ĐẠI HỌC QUỐC GIA THÀNH PHỐ HỒ CHÍ MINH
TRƯỜNG ĐẠI HỌC BÁCH KHOA
KHOA CƠ KHÍ
BỘ MÔN CƠ ĐIỆN TỬ

BÁO CÁO BÀI TẬP LỚN

KỸ THUẬT ROBOT

GVHD: TS. PHÙNG TRÍ CÔNG

SINH VIÊN THỰC HIỆN:

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Mục lục

Chương 1

KINEMATIC PROBLEM

1.1 Topic

Formulate the forward kinematic problem. Then determine the coordinates of the end-point according to the three joint variables. Make a plot for a certain case.

1.2 Theory

The transformaytion matrix ${}^{i-1}_{i}T$ to transform coordinate frames B_i to B_i-1 is represented as a product of four basic transformations using paramaters of link (i) and joint i

$$
{}^{i-1}_{i}T = \begin{bmatrix} \cos(\theta_i) & -\sin(\theta_i)\cos(\alpha_i) & \sin(\theta_i)\sin(\alpha_i) & a_i\cos(\theta_i) \\ \sin(\theta_i) & \cos(\theta_i)\cos(\alpha_i) & -\cos(\theta_i)\sin(\alpha_i) & a_i\sin(\theta_i) \\ 0 & \sin(\alpha_i) & \cos(\alpha_i) & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.1}
$$

To find the single transformation that relates frame $\{i\}$ to frame $\{0\}$, the transformation matrices of every link are then multiplied together:

$$
{}^{0}_{i}T ={}^{0}_{1} T \, {}^{1}_{2}T \, ... \, {}^{i-1}_{i}T \tag{1.2}
$$

This transformation ${}^{0}_{i}T$ is a function of all i joint variables. If the robot's joint-position sensors are queried, the Cartesian position and orientation of the end effector could be computed by ${}^{0}_{i}T$.

1.3 Application

We have:

$$
{}^{0}T_1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.3}
$$

$$ {}^{1}T_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.4} $$

$$ {}^{2}T_3 = \begin{bmatrix} 1 & 0 & 0 & \ell_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.5} $$

$$ {}^{3}T_4 = \begin{bmatrix} \cos\theta_4 & -\sin\theta_4 & 0 & \ell_4 \cdot \cos\theta_4 \\ \sin\theta_4 & \cos\theta_4 & 0 & \ell_4 \cdot \sin\theta_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.6} $$

Thus

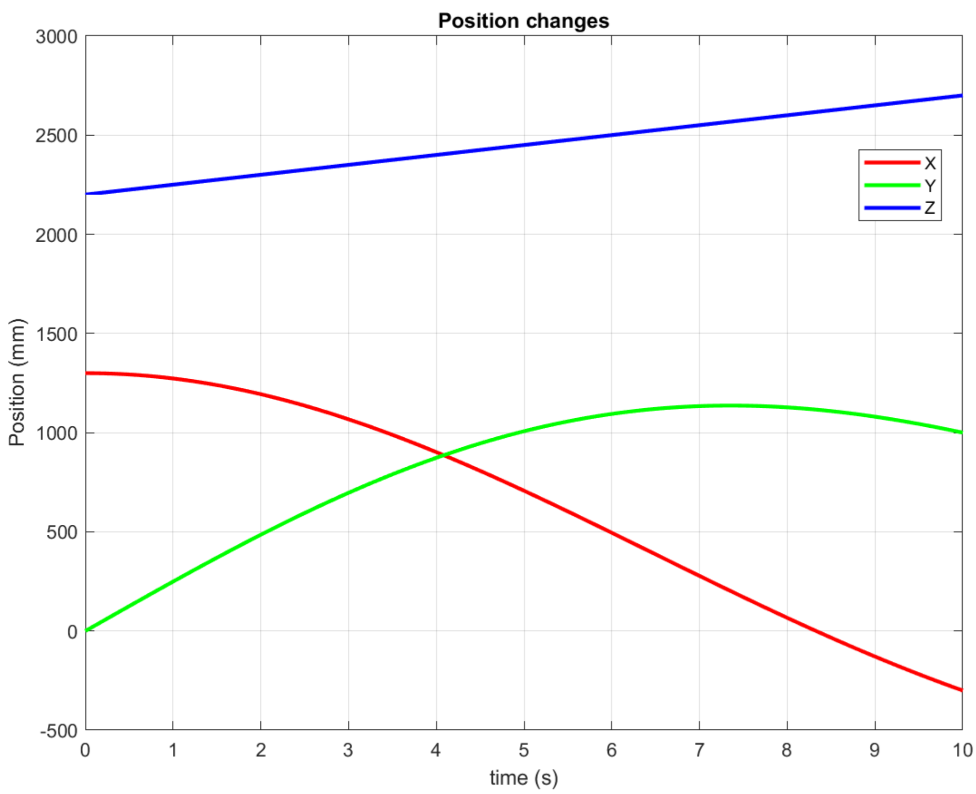
$$ {}^{0}T_4 = {}^{0}T_1.{}^{1}T_2.{}^{2}T_3.{}^{3}T_4 = \begin{bmatrix} \cos(\theta_1+\theta_4) & -\sin(\theta_1+\theta_4) & 0 & \ell_4.\cos(\theta_1+\theta_4)+\ell_3.\cos\theta_1 \\ \sin(\theta_1+\theta_4) & \cos(\theta_1+\theta_4) & 0 & \ell_4.\sin(\theta_1+\theta_4)+\ell_3.\sin\theta_1 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

In conclusion, we have the solution for the forward kinematic problem as below

$$ \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \ell_4.\cos(\theta_1+\theta_4)+\ell_3.\cos\theta_1 \\ \ell_4.\sin(\theta_1+\theta_4)+\ell_3.\sin\theta_1 \\ d_2 \end{bmatrix} \tag{1.7} $$

Make a plot for a certain case:

During the time t from 0s to 10s, the input $(\theta_1, d_2, \theta_4)$ gradually increases from $(0°, 2200, 0°)$ to $(90°, 2700, 90°)$. Using MATLAB, the position $(x, y, z$ of the end-effector can be plotted as below:



Hình 1.1: The position of the end-effector (x, y, z) with respect to time

Solution check:

- At time $t = 0$ s, the end-effector is at position $(x, y, z) = (1300, 0, 2200)$ mm. Correct!
- At time $t = 10$ s, the end-effector is at position $(x, y, z) = (-300, 1000, 2700)$ mm. Correct!

So we can confirm the accurate of the solution.

Chương 2

INVERSE KINEMATIC PROBLEM

2.1 Topic

Formulate the inverse kinematic problem. Then determine the three joint values according to the coordinates of the end-point. Make a plot for a certain case.

2.2 Theory

Inverse kinematics is the mathematical process of calculating the variable joint parameters needed to place the end of a kinematic chain which is, in this project, the position of the end effector of a robot arm [1].

2.3 Application

We have the forward kinematic equation:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \ell_4.\cos(\theta_1+\theta_4)+\ell_3.\cos\theta_1 \\ \ell_4.\sin(\theta_1+\theta_4)+\ell_3.\sin\theta_1 \\ d_2 \end{bmatrix}$$

The position of the end-effector (X,Y,Z) can be extracted from the matrix as below:

$$x = \ell_4.\cos(\theta_1+\theta_4)+\ell_3.\cos\theta_1 \tag{2.1}$$

$$y = \ell_4.\sin(\theta_1+\theta_4)+\ell_3.\sin\theta_1 \tag{2.2}$$

$$z = d_2 \tag{2.3}$$

$$\tag{2.4}$$

From (2.1) and (2.2) we have

$$\begin{aligned} x^2+y^2 &= \ell_4^2+\ell_3^2+2\ell_3\cdot\ell_4\left[\cos(\theta_1+\theta_4)\cdot\cos\theta_1+\sin(\theta_1+\theta_4)\cdot\sin\theta_1\right] \\ &= \ell_4^2+\ell_3^2+2\ell_3\cdot\ell_4\cos(\theta_1+\theta_4-\theta_1) \\ &= \ell_4^2+\ell_3^2+2\ell_3\cdot\ell_4\cos(\theta_4) \end{aligned}$$

Set

$$a = \cos(\theta_4) = \frac{x^2+y^2-\ell_4^2-\ell_3^2}{2\ell_3\cdot\ell_4}$$

Thus

$$\theta_4 = \operatorname{atan2}\left(\pm\sqrt{1-a^2}, a\right)$$

The x and y components can be expressed as:

$$\begin{aligned}
x\cos\theta_1 + y\sin\theta_1 &= \ell_4 \cdot \cos(\theta_1+\theta_4)\cos\theta_1 + \ell_4 \cdot \sin(\theta_1+\theta_4)\cdot\sin\theta_1 + \ell_3 \\
&= \ell_4 \cdot \cos(\theta_4) + \ell_3
\end{aligned}$$

We can rewrite

$$x\cos\theta_1 + y\sin\theta_1 = \sqrt{x^2+y^2}\left(\frac{x}{\sqrt{x^2+y^2}}\cos\theta_1 + \frac{y}{\sqrt{x^2+y^2}}\sin\theta_1\right)$$

Set

$$\begin{aligned}
\cos\phi &= \frac{x}{\sqrt{x^2+y^2}} \\
\sin\phi &= \frac{y}{\sqrt{x^2+y^2}} \\
\phi &= \operatorname{atan2}(y, x)
\end{aligned}$$

Thus we have

$$\sqrt{x^2+y^2}\left(\cos\phi\cdot\cos\theta_1 + \sin\phi\cdot\sin\theta_1\right) = \ell_4\cdot\cos(\theta_4) + \ell_3$$

Infer the following:

$$\begin{aligned}
\cos(\theta_1-\phi) &= \frac{\ell_4\cdot a+\ell_3}{\sqrt{x^2+y^2}} \\
\sin(\theta_1-\phi) &= \pm\frac{\sqrt{x^2+y^2-(\ell_4\cdot a+\ell_3)^2}}{\sqrt{x^2+y^2}} \\
\Rightarrow \theta_1-\phi &= \operatorname{atan2}\left(\pm\sqrt{x^2+y^2-(\ell_4\cdot a+\ell_3)^2}, \ell_4\cdot a+\ell_3\right) \\
\Rightarrow \theta_1 &= \phi + \operatorname{atan2}\left(\pm\sqrt{x^2+y^2-(\ell_4\cdot a+\ell_3)^2}, \ell_4\cdot a+\ell_3\right) \\
&= \operatorname{atan2}(y,x) + \operatorname{atan2}\left(\pm\sqrt{x^2+y^2-(\ell_4\cdot\cos(\theta_4)+\ell_3)^2}, \ell_4\cdot\cos(\theta_4)+\ell_3\right)
\end{aligned}$$

Conclusion, we have the following solution for the inverse kinematic problem:

$$\begin{cases}
\theta_1 = \operatorname{atan2}(y,x) + \operatorname{atan2}\left(\pm\sqrt{x^2+y^2-(\ell_4\cdot\cos(\theta_4)+\ell_3)^2}, \ell_4\cdot\cos(\theta_4)+\ell_3\right) \\
d_2 = z \\
\theta_4 = \operatorname{atan2}\left(\pm\sqrt{1-a^2}, a\right)
\end{cases}$$

or can rewrite as:

$$\begin{bmatrix}\theta_1\\ d_2\\ \theta_4\end{bmatrix} = \begin{bmatrix}\text{atan2}(y,x) + \text{atan2}\left(\pm\sqrt{x^2+y^2-(\ell_4\cdot\cos(\theta_4)+\ell_3)^2}, \ell_4\cdot\cos(\theta_4)+\ell_3\right)\\ Z\\ \text{atan2}\left(\pm\sqrt{1-a^2}, a\right)\end{bmatrix}$$

with

$$a = \frac{x^2+y^2-\ell_4^2-\ell_3^2}{2\ell_3\cdot\ell_4}$$

Solution check:

As forward kinematic solution, with the input $(\theta_1, d_2, \theta_4) = (0°, 2200, 0°)$, the position of the end-effector is $(x, y, z) = (1300, 0, 2200)$.

Using $(x, y, z) = (1300, 0, 2200)$ as an input to inverse function, we got $(\theta_1, d_2, \theta_4) = (0°, 2200, 0°)$.

Similarly, using $(x, y, z) = (-300, 1000, 2700)$ as an input to inverse function, we got $(\theta_1, d_2, \theta_4) = (90°, 2700, 90°)$. Correct.

So we can confirm the accurate of the solution.

Chương 3

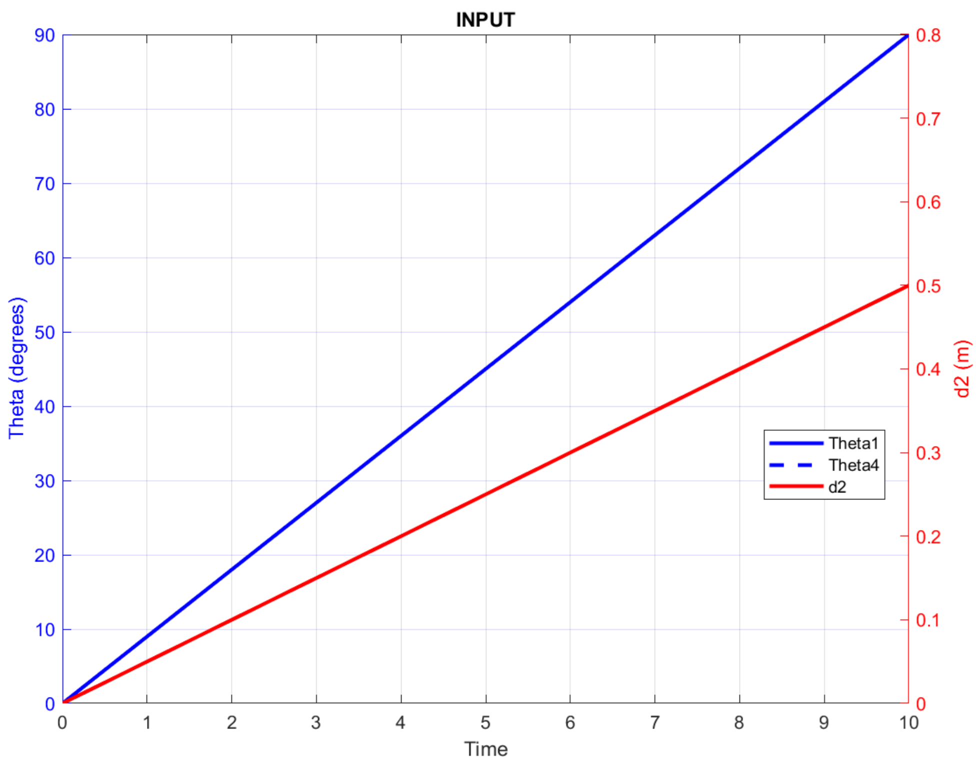
SIMULATING THE MOTION OF ROBOT

3.1 Topic

Simulate the motion of this robot to plot initial letters from the first names of your team members on a certain plane that is perpendicular to axis z_0.

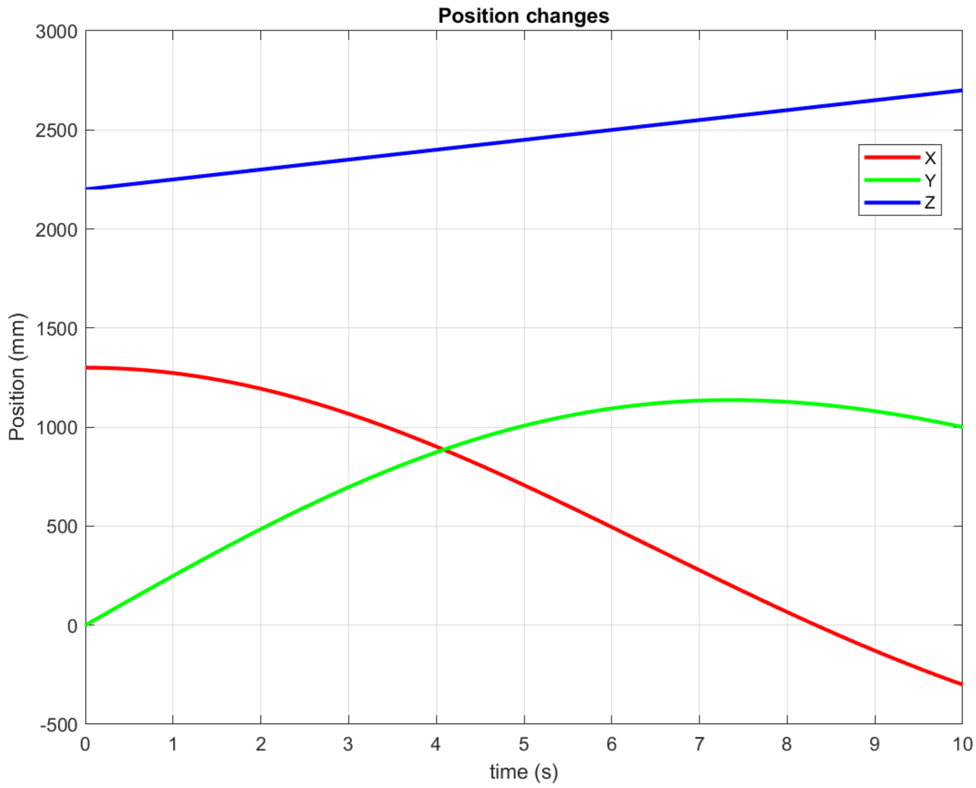
3.2 Theory

Simulate robot in simscape $(\theta_1, d_2, \theta_4)$ change from $(0, 2200, 0)$ to $(90°, 2700, 90°)$.



Hình 3.1: Simulation of the robot

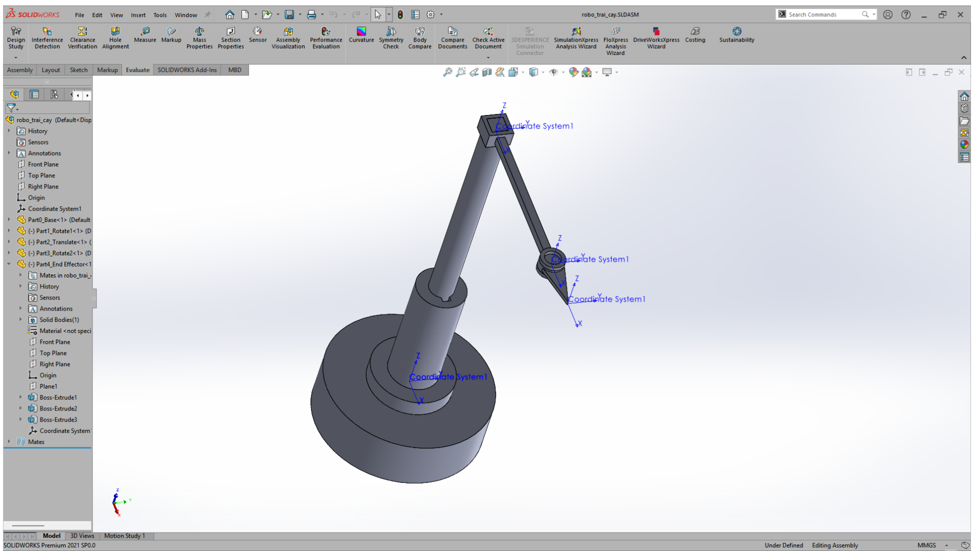
Apply to forward, get position of end-effector (x, y, z) with respect to time t



Hình 3.2: Simulation of the robot

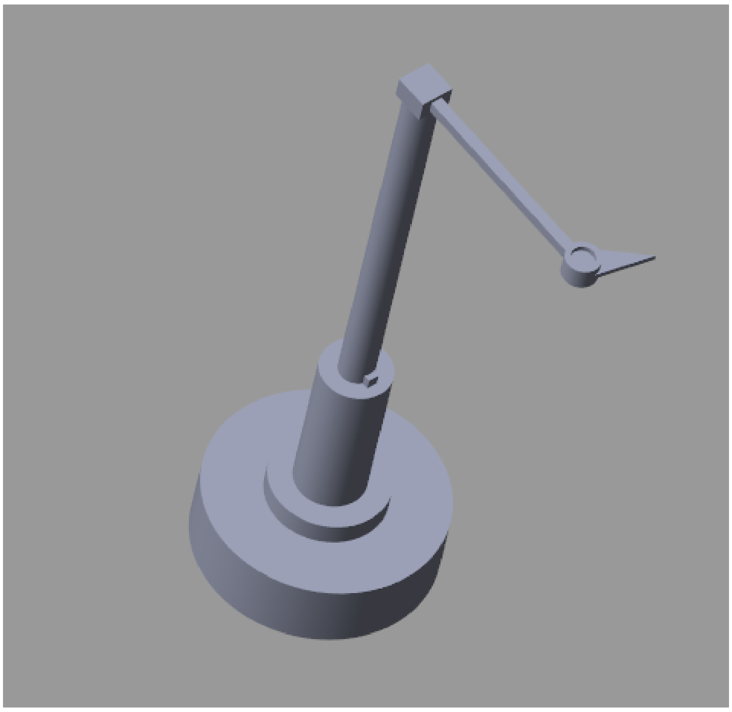
3.3 Application

Modeling robot by Solidworks then convert to step file, this is suitable for Simscape Multibody Link.



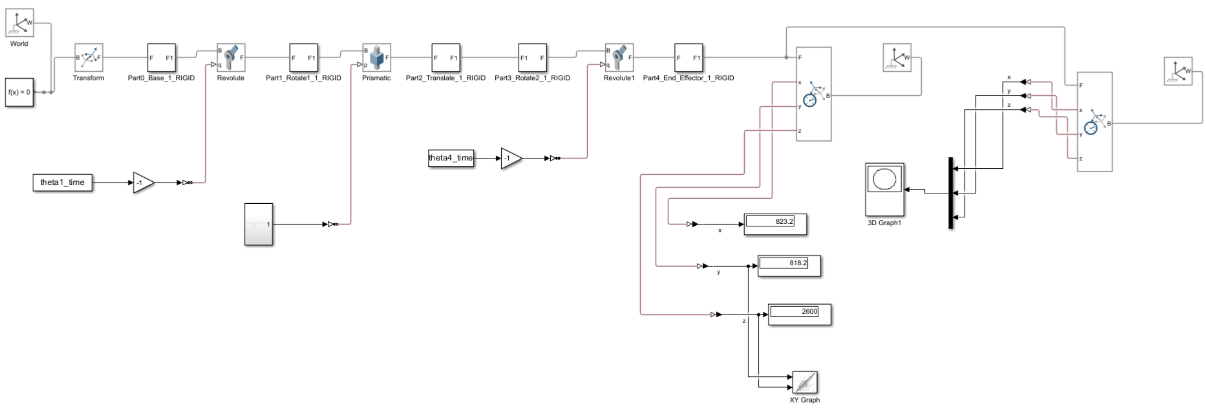
Hình 3.3: Robot model in Solidworks

Model in Simscape Multibody Link



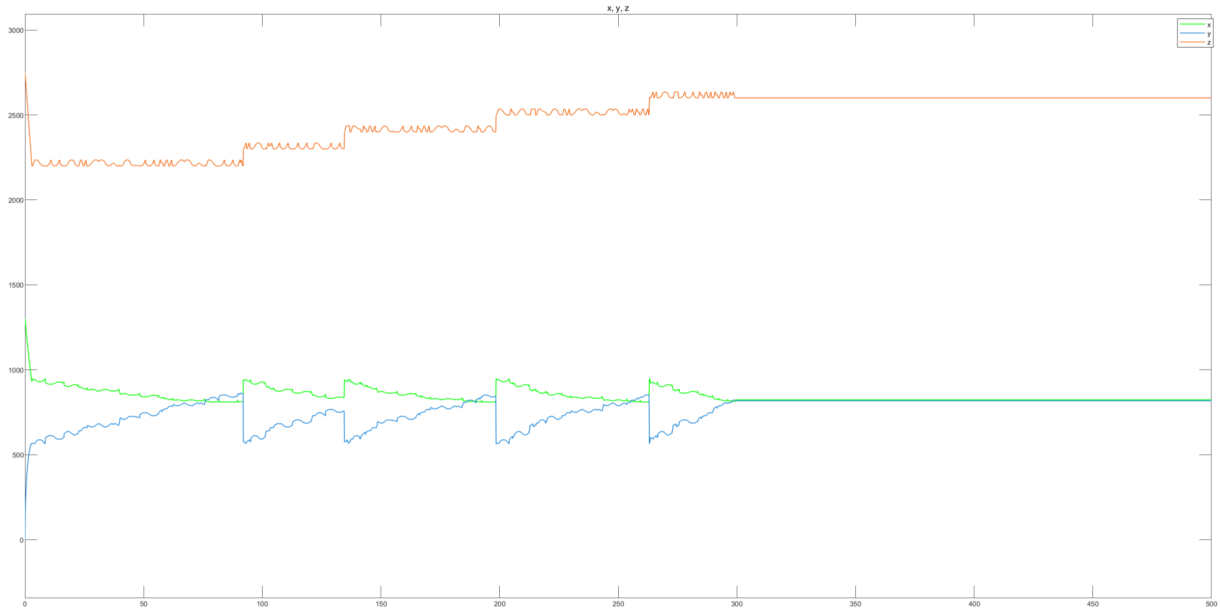
Hình 3.4: Robot model in Simscape Multibody Link

Block diagrams of Matlab Simulink is as the follow:



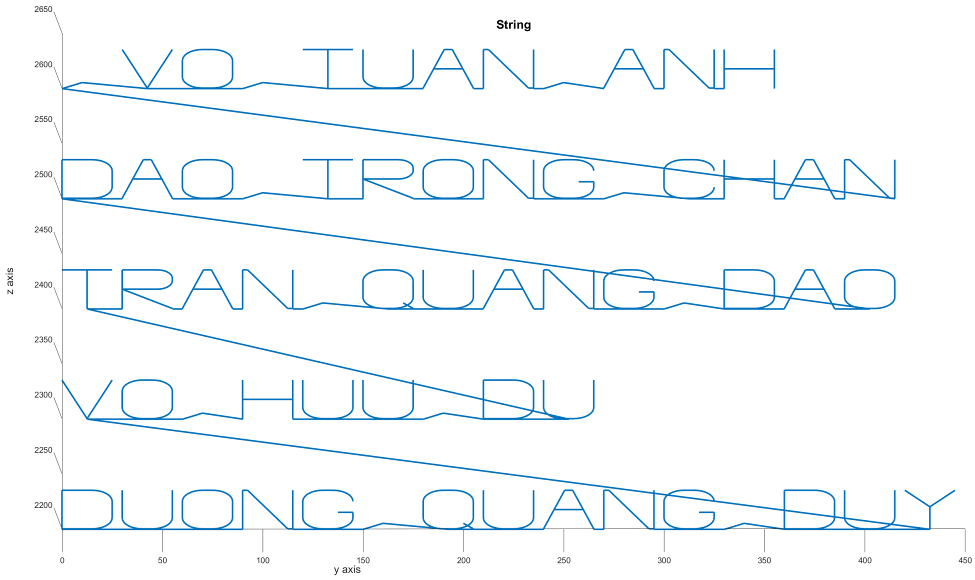
Hình 3.5: Block diagram of Matlab Simulink

Results of simulation:
Position (x, y, z) of robot from transsform sensor



Hình 3.6: Position of robot

Result on YZ plane



Hình 3.7: Result on YZ plane

Appendices