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HO CHI MINH CITY UNIVERSITY OF TECHNOLOGY
FACULTY OF MECHANICAL ENGINEERING
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GROUP PROJECT REPORT

ROBOTICS

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HCM City - May 2025

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Chapter 1

SETTING COORDINATE FRAMES

1.1 Topic

Set coordinate frames for the first four links (link 1, link 2, link 3).

1.2 Theory

Based on “DENAVIT-HARTENBERG NOTATION” (Lecture 4: Forward Kinematics [1]): Local frame B_i to each link (i) at joint $i + 1$ is defined as:

- The z_i axis is aligned with the $i + 1$ joint axis.
- The x_i axis is defined along the common normal between the $z_i - 1$ and z_i axes, pointing from the $z_i - 1$ to the z_i axis.
- The y_i axis is defined by the right-hand rule.
- The origin o_i of the i frame is located at the intersection of the joint axis $i + 1$ with the common normal between the $z_i - 1$ and z_i axes.

1.3 Application

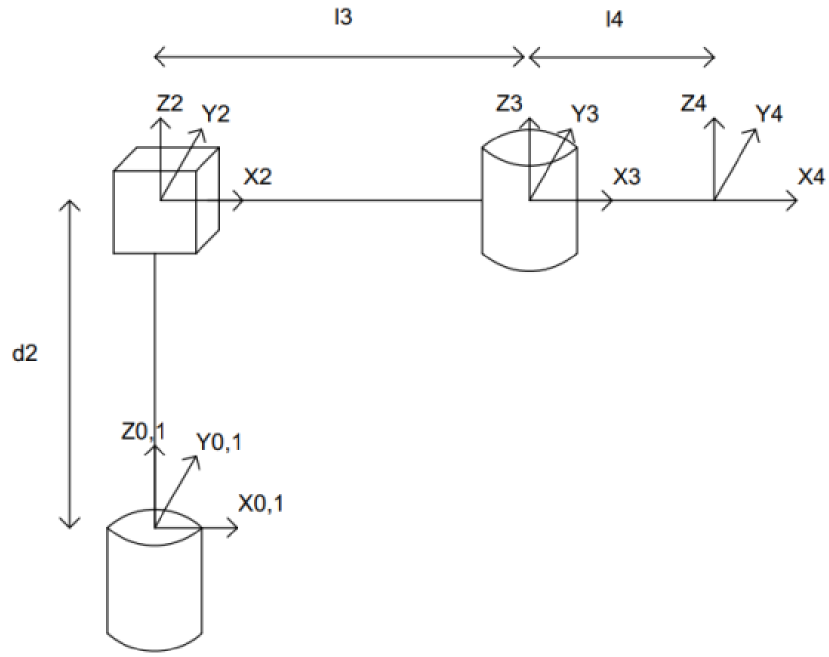

Figure 1.1: Setting coordinate frames for robot

Chapter 2

DETERMINING D-H PARAMETERS

2.1 Topic

Determine the Denavit-Hartenberg parameters for the robot model.

2.2 Theory

The Denavit-Hartenberg notation is introduced as a systematic method of describing the kinematic relationship ${}^{i-1}T_i$ using only four parameters [**1**]:

α	Link twist	Describe the link itself
a	Link length	
d	Link offset	Describe the link's connection to neighboring link
θ	Joint angle	
If the joint is:		
Revolute: θ joint variable		The other three are fixed link parameters
Prismatic: d joint variable		

2.3 Application

We got the D-H table:

i	a_i	α_i	d_i	θ_i
1	0	0	0	θ_1
2	0	0	d_2	0
3	ℓ_3	0	0	0
4	ℓ_4	0	0	θ_4

Limitations of the D-H notation:

$$\ell_3 = 1000\,\text{mm}$$
$$\ell_4 = 300\,\text{mm}$$
$$d_2 \in [2150; 2750]$$
$$\theta_1 \in [0^\circ; 360^\circ]$$
$$\theta_4 \in [-90^\circ; 90^\circ]$$

Chapter 3

KINEMATIC PROBLEM

3.1 Topic

Formulate the forward kinematic problem. Then determine the coordinates of the end-point according to the three joint variables. Make a plot for a certain case.

3.2 Theory

The transformaytion matrix ${}^{i-1}_{i}T$ to transform coordinate frames B_i to $B_i{-}1$ is represented as a product of four basic transformations using paramaters of link (i) and joint i

$$
{}^{i-1}_{i}T = \begin{bmatrix} \cos(\theta_i) & -\sin(\theta_i)\cos(\alpha_i) & \sin(\theta_i)\sin(\alpha_i) & a_i\cos(\theta_i) \\ \sin(\theta_i) & \cos(\theta_i)\cos(\alpha_i) & -\cos(\theta_i)\sin(\alpha_i) & a_i\sin(\theta_i) \\ 0 & \sin(\alpha_i) & \cos(\alpha_i) & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.1}
$$

To find the single transformation that relates frame $\{i\}$ to frame $\{0\}$, the transformation matrices of every link are then multiplied together:

$$
{}^{0}_{i}T =^{0}_{1} T \; {}^{1}_{2}T \; ... \; {}^{i-1}_{i}T \tag{3.2}
$$

This transformation ${}^{0}_{i}T$ is a function of all i joint variables. If the robot's joint-position sensors are queried, the Cartesian position and orientation of the end effector could be computed by ${}^{0}_{i}T$.

3.3 Application

We have:

$$
{}^{0}T_1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.3}
$$

$$^{1}T_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.4}$$

$$^{2}T_3 = \begin{bmatrix} 1 & 0 & 0 & \ell_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.5}$$

$$^{3}T_4 = \begin{bmatrix} \cos\theta_4 & -\sin\theta_4 & 0 & \ell_4 \cdot \cos\theta_4 \\ \sin\theta_4 & \cos\theta_4 & 0 & \ell_4 \cdot \sin\theta_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.6}$$

Thus

$$^{0}T_4 =^{0} T_1.^{1}T_2.^{2}T_3.^{3}T_4 = \begin{bmatrix} \cos(\theta_1+\theta_4) & -\sin(\theta_1+\theta_4) & 0 & \ell_4.\cos(\theta_1+\theta_4)+\ell_3.\cos\theta_1 \\ \sin(\theta_1+\theta_4) & \cos(\theta_1+\theta_4) & 0 & \ell_4.\sin(\theta_1+\theta_4)+\ell_3.\sin\theta_1 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In conclusion, we have the solution for the forward kinematic problem as below

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \ell_4.\cos(\theta_1+\theta_4)+\ell_3.\cos\theta_1 \\ \ell_4.\sin(\theta_1+\theta_4)+\ell_3.\sin\theta_1 \\ d_2 \end{bmatrix} \tag{3.7}$$

Make a plot for a certain case:

During the time t from 0s to 10s, the input $(\theta_1, d_2, \theta_4)$ gradually increases from $(0°, 2200, 0°)$ to $(90°, 2700, 90°)$. Using MATLAB, the position $(x, y, z$ of the end-effector can be plotted as below:

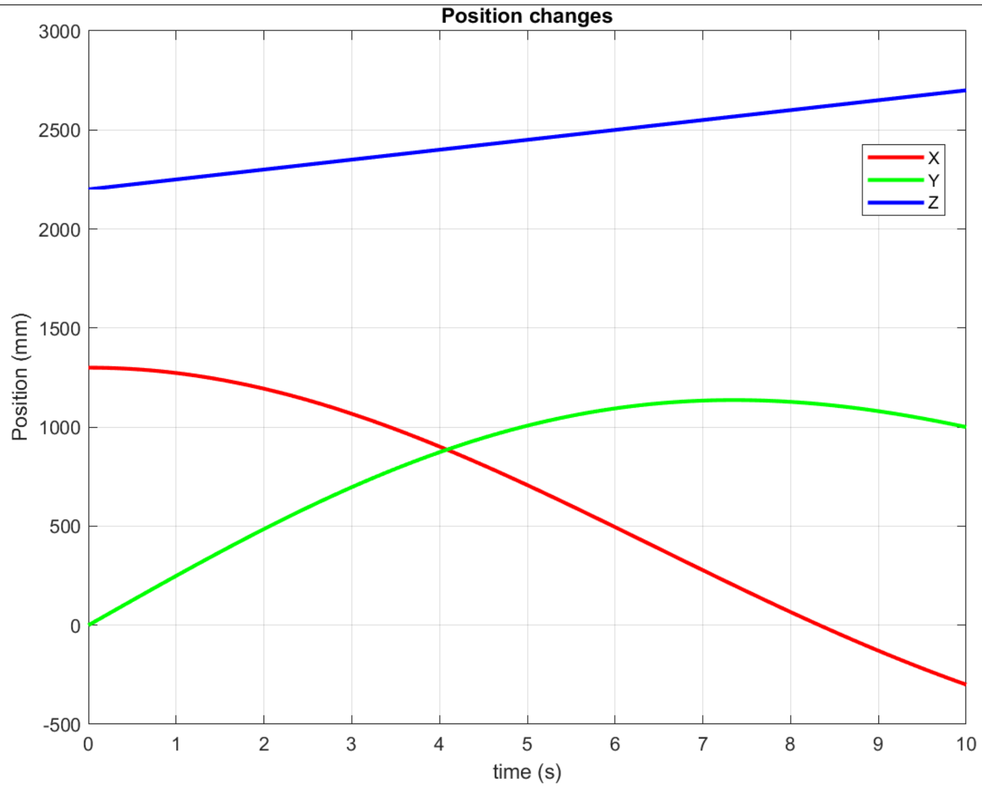

Figure 3.1: The position of the end-effector (x, y, z) with respect to time

Solution check:

- At time $t = 0$ s, the end-effector is at position $(x, y, z) = (1300, 0, 2200)$ mm. Correct!
- At time $t = 10$ s, the end-effector is at position $(x, y, z) = (-300, 1000, 2700)$ mm. Correct!

So we can confirm the accurate of the solution.

Chapter 4

INVERSE KINEMATIC PROBLEM

4.1 Topic

Formulate the inverse kinematic problem. Then determine the three joint values according to the coordinates of the end-point. Make a plot for a certain case.

4.2 Theory

Inverse kinematics is the mathematical process of calculating the variable joint parameters needed to place the end of a kinematic chain which is, in this project, the position of the end effector of a robot arm [1].

4.3 Application

We have the forward kinematic equation:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \ell_4 . \cos(\theta_1 + \theta_4) + \ell_3 . \cos\theta_1 \\ \ell_4 . \sin(\theta_1 + \theta_4) + \ell_3 . \sin\theta_1 \\ d_2 \end{bmatrix}$$

The position of the end-effector (X,Y,Z) can be extracted from the matrix as below:

$$x = \ell_4 . \cos(\theta_1 + \theta_4) + \ell_3 . \cos\theta_1 \tag{4.1}$$

$$y = \ell_4 . \sin(\theta_1 + \theta_4) + \ell_3 . \sin\theta_1 \tag{4.2}$$

$$z = d_2 \tag{4.3}$$

$$\tag{4.4}$$

From (2.1) and (2.2) we have

$$\begin{aligned} x^2 + y^2 &= \ell_4^2 + \ell_3^2 + 2\ell_3 \cdot \ell_4 \left[\cos(\theta_1 + \theta_4) \cdot \cos\theta_1 + \sin(\theta_1 + \theta_4) \cdot \sin\theta_1\right] \\ &= \ell_4^2 + \ell_3^2 + 2\ell_3 \cdot \ell_4 \cos(\theta_1 + \theta_4 - \theta_1) \\ &= \ell_4^2 + \ell_3^2 + 2\ell_3 \cdot \ell_4 \cos(\theta_4) \end{aligned}$$

Set

$$a = \cos(\theta_4) = \frac{x^2 + y^2 - \ell_4^2 - \ell_3^2}{2\ell_3 \cdot \ell_4}$$

Thus

$$\theta_4 = \text{atan2}\left(\pm\sqrt{1-a^2}, a\right)$$

The x and y components can be expressed as:

$$\begin{aligned} x\cos\theta_1 + y\sin\theta_1 &= \ell_4 \cdot \cos(\theta_1+\theta_4)\cos\theta_1 + \ell_4 \cdot \sin(\theta_1+\theta_4) \cdot \sin\theta_1 + \ell_3 \\ &= \ell_4 \cdot \cos(\theta_4) + \ell_3 \end{aligned}$$

We can rewrite

$$x\cos\theta_1 + y\sin\theta_1 = \sqrt{x^2+y^2}\left(\frac{x}{\sqrt{x^2+y^2}}\cos\theta_1 + \frac{y}{\sqrt{x^2+y^2}}\sin\theta_1\right)$$

Set

$$\begin{aligned} \cos\phi &= \frac{x}{\sqrt{x^2+y^2}} \\ \sin\phi &= \frac{y}{\sqrt{x^2+y^2}} \\ \phi &= \text{atan2}(y, x) \end{aligned}$$

Thus we have

$$\sqrt{x^2+y^2}\,(\cos\phi \cdot \cos\theta_1 + \sin\phi \cdot \sin\theta_1) = \ell_4 \cdot \cos(\theta_4) + \ell_3$$

Infer the following:

$$\begin{aligned} \cos(\theta_1-\phi) &= \frac{\ell_4 \cdot a + \ell_3}{\sqrt{x^2+y^2}} \\ \sin(\theta_1-\phi) &= \pm\frac{\sqrt{x^2+y^2-(\ell_4 \cdot a+\ell_3)^2}}{\sqrt{x^2+y^2}} \\ \Rightarrow \theta_1 - \phi &= \text{atan2}\left(\pm\sqrt{x^2+y^2-(\ell_4 \cdot a+\ell_3)^2}, \ell_4 \cdot a + \ell_3\right) \\ \Rightarrow \theta_1 &= \phi + \text{atan2}\left(\pm\sqrt{x^2+y^2-(\ell_4 \cdot a+\ell_3)^2}, \ell_4 \cdot a + \ell_3\right) \\ &= \text{atan2}(y,x) + \text{atan2}\left(\pm\sqrt{x^2+y^2-(\ell_4 \cdot \cos(\theta_4)+\ell_3)^2}, \ell_4 \cdot \cos(\theta_4) + \ell_3\right) \end{aligned}$$

Conclusion, we have the following solution for the inverse kinematic problem:

$$\begin{cases} \theta_1 = \text{atan2}(y,x) + \text{atan2}\left(\pm\sqrt{x^2+y^2-(\ell_4 \cdot \cos(\theta_4)+\ell_3)^2}, \ell_4 \cdot \cos(\theta_4) + \ell_3\right) \\ d_2 = z \\ \theta_4 = \text{atan2}\left(\pm\sqrt{1-a^2}, a\right) \end{cases}$$

or can rewrite as:

$$\begin{bmatrix}\theta_1 \\ d_2 \\ \theta_4\end{bmatrix} = \begin{bmatrix}\text{atan2}(y,x) + \text{atan2}\left(\pm\sqrt{x^2+y^2-(\ell_4\cdot\cos(\theta_4)+\ell_3)^2}, \ell_4\cdot\cos(\theta_4)+\ell_3\right) \\ Z \\ \text{atan2}\left(\pm\sqrt{1-a^2}, a\right)\end{bmatrix}$$

with

$$a = \frac{x^2+y^2-\ell_4^2-\ell_3^2}{2\ell_3\cdot\ell_4}$$

Solution check:

As forward kinematic solution, with the input $(\theta_1, d_2, \theta_4) = (0°, 2200, 0°)$, the position of the end-effector is $(x, y, z) = (1300, 0, 2200)$.

Using $(x, y, z) = (1300, 0, 2200)$ as an input to inverse function, we got $(\theta_1, d_2, \theta_4) = (0°, 2200, 0°)$.

Similarly, using $(x, y, z) = (-300, 1000, 2700)$ as an input to inverse function, we got $(\theta_1, d_2, \theta_4) = (90°, 2700, 90°)$. Correct.

So we can confirm the accurate of the solution.

Chapter 5

WORKSPACE

5.1 Topic

Give a comment on the workspace.

5.2 Theory

Existing of any solution raises the question of the manipulator's workspace, which is the volume of space that the end-effector of the manipulator can reach.

5.3 Application

We have:

$$\begin{cases} x = \ell_4 \cdot \cos(\theta_1 + \theta_4) + \ell_3 \cdot \cos\theta_1 \\ y = \ell_4 \cdot \sin(\theta_1 + \theta_4) + \ell_3 \cdot \sin\theta_1 \\ z = d_2 \end{cases}$$

With:

- $\ell_3 = 1000\,\text{mm}$
- $\ell_4 = 300\,\text{mm}$
- $d_2 \in [2150; 2750]$
- $\theta_1 \in [0°; 360°]$
- $\theta_4 \in [-90°; 90°]$

Next:

$$x^2 + y^2 = \ell_4^2 + \ell_3^2 + 2\ell_3\ell_4\cos(\theta_4)$$

$$\Leftrightarrow \cos(\theta_4) = \frac{x^2 + y^2 - \ell_4^2 - \ell_3^2}{2\ell_3\ell_4}$$

We also have:

$$0 \leq \cos(\theta_4) \leq 1$$

$$\Rightarrow 0 \leq x^2 + y^2 \leq (\ell_4 + \ell_3)^2$$

$$\Rightarrow \begin{cases} 0 \leq x^2 + y^2 \leq 1690000 \\ d_2 \in [2150; 2750] \\ \theta_1 \in [0°; 360°] \\ \theta_4 \in [-90°; 90°] \end{cases}$$

To simplify matters, we remove the mechanical constraint, the joints can be fully rotated. Therefore, our workspace is idealized.

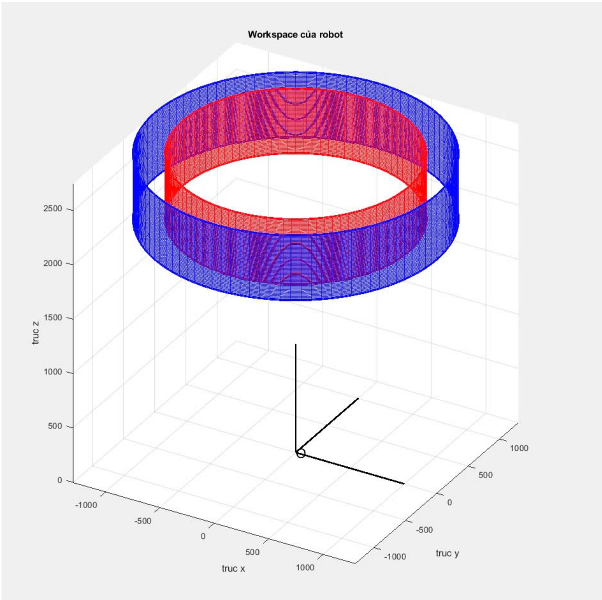

Figure 5.1: Workspace of the robot

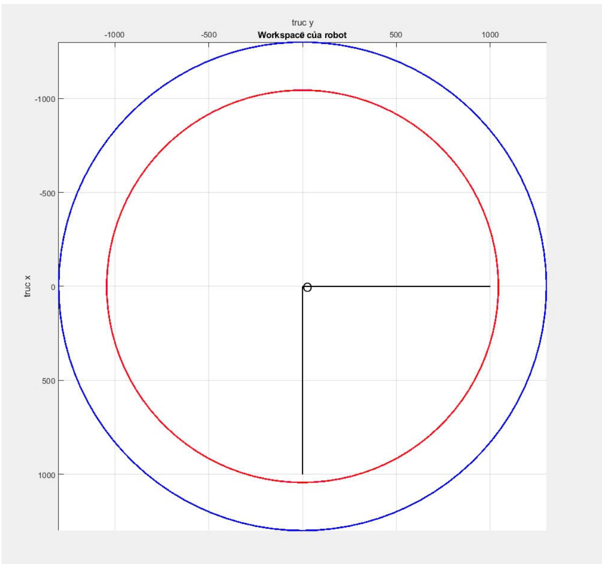

Figure 5.2: Workspace of the robot another view

Chapter 6

JACOBIAN MATRIX

6.1 Topic

Formulate the Jacobian matrix for this robot. Is there any singularity?

6.2 Theory

The Jacobian is a multidimensional form of the derivative. The 6 x 6 matrix of partial derivatives is the Jacobian J, as mapping velocities in X to those in Y.
In the field of robotics, Jacobians are used to relate joint velocities to Cartesian velocities of the tip of the arm.

$$^{0}\mathbf{V} = {}^{0}J(\theta)\,\dot{\theta}$$

All manipulators have singularities at:

- The boundary of their workspace.
- Most have loci of singularities inside their workspace.

6.3 Application

Vector of joint variables:

$$\mathbf{Q} = \begin{bmatrix}\theta_1 & d_2 & \theta_4\end{bmatrix}^T$$

Position-orientation state vector $\mathbf{X}$:

$$\mathbf{X} = \begin{bmatrix}p_x & p_y & d_2 & 0 & 0 & \theta_1 + \theta_4\end{bmatrix}^T$$

$$= \begin{bmatrix}\ell_4 \cos(\theta_1 + \theta_4) + \ell_3 \cos\theta_1 & \ell_4 \sin(\theta_1 + \theta_4) + \ell_3 \sin\theta_1 & d_2 & 0 & 0 & \theta_1 + \theta_4\end{bmatrix}^T$$

The Jacobian matrix J is the partial derivative of the position vector $\mathbf{X}$ with respect to the joint variable vector $\mathbf{q}$:

$$J = \frac{\partial \mathbf{X}}{\partial \mathbf{q}} = \begin{bmatrix} \frac{\partial p_x}{\partial \theta_1} & \frac{\partial p_x}{\partial d_2} & \frac{\partial p_x}{\partial \theta_4} \\ \frac{\partial p_y}{\partial \theta_1} & \frac{\partial p_y}{\partial d_2} & \frac{\partial p_y}{\partial \theta_4} \\ \frac{\partial d_2}{\partial \theta_1} & \frac{\partial d_2}{\partial d_2} & \frac{\partial d_2}{\partial \theta_4} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{\partial(\theta_1+\theta_4)}{\partial \theta_1} & \frac{\partial(\theta_1+\theta_4)}{\partial d_2} & \frac{\partial(\theta_1+\theta_4)}{\partial \theta_4} \end{bmatrix}$$

$$= \begin{bmatrix} -\ell_4 \sin(\theta_1+\theta_4) - \ell_3 \sin\theta_1 & 0 & -\ell_4 \sin(\theta_1+\theta_4) \\ \ell_4 \cos(\theta_1+\theta_4) + \ell_3 \cos\theta_1 & 0 & \ell_4 \cos(\theta_1+\theta_4) \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \text{ Control matrix } \mathbf{J} = \begin{bmatrix} -\ell_4 \sin(\theta_1+\theta_4) - \ell_3 \sin\theta_1 & 0 & -\ell_4 \sin(\theta_1+\theta_4) \\ \ell_4 \cos(\theta_1+\theta_4) + \ell_3 \cos\theta_1 & 0 & \ell_4 \cos(\theta_1+\theta_4) \\ 0 & 1 & 0 \end{bmatrix}$$

Case of Jacobian Singularity

$$J = \begin{bmatrix} -l_4 \sin(\theta_0+\theta_4) & -l_3 \sin\theta_1 & 0 & -l_4 \sin(\theta_1+\theta_4) \\ l_4 \cos(\theta_0+\theta_4) + l_3 \cos\theta_1 & 0 & 0 & l_4 \cos(\theta_1+\theta_4) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\det(J_a) = a_{11} \cdot (a_{22} \cdot a_{33} - a_{23} \cdot a_{32}) - a_{12} \cdot (a_{21} \cdot a_{33} - a_{23} \cdot a_{31}) + a_{13} \cdot (a_{21} \cdot a_{32} - a_{22} \cdot a_{31})$$

$$= [-l_4 \sin(\theta_0+\theta_4) - l_3 \sin\theta_1][l_4 \cos(\theta_1+\theta_4)] + [-l_4 \sin(\theta_1+\theta_4)][l_4 \cos(\theta_0+\theta_4) + l_3 \cos\theta_1]$$
$$-l_4^2 \sin(\theta_0+\theta_4)\cos(\theta_1+\theta_4) + l_3 \sin\theta_1 l_4 \cos(\theta_1+\theta_4) - l_4^2 \sin(\theta_1+\theta_4)\cos(\theta_0+\theta_4)$$
$$-l_3 l_4 \sin(\theta_1+\theta_4)\cos\theta_1$$

$$= l_3 l_4 [\sin\theta_1 \cdot \cos(\theta_1+\theta_4) - l_4 l_3 \sin(\theta_1+\theta_4) \cdot \cos\theta_1]$$

$$= -l_3 \cdot l_4 \cdot \sin(-\theta_4)$$

$$\det(J_a) = 0 \iff -l_4 \cdot l_3 \cdot \sin(-\theta_4) = 0$$

$$\iff -l_3 \cdot l_4 \cdot \sin(\theta_4) = 0$$

$$\iff \theta_4 = k\pi (k \in \mathbb{Z}); \theta_4 \in [-90^\circ, 90^\circ]$$

$$\Rightarrow \text{The point is within the range where } \theta_4 = 0$$

Chapter 7

SIMULATING THE MOTION OF ROBOT

7.1 Topic

Simulate the motion of this robot to plot initial letters from the first names of your team members on a certain plane that is perpendicular to axis z_0.

7.2 Theory

Simulate robot in simscape $(\theta_1, d_2, \theta_4)$ change from $(0, 2200, 0)$ to $(90°, 2700, 90°)$.

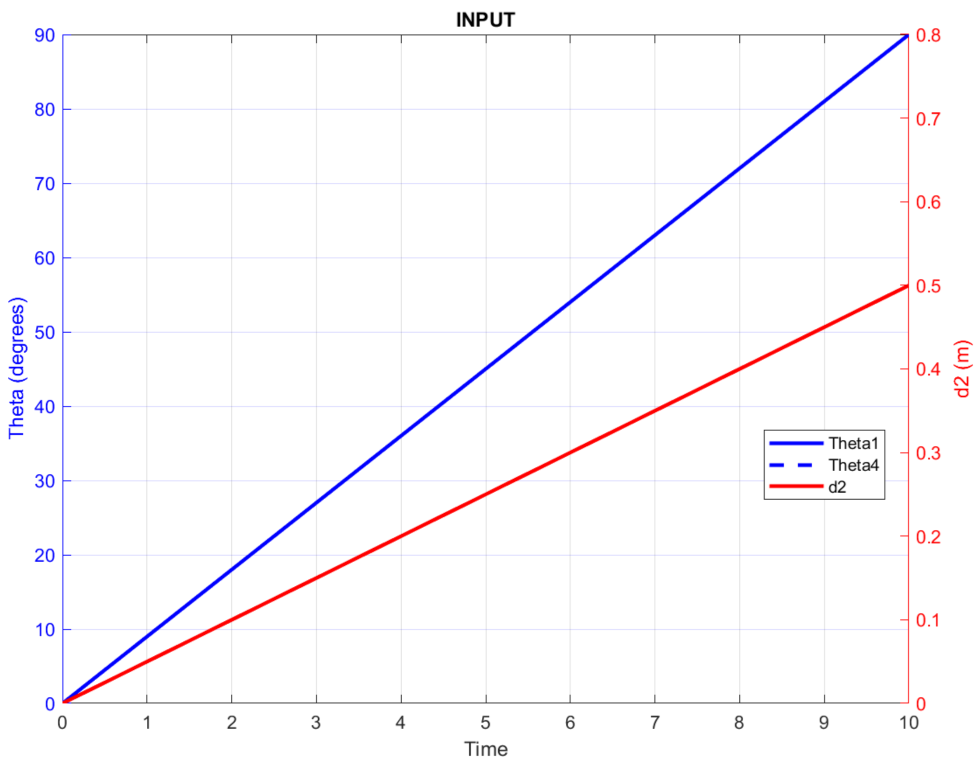

Figure 7.1: Simulation of the robot

Apply to forward, get position of end-effector (x, y, z) with respect to time t

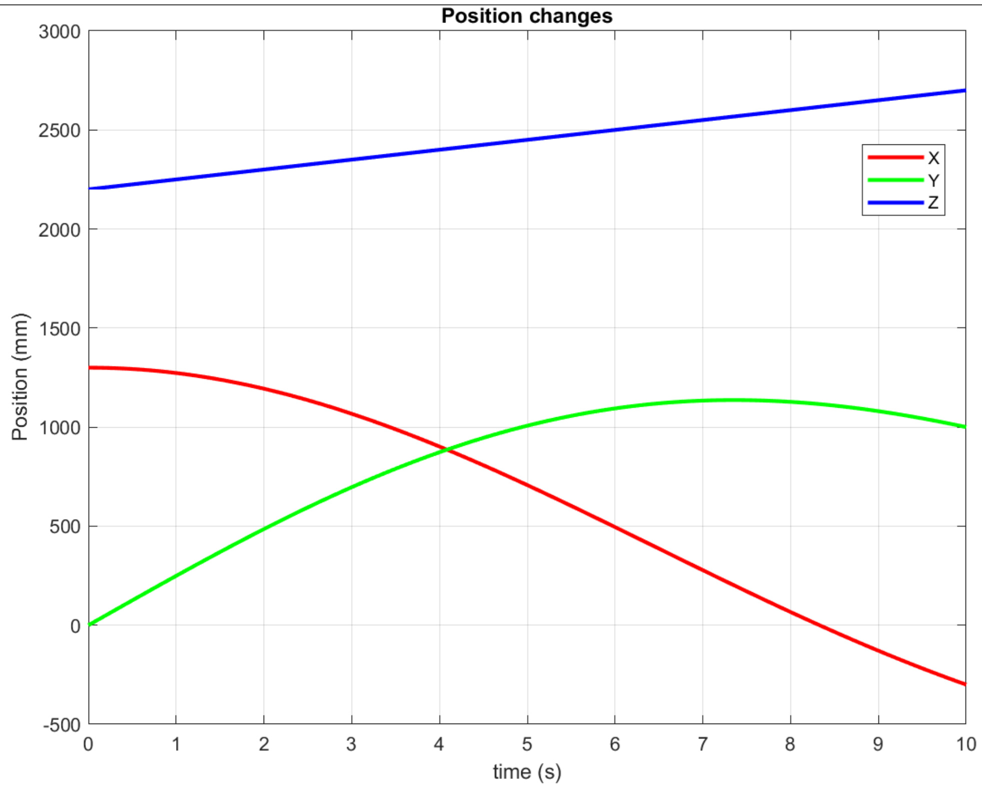

Figure 7.2: Simulation of the robot

7.3 Application

Modeling robot by Solidworks then convert to step file, this is suitable for Simscape Multibody Link.

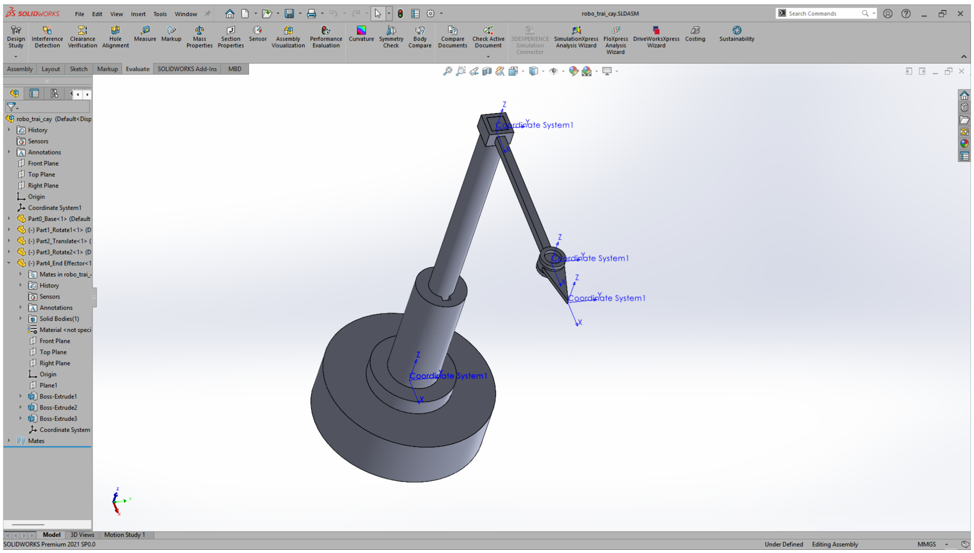

Figure 7.3: Robot model in Solidworks

Model in Simscape Multibody Link

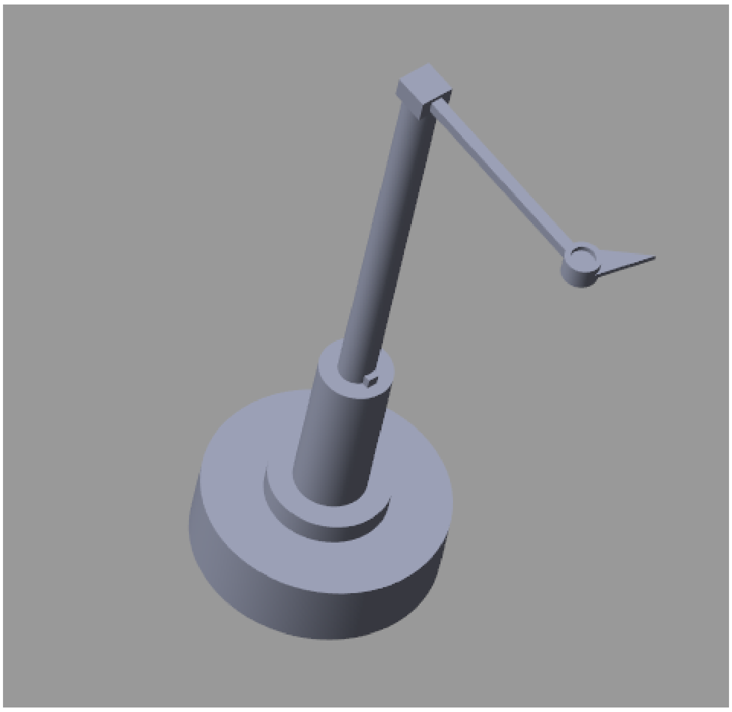

Figure 7.4: Robot model in Simscape Multibody Link

Block diagrams of Matlab Simulink is as the follow:

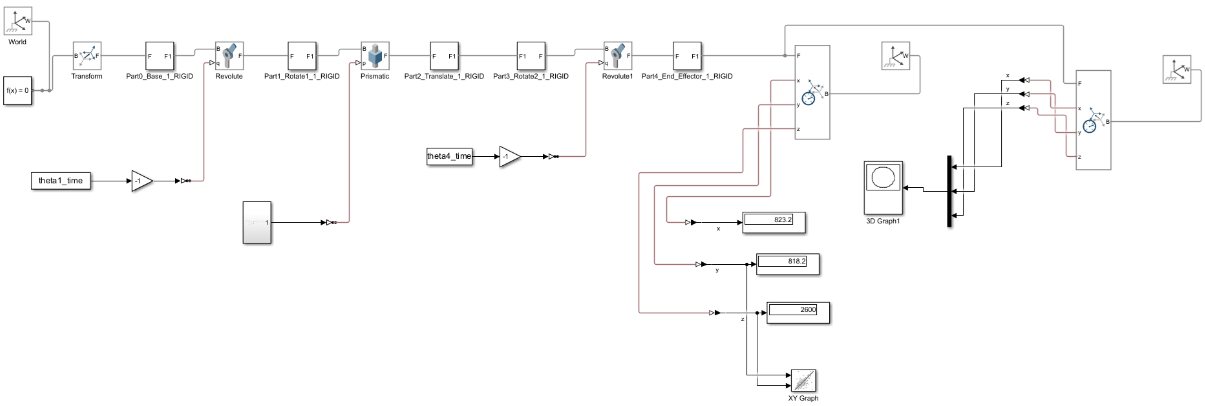

Figure 7.5: Block diagram of Matlab Simulink

Results of simulation:
Position (x, y, z) of robot from transsform sensor

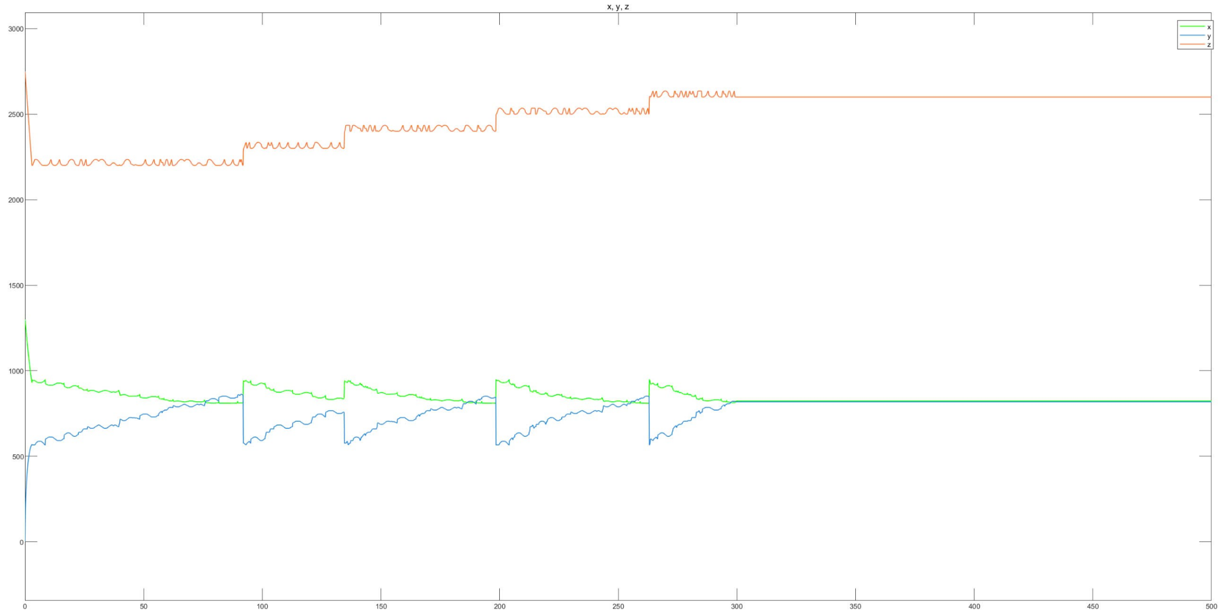

Figure 7.6: Position of robot

Result on YZ plane

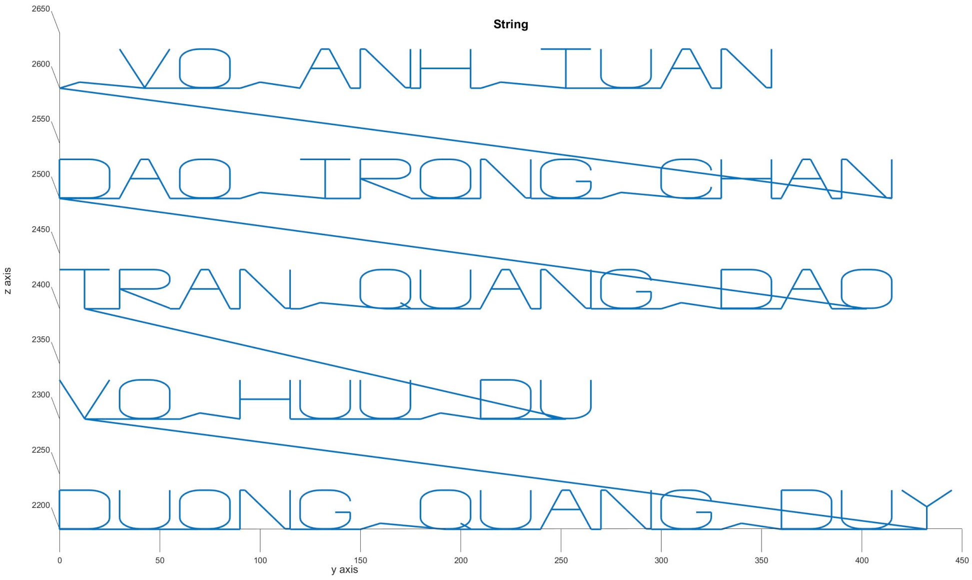

Figure 7.7: Result on YZ plane

Chapter 8

RESULTS

Kết quả mô phỏng được đăng trên Youtube tại đây

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