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GROUP PROJECT REPORT

ROBOTICS

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Chapter 1

SETTING COORDINATE FRAMES

1.1 Topic

Set coordinate frames for the first four links (link 1, link 2, link 3).

1.2 Theory

Based on “DENAVID-HARTENBERG NOTATION” (Lecture 4: Forward Kinematics [1]): Local frame B_i to each link (i) at joint $i + 1$ is defined as:

- The z_i axis is aligned with the $i + 1$ joint axis.
- The x_i axis is defined along the common normal between the z_{i-1} and z_i axes, pointing from the z_{i-1} to the z_i axis.
- The y_i axis is defined by the right-hand rule.
- The origin o_i of the i frame is located at the intersection of the joint axis $i + 1$ with the common normal between the z_{i-1} and z_i axes.

1.3 Application

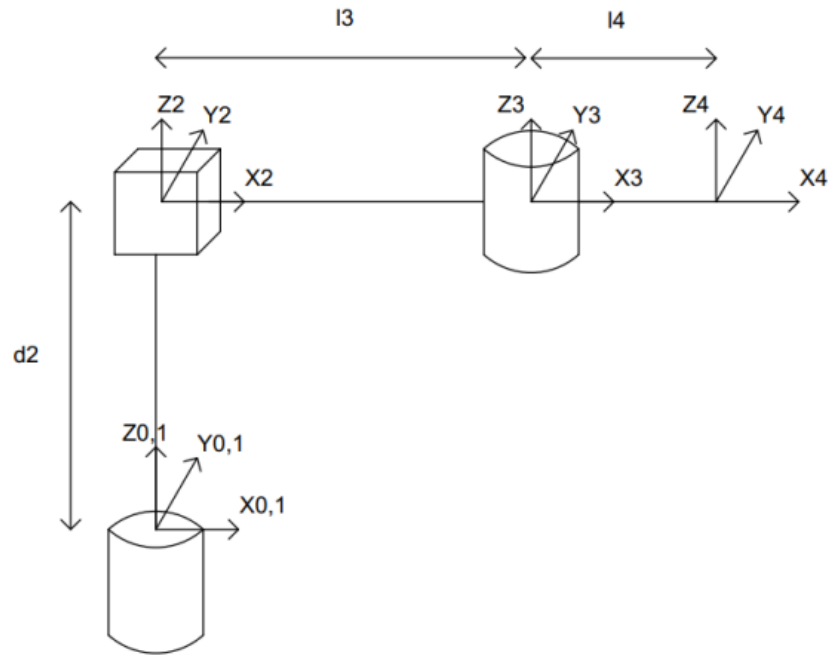


Figure 1.1: Setting coordinate frames for robot

Chapter 2

DETERMINING D-H PARAMETERS

2.1 Topic

Determine the Denavit-Hartenberg parameters for the robot model.

2.2 Theory

The Denavit-Hartenberg notation is introduced as a systematic method of describing the kinematic relationship ${}^{i-1}T_i$ using only four parameters [1]:

α	Link twist	Describe the link itself
a	Link length	
d	Link offset	Describe the link's connection to neighboring link
θ	Joint angle	
If the joint is:		
Revolute: θ joint variable		The other three are fixed link parameters
Prismatic: d joint variable		

2.3 Application

We got the D-H table:

i	a_i	α_i	d_i	θ_i
1	0	0	0	θ_1
2	0	0	d_2	0
3	ℓ_3	0	0	0
4	ℓ_4	0	0	θ_4

Limitations of the D-H notation:

$$\ell_3 = 1000 \text{ mm}$$

$$\ell_4 = 300 \text{ mm}$$

$$d_2 \in [2150; 2750]$$

$$\theta_1 \in [0^\circ; 360^\circ]$$

$$\theta_4 \in [-90^\circ; 90^\circ]$$

Chapter 3

KINEMATIC PROBLEM

3.1 Topic

Formulate the forward kinematic problem. Then determine the coordinates of the end-point according to the three joint variables. Make a plot for a certain case.

3.2 Theory

The transformation matrix ${}^i{}^{i-1}T$ to transform coordinate frames B_i to B_{i-1} is represented as a product of four basic transformations using parameters of link (i) and joint i

$${}^i{}^{i-1}T = \begin{bmatrix} \cos(\theta_i) & -\sin(\theta_i) \cos(\alpha_i) & \sin(\theta_i) \sin(\alpha_i) & a_i \cos(\theta_i) \\ \sin(\theta_i) & \cos(\theta_i) \cos(\alpha_i) & -\cos(\theta_i) \sin(\alpha_i) & a_i \sin(\theta_i) \\ 0 & \sin(\alpha_i) & \cos(\alpha_i) & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.1)$$

To find the single transformation that relates frame $\{i\}$ to frame $\{0\}$, the transformation matrices of every link are then multiplied together:

$${}^0T = {}^0T_1 T_2 \dots {}^i{}^{i-1}T \quad (3.2)$$

This transformation 0T is a function of all i joint variables. If the robot's joint-position sensors are queried, the Cartesian position and orientation of the end effector could be computed by 0T .

3.3 Application

We have:

$${}^0T_1 = \begin{bmatrix} \cos \theta_1 & -\sin \theta_1 & 0 & 0 \\ \sin \theta_1 & \cos \theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.3)$$

$${}^1T_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.4)$$

$${}^2T_3 = \begin{bmatrix} 1 & 0 & 0 & \ell_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.5)$$

$${}^3T_4 = \begin{bmatrix} \cos \theta_4 & -\sin \theta_4 & 0 & \ell_4 \cdot \cos \theta_4 \\ \sin \theta_4 & \cos \theta_4 & 0 & \ell_4 \cdot \sin \theta_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.6)$$

Thus

$${}^0T_4 = {}^0T_1 \cdot {}^1T_2 \cdot {}^2T_3 \cdot {}^3T_4 = \begin{bmatrix} \cos(\theta_1 + \theta_4) & -\sin(\theta_1 + \theta_4) & 0 & \ell_4 \cdot \cos(\theta_1 + \theta_4) + \ell_3 \cdot \cos \theta_1 \\ \sin(\theta_1 + \theta_4) & \cos(\theta_1 + \theta_4) & 0 & \ell_4 \cdot \sin(\theta_1 + \theta_4) + \ell_3 \cdot \sin \theta_1 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In conclusion, we have the solution for the forward kinematic problem as below

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \ell_4 \cdot \cos(\theta_1 + \theta_4) + \ell_3 \cdot \cos \theta_1 \\ \ell_4 \cdot \sin(\theta_1 + \theta_4) + \ell_3 \cdot \sin \theta_1 \\ d_2 \end{bmatrix} \quad (3.7)$$

Make a plot for a certain case:

During the time t from 0s to 10s, the input $(\theta_1, d_2, \theta_4)$ gradually increases from $(0^\circ, 2200, 0^\circ)$ to $(90^\circ, 2700, 90^\circ)$. Using MATLAB, the position (x, y, z) of the end-effector can be plotted as below:

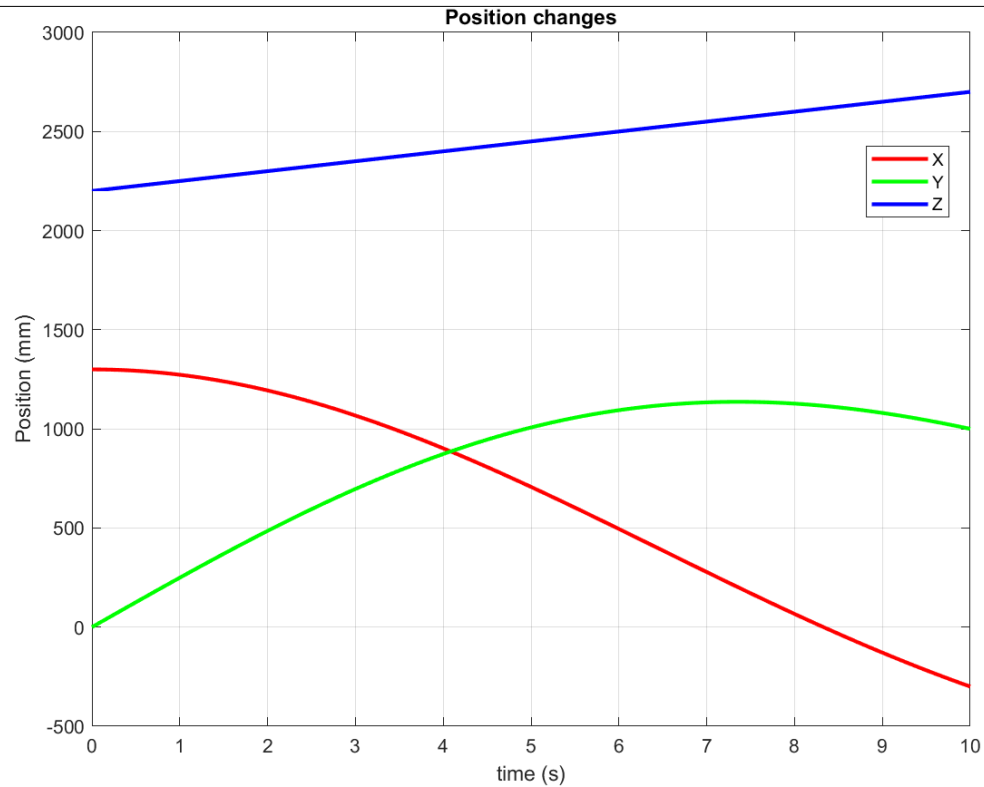


Figure 3.1: The position of the end-effector (x, y, z) with respect to time

Solution check:

- At time $t = 0$ s, the end-effector is at position $(x, y, z) = (1300, 0, 2200)$ mm. Correct!
- At time $t = 10$ s, the end-effector is at position $(x, y, z) = (-300, 1000, 2700)$ mm. Correct!

So we can confirm the accurate of the solution.

Chapter 4

INVERSE KINEMATIC PROBLEM

4.1 Topic

Formulate the inverse kinematic problem. Then determine the three joint values according to the coordinates of the end-point. Make a plot for a certain case.

4.2 Theory

Inverse kinematics is the mathematical process of calculating the variable joint parameters needed to place the end of a kinematic chain which is, in this project, the position of the end effector of a robot arm [1].

4.3 Application

We have the forward kinematic equation:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \ell_4 \cdot \cos(\theta_1 + \theta_4) + \ell_3 \cdot \cos \theta_1 \\ \ell_4 \cdot \sin(\theta_1 + \theta_4) + \ell_3 \cdot \sin \theta_1 \\ d_2 \end{bmatrix}$$

The position of the end-effector (X,Y,Z) can be extracted from the matrix as below:

$$x = \ell_4 \cdot \cos(\theta_1 + \theta_4) + \ell_3 \cdot \cos \theta_1 \quad (4.1)$$

$$y = \ell_4 \cdot \sin(\theta_1 + \theta_4) + \ell_3 \cdot \sin \theta_1 \quad (4.2)$$

$$z = d_2 \quad (4.3)$$

$$(4.4)$$

From (2.1) and (2.2) we have

$$\begin{aligned} x^2 + y^2 &= \ell_4^2 + \ell_3^2 + 2\ell_3 \cdot \ell_4 [\cos(\theta_1 + \theta_4) \cdot \cos \theta_1 + \sin(\theta_1 + \theta_4) \cdot \sin \theta_1] \\ &= \ell_4^2 + \ell_3^2 + 2\ell_3 \cdot \ell_4 \cos(\theta_1 + \theta_4 - \theta_1) \\ &= \ell_4^2 + \ell_3^2 + 2\ell_3 \cdot \ell_4 \cos(\theta_4) \end{aligned}$$

Set

$$a = \cos(\theta_4) = \frac{x^2 + y^2 - \ell_4^2 - \ell_3^2}{2\ell_3 \cdot \ell_4}$$

Thus

$$\theta_4 = \text{atan2} \left(\pm \sqrt{1 - a^2}, a \right)$$

The x and y components can be expressed as:

$$\begin{aligned} x \cos \theta_1 + y \sin \theta_1 &= \ell_4 \cdot \cos(\theta_1 + \theta_4) \cos \theta_1 + \ell_4 \cdot \sin(\theta_1 + \theta_4) \cdot \sin \theta_1 + \ell_3 \\ &= \ell_4 \cdot \cos(\theta_4) + \ell_3 \end{aligned}$$

We can rewrite

$$x \cos \theta_1 + y \sin \theta_1 = \sqrt{x^2 + y^2} \left(\frac{x}{\sqrt{x^2 + y^2}} \cos \theta_1 + \frac{y}{\sqrt{x^2 + y^2}} \sin \theta_1 \right)$$

Set

$$\begin{aligned} \cos \phi &= \frac{x}{\sqrt{x^2 + y^2}} \\ \sin \phi &= \frac{y}{\sqrt{x^2 + y^2}} \\ \phi &= \text{atan2}(y, x) \end{aligned}$$

Thus we have

$$\sqrt{x^2 + y^2} (\cos \phi \cdot \cos \theta_1 + \sin \phi \cdot \sin \theta_1) = \ell_4 \cdot \cos(\theta_4) + \ell_3$$

Infer the following:

$$\begin{aligned} \cos(\theta_1 - \phi) &= \frac{\ell_4 \cdot a + \ell_3}{\sqrt{x^2 + y^2}} \\ \sin(\theta_1 - \phi) &= \pm \frac{\sqrt{x^2 + y^2 - (\ell_4 \cdot a + \ell_3)^2}}{\sqrt{x^2 + y^2}} \\ \Rightarrow \theta_1 - \phi &= \text{atan2} \left(\pm \sqrt{x^2 + y^2 - (\ell_4 \cdot a + \ell_3)^2}, \ell_4 \cdot a + \ell_3 \right) \\ \Rightarrow \theta_1 &= \phi + \text{atan2} \left(\pm \sqrt{x^2 + y^2 - (\ell_4 \cdot a + \ell_3)^2}, \ell_4 \cdot a + \ell_3 \right) \\ &= \text{atan2}(y, x) + \text{atan2} \left(\pm \sqrt{x^2 + y^2 - (\ell_4 \cdot \cos(\theta_4) + \ell_3)^2}, \ell_4 \cdot \cos(\theta_4) + \ell_3 \right) \end{aligned}$$

Conclusion, we have the following solution for the inverse kinematic problem:

$$\begin{cases} \theta_1 = \text{atan2}(y, x) + \text{atan2} \left(\pm \sqrt{x^2 + y^2 - (\ell_4 \cdot \cos(\theta_4) + \ell_3)^2}, \ell_4 \cdot \cos(\theta_4) + \ell_3 \right) \\ d_2 = z \\ \theta_4 = \text{atan2} \left(\pm \sqrt{1 - a^2}, a \right) \end{cases}$$

or can rewrite as:

$$\begin{bmatrix} \theta_1 \\ d_2 \\ \theta_4 \end{bmatrix} = \begin{bmatrix} \text{atan2}(y, x) + \text{atan2} \left(\pm \sqrt{x^2 + y^2 - (\ell_4 \cdot \cos(\theta_4) + \ell_3)^2}, \ell_4 \cdot \cos(\theta_4) + \ell_3 \right) \\ Z \\ \text{atan2}(\pm \sqrt{1 - a^2}, a) \end{bmatrix}$$

with

$$a = \frac{x^2 + y^2 - \ell_4^2 - \ell_3^2}{2\ell_3 \cdot \ell_4}$$

Solution check:

As forward kinematic solution, with the input $(\theta_1, d_2, \theta_4) = (0^\circ, 2200, 0^\circ)$, the position of the end-effector is $(x, y, z) = (1300, 0, 2200)$.

Using $(x, y, z) = (1300, 0, 2200)$ as an input to inverse function, we got $(\theta_1, d_2, \theta_4) = (0^\circ, 2200, 0^\circ)$.

Similarly, using $(x, y, z) = (-300, 1000, 2700)$ as an input to inverse function, we got $(\theta_1, d_2, \theta_4) = (90^\circ, 2700, 90^\circ)$. Correct.

So we can confirm the accurate of the solution.

Chapter 5

WORKSPACE

5.1 Topic

Give a comment on the workspace.

5.2 Theory

Existing of any solution raises the question of the manipulator's workspace, which is the volume of space that the end-effector of the manipulator can reach.

5.3 Application

We have:

$$\begin{cases} x = \ell_4 \cdot \cos(\theta_1 + \theta_4) + \ell_3 \cdot \cos \theta_1 \\ y = \ell_4 \cdot \sin(\theta_1 + \theta_4) + \ell_3 \cdot \sin \theta_1 \\ z = d_2 \end{cases}$$

With:

- $\ell_3 = 1000 \text{ mm}$
- $\ell_4 = 300 \text{ mm}$
- $d_2 \in [2150; 2750]$
- $\theta_1 \in [0^\circ; 360^\circ]$
- $\theta_4 \in [-90^\circ; 90^\circ]$

Next:

$$\begin{aligned} x^2 + y^2 &= \ell_4^2 + \ell_3^2 + 2\ell_3\ell_4 \cos(\theta_4) \\ \Leftrightarrow \cos(\theta_4) &= \frac{x^2 + y^2 - \ell_4^2 - \ell_3^2}{2\ell_3\ell_4} \end{aligned}$$

We also have:

$$\begin{aligned} 0 &\leq \cos(\theta_4) \leq 1 \\ \Rightarrow 0 &\leq x^2 + y^2 \leq (\ell_4 + \ell_3)^2 \end{aligned}$$

$$\Rightarrow \begin{cases} 0 \leq x^2 + y^2 \leq 1690000 \\ d_2 \in [2150; 2750] \\ \theta_1 \in [0^\circ; 360^\circ] \\ \theta_4 \in [-90^\circ; 90^\circ] \end{cases}$$

To simplify matters, we remove the mechanical constraint, the joints can be fully rotated. Therefore, our workspace is idealized.

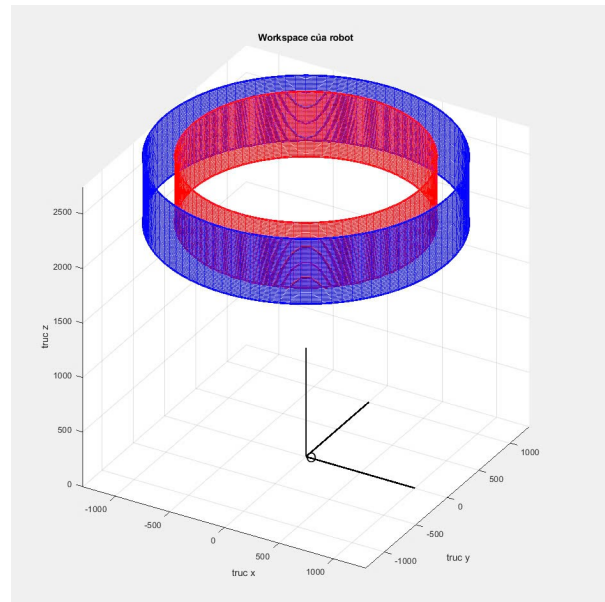


Figure 5.1: Workspace of the robot

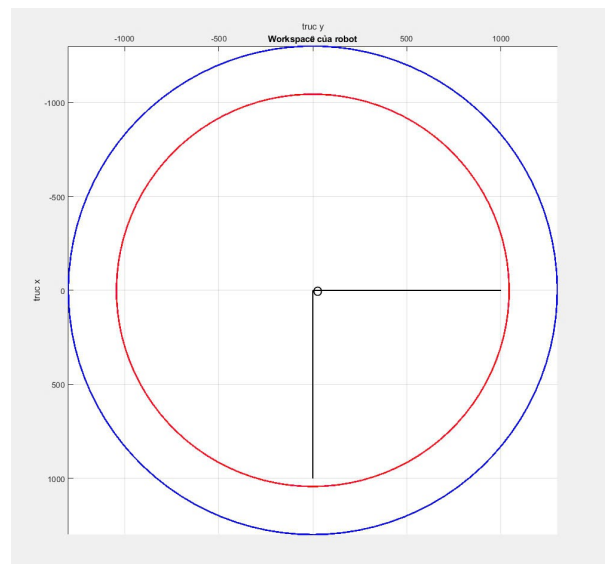


Figure 5.2: Workspace of the robot another view

Chapter 6

JACOBIAN MATRIX

6.1 Topic

Formulate the Jacobian matrix for this robot. Is there any singularity?

6.2 Theory

The Jacobian is a multidimensional form of the derivative. The 6 x 6 matrix of partial derivatives is the Jacobian J , as mapping velocities in X to those in Y .

In the field of robotics, Jacobians are used to relate joint velocities to Cartesian velocities of the tip of the arm.

$${}^0\mathbf{V} = {}^0J(\theta) \dot{\theta}$$

All manipulators have singularities at:

- The boundary of their workspace.
- Most have loci of singularities inside their workspace.

6.3 Application

Vector of joint variables:

$$\mathbf{Q} = [\theta_1 \quad d_2 \quad \theta_4]^T$$

Position-orientation state vector \mathbf{X} :

$$\begin{aligned} \mathbf{X} &= [p_x \quad p_y \quad d_2 \quad 0 \quad 0 \quad \theta_1 + \theta_4]^T \\ &= [\ell_4 \cos(\theta_1 + \theta_4) + \ell_3 \cos \theta_1 \quad \ell_4 \sin(\theta_1 + \theta_4) + \ell_3 \sin \theta_1 \quad d_2 \quad 0 \quad 0 \quad \theta_1 + \theta_4]^T \end{aligned}$$

The Jacobian matrix J is the partial derivative of the position vector \mathbf{X} with respect to the joint variable vector \mathbf{q} :

$$\begin{aligned}
 J = \frac{\partial \mathbf{X}}{\partial \mathbf{q}} &= \begin{bmatrix} \frac{\partial p_x}{\partial \theta_1} & \frac{\partial p_x}{\partial d_2} & \frac{\partial p_x}{\partial \theta_4} \\ \frac{\partial p_y}{\partial \theta_1} & \frac{\partial p_y}{\partial d_2} & \frac{\partial p_y}{\partial \theta_4} \\ \frac{\partial d_2}{\partial \theta_1} & \frac{\partial d_2}{\partial d_2} & \frac{\partial d_2}{\partial \theta_4} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{\partial(\theta_1+\theta_4)}{\partial \theta_1} & \frac{\partial(\theta_1+\theta_4)}{\partial d_2} & \frac{\partial(\theta_1+\theta_4)}{\partial \theta_4} \end{bmatrix} \\
 &= \begin{bmatrix} -\ell_4 \sin(\theta_1 + \theta_4) - \ell_3 \sin \theta_1 & 0 & -\ell_4 \sin(\theta_1 + \theta_4) \\ \ell_4 \cos(\theta_1 + \theta_4) + \ell_3 \cos \theta_1 & 0 & \ell_4 \cos(\theta_1 + \theta_4) \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} \\
 \Rightarrow \text{Control matrix } \mathbf{J} &= \begin{bmatrix} -\ell_4 \sin(\theta_1 + \theta_4) - \ell_3 \sin \theta_1 & 0 & -\ell_4 \sin(\theta_1 + \theta_4) \\ \ell_4 \cos(\theta_1 + \theta_4) + \ell_3 \cos \theta_1 & 0 & \ell_4 \cos(\theta_1 + \theta_4) \\ 0 & 1 & 0 \end{bmatrix}
 \end{aligned}$$

Case of Jacobian Singularity

$$J = \begin{bmatrix} -\ell_4 \sin(\theta_0 + \theta_4) & -\ell_3 \sin \theta_1 & 0 & -\ell_4 \sin(\theta_1 + \theta_4) \\ \ell_4 \cos(\theta_0 + \theta_4) + \ell_3 \cos \theta_1 & 0 & 0 & \ell_4 \cos(\theta_1 + \theta_4) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{aligned}
 \det(J_a) &= a_{11} \cdot (a_{22} \cdot a_{33} - a_{23} \cdot a_{32}) - a_{12} \cdot (a_{21} \cdot a_{33} - a_{23} \cdot a_{31}) + a_{13} \cdot (a_{21} \cdot a_{32} - a_{22} \cdot a_{31}) \\
 &= [-\ell_4 \sin(\theta_0 + \theta_4) - \ell_3 \sin \theta_1][\ell_4 \cos(\theta_1 + \theta_4)] + [-\ell_4 \sin(\theta_1 + \theta_4)][\ell_4 \cos(\theta_0 + \theta_4) + \ell_3 \cos \theta_1] \\
 &\quad - \ell_4^2 \sin(\theta_0 + \theta_4) \cos(\theta_1 + \theta_4) + \ell_3 \sin \theta_1 \ell_4 \cos(\theta_1 + \theta_4) - \ell_4^2 \sin(\theta_1 + \theta_4) \cos(\theta_0 + \theta_4) \\
 &\quad - \ell_3 \ell_4 \sin(\theta_1 + \theta_4) \cos \theta_1
 \end{aligned}$$

$$= \ell_3 \ell_4 [\sin \theta_1 \cdot \cos(\theta_1 + \theta_4) - \ell_4 \ell_3 \sin(\theta_1 + \theta_4) \cdot \cos \theta_1]$$

$$= -\ell_3 \cdot \ell_4 \cdot \sin(-\theta_4)$$

$$\det(J_a) = 0 \iff -\ell_4 \cdot \ell_3 \cdot \sin(-\theta_4) = 0$$

$$\iff -\ell_3 \cdot \ell_4 \cdot \sin(\theta_4) = 0$$

$$\iff \theta_4 = k\pi (k \in \mathbb{Z}); \theta_4 \in [-90^\circ, 90^\circ]$$

\Rightarrow The point is within the range where $\theta_4 = 0$

Chapter 7

SIMULATING THE MOTION OF ROBOT

7.1 Topic

Simulate the motion of this robot to plot initial letters from the first names of your team members on a certain plane that is perpendicular to axis z_0 .

7.2 Theory

Simulate robot in Simscape (θ_1, d_2, θ_4) change from $(0, 2200, 0)$ to $(90^\circ, 2700, 90^\circ)$.

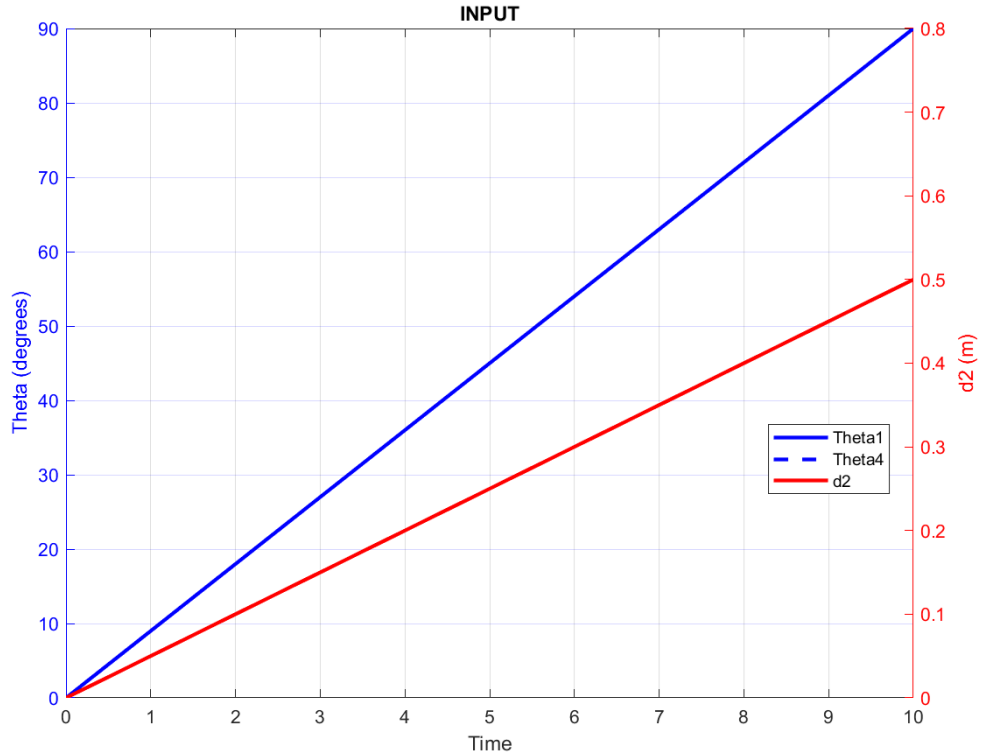


Figure 7.1: Simulation of the robot

Apply to forward, get position of end-effector (x, y, z) with respect to time t

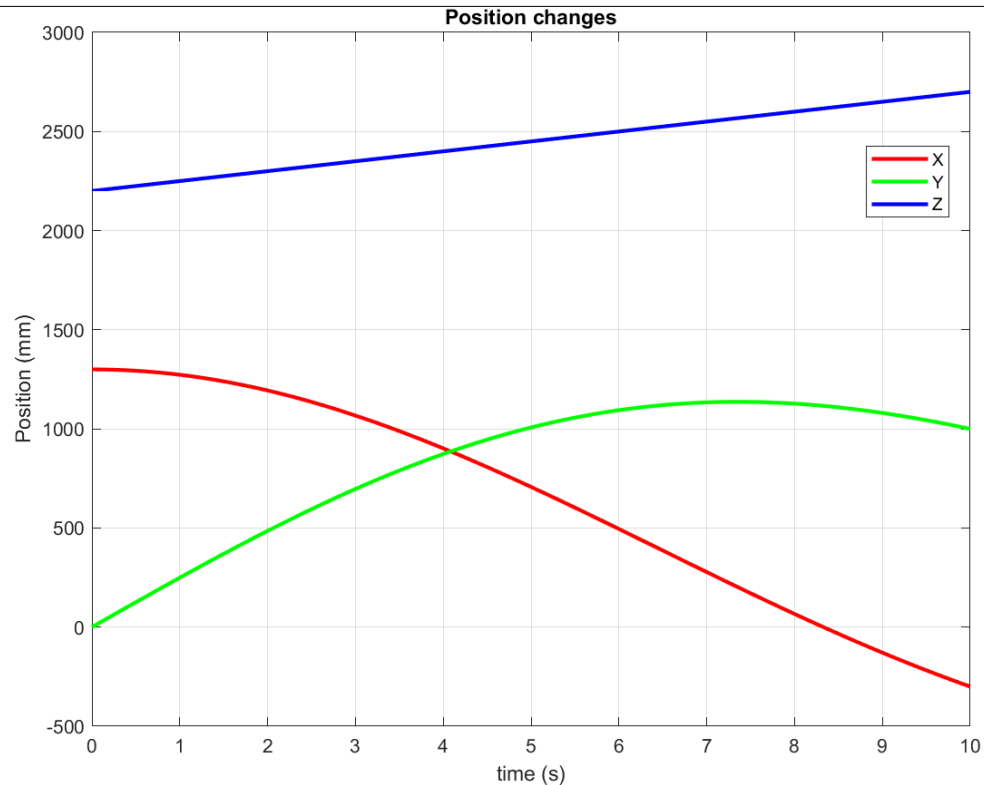


Figure 7.2: Simulation of the robot

7.3 Application

Modeling robot by Solidworks then convert to step file, this is suitable for Simscape Multibody Link.

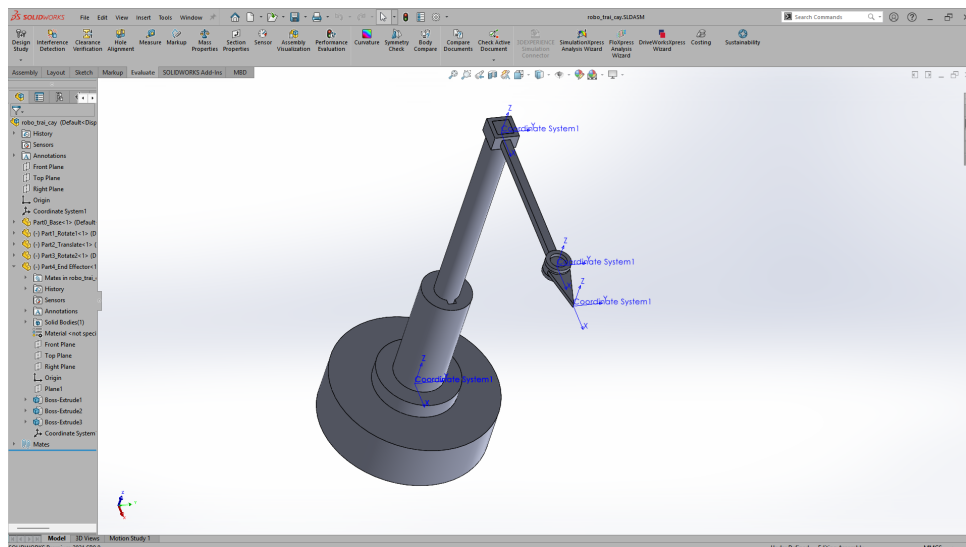


Figure 7.3: Robot model in Solidworks

Model in Simscape Multibody Link

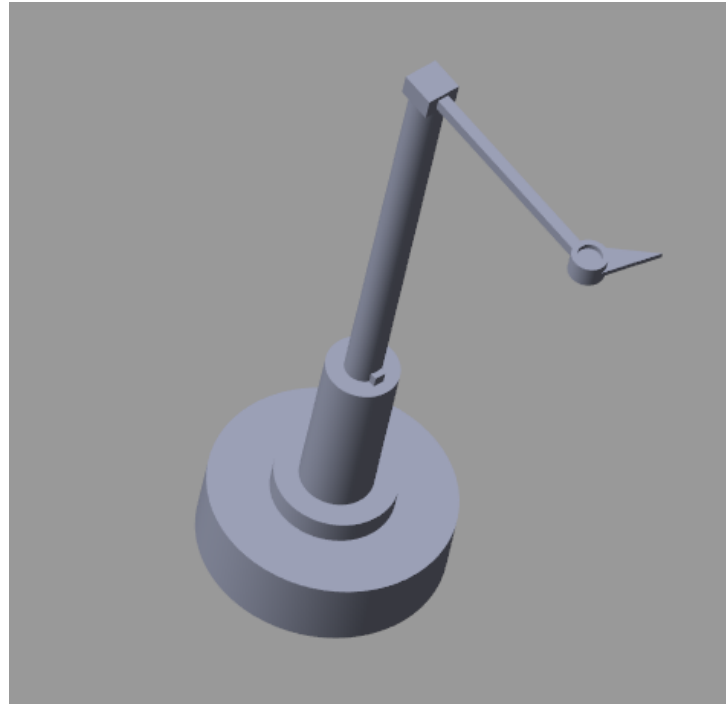


Figure 7.4: Robot model in Simscape Multibody Link

Block diagrams of Matlab Simulink is as the follow:

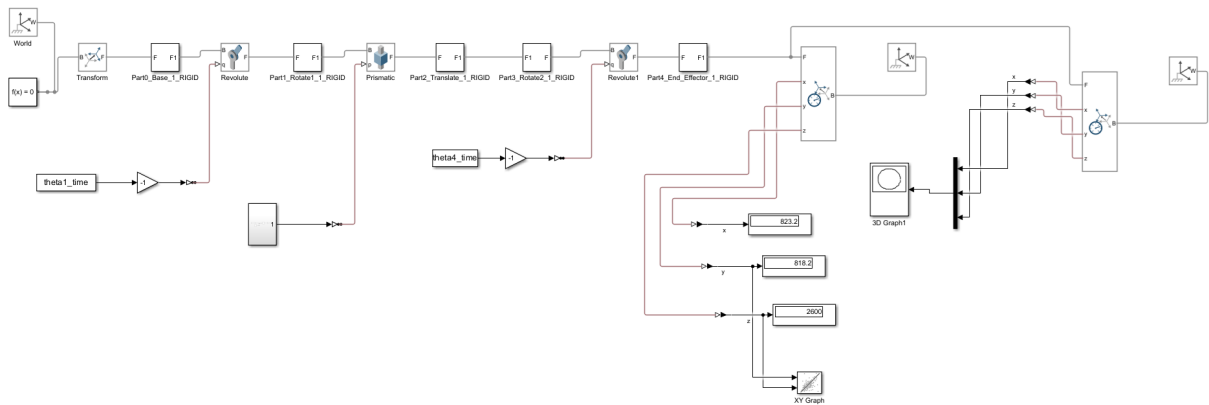


Figure 7.5: Block diagram of Matlab Simulink

Results of simulation:

Position (x, y, z) of robot from transsform sensor

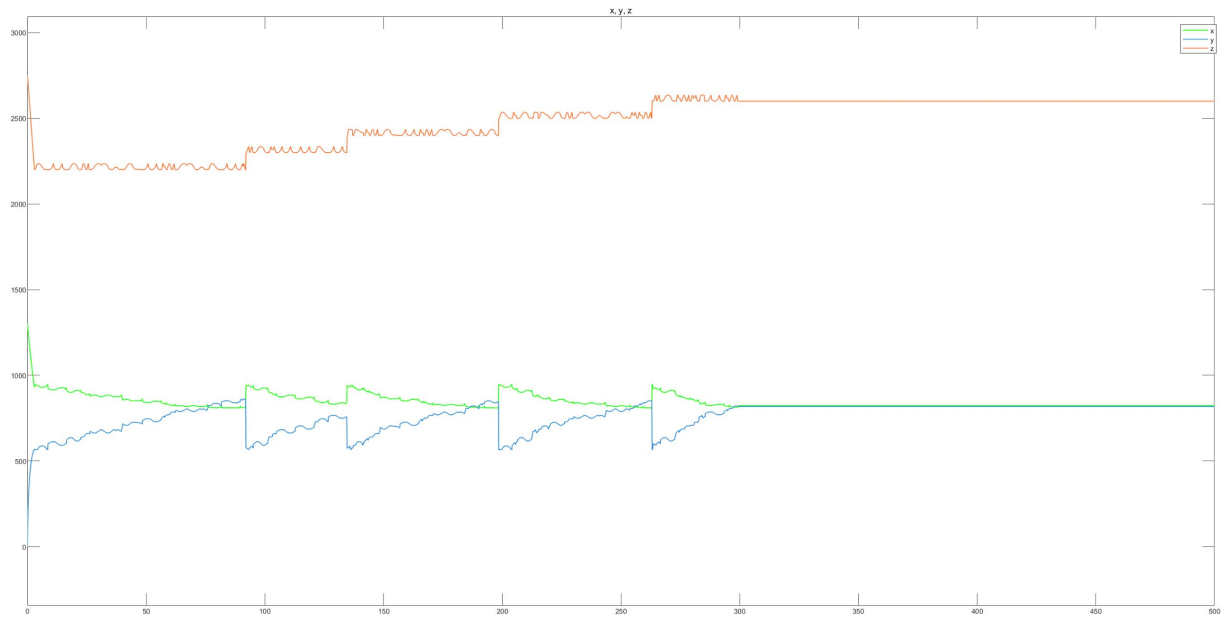


Figure 7.6: Position of robot

Result on YZ plane

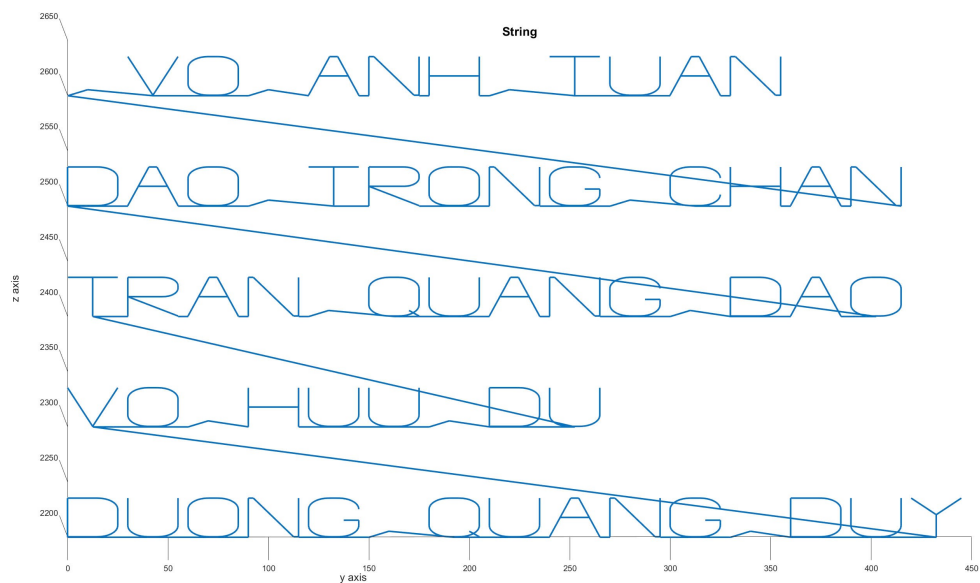


Figure 7.7: Result on YZ plane

Chapter 8

RESULTS

Kết quả mô phỏng được đăng trên Youtube tại đây

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