ĐẠI HỌC QUỐC GIA THÀNH PHỐ HỒ CHÍ MINH
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BÁO CÁO BÀI TẬP LỚN

KỸ THUẬT ROBOT

GVHD: TS. PHÙNG TRÍ CÔNG

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Mục lục

Chương 1

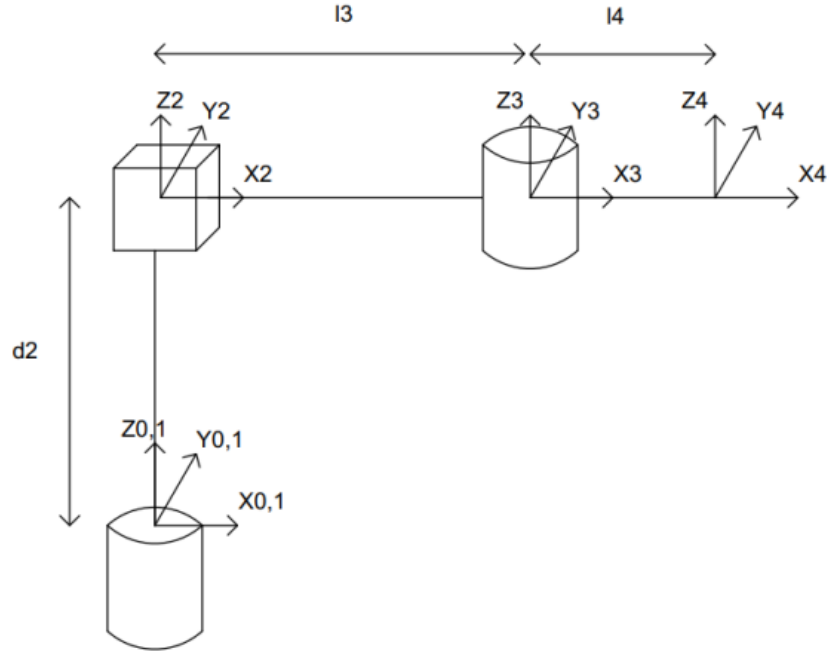# SETTING COORDINATE FRAMES

1.1 Topic

Set coordinate frames for the first four links (link 1, link 2, link 3).

1.2 Theory

Based on "DENAVIT-HARTENBERG NOTATION" (Lecture 4: Forward Kinematics [1]): Local frame B_i to each link (i) at joint $i + 1$ is defined as:

- The z_i axis is aligned with the $i + 1$ joint axis.

- The x_i axis is defined along the common normal between the $z_i - 1$ and z_i axes, pointing from the $z_i - 1$ to the z_i axis.

- The y_i axis is defined by the right-hand rule.

- The origin o_i of the i frame is located at the intersection of the joint axis $i + 1$ with the common normal between the $z_i - 1$ and z_i axes.

1.3 Application

Hình 1.1: Setting coordinate frames for robot

Phương trình Lagrange tổng quát cho cơ hệ

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i \tag{1.1}$$

Trong đó

- $L = K - V$: hàm Lagrange

- q_i: tọa độ suy rộng, trong phạm vi bài báo cáo ta có $q = [x\ \theta]^T$

- $\dot{q}_i$: đạo hàm theo thời gian của q_i

- Q_i: moment hoặc lực tổng quát (nếu có). Trong phạm vi bài báo cáo ta có $Q_i = [F_x\ \tau_\theta]$.

Hàm Lagrange

$$L = K - V = \frac{1}{2}M_\Sigma \dot{x}^2 + \frac{1}{2}J\dot{\theta}^2 + M_p\ell\dot{x}\dot{\theta}\cos\theta - M_p g\ell\cos\theta \tag{1.2}$$

Đối với tọa độ suy rộng x:

$$\frac{\partial L}{\partial \dot{x}} = M_\Sigma \dot{x} + M_p\ell\dot{\theta}\cos\theta$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = M_\Sigma \ddot{x} + M_p\ell\ddot{\theta}\cos\theta - M_p\ell\dot{\theta}^2\sin\theta$$

$$\frac{\partial L}{\partial x} = 0$$

Do đó

$$M_\Sigma \ddot{x} + M_p \ell \ddot{\theta} \cos\theta - M_p \ell \dot{\theta}^2 \sin\theta = F_x \tag{1.3}$$

Đối với tọa độ suy rộng θ:

$$\frac{\partial L}{\partial \dot{\theta}} = J\dot{\theta} + M_p \ell \dot{x} \cos\theta$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = J\ddot{\theta} + M_p \ell \ddot{x} \cos\theta - M_p \ell \dot{x}\dot{\theta} \sin\theta$$

$$\frac{\partial L}{\partial \theta} = -M_p \ell \dot{x}\dot{\theta} \sin\theta - M_p g\ell(-\sin\theta) = -M_p \ell \dot{x}\dot{\theta} \sin\theta + M_p g\ell \sin\theta$$

Do đó

$$J\ddot{\theta} + M_p \ell \ddot{x} \cos\theta - M_p \ell \dot{x}\dot{\theta} \sin\theta + M_p \ell \dot{x}\dot{\theta} \sin\theta - M_p g\ell \sin\theta = \tau_\theta$$

$$\Leftrightarrow J\ddot{\theta} + M_p \ell \ddot{x} \cos\theta - M_p g\ell \sin\theta = \tau_\theta \tag{1.4}$$

Từ phương trình (1.13) và (1.15) ta có hệ

$$\begin{cases} M_\Sigma \ddot{x} + M_p \ell \ddot{\theta} \cos\theta - M_p \ell \dot{\theta}^2 \sin\theta = F_x \\ J\ddot{\theta} + M_p \ell \ddot{x} \cos\theta - M_p g\ell \sin\theta = \tau_\theta \end{cases} \tag{1.5}$$

Ta đưa về dạng tổng quát

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau \tag{1.6}$$

Trong đó

- $q = [x\ \theta]^T$: vector trạng thái

- $M(q)$: ma trận quán tính

- $C(q,\dot{q})$: ma trận Coriolis và ly tâm

- $G(q)$: vector trọng lực

- τ: moment xoắn điều khiển từ động cơ

Ta được

$$\underbrace{\begin{bmatrix} M_\Sigma & M_p \ell \cos\theta \\ M_p \ell \cos\theta & J \end{bmatrix}}_{M(q)} \begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & -M_p \ell \dot{\theta} \sin\theta \\ 0 & 0 \end{bmatrix}}_{C(q,\dot{q})} \begin{bmatrix} \dot{x} \\ \dot{\theta} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ -M_p g\ell \sin\theta \end{bmatrix}}_{G(q)} = \underbrace{\begin{bmatrix} F_x \\ \tau \end{bmatrix}}$$

Trong chương tiếp theo, ta sẽ tiến hành thiết kế và mô phỏng các bộ điều khiển dựa trên hàm truyền đã thu được.

Chương 2

DETERMINING D-H PARAMETERS

2.1 Topic

Determine the Denavit-Hartenberg parameters for the robot model.

2.2 Theory

The Denavit-Hartenberg notation is introduced as a systematic method of describing the kinematic relationship $^{i-1}T_i$ using only four parameters [1]:

α	Link twist	Describe the link itself
a	Link length	
d	Link offset	Describe the link's connection to neighboring link
θ	Joint angle	
If the joint is:		
Revolute: θ joint variable	The other three are fixed link parameters	
Prismatic: d joint variable		

2.3 Application

We got the D-H table:

i	a_i	α_i	d_i	θ_i
1	0	0	0	θ_1
2	0	0	d_2	0
3	ℓ_3	0	0	0
4	ℓ_4	0	0	θ_4

Limitations of the D-H notation:

$$\ell_3 = 1000\,\mathrm{mm}$$
$$\ell_4 = 300\,\mathrm{mm}$$
$$d_2 \in [2150; 2750]$$
$$\theta_1 \in [0°; 360°]$$
$$\theta_4 \in [-90°; 90°]$$

Chương 3

KINEMATIC PROBLEM

3.1 Topic

Formulate the forward kinematic problem. Then determine the coordinates of the end-point according to the three joint variables. Make a plot for a certain case.

3.2 Theory

The transformaytion matrix $_i^{i-1}T$ to transform coordinate frames B_i to B_i-1 is represented as a product of four basic transformations using paramaters of link (i) and joint i

$$_i^{i-1}T = \begin{bmatrix} \cos(\theta_i) & -\sin(\theta_i)\cos(\alpha_i) & \sin(\theta_i)\sin(\alpha_i) & a_i\cos(\theta_i) \\ \sin(\theta_i) & \cos(\theta_i)\cos(\alpha_i) & -\cos(\theta_i)\sin(\alpha_i) & a_i\sin(\theta_i) \\ 0 & \sin(\alpha_i) & \cos(\alpha_i) & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.1}$$

To find the single transformation that relates frame $\{i\}$ to frame $\{0\}$, the transformation matrices of every link are then multiplied together:

$$_i^0T =_1^0 T \, _2^1T \, ... \, _i^{i-1}T \tag{3.2}$$

This transformation $_i^0T$ is a function of all i joint variables. If the robot's joint-position sensors are queried, the Cartesian position and orientation of the end effector could be computed by $_i^0T$.

3.3 Application

We have:

$$^0T_1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.3}$$

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$$
{}^1T_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.4}
$$

$$
{}^2T_3 = \begin{bmatrix} 1 & 0 & 0 & \ell_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.5}
$$

$$
{}^3T_4 = \begin{bmatrix} \cos\theta_4 & -\sin\theta_4 & 0 & \ell_4 \cdot \cos\theta_4 \\ \sin\theta_4 & \cos\theta_4 & 0 & \ell_4 \cdot \sin\theta_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.6}
$$

Thus

$$
{}^0T_4 = {}^0T_1.{}^1T_2.{}^2T_3.{}^3T_4 = \begin{bmatrix} \cos(\theta_1+\theta_4) & -\sin(\theta_1+\theta_4) & 0 & \ell_4.\cos(\theta_1+\theta_4)+\ell_3.\cos\theta_1 \\ \sin(\theta_1+\theta_4) & \cos(\theta_1+\theta_4) & 0 & \ell_4.\sin(\theta_1+\theta_4)+\ell_3.\sin\theta_1 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

In conclusion, we have the solution for the forward kinematic problem as below

$$
\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \ell_4.\cos(\theta_1+\theta_4)+\ell_3.\cos\theta_1 \\ \ell_4.\sin(\theta_1+\theta_4)+\ell_3.\sin\theta_1 \\ d_2 \end{bmatrix} \tag{3.7}
$$

Make a plot for a certain case:

During the time t from 0s to 10s, the input $(\theta_1, d_2, \theta_4)$ gradually increases from $(0°, 2200, 0°)$ to $(90°, 2700, 90°)$. Using MATLAB, the position $(x, y, z$ of the end-effector can be plotted as below:
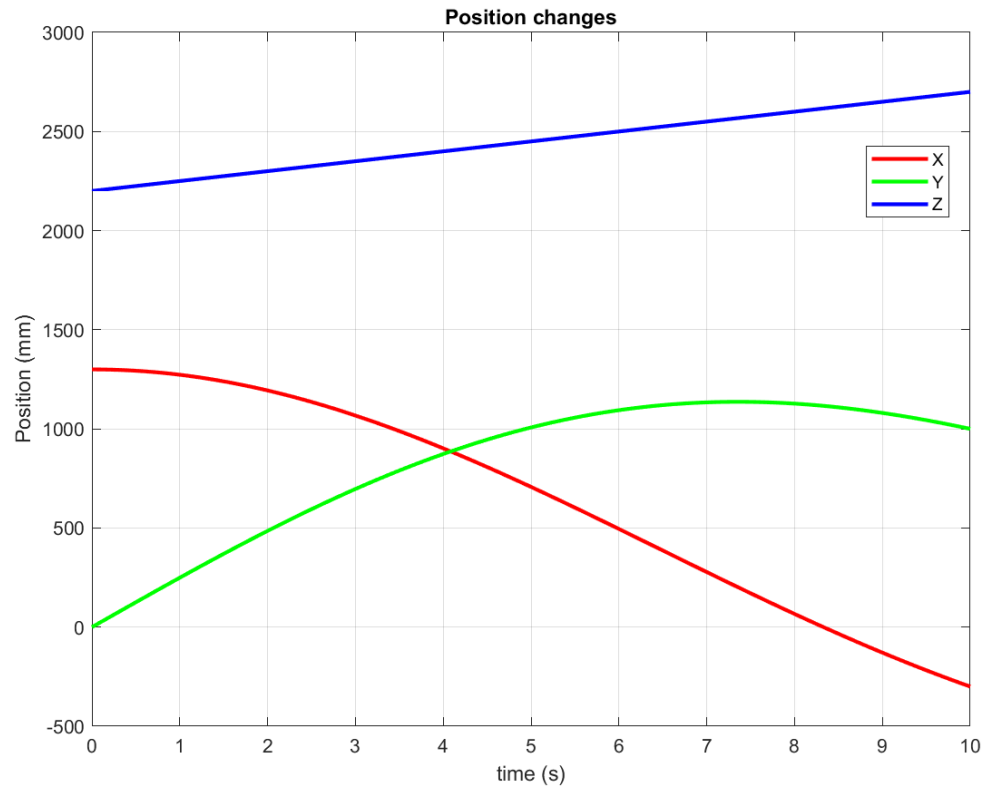
Hình 3.1: The position of the end-effector (x, y, z) with respect to time

Solution check:

- At time $t = 0$ s, the end-effector is at position $(x, y, z) = (1300, 0, 2200)$ mm. Correct!

- At time $t = 10$ s, the end-effector is at position $(x, y, z) = (-300, 1000, 2700)$ mm. Correct!

So we can confirm the accurate of the solution.

Chương 4

INVERSE KINEMATIC PROBLEM

4.1 Topic

Formulate the inverse kinematic problem. Then determine the three joint values according to the coordinates of the end-point. Make a plot for a certain case.

4.2 Theory

Inverse kinematics is the mathematical process of calculating the variable joint parameters needed to place the end of a kinematic chain which is, in this project, the position of the end effector of a robot arm [1].

4.3 Application

We have the forward kinematic equation:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \ell_4.\cos(\theta_1 + \theta_4) + \ell_3.\cos\theta_1 \\ \ell_4.\sin(\theta_1 + \theta_4) + \ell_3.\sin\theta_1 \\ d_2 \end{bmatrix}$$

The position of the end-effector (X,Y,Z) can be extracted from the matrix as below:

$$x = \ell_4.\cos(\theta_1 + \theta_4) + \ell_3.\cos\theta_1 \tag{4.1}$$
$$y = \ell_4.\sin(\theta_1 + \theta_4) + \ell_3.\sin\theta_1 \tag{4.2}$$
$$z = d_2 \tag{4.3}$$
$$\tag{4.4}$$

From (2.1) and (2.2) we have

$$\begin{aligned} x^2 + y^2 &= \ell_4^2 + \ell_3^2 + 2\ell_3 \cdot \ell_4 \left[\cos(\theta_1 + \theta_4) \cdot \cos\theta_1 + \sin(\theta_1 + \theta_4) \cdot \sin\theta_1\right] \\ &= \ell_4^2 + \ell_3^2 + 2\ell_3 \cdot \ell_4 \cos(\theta_1 + \theta_4 - \theta_1) \\ &= \ell_4^2 + \ell_3^2 + 2\ell_3 \cdot \ell_4 \cos(\theta_4) \end{aligned}$$

Set

$$a = \cos(\theta_4) = \frac{x^2 + y^2 - \ell_4^2 - \ell_3^2}{2\ell_3 \cdot \ell_4}$$

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Thus

$$\theta_4 = \text{atan2}\left(\pm\sqrt{1-a^2}, a\right)$$

The x and y components can be expressed as:

$$x\cos\theta_1 + y\sin\theta_1 = \ell_4 \cdot \cos(\theta_1 + \theta_4)\cos\theta_1 + \ell_4 \cdot \sin(\theta_1 + \theta_4) \cdot \sin\theta_1 + \ell_3$$
$$= \ell_4 \cdot \cos(\theta_4) + \ell_3$$

We can rewrite

$$x\cos\theta_1 + y\sin\theta_1 = \sqrt{x^2+y^2}\left(\frac{x}{\sqrt{x^2+y^2}}\cos\theta_1 + \frac{y}{\sqrt{x^2+y^2}}\sin\theta_1\right)$$

Set

$$\cos\phi = \frac{x}{\sqrt{x^2+y^2}}$$
$$\sin\phi = \frac{y}{\sqrt{x^2+y^2}}$$
$$\phi = \text{atan2}(y, x)$$

Thus we have

$$\sqrt{x^2+y^2}\left(\cos\phi \cdot \cos\theta_1 + \sin\phi \cdot \sin\theta_1\right) = \ell_4 \cdot \cos(\theta_4) + \ell_3$$

Infer the following:

$$\cos(\theta_1 - \phi) = \frac{\ell_4 \cdot a + \ell_3}{\sqrt{x^2+y^2}}$$

$$\sin(\theta_1 - \phi) = \pm\frac{\sqrt{x^2+y^2 - (\ell_4 \cdot a + \ell_3)^2}}{\sqrt{x^2+y^2}}$$

$$\Rightarrow \theta_1 - \phi = \text{atan2}\left(\pm\sqrt{x^2+y^2 - (\ell_4 \cdot a + \ell_3)^2}, \ell_4 \cdot a + \ell_3\right)$$

$$\Rightarrow \theta_1 = \phi + \text{atan2}\left(\pm\sqrt{x^2+y^2 - (\ell_4 \cdot a + \ell_3)^2}, \ell_4 \cdot a + \ell_3\right)$$

$$= \text{atan2}(y, x) + \text{atan2}\left(\pm\sqrt{x^2+y^2 - (\ell_4 \cdot \cos(\theta_4) + \ell_3)^2}, \ell_4 \cdot \cos(\theta_4) + \ell_3\right)$$

Conclusion, we have the following solution for the inverse kinematic problem:

$$\begin{cases} \theta_1 = \text{atan2}(y, x) + \text{atan2}\left(\pm\sqrt{x^2+y^2 - (\ell_4 \cdot \cos(\theta_4) + \ell_3)^2}, \ell_4 \cdot \cos(\theta_4) + \ell_3\right) \\ d_2 = z \\ \theta_4 = \text{atan2}\left(\pm\sqrt{1-a^2}, a\right) \end{cases}$$

or can rewrite as:

$$
\begin{bmatrix} \theta_1 \\ d_2 \\ \theta_4 \end{bmatrix} = \begin{bmatrix} \operatorname{atan2}(y,x) + \operatorname{atan2}\left(\pm\sqrt{x^2 + y^2 - (\ell_4 \cdot \cos(\theta_4) + \ell_3)^2}, \ell_4 \cdot \cos(\theta_4) + \ell_3 \right) \\ Z \\ \operatorname{atan2}\left(\pm\sqrt{1 - a^2}, a \right) \end{bmatrix}
$$

with

$$
a = \frac{x^2 + y^2 - \ell_4^2 - \ell_3^2}{2\ell_3 \cdot \ell_4}
$$

Solution check:

As forward kinematic solution, with the input $(\theta_1, d_2, \theta_4) = (0°, 2200, 0°)$, the position of the end-effector is $(x, y, z) = (1300, 0, 2200)$.

Using $(x, y, z) = (1300, 0, 2200)$ as an input to inverse function, we got $(\theta_1, d_2, \theta_4) = (0°, 2200, 0°)$.

Similarly, using $(x, y, z) = (-300, 1000, 2700)$ as an input to inverse function, we got $(\theta_1, d_2, \theta_4) = (90°, 2700, 90°)$. Correct.

So we can confirm the accurate of the solution.

Chương 5

Workspace

5.1 Topic

Give a comment on the workspace.

5.2 Theory

Existing of any solution raises the question of the manipulator's workspace, which is the volume of space that the end-effector of the manipulator can reach.

5.3 Application

We have:

$$\begin{cases} x = \ell_4 \cdot \cos(\theta_1 + \theta_4) + \ell_3 \cdot \cos\theta_1 \\ y = \ell_4 \cdot \sin(\theta_1 + \theta_4) + \ell_3 \cdot \sin\theta_1 \\ z = d_2 \end{cases}$$

With:

- $\ell_3 = 1000\,\text{mm}$

- $\ell_4 = 300\,\text{mm}$

- $d_2 \in [2150; 2750]$

- $\theta_1 \in [0°; 360°]$

- $\theta_4 \in [-90°; 90°]$

Next:

$$x^2 + y^2 = \ell_4^2 + \ell_3^2 + 2\ell_3\ell_4 \cos(\theta_4)$$

$$\Leftrightarrow \cos(\theta_4) = \frac{x^2 + y^2 - \ell_4^2 - \ell_3^2}{2\ell_3\ell_4}$$

We also have:

$$0 \leq \cos(\theta_4) \leq 1$$

$$\Rightarrow 0 \leq x^2 + y^2 \leq (\ell_4 + \ell_3)^2$$

$$\Rightarrow \begin{cases} 0 \le x^2 + y^2 \le 1690000 \\ d_2 \in [2150; 2750] \\ \theta_1 \in [0°; 360°] \\ \theta_4 \in [-90°; 90°] \end{cases}$$

To simplify matters, we remove the mechanical constraint, the joints can be fully rotated. Therefore, our workspace is idealized.

Chương 6

JACOBIAN MATRIX

6.1 Topic

Formulate the Jacobian matrix for this robot. Is there any singularity?

6.2 Theory

The Jacobian is a multidimensional form of the derivative. The 6 x 6 matrix of partial derivatives is the Jacobian J, as mapping velocities in X to those in Y.
In the field of robotics, Jacobians are used to relate joint velocities to Cartesian velocities of the tip of the arm.

$$^0\mathbf{V} = {}^0J(\theta)\,\dot{\theta}$$

All manipulators have singularities at:

- The boundary of their workspace.

- Most have loci of singularities inside their workspace.

6.3 Application

Vector of joint variables:
$$\mathbf{Q} = \begin{bmatrix} \theta_1 & d_2 & \theta_4 \end{bmatrix}^T$$

Position vector $\mathbf{X}$:
$$\mathbf{X} = \begin{bmatrix} p_x & p_y & d_2 & 0 & 0 & \theta_1 + \theta_4 \end{bmatrix}^T$$
$$= \begin{bmatrix} \ell_4 \cos(\theta_1 + \theta_4) + \ell_3 \cos\theta_1 & \ell_4 \sin(\theta_1 + \theta_4) + \ell_3 \sin\theta_1 & d_2 & 0 & 0 & \theta_1 + \theta_4 \end{bmatrix}^T$$

The Jacobian matrix J is the partial derivative of the position vector $\mathbf{X}$ with respect to the joint variable vector $\mathbf{q}$:

$$J = \frac{\partial \mathbf{X}}{\partial \mathbf{q}} = \begin{bmatrix} \frac{\partial p_x}{\partial \theta_1} & \frac{\partial p_x}{\partial d_2} & \frac{\partial p_x}{\partial \theta_4} \\ \frac{\partial p_y}{\partial \theta_1} & \frac{\partial p_y}{\partial d_2} & \frac{\partial p_y}{\partial \theta_4} \\ \frac{\partial d_2}{\partial \theta_1} & \frac{\partial d_2}{\partial d_2} & \frac{\partial d_2}{\partial \theta_4} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{\partial(\theta_1+\theta_4)}{\partial \theta_1} & \frac{\partial(\theta_1+\theta_4)}{\partial d_2} & \frac{\partial(\theta_1+\theta_4)}{\partial \theta_4} \end{bmatrix}$$

$$
= \begin{bmatrix}
-\ell_4 \sin(\theta_1 + \theta_4) - \ell_3 \sin\theta_1 & 0 & -\ell_4 \sin(\theta_1 + \theta_4) \\
\ell_4 \cos(\theta_1 + \theta_4) + \ell_3 \cos\theta_1 & 0 & \ell_4 \cos(\theta_1 + \theta_4) \\
0 & 1 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
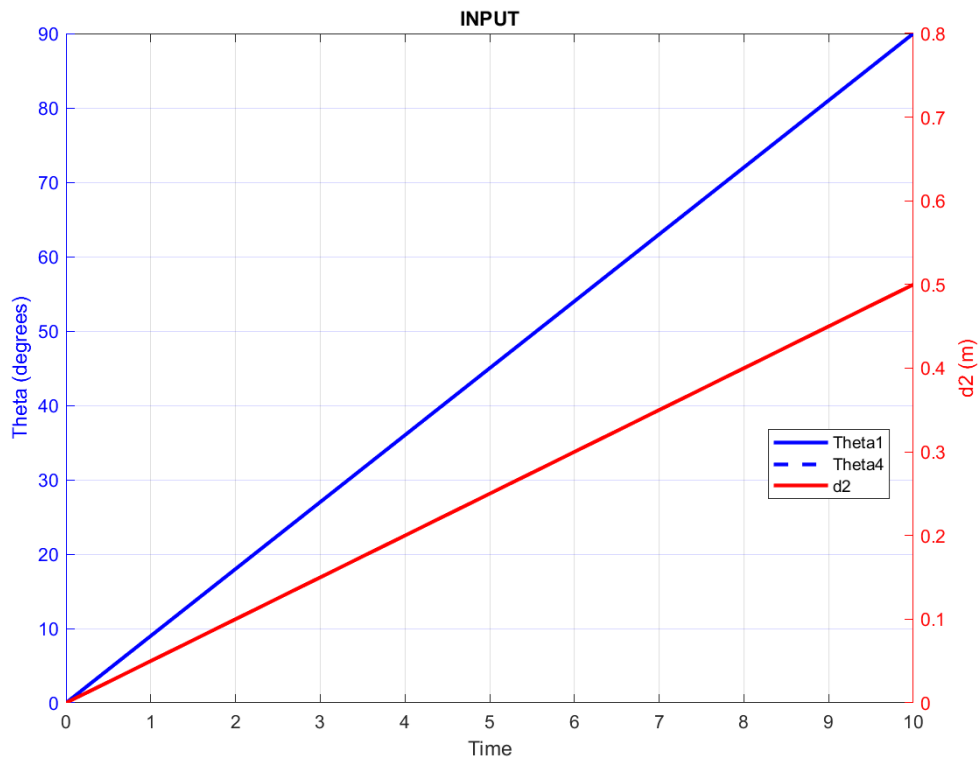1 & 0 & 1
\end{bmatrix}
$$

$$
\Rightarrow \text{ Ma trận điều khiển } \mathbf{J} = \begin{bmatrix}
-\ell_4 \sin(\theta_1 + \theta_4) - \ell_3 \sin\theta_1 & 0 & -\ell_4 \sin(\theta_1 + \theta_4) \\
\ell_4 \cos(\theta_1 + \theta_4) + \ell_3 \cos\theta_1 & 0 & \ell_4 \cos(\theta_1 + \theta_4) \\
0 & 1 & 0
\end{bmatrix}
$$

Chương 7

SIMULATING THE MOTION OF ROBOT

7.1 Topic

Simulate the motion of this robot to plot initial letters from the first names of your team members on a certain plane that is perpendicular to axis z_0.

7.2 Theory

Simulate robot in simscape $(\theta_1, d_2, \theta_4)$ change from $(0, 2200, 0)$ to $(90°, 2700, 90°)$.

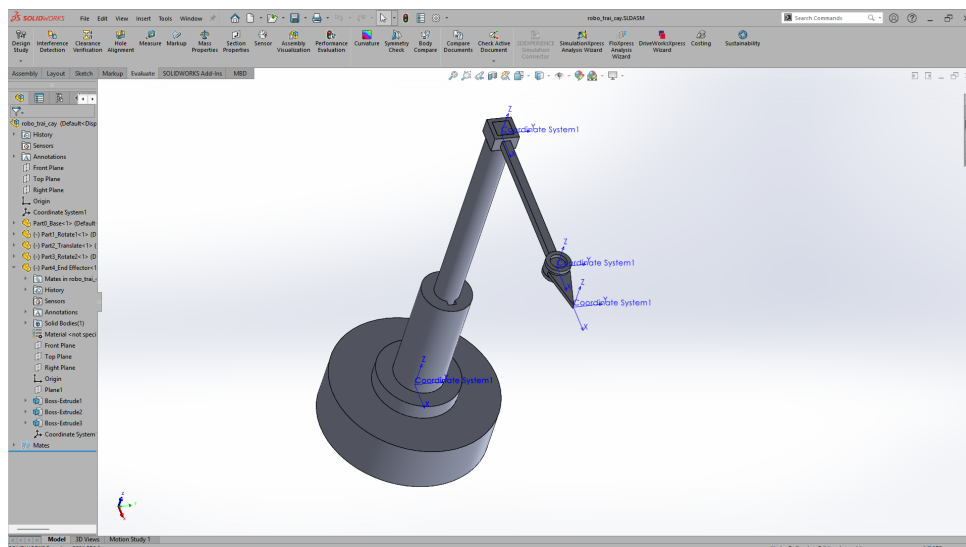Hình 7.1: Simulation of the robot

Apply to forward, get position of end-effector (x, y, z) with respect to time t
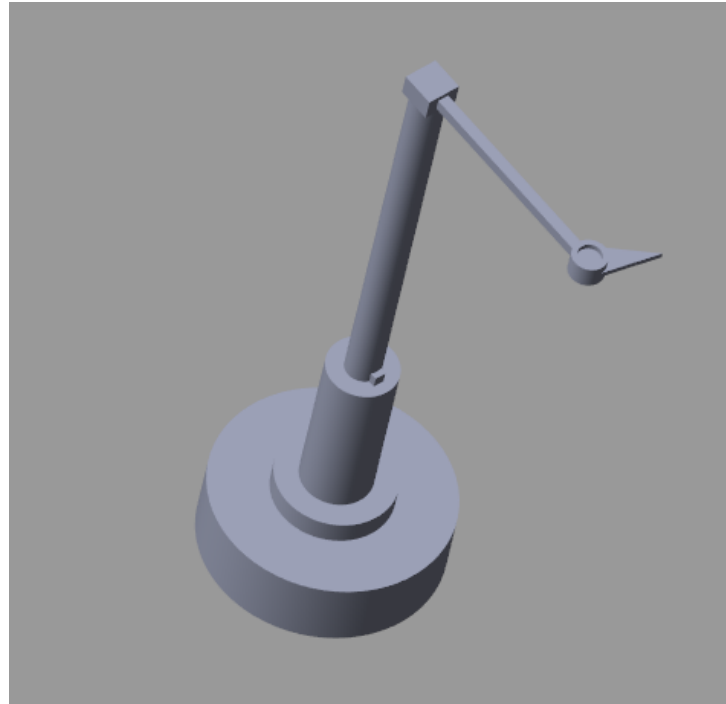
16
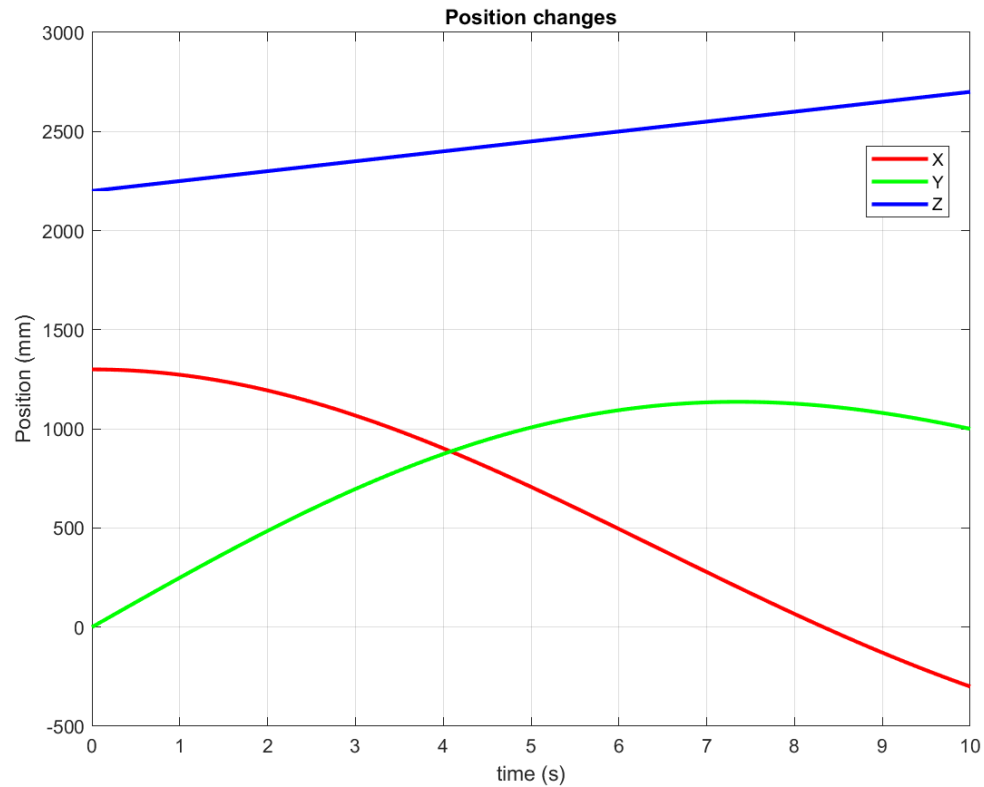
Hình 7.2: Simulation of the robot

7.3 Application

Modeling robot by Solidworks then convert to step file, this is suitable for Simscape Multibody Link.
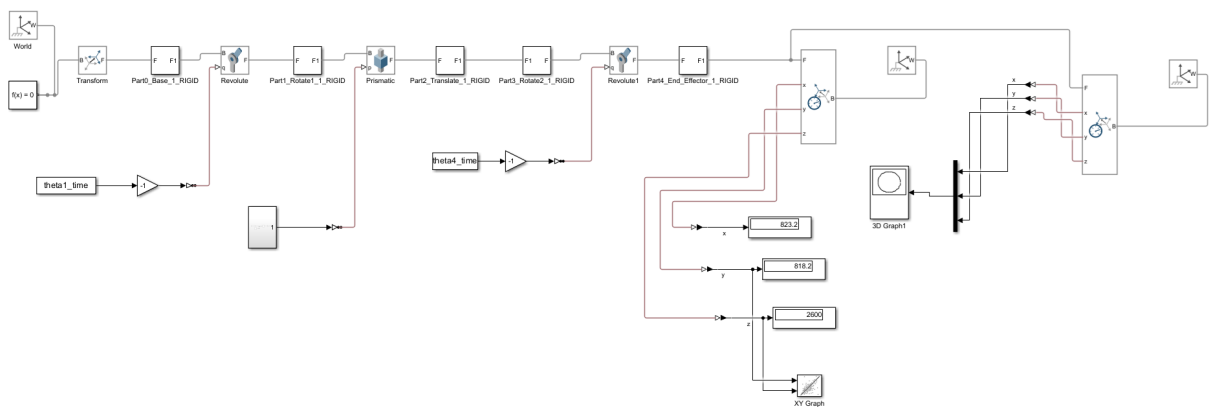

Hình 7.3: Robot model in Solidworks

Model in Simscape Multibody Link

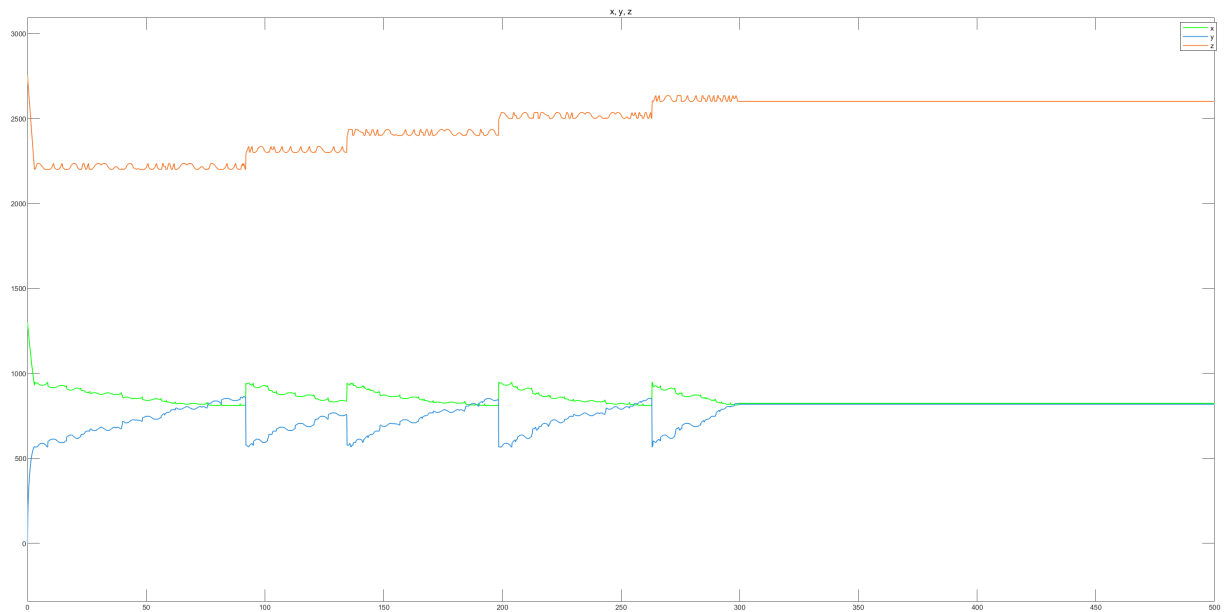Hình 7.4: Robot model in Simscape Multibody Link

Block diagrams of Matlab Simulink is as the follow:

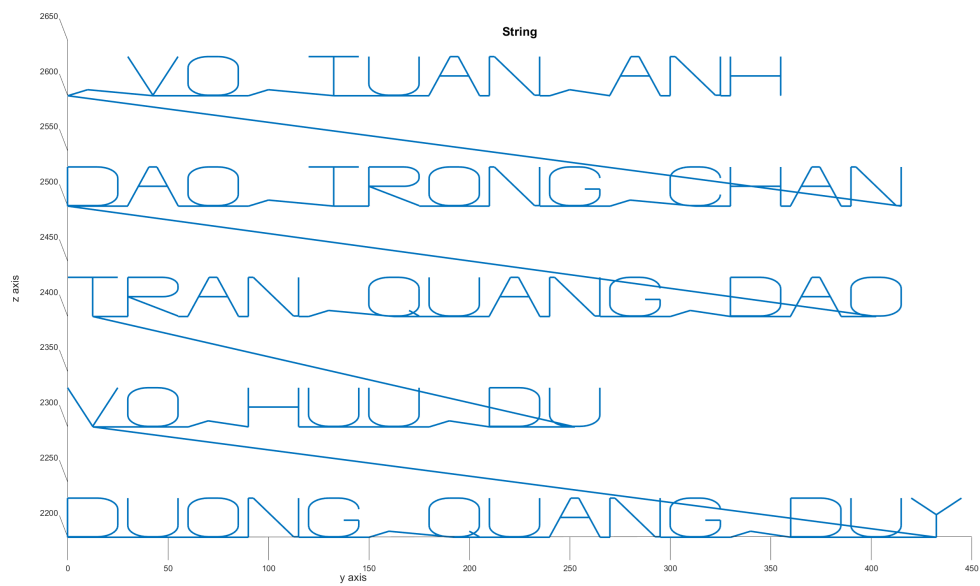Hình 7.5: Block diagram of Matlab Simulink

Results of simulation:
Position (x, y, z) of robot from transsform sensor

Hình 7.6: Position of robot

Result on YZ plane

Hình 7.7: Result on YZ plane

Appendices