		4/1/1	6/0/0	6/0/0	6/0/0	6/0/0	6/0/0	6/0/0	–

TABLE S3: IGD values obtained by DK-SAMOEA and 7 comparison algorithms on DTLZs when D=10

Problem	Nf	ABSAEA	CSEA	EDN-ARMOEA	He-EMOEA	MCEAD	ParEGO	REMO	DK-SAMOEA
DTLZ2	2	1.4677e-2	7.7719e-2	1.0735e-1	1.1526e-1	5.3867e-2	3.8890e-2	7.1117e-2	**1.4049e-2**
		1.32e-3≈	1.75e-2+	2.39e-2+	2.46e-2+	1.00e-2+	4.69e-3+	2.00e-2+	6.16e-4
	3	2.2670e-1	3.2945e-1	7.4434e-1	2.5361e-1	3.1671e-1	5.9504e-1	2.8752e-1	**7.6875e-2**
		3.68e-2+	4.29e-2+	4.85e-2+	2.19e-2+	5.60e-2+	4.60e-2+	4.72e-2+	5.60e-3
DTLZ5	2	1.8110e-2	8.3195e-2	1.2729e-1	1.0533e-1	3.6982e-2	3.8427e-2	7.2668e-2	**1.3873e-2**
		6.59e-3+	1.40e-2+	2.97e-2+	1.86e-2+	5.84e-3+	6.06e-3+	1.61e-2+	1.10e-3
	3	**5.6753e-2**	1.7716e-1	4.9584e-1	1.9168e-1	1.1071e-1	2.1053e-1	1.1538e-1	7.4509e-2
		7.66e-3-	4.20e-2+	2.94e-2	3.77e-2+	1.74e-2+	5.69e-2+	6.63e-3+	2.37e-3
DTLZ7	2	2.7583e-2	3.4843e-1	7.2872e-1	1.5230e+0	6.7968e-1	6.4911e-2	1.9750e-1	**2.3195e-2**
		4.46e-3+	2.98e-1+	2.58e-1+	3.07e-1+	4.15e-1+	8.25e-3+	6.15e-2+	1.05e-2
	3	2.9323e-1	3.5351e-1	1.0111e+0	2.2453e+0	3.2980e+0	2.8353e-1	3.4787e-1	**2.7635e-1**
		2.51e-1+	6.14e-1+	4.54e-1+	2.29e-1+	7.07e-1+	1.36e-1≈	6.82e-2+	9.39e-2
win/tie/lose		4/1/1	6/0/0	6/0/0	6/0/0	6/0/0	5/1/0	6/0/0	–

TABLE S4: HV values obtained by DK-SAMOEA and 7 comparison algorithms on DTLZs when D=20

Problem	Nf	ABSAEA	CSEA	EDN-ARMOEA	He-EMOEA	MCEAD	ParEGO	REMO	DK-SAMOEA
DTLZ2	2	1.9124e-1	6.3787e-2	1.6248e-2	8.2663e-2	1.6541e-1	4.5133e-2	1.7208e-1	**3.1775e-1**
		7.39e-2+	4.29e-2+	1.15e-2+	3.49e-2+	9.60e-3+	4.10e-2+	2.24e-2+	2.87e-3
	3	2.4667e-1	1.4782e-1	3.6075e-4	1.9124e-1	2.0423e-1	1.0723e-2	1.9377e-1	**4.7791e-1**
		6.97e-2+	5.82e-2+	5.37e-4+	4.43e-2+	4.18e-2+	1.36e-2+	5.64e-2+	1.30e-2
DTLZ5	2	1.6251e-1	6.5450e-2	1.4740e-2	5.7180e-2	1.7129e-1	9.1738e-2	1.6272e-1	**3.1910e-1**
		2.85e-2+	2.36e-2	1.51e-2+	3.48e-2+	2.24e-2 +	3.61e-2 +	1.91e-2+	1.82e-3
	3	**2.6804e-1**	1.2164e-1	1.3520e-2	1.0134e-1	2.1504e-1	9.3121e-2	1.8757e-1	2.6088e-1
		1.18e-2≈	5.03e-2+	8.06e-3+	4.36e-2+	2.72e-2+	5.20e-2+	1.22e-2+	4.37e-3
DTLZ7	2	1.8304e-1	2.3945e-2	1.2755e-2	/	/	3.9436e-2	1.0033e-1	**8.0682e-1**
		3.14e-2+	1.51e-2+	8.32e-3+	/	/	4.25e-2+	2.97e-2+	8.10e-4
	3	1.9459e-1	4.7493e-2	1.6158e-2	/	/	8.9808e-2	1.3412e-1	**8.6212e-1**
		3.79e-2+	4.26e-2+	1.03e-2+	/	/	3.89e-2+	2.32e-2+	8.51e-4
win/tie/lose		5/1/0	6/0/0	6/0/0	6/0/0	6/0/0	6/0/0	6/0/0	–

TABLE S5: IGD values obtained by DK-SAMOEA and 7 comparison algorithms on DTLZs when D=20

Problem	Nf	ABSAEA	CSEA	EDN-ARMOEA	He-EMOEA	MCEAD	ParEGO	REMO	DK-SAMOEA
DTLZ2	2	1.1730e-1	2.5244e-1	5.0090e-1	2.3086e-1	1.6917e-1	2.9260e-1	1.2797e-1	**2.4687e-2**
		6.24e-2+	7.95e-2+	6.24e-2 +	4.12e-2+	2.10e-2+	4.99e-2	1.83e-2+	1.38e-3
	3	2.2670e-1	3.2945e-1	7.4434e-1	2.5361e-1	3.1671e-1	5.9504e-1	2.8752e-1	**8.9483e-2**
		3.68e-2+	4.29e-2+	4.85e-2+	2.19e-2+	5.60e-2+	4.60e-2+	4.72e-2+	5.61e-3
DTLZ5	2	1.3539e-1	2.4917e-1	4.9200e-1	2.2072e-1	1.5847e-1	2.1795e-1	1.3926e-1	**2.3141e-2**
		2.49e-2+	2.59e-2+	4.49e-2+	3.37e-2+	3.01e-2+	4.41e-2+	2.25e-2+	2.18e-3
	3	5.6753e-2	1.7716e-1	4.9584e-1	1.9168e-1	1.1071e-1	2.1053e-1	1.1538e-1	**5.4464e-2**
		7.66e-3≈	4.20e-2+	2.94e-2+	3.77e-2+	1.74e-2+	5.69e-2+	6.63e-3+	7.77e-3
DTLZ7	**2**	**3.4086e-1**	1.0782e+0	9.9448e-1	2.6858e+0	3.3967e+0	6.8842e-1	3.5514e-1	1.1452e+0
		2.23e-1-	7.02e-1 -	2.20e-1 -	2.33e-1+	1.22e+0+	4.10e-1-	1.06e-1-	9.16e-2
	3	2.9323e-1	8.5351e-1	1.0111e+0	2.2453e+0	3.2980e+0	3.7353e-1	2.4787e-1	**1.7611e-1**
		2.51e-1+	6.14e-1+	4.54e-1+	2.29e-1+	7.07e-1+	1.36e-1+	6.82e-2+	1.77e-01
win/tie/lose		4/1/1	5/0/1	5/0/1	6/0/0	6/0/0	5/0/1	5/0/1	–

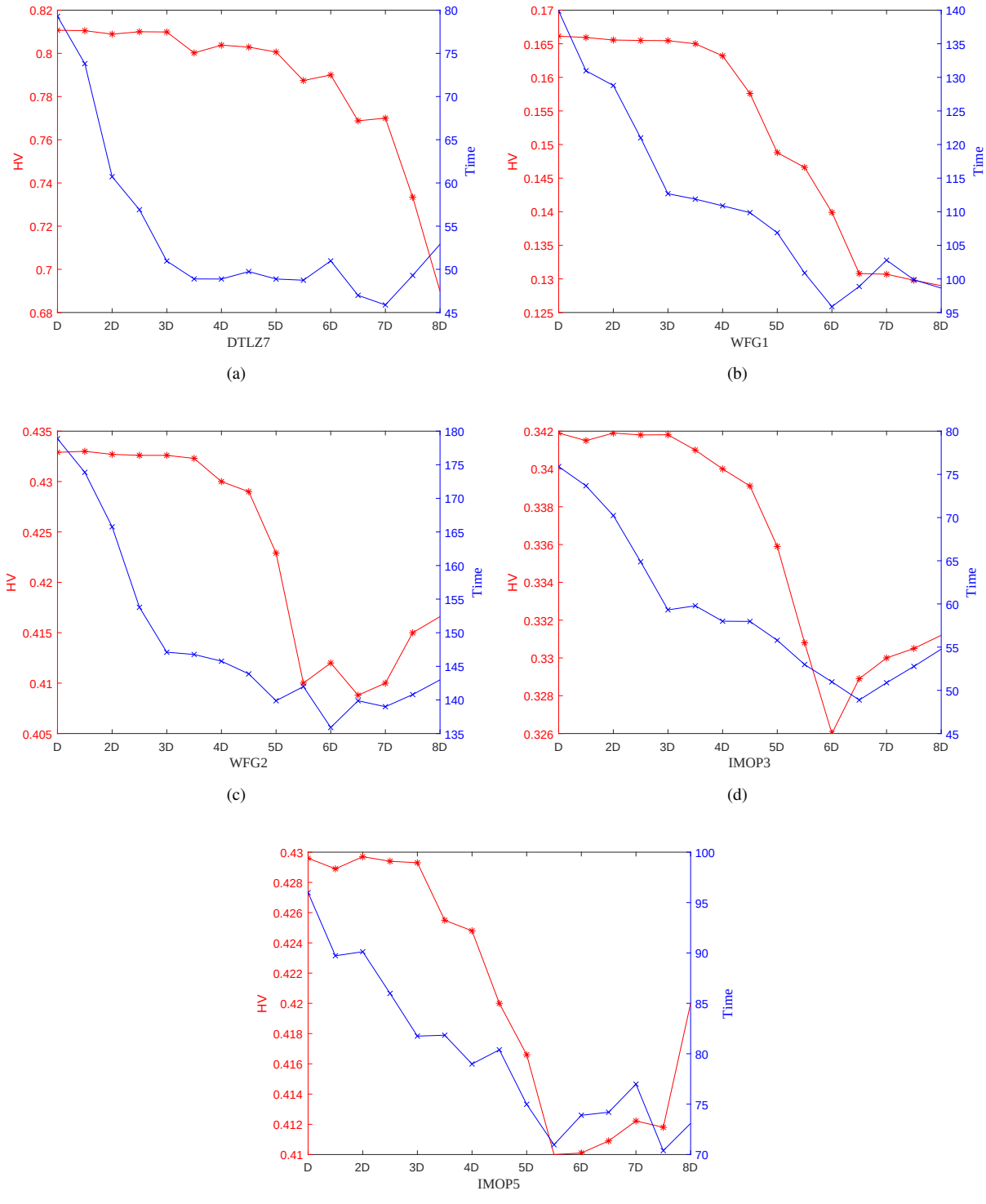
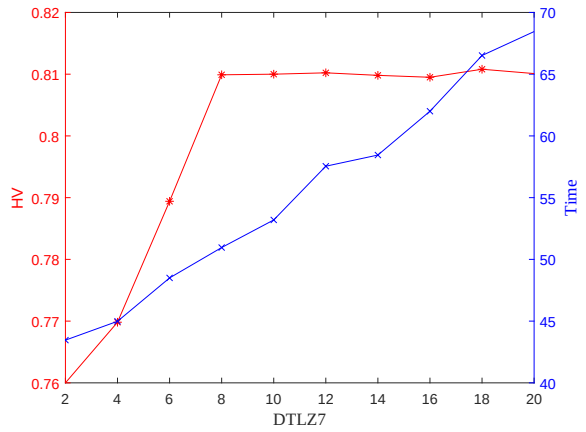
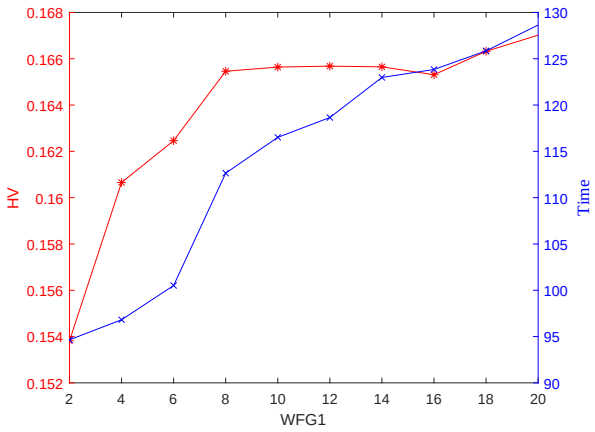


Fig. S2: HV values and running time obtained by DK-SAMOEA under different N_{SA}



(a)



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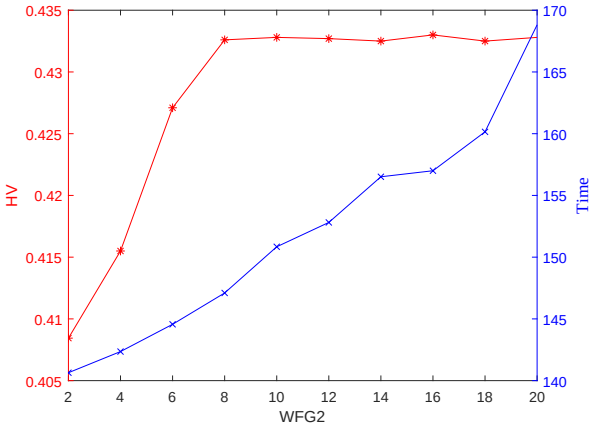

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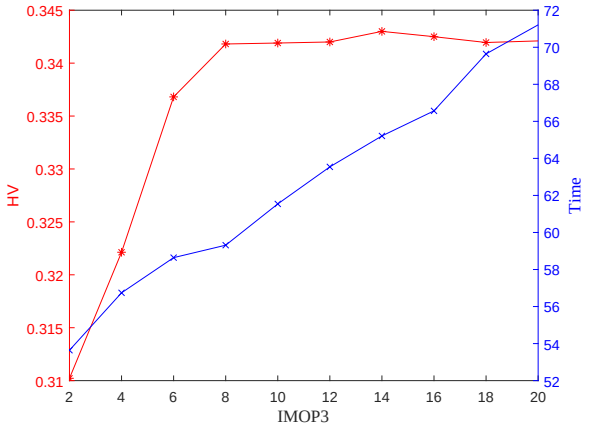

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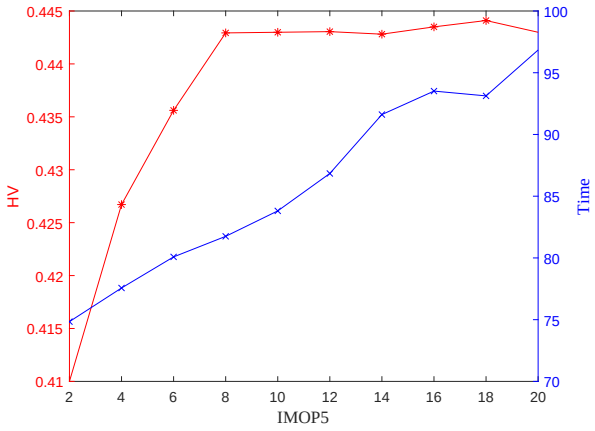

(e)

Fig. S3: HV values and running time obtained by DK-SAMOEA under different max_it

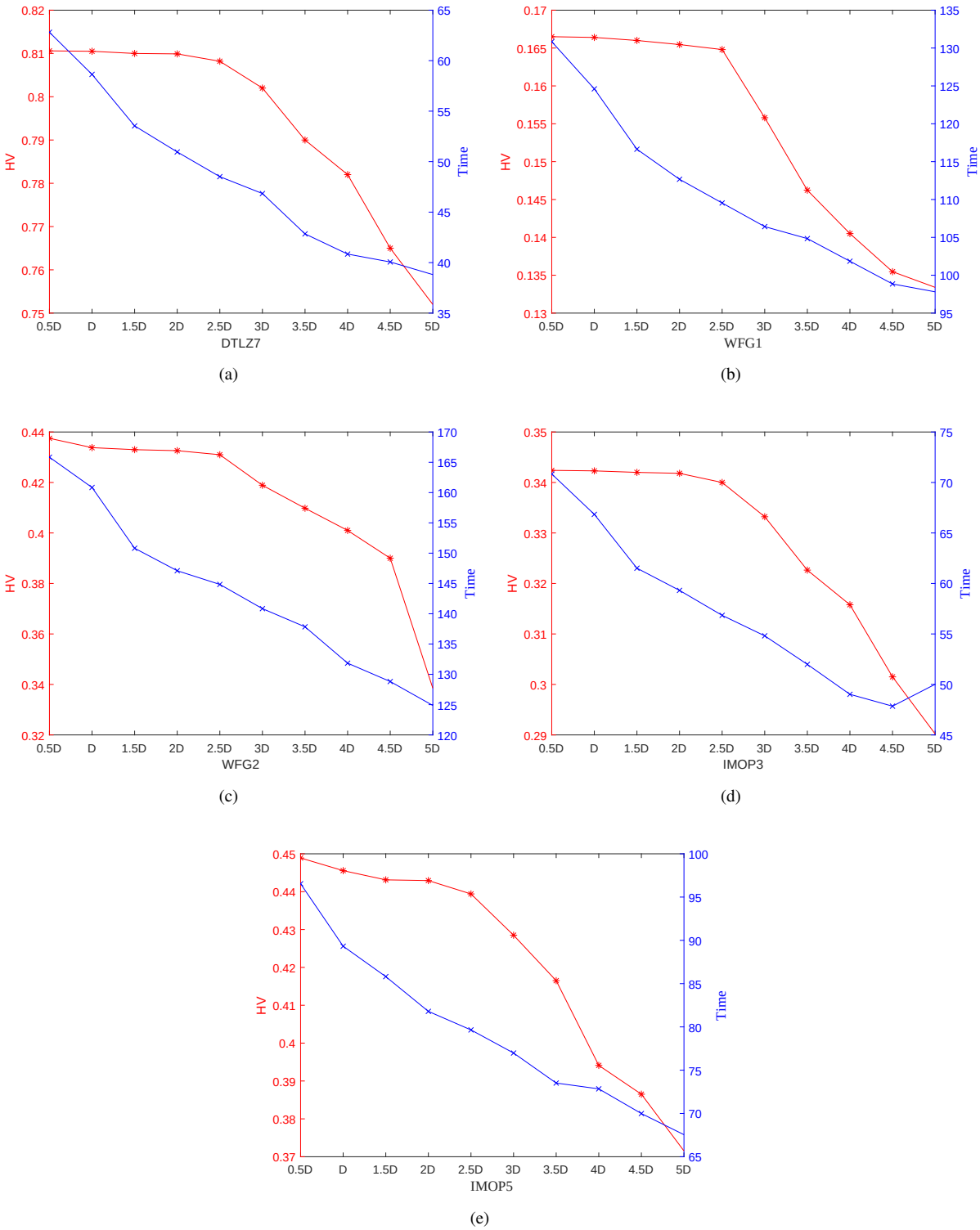

Fig. S4: HV values and running time obtained by DK-SAMOEA under different $NumUp$

TABLE S6: HV values obtained by DK-SAMOEA and 7 comparison algorithms on WFGs when D=10

Problem	Nf	ABSAEA	CSEA	EDN-ARMOEA	He-EMOEA	MCEAD	ParEGO	REMO	DK-SAMOEA
WFG1	2	1.4000e-1	1.5330e-1	1.2843e-1	8.4644e-2	1.5369e-1	1.5135e-1	1.5551e-1	**1.6546e-1**
		5.52e-2+	7.24e-3≈	1.78e-2+	6.82e-3+	2.85e-3≈	9.85e-3+	2.82e-2≈	2.14e-03
	3	2.7658e-1	3.0308e-1	2.6227e-1	1.7267e-1	2.7314e-1	2.5485e-1	3.0958e-1	**3.1113e-1**
		6.19e-2+	2.89e-2≈	1.07e-2+	3.52e-2+	1.96e-3+	1.22e-2+	4.52e-2≈	1.24e-2
WFG2	2	**4.4631e-1**	4.5142e-1	3.5653e-1	3.9283e-1	3.6630e-1	3.5607e-1	3.9701e-1	4.3026e-1
		3.43e-2-	4.02e-2-	1.14e-2+	1.62e-2+	6.24e-3+	6.93e-3+	1.66e-2+	3.83e-2
	3	**8.1362e-1**	7.5195e-1	6.6869e-1	6.8389e-1	7.0555e-1	7.9370e-1	7.6476e-1	7.5542e-1
		1.77e-2-	3.14e-2≈	2.93e-2+	2.06e-2+	5.06e-2+	2.57e-2+	4.75e-2≈	3.96e-02
WFG3	2	5.2817e-1	4.7442e-1	4.5131e-1	4.5284e-1	4.1823e-1	4.6670e-1	4.3287e-1	**5.3448e-1**
		1.49e-2≈	2.38e-2+	1.05e-2+	1.08e-2+	2.84e-1+	6.29e-3+	3.7e-2+	3.89e-3
	3	2.7855e-1	2.0487e-1	1.6487e-1	2.2773e-1	1.9077e-1	2.4377e-1	2.4923e-1	**2.9282e-1**
		1.36e-2≈	2.79e-2+	1.12e-2+	6.73e-2+	1.61e-2+	1.90e-2+	2.60e-2+	4.90e-3
WFG4	2	2.4213e-1	2.3344e-1	2.2579e-1	2.0920e-1	2.2355e-1	2.4055e-1	2.4200e-1	**2.6438e-1**
		5.08e-3+	3.27e-3+	1.06e-2+	4.84e-3+	1.14e-2+	1.00e-2+	1.23e-2+	4.15e-3
	3	4.4164e-1	3.9514e-1	3.7394e-1	2.9587e-1	3.5755e-1	3.4071e-1	4.2146e-1	**4.5170e-1**
		1.51e-2≈	2.37e-2+	1.47e-2+	1.93e-2+	2.77e-2+	9.14e-3+	1.09e-2≈	9.82e-3
WFG5	2	**2.9611e-1**	2.4898e-1	2.3028e-1	1.9615e-1	2.6692e-1	2.8860e-1	2.956e-1	2.8790e-1
		7.62e-3≈	2.10e-2+	8.35e-3+	1.01e-2+	1.118e-2+	1.43e-2+	1.15e-2+	2.16e-3
	3	4.5186e-1	4.4084e-1	3.0392e-1	2.9216e-1	3.7643e-1	3.4312e-1	4.0700e-1	**4.6012e-1**
		1.70e-2≈	2.31e-2≈	1.19e-2+	2.95e-2+	2.18e-2+	1.50e-2+	1.77e-2+	1.00e-2
WFG6	2	1.9378e-1	2.0530e-1	1.7624e-1	1.7285e-1	1.8719e-1	1.7307e-1	1.8146e-1	**2.5639e-1**
		3.62e-2+	2.18e-2+	2.41e-2+	9356e-3+	2.84e-2+	1.68e-2+	2.22e-2+	1.40e-2
	3	3.5716e-1	2.7763e-1	2.3313e-1	2.3630e-1	2.5870e-1	2.6160e-1	2.7590e-1	**4.1518e-1**
		5.27e-2+	2.15e-2+	7.49e-3+	5.06e-2+	2.96e-2+	2084e-2+	2.65e-2+	2.43e-2
WFG7	2	**2.4479e-1**	2.1463e-1	1.9994e-1	1.5865e-1	2.3984e-1	2.3382e-1	2.4335e-1	2.3965e-1
		1.06e-2≈	7.82e-2+	7.99e-3+	8.20e-3+	1.46e-2≈	1.53e-2≈	9.32e-3+	3.50e-3
	3	4.1138e-1	3.2554e-1	2.8203e-1	3.0333e-1	3.6009e-1	3.1184e-1	3.5941e-1	**4.2139e-1**
		2.77e-2≈	1..46e-2+	6.73e-3+	2.29e-2+	3.47e-2+	1.27e-2+	2.04e-2+	5.76e-3
WFG8	2	1.5393e-1	1.5829e-1	1.4157e-1	1.616e-1	1.5071e-1	1.4745e-1	1.5235e-1	**1.5985e-1**
		1.02e-1≈	3.78e-3≈	1.94e-2+	5.14e-3≈	5.41e-1≈	1.08e-2+	16.89e-3≈	7.49e-3
	3	1.5393e-1	1.5829e-1	1.4157e-1	1.4516e-1	1.5071e-1	1.4745e-1	1.5234e-1	**2.3157e-1**
		1.02e-2+	3.78e-3+	1.94e-2+	5.14e-3+	5.41e-3+	1.08e-2+	6.89e-3+	2.16e-2
WFG9	2	2.0708e-1	2.4044e-1	1.9142e-1	1.9037e-1	2.4032e-1	2.2239e-1	2.1067e-1	**2.6479e-1**
		2.90e-2+	2.49e-2+	3.04e-2+	1.41e-2+	1.97e-2+	1.97e-2+	2.23e-2+	2.15e-2
	3	**3.4922e-1**	3.3319e-1	2.8334e-1	2.7711e-1	3.3060e-1	2.7590e-1	3.2850e-1	3.2811e-1
		6.61e-2-	2.46e-2-	2.78e-2+	1.97e-2+	1.10e-2≈	2.74e-2+	5.20e-2≈	3.72e-2
win/tie/lose		7/8/3	11/5/2	18/0/0	17/1/0	14/4/0	17/1/0	12/6/0	–

TABLE S7: IGD values obtained by DK-SAMOEA and 7 comparison algorithms on WFGs when D=10

Problem	Nf	ABSAEA	CSEA	EDN-ARMOEA	He-EMOEA	MCEAD	ParEGO	REMO	DK-SAMOEA
WFG1	2	1.3372e+0	1.3421e+0	1.3188e+0	1.3936e+0	1.3954e+0	1.300e+0	1.2823e+0	**1.2469e+0**
		1.54e-1+	4.79e-2+	2.84e-2+	3.91e-2+	5.71e-3+	1.14e-2+	1.05e-1+	3.23e-2
	3	1.5997e+0	**1.4386e+0**	1.6216e+0	1.8220e+0	1.6067e+0	1.6388e+0	1.4783e+0	1.4525e+0
		2.11e-1+	8.83e-2-	3.12e-2+	1.47e-1+	1.98e-2+	3.53e-2+	1.37e-1≈	3.31e-2
WFG2	2	4.0251e-1	4.0248e-1	4.1253e-1	**3.8441e-1**	5.8842e-1	4.0670e-1	4.0820e-1	4.1067e-1
		2.23e-1-	7.17e-2≈	3.38e-2+	2.69e-2-	1.06e-1+	4.14e-2≈	3.11e-2≈	1.24e-1
	3	**3.0270e-1**	4.5656e-1	5.1076e-1	5.2008e-1	5.5825e-1	5.4158e-1	5.5211e-1	5.6696e-1
		3.93e-2-	5.00e-2-	5.21e-2+	4.99e-2+	1.12e-1≈	4.27e-2-	6.24e-2≈	1.36e-1
WFG3	2	9.6905e-2	2.0925e-1	2.8745e-1	2.3765e-1	2.5681e-1	2.1397e-1	2.8380e-1	**9.1890e-2**
		2.94e-2≈	3.44e-2+	3.34e-2+	1.74e-2+	8369e-2+	1.07e-2+	4.52e-2+	5.27e-3
	3	2.7976e-1	4.9179e-1	5.8944e-1	4.00e-1	4.5565e-	4.3874e-1	3.8245e-1	**2.7151e-1**
		3.41e-2≈	5.97e-2+	2.22e-2+	1.69e-2+	7.21e-2+	6.49e-2+	5.82e-2+	1.04e-2
WFG4	2	1.4930e-1	1.7442e-1	2.3334e-1	5.3449e-1	2.0522e-1	2.0843e-1	2.0040e-1	**1.4411e-1**
		1.14e-2≈	1.79e-2+	2.16e-2+	6.80e-2+	3.12e-2+	1.49e-2+	3.16e-2+	8.26e-3
	3	**3.2263e-1**	4.2865e-1	4.3940e-1	8.0185e-1	4.6971e-1	4.9625e-1	3.7330e-1	3.5186e-1
		1.22e-2-	9.68e-2+	2.85e-2+	1.32e-1+	5.07e-2+	2.91e-2+	2.14e-2≈	1.77e-2
WFG5	2	1.0940e-1	2.1609e-1	2.6941e-1	4.3977e-1	1.9231e-1	2.2139e-1	1.6541e-1	**1.0442e-1**
		1.42e-2≈	4.78e-2+	3.79e-2+	4.60e-2+	3.45e-2+	2.74e-2+	2.76e-2+	4.80e-3
	3	**2.8297e-1**	3.1758e-1	5.6863e-1	6.4061e-1	3.3751e-1	3.6900e-1	3.8116e-1	3.8714e-1
		4.04e-2-	2.38e-2+	2.17e-2+	7.32e-2+	6.14e-2+	3.16e-2+	3.60e-2≈	1.20e-2
WFG6	2	3.3563e-1	3.4141e-1	4.2978e-1	5.0147e-1	3.6704e-1	3.5890e-1	3.2307e-1	**1.6694e-1**
		1.09e-1+	8.28e-2+	5.36e-2+	2.83e-2+	8.85e-2+	4.90e-2+	4.48e-2+	1.72e-2
	3	4.8620e-1	6.1400e-1	7.3018e-1	8.9781e-1	6.6203e-1	6.7517e-1	6.3255e-1	**4.1673e-1**
		8.28e-2+	4.33e-2+	2.50e-2+	4.33e-2+	6.15e-2+	2.10e-2+	3.96e-2+	4.56e-2
WFG7	2	2.6248e-1	3.4489e-1	3.4774e-1	4.5996e-1	3.5570e-1	2.4646e-1	3.9161e-1	**2.3470e-1**
		2.36e-2+	5.14e-2+	4.57e-2+	6.47e-2+	1.08e-1+	2.24e-2≈	9.21e-2+	5.91e-3
	3	**4.0078e-1**	5.7975e-1	6.2847e-1	6.6638e-1	4.9782e-1	5.8943e-1	4.9143e-1	4.0307e-1
		4.25e-2≈	5.49e-2+	2.10e-2+	1.25e-1+	8.52e-2+	4.64e-2+	4.05e-2+	1.04e-2
WFG8	2	5.6896e-1	6.8086e-1	7.4772e-1	6.5981e-1	6.4492e-1	5.4051e-1	6.3567e-1	**5.3325e-1**
		4.59e-2+	9.28e-2+	6.63e-2+	3.41e-2+	3.12e-2+	4.53e-2≈	5.28e-2+	2.19e-2
	3	5.6896e-1	6.0886e-1	7.4772e-1	6.5981e-1	6.4492e-1	**5.4236e-1**	6.3567e-1	5.9654e-1
		4.59e-2-	9.28e-2≈	6.63e-2+	3.49e-2+	3.12e-2+	4.93e-2-	5.28e-2+	4.06e-2
WFG9	2	3.1423e-1	3.7676e-1	3.6744e-1	4.1591e-1	2.4703e-1	3.1525e-1	3.3393e-1	**1.6141e-1**
		3.86e-2+	5.69e-2+	7.59e-2+	1.65e-2+	3.25e-2+	7.67e-2+	7.16e-2+	4.95e-2
	3	5.0693e-1	5.3014e-1	6.7512e-1	7.4299e-1	5.7219e-1	6.7922e-1	5.8123e-1	**5.0149e-1**
		1.25e-1≈	4.39e-2+	8.24e-2+	8.15e-2+	3.97e-+2+	8.14e-2+	1.38e-1+	5.96e-2
win/tie/lose		7/6/5	14/2/2	18/0/0	17/0/1	17/1/0	13/3/2	13/5/0	–

TABLE S8: HV values obtained by DK-SAMOEA and 7 comparison algorithms on WFGs when D=20

Problem	Nf	ABSAEA	CSEA	EDN-ARMOEA	He-EMOEA	MCEAD	ParEGO	REMO	DK-SAMOEA
WFG1	2	1.1501e-1	1.6750e-1	1.1594e-1	5.9296e-2	1.5364e-1	1.2848e-1	1.8469e-1	**1.8679e-1**
		3.98e-2+	5.65e-3+	1.99e-2+	3.77e-2+	5.85e-3+	2.02e-2+	9.94e-3≈	2.85e-3
	3	2.4358e-1	2.5420e-1	2.4934e-1	1.3947e-1	2.4804e-1	2.4158e-1	2.6910e-1	**2.8089e-1**
		1.24e-2+	1.45e-2+	5.17e-3+	5.17e-2+	7.28e-3+	2.00e-2+	3.70e-2+	2.74e-3
WFG2	2	**4.5887e-1**	4.2464e-1	3.6844e-1	4.2390e-1	4.3429e-1	3.8600e-1	4.4958e-1	4.1500e-1
		3.90e-2-	3.22e-2+	1.71e-2+	1.42e-2-	1.68e-2-	1.64e-2+	1.50e-2-	2.95e-2
	3	**6.9980e-1**	5.9907e-1	5.3965e-1	6.4685e-1	6.0792e-1	5.8792e-1	5.9715e-1	5.5981e-1
		5.72e-2-	4.03e-2-	1.32e-2+	1.73e-2-	4.99e-2-	1.13e-2-	5.46e-2-	4.80e-2
WFG3	2	4.3725e-1	4.1207e-1	3.1333e-1	3.7379e-1	4.0248e-1	3.4546e-1	3.7513e-1	**5.1634e-1**
		1.81e-2+	1.20e-2+	1.09e-2+	7.59e-3+	2.56e-2+	9.12e-3+	2.98e-2+	4.16e-3
	3	2.1580e-1	1.7182e-1	1.3686e-1	1.9885e-1	1.7998e-1	1.6346e-1	2.0094e-1	**2.2275e-1**
		1.78e-2≈	1.20e-2+	8.67e-3+	4.25e-3+	2.39e-2+	6.32e-3+	1.76e-2+	1.29e-2
WFG4	2	2.3670e-1	2.3798e-1	1.9006e-1	2.3785e-1	2.3540e-1	1.9191e-1	2.4259e-1	**2.4299e-1**
		7.24e-3≈	3.48e-3≈	6.07e-3+	7.25e-3≈	1.43e-2≈	7.12e-3+	1.43e-2≈	5.66e-3
	3	3.43686e-1	34785e-1	3.4127e-1	2.9786e-1	3.4085e-1	3.4093e-1	3.4785e-1	**3.4996e-1**
		1.51e-2≈	2.13e-2≈	6.13e-3≈	1.20e-2+	2.54e-2≈	4.21e-3≈	2.17e-2≈	9.35e-3
WFG5	2	**2.6440e-1**	1.9564e-1	1.6413e-1	1.3429e-1	2.5364e-1	1.3316e-1	2.5356e-1	2.5027e-1
		8.80e-2-	1.26e-2+	6.10e-3+	3.69e-3+	1.38e-2≈	5.93e-3+	1.55e-2≈	1.43e-2
	3	3.5247e-1	**3.5317e-1**	2.3960e-1	2.3471e-1	3.4931e-1	2.4818e-1	3.2849e-1	3.4925e-1
		1.10e-2-	2.31e-2-	6.80e-3+	5.50e-3+	2.10e-2≈	9.92e-3+	1.18e-2+	8.20e-3
WFG6	2	1.3511e-1	1.5311e-1	7.3918e-2	1.1696e-1	1.7164e-1	8.4697e-2	1.4312e-1	**1.7258e-1**
		1.15e-2+	1.95e-2+	3.02e-3+	8.57e-3+	1.51e-2≈	8.11e-3+	1.78e-2+	2.31e-2
	3	1.8041e-1	1.5422e-1	1.2739e-1	1.2762e-1	1.8748e-1	1.3765e-1	1.5817e-1	**1.8860e-1**
		1.41e-2≈	2.32e-2+	3.11e-3+	8.48e-3+	4.46e-2≈	3.35e-3+	2.91e-2+	2.31e-2
WFG7	2	**1.9922e-1**	1.7144e-1	1.2212e-1	1.6467e-1	1.8613e-1	1.2869e-1	1.7301e-1	1.8526e-1
		8.59e-3-	1.04e-2+	2.98e-3+	3.72e-3+	1.04e-2≈	6.50e-3+	1.89e-2+	1.19e-2
	3	**3.2186e-1**	2.5997e-1	2.2802e-1	2.9257e-1	2.9476e-1	2.3747e-1	2.9860e-1	2.9311e-1
		1.60e-2-	1.62e-2+	4.96e-3+	8.49e-3≈	3.36e-2≈	4.95e-3+	1.55e-2≈	1.61e-2
WFG8	2	**1.6059e-1**	1.5662e-1	1.2139e-1	1.3771e-1	1.4304e-1	1.3346e-1	1.5273e-1	1.5118e-1
		1.56e-2-	6.82e-3≈	5.17e-3+	3.02e-3+	6.14e-3+	5.28e-3+	7.83e-3≈	1.24e-2
	3	**2.2086e-1**	1.8718e-1	2.1083e-1	2.1362e-1	2.1579e-1	1.8733e-1	2.1530e-1	2.1492e-1
		1.56e-2-	1.37e-2+	8.95e-3≈	6.34e-3≈	1.64e-2≈	8.16e-3+	1.56e-2≈	7.79e-3
WFG9	2	1.5972e-1	1.4891e-1	1.0281e-1	1.5984e-1	1.4632e-1	1.2539e-1	2.1003e-1	**2.1051e-1**
		3.18e-2+	3.80e-2+	1.34e-2+	2.06e-2+	9.39e-3+	1.19e-2+	4.11e-2≈	1.57e-2
	3	2.4758e-1	2.2969e-1	1.9399e-1	2.1295e-1	2.4921e-1	2.0827e-1	2.5866e-1	**2.6090e-1**
		4.02e-2+	3.89e-2+	1.47e-2+	9.18e-3+	1.49e-2+	2.68e-2+	3.17e-2≈	2.49e-2
win/tie/lose		6/4/8	13/3/2	16/2/0	13/3/2	7/9/2	16/1/1	7/9/2	–

TABLE S9: IGD values obtained by DK-SAMOEA and 7 comparison algorithms on WFGs when D=20

Problem	Nf	ABSAEA	CSEA	EDN-ARMOEA	He-EMOEA	MCEAD	ParEGO	REMO	DK-SAMOEA
WFG1	2	1.3570e+0	1.2514e+0	1.3426e+0	1.4849e+0	1.2937e+0	1.3402e+0	1.1919e+0	**1.1400e+0**
		5.84e-2+	4.19e-2+	3.34e-2+	1.07e-1+	1.52e-2+	3.34e-2+	3.70e-2≈	4.38e-3
	3	1.6072e+0	1.5567e+0	1.6358e+0	2.0171e+0	1.6274e+0	1.6831e+0	1.4573e+0	**1.4368e+0**
		5.30e-2+	4.47e-2+	6.39e-3+	2.12e-1+	1.87e-2+	6.79e-2+	9.59e-2≈	6.31e-3
WFG2	2	**4.4759e-1**	5.3211e-1	6.5784e-1	5.3627e-1	6.0951e-1	5.6606e-1	4.4903e-1	6.3326e-1
		6.95e-2-	3.54e-2-	5.67e-2+	4.09e-2-	1.50e-1-	3.44e-2-	2.47e-2-	2.52e-1
	3	**5.3295e-1**	7.6871e-1	9.2093e-1	6.4048e-1	8.3783e-1	7.1584e-1	7.7486e-1	8.1061e-1
		1.54e-1-	1.05e-1-	4.08e-2+	5.67e-2-	2.94e-1+	3.54e-2-	1.46e-1-	4.55e-1
WFG3	2	2.7685e-1	3.2878e-1	5.7026e-1	4.0121e-1	3.6198e-1	4.6744e-1	4.1080e-1	**1.3999e-1**
		4.26e-2+	3.22e-2+	3.24e-2+	1.46e-2+	4.99e-2+	2.31e-2+	5.60e-2+	7.69e-3
	3	4.4690e-1	5.7503e-1	6.7603e-1	4.4133e-1	5.1735e-1	6.2695e-1	4.4115e-1	**4.4005e-1**
		4.72e-2≈	6.20e-2+	3.13e-2+	2.16e-2≈	6.03e-2+	1.46e-2+	3.74e-2≈	3.74e-2
WFG4	2	**2.1706e-1**	2.5472e-1	3.8506e-1	4.5789e-1	3.2019e-1	3.3345e-1	2.7325e-1	2.1943e-1
		3.07e-2≈	2.87e-2+	1.94e-2+	7.53e-1+	5.42e-2+	2.51e-2+	5.78e-2+	1.64e-2
	3	4.5981e-1	4.5698e-1	5.3182e-1	7.7486e-1	5.9905e-1	5.9435e-1	4.9128e-1	**4.5664e-1**
		2.29e-2≈	4.25e-2≈	1.57e-2+	8.78e-2+	8.62e-2+	4.09e-2+	4.21e-2+	2.09e-2
WFG5	2	3.1858e-1	3.2260e-1	4.1347e-1	5.8002e-1	2.3197e-1	4.4611e-1	2.7256e-1	**1.7952e-1**
		2.18e-1+	4.95e-1+	2.68e-2+	1.97e-2+	4.26e-2+	2.34e-2+	6.76e-2+	2.83e-2
	3	**4.9247e-1**	5.0221e-1	7.1071e-1	8.0205e-1	5.4646e-1	6.7099e-1	5.0138e-1	5.1818e-1
		3.62e-2-	2.64e-2-	2.44e-2+	3.70e-2+	4.78e-2+	2.11e-2+	2.87e-2-	4.00e-2
WFG6	2	4.0458e-1	**3.8763e-1**	6.9271e-1	6.5182e-1	4.0322e-1	5.7987e-1	4.0075e-1	4.1765e-1
		2.68e-2-	4.893e-2-	4.78e-2+	2.23e-2+	4.74e-2-	2.21e-2+	4.52-2-	6.87e-2;
	3	6.0576e-1	**6.0352e-1**	8.5282e-1	9.4140e-1	6.1477e-1	8.1883e-1	6.1170e-1	6.3587e-1
		2.90e-2-	4.36e-2-	1.31e-2+	3.87e-2+	1.23e-1-	2.11e-2+	4.93e-2-	3.2787e-2
WFG7	2	**3.3378e-1**	4.1148e-1	4.9051e-1	3.4018e-1	3.5594e-1	4.6931e-1	4.4075e-1	3.5306e-1
		2.03e-2-	3.83e-2+	1.54e-2+	2.51e-2-	6.62e-1≈	2.31e-2+	52e-2+	2.20e-2
	3	**5.0261e-1**	5.0873e-1	6.7441e-1	6.0824e-1	6.2330e-1	6.7117e-1	5.0599e-1	5.1160e-1
		3.58e-2-	4.79e-2-	1.63e-2+	4.27e-2+	1.06e-1+	2.68e-2+	4.48e-2-	3.20e-2
WFG8	2	**4.4261e-1**	4.5662e-1	6.2364e-1	4.5874e-1	5.4518e-1	5.1683e-1	5.0522e-1	4.8375e-1
		1.74e-2-	7.22e-2-	4.61e-2+	2.15e-2-	4.82e-2+	1.27e-2+	3.78e-2+	1.62e-2
	3	**7.0105e-1**	7.1903e-1	8.1258e-1	7.9863e-1	8.8053e-1	7.1722e-1	7.0466e-1	7.1058e-1
		3.02e-2-	5.83e-2≈	2.96e-2+	4.42e-2+	4.28e-2+	2.36e-2≈	4.32e-2-	2.25e-2
WFG9	2	4.3200e-1	**3.0441e-1**	3.8506e-1	5.7340e-1	4.5522e-1	5.1212e-1	3.8135e-1	3.9074e-1
		6.44e-2+	9.22e-2-	1.94e-2-	5.35e-2+	3.56e-2+	4.31e-2+	1.00e-1-	4.08e-2
	3	6.5230e-1	6.5777e-1	9.1644e-1	9.1981e-1	6.4855e-1	8.5918e-1	**6.4168e-1**	6.5592e-1
		1.10e-1≈	1.19e-1≈	4.86e-2+	3.92e-2+	4.35e-2-	4.46e-2+	8.69e-2-	4.74e-2
win/tie/lose		5/4/9	7/3/8	17/1/0	13/1/4	13/1/4	15/1/2	6/3/9	–

TABLE S10: HV values obtained by DK-SAMOEA and 7 comparison algorithms on IMOPs

Problem	Nf	ABSAEA	CSEA	EDN-ARMOEA	He-EMOEA	MCEAD	ParEGO	REMO	DK-SAMOEA
IMOP1	2	5.2851e-1	1.4233e-1	1.2411e-1	9.5763e-2	2.5306e-1	4.5440e-1	2.0957e-1	**7.4001e-1**
		1.12e-1+	5.26e-2+	2.00e-2+	4.45e-3+	1.26e-1+	1.18e-1+	1.40e-1+	5.66e-2
IMOP2	2	8.8552e-2	7.9314e-2	7.6712e-2	6.8642e-2	7.5269e-2	7.5903e-2	7.7259e-2	**9.2960e-2**
		1.44e-3+	4.41e-3+	2.21e-3+	5.02e-3+	3.67e-3 +	8.79e-3+	3.13e-3+	1.39e-3
IMOP3	2	2.7070e-1	1.0917e-1	1.0588e-1	3.8042e-2	1.4703e-1	1.5430e-1	1.4333e-1	**3.4181e-1**
		5.93e-2+	3.68e-2+	9.99e-3+	2.32e-2+	6.56e-2+	1.72e-2+	3.40e-2+	3.86e-2
IMOP4	3	4.0669e-2	2.7245e-2	2.5803e-2	2.1866e-2	9.9591e-2	1.1430e-1	3.4495e-2	**1.1215e-1**
		2.05e-3+	3.71e-3+	2.27e-3+	1.49e-3+	1.06e-1+	1.93e-3≈	2.40e-3+	3.53e-2
IMOP5	3	4.0483e-1	3.3962e-1	2.4950e-1	2.4202e-1	2.6730e-1	3.6798e-1	**4.6595e-1**	4.4293e-1
		8.31e-2+	5.99e-2+	2.43e-3+	9.40e-3+	2.56e-2+	1.87e-1+	1.84e-2-	7.54e-2
IMOP6	3	2.1093e-1	9.8469e-2	7.3862e-2	5.6338e-2	1.8066e-1	2.0907e-1	8.9586e-2	**2.1263e-1**
		9.99e-2≈	2.00e-2+	6.49e-3+	4.89e-3+	7.53e-2+	1.67e-2+	8.44e-3+	5.56e-2
IMOP7	3	9.0927e-2	9.0206e-2	7.6389e-2	8.1386e-2	9.1756e-2	9.1084e-2	8.9928e-2	**9.1949e-2**
		1.78e-5+	6.71e-4+	6.94e-3+	7.89e-3+	1.73e-3	1.98e-3	1.18e-3+	4.54e-4
IMOP8	3	1.5936e-1	1.4924e-1	1.0836e-1	1.0689e-1	1.0451e-1	1.5033e-1	1.3370e-1	**1.6317e-1**
		3.42e-2≈	3.31e-2+	1.71e-2+	1.64e-2+	1.76e-2+	2.98e-2+	3.69e-2+	2.10e-2
win/tie/lose		6/2/0	8/0/0	8/0/0	8/0/0	7/1/0	6/2/0	7/0/1	–

TABLE S11: IGD values obtained by DK-SAMOEA and 7 comparison algorithms on IMOPs

Problem	Nf	ABSAEA	CSEA	EDN-ARMOEA	He-EMOEA	MCEAD	ParEGO	REMO	DK-SAMOEA
IMOP1	2	2.8671e-1 5.19e-2+	7.5721e-1 5.05e-2+	7.7567e-1 2.05e-2+	8.0291e-1 4.03e-3+	6.1438e-1 1.78e-1+	3.6767e-1 5.12e-2+	7.0167e-1 1.19e-1+	**2.0519e-1** 2.38e-2
IMOP2	2	7.1564e-1 1.31e-1+	6.5646e-1 2.46e-2+	6.2107e-1 3.64e-2+	5.9451e-1 7.27e-3+	6.4284e-1 3.81e-2+	5.9729e-1 1.07e-1+	7.4094e-1 2.83e-2+	**5.1007e-1** 4.79e-2
IMOP3	2	4.5367e-1 1.16e-1+	6.7177e-1 1.33e-1+	7.0978e-1 9.44e-2+	7.5131e-1 1.50e-2+	5.9969e-1 2.10e-1+	5.5175e-1 4.01e-2+	6.0577e-1 1.28e-1+	**2.2719e-1** 2.91e-2
IMOP4	3	7.0217e-1 3.21e-3+	7.5336e-1 1.85e-2+	7.5961e-1 1.12e-2+	7.7580e-1 6.97e-3+	5.0518e-1 2.82e-1+	3.7309e-1 3.12e-3≈	3.1842e-1 8.80e-3+	**3.6938e-1** 7.87e-3
IMOP5	3	4.7627e-1 1.33e-1+	5.8873e-1 1.06e-1+	7.1299e-1 1.68e-5+	7.1305e-1 1.45e-4+	6.9414e-1 3.57e-2+	**4.2929e-1** 2.87e-1≈	5.5388e-1 8.09e-2+	4.3627e-1 8.48e-2
IMOP6	3	4.8121e-1 1.16e-1≈	6.2554e-1 1.03e-1+	7.2180e-1 8.22e-2+	7.0514e-1 6.93e-2+	4.8933e-1 8.24e-2+	4.8288e-1 9.72e-2≈	5.8117e-1 9.23e-2+	**4.8010e-1** 7.21e-2
IMOP7	3	9.3559e-1 1.70e-3+	9.3645e-1 2.53e-3+	9.2451e-1 5.28e-3+	9.2822e-1 7.77e-3+	9.1419e-1 1.78e-2+	8.9261e-1 9.87e-3+	9.2050e-1 1.25e-2+	**8.7127e-1** 5.62e-3
IMOP8	3	5.7957e-1 3.76e-2+	6.0634e-1 9.23e-2+	6.4068e-1 8.21e-2+	7.0621e-1 7.28e-2+	7.1336e-1 5.80e-2+	5.3509e-1 1.98e-1+	5.8730e-1 1.29e-1+	**5.2500e-1** 4.45e-2
win/tie/lose		7/1/0	8/0/0	8/0/0	8/0/0	7/1/0	6/2/0	8/0/0	–

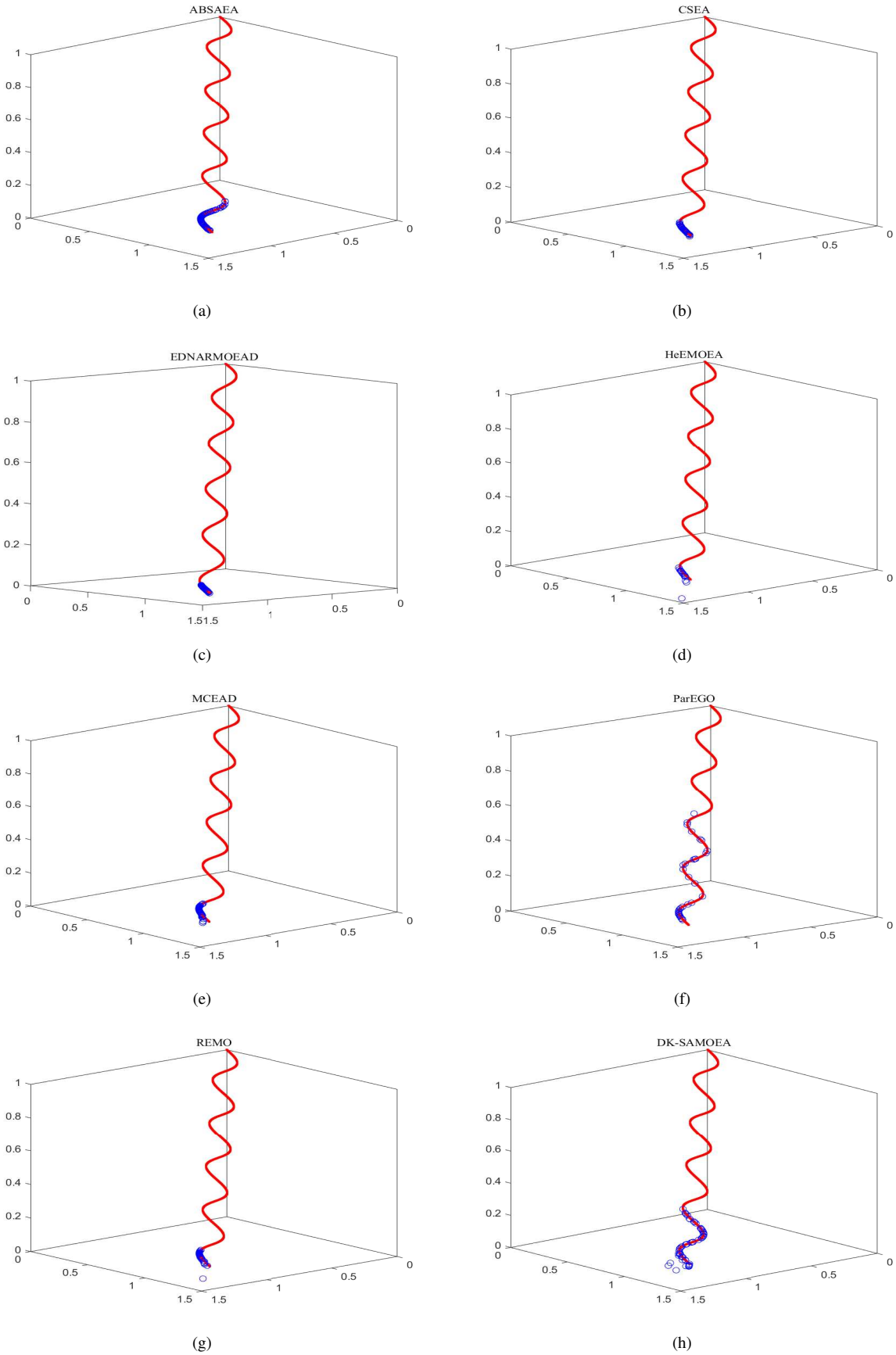

Fig. S5: Non-dominated solutions obtained by DK-SAMOEA and the 7 comparison algorithms on IMOP4

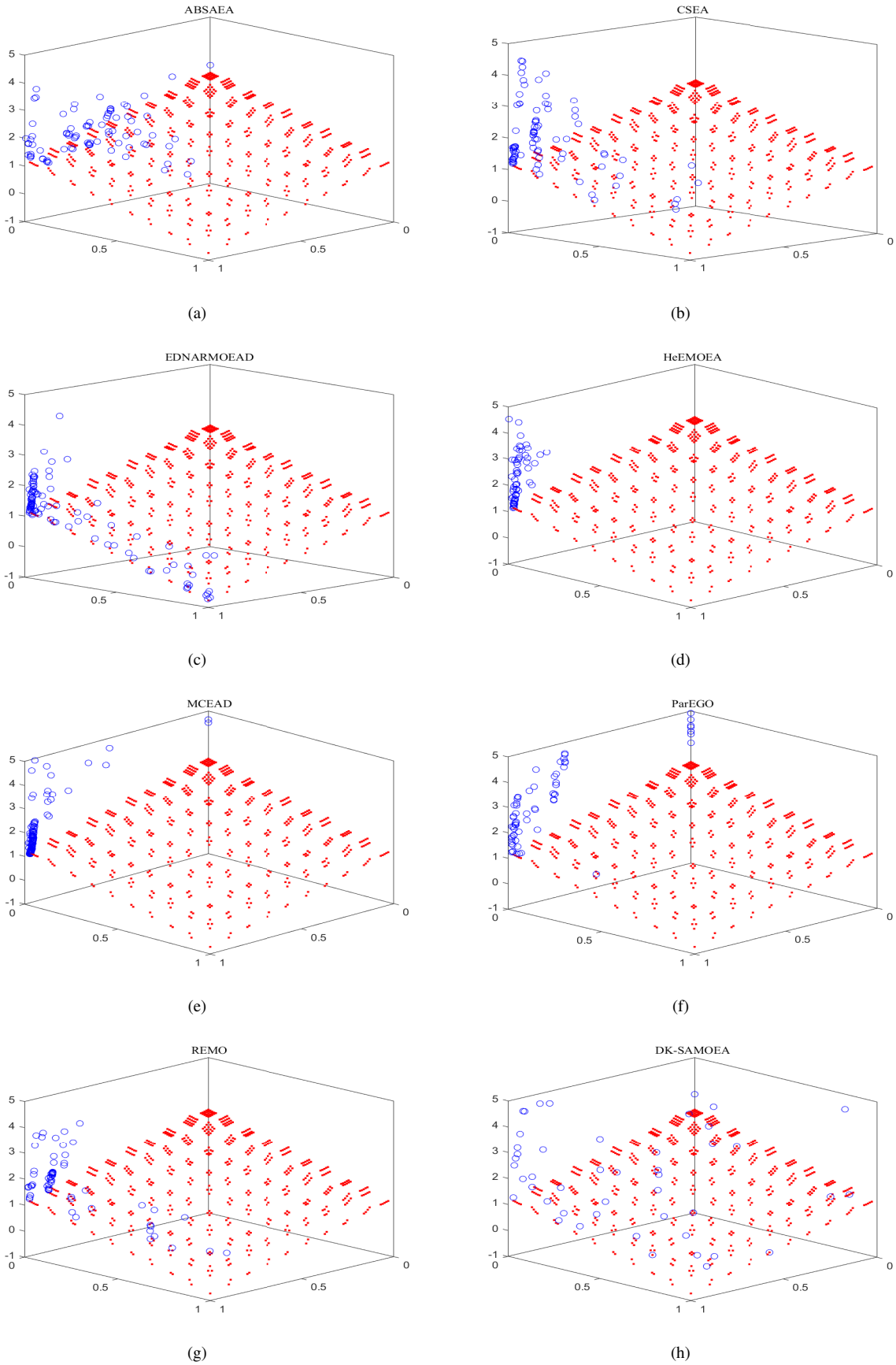

Fig. S6: Non-dominated solutions obtained by DK-SAMOEA and the 7 comparison algorithms on IMOP8

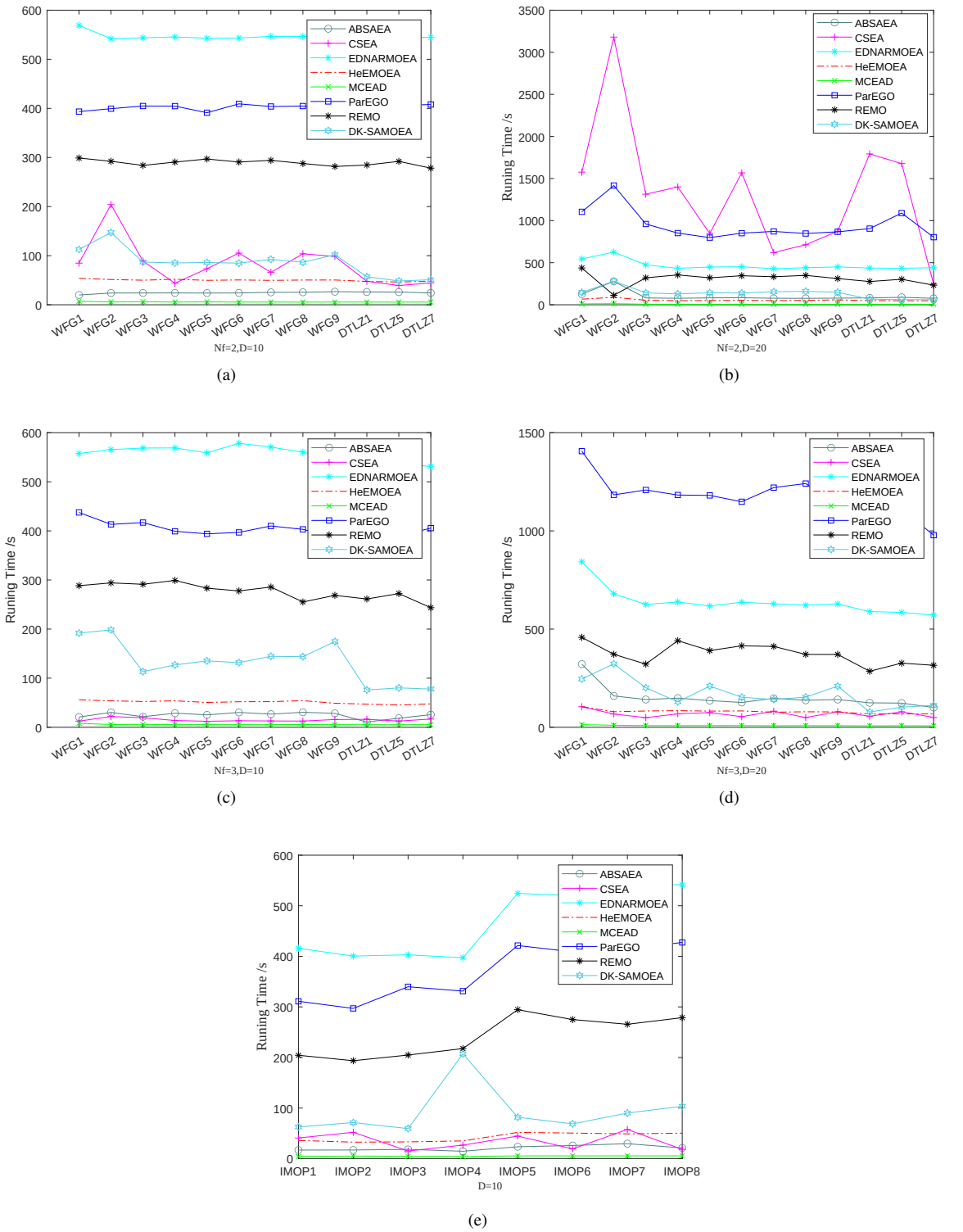

Fig. S7: Running time of DK-SAMOEA and the 7 comparison algorithms (unit/s)

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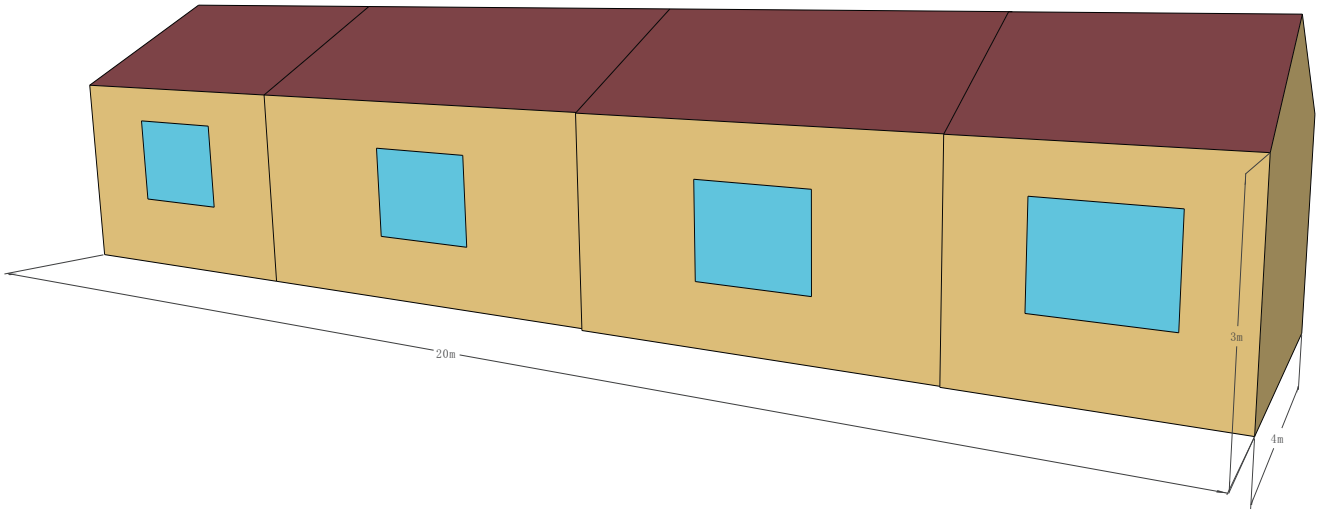

Fig. S8: Basic appearance of a residential building

TABLE S12: Decision variables and their ranges

Decision variables	Unit	Range
Building orientation		[0,360]
Insulation thickness	m	(0.0001,0.1)
Solar absorption rate of exterior wall	–	(0.1,1)
Glazing u-factor	$w/(m^2k)$	(2,6)
Glazing solar heat gain coefficient	–	(0.0001,0.7)
Living room window length	m	(0,4.9)
Living room window width	m	(0,3.19)
Bedroom window length	m	(0,6.9)
Bedroom window width	m	(0,3.19)
Kitchen window length	m	(0,5.9)
Kitchen window width	m	(0,3.19)
Toilet window length	m	(0,3.9)
Toilet window width	m	(0,2.19)
Living room lighting density	w/m^2	[4.5,6]
Bedroom lighting density	w/m^2	[5,8]
Kitchen lighting density	w/m^2	[5,6]
Toilet lighting density	w/m^2	[5.5,8]
Heating setpoint temperature		[18,23]
Cooling setpoint temperature		[24,28]