

1 Grammar

$e ::=$	exprs.	$\tau ::=$	types
x	<i>variable</i>	X	<i>type variable</i>
v	<i>value</i>	$\{\bar{r}\}$	<i>effect set</i>
$e.\pi$	<i>operation call</i>	$\tau \rightarrow \tau$	<i>arrow</i>
$e e$	<i>application</i>	$\forall X <: \tau.\tau$	<i>universal type</i>
$e \tau$	<i>type application</i>		
$v ::=$	values	$\hat{\tau} ::=$	annotated types
r	<i>resource literal</i>	X	<i>type variable</i>
$\lambda x : \tau.e$	<i>abstraction</i>	$\{\bar{r}\}$	<i>resource set</i>
$\lambda X <: \tau.e$	<i>type polymorphism</i>	$\hat{\tau} \rightarrow_{\varepsilon} \hat{\tau}$	<i>annotated arrow</i>
		$\forall X <: \hat{\tau}.\hat{\tau} \text{ caps } \varepsilon$	<i>universal type</i>
		$\forall \phi \subseteq \varepsilon.\hat{\tau} \text{ caps } \varepsilon$	<i>universal effect set</i>
$\hat{e} ::=$	annotated exprs.	$\varepsilon ::=$	effects
x	<i>variable</i>	ϕ	<i>effect variable</i>
\hat{v}	<i>value</i>	$\{\bar{r}.\pi\}$	<i>effect set</i>
$\hat{e}.\pi$	<i>operation call</i>		
$\hat{e} \hat{e}$	<i>application</i>	$\Gamma ::=$	contexts
$e \hat{\tau}$	<i>type application</i>	\emptyset	<i>empty ctx.</i>
$e \varepsilon$	<i>effect application</i>	$\Gamma, x : \tau$	<i>var. binding</i>
$\text{import}(\varepsilon_s) \overline{x = \hat{e}} \text{ in } e$		$\Gamma, X <: \tau$	<i>type var. binding</i>
$\hat{v} ::=$	annotated values	$\hat{\Gamma} ::=$	annotated contexts
r	<i>resource literal</i>	\emptyset	<i>empty ctx.</i>
$\lambda x : \hat{\tau}.\hat{e}$	<i>abstraction</i>	$\hat{\Gamma}, x : \hat{\tau}$	<i>var. binding</i>
$\lambda X <: \hat{\tau}.\hat{e}$	<i>type polymorphism</i>	$\hat{\Gamma}, X <: \hat{\tau}$	<i>type var. binding</i>
$\lambda \phi \subseteq \varepsilon.\hat{e}$	<i>effect polymorphism</i>	$\hat{\Gamma}, \phi \subseteq \varepsilon$	<i>effect var. binding</i>

2 Functions

Definition ($\text{annot} :: \tau \times \varepsilon \rightarrow \hat{\tau}$)

1. $\text{annot}(X, _) = X$
2. $\text{annot}(\{\bar{r}\}, _) = \{\bar{r}\}$
3. $\text{annot}(\tau_1 \rightarrow \tau_2, \varepsilon) = \text{annot}(\tau_1, \varepsilon) \rightarrow_{\varepsilon} \text{annot}(\tau_2, \varepsilon)$
4. $\text{annot}(\forall X <: \tau_1.\tau_2, \varepsilon) = \forall X <: \text{annot}(\tau_1, \varepsilon).\text{annot}(\tau_2, \varepsilon) \text{ caps } \varepsilon$

Definition ($\text{annot} :: e \times \varepsilon \rightarrow \hat{e}$)

1. $\text{annot}(x, _) = x$
2. $\text{annot}(r, _) = r$
3. $\text{annot}(\lambda x : \tau.e, \varepsilon) = \lambda x : \text{annot}(\tau, \varepsilon).\text{annot}(e, \varepsilon)$
4. $\text{annot}(e_1 e_2, \varepsilon) = \text{annot}(e_1, \varepsilon) \text{ annot}(e_2, \varepsilon)$
5. $\text{annot}(e.\pi, \varepsilon) = \text{annot}(e, \varepsilon).\pi$
6. $\text{annot}(\lambda X <: \tau_1.e, \varepsilon) = \lambda X <: \text{annot}(\tau_1, \varepsilon).\text{annot}(e, \varepsilon)$
7. $\text{annot}(e \tau, \varepsilon) = \text{annot}(e, \varepsilon) \text{ annot}(\tau, \varepsilon)$

Definition ($\text{annot} :: \Gamma \times \varepsilon \rightarrow \hat{\Gamma}$)

1. $\text{annot}(\emptyset, _) = \emptyset$
2. $\text{annot}((\Gamma, x : \tau), \varepsilon) = \text{annot}(\Gamma, \varepsilon), x : \text{annot}(\tau, \varepsilon)$
3. $\text{annot}((\Gamma, X <: \tau), \varepsilon) = \text{annot}(\Gamma, \varepsilon), X <: \text{annot}(\tau, \varepsilon)$

Definition ($\text{erase} :: \hat{\tau} \rightarrow \tau$)

1. $\text{erase}(X) = X$
2. $\text{erase}(\{\bar{r}\}) = \{\bar{r}\}$
3. $\text{erase}(\hat{\tau}_1 \rightarrow_{\varepsilon} \hat{\tau}_2) = \text{erase}(\hat{\tau}_1) \rightarrow \text{erase}(\hat{\tau}_2)$
4. $\text{erase}(\forall X <: \hat{\tau}_1. \hat{\tau}_2 \text{ caps } \varepsilon) = \forall X <: \text{erase}(\hat{\tau}_1). \text{erase}(\hat{\tau}_2)$

Definition ($\text{erase} :: \hat{e} \rightarrow e$)

1. $\text{erase}(x) = x$
2. $\text{erase}(r) = r$
3. $\text{erase}(\lambda x : \hat{\tau}. \hat{e}) = \lambda x : \text{erase}(\hat{\tau}). \text{erase}(\hat{e})$
4. $\text{erase}(\hat{e}_1 \hat{e}_2) = \text{erase}(\hat{e}_1) \text{erase}(\hat{e}_2)$
5. $\text{erase}(\hat{e}. \pi) = \text{erase}(\hat{e}). \pi$
6. $\text{erase}(\lambda X <: \hat{\tau}. \hat{e}) = \lambda X <: \text{erase}(\hat{\tau}). \text{erase}(\hat{e})$

Definition ($\text{erase} :: \hat{\Gamma} \rightarrow \Gamma$)

1. $\text{erase}(\emptyset) = \emptyset$
2. $\text{erase}(\hat{I}, x : \hat{\tau}) = \text{erase}(\hat{I}), x : \text{erase}(\hat{\tau})$
3. $\text{erase}(\hat{I}, X <: \hat{\tau}) = \text{erase}(\hat{I}), X <: \text{erase}(\hat{\tau})$

Definition ($\text{effects} :: \hat{\tau} \rightarrow \varepsilon$)

1. $\text{effects}(X) = \emptyset$
2. $\text{effects}(\{\bar{r}\}) = \{r. \pi \mid r \in \bar{r}, \pi \in \Pi\}$
3. $\text{effects}(\hat{\tau}_1 \rightarrow_{\varepsilon} \hat{\tau}_2) = \text{ho-effects}(\hat{\tau}_1) \cup \varepsilon \cup \text{effects}(\hat{\tau}_2)$
4. $\text{effects}(\forall X <: \hat{\tau}_1. \hat{\tau}_2 \text{ caps } \varepsilon_1) = \text{ho-effects}(\hat{\tau}_1) \cup \text{effects}([\hat{\tau}_1/X]\hat{\tau}_2) \cup \varepsilon_1$
5. $\text{effects}(\forall \phi \subseteq \varepsilon. \hat{\tau} \text{ caps } \varepsilon_1) = \text{effects}([\varepsilon/\phi]\hat{\tau}) \cup \varepsilon_1$

Defintion ($\text{effects} :: \hat{\tau} \times \bar{\tau} \rightarrow \varepsilon$)

1. $\text{effects}(\hat{\tau}, \bar{\tau}) = \text{effects}(\hat{\tau}) \cap \bigcup_i \text{effects}(\hat{\tau}_i)$, if $\hat{\tau}$ is polymorphic.
2. $\text{effects}(\hat{\tau}, _) = \text{effects}(\hat{\tau})$, otherwise.

Note: the definitions given for non-polymorphic types could also be refined to give a more precise upper-bound on the effects they capture. This definition is also probably deficient when there is another polymorphic function in scope.

Definition ($\text{ho-effects} :: \hat{\tau} \rightarrow \varepsilon$)

1. $\text{ho-effects}(X) = \emptyset$
2. $\text{ho-effects}(\{\bar{r}\}) = \emptyset$
3. $\text{ho-effects}(\hat{\tau}_1 \rightarrow_{\varepsilon} \hat{\tau}_2) = \text{effects}(\hat{\tau}_1) \cup \text{ho-effects}(\hat{\tau}_2)$
4. $\text{ho-effects}(\forall X <: \hat{\tau}_1. \hat{\tau}_2 \text{ caps } \varepsilon) = \text{effects}(\hat{\tau}_1) \cup \text{ho-effects}([\hat{\tau}_1/X]\hat{\tau}_2)$
5. $\text{ho-effects}(\forall \phi \subseteq \varepsilon. \hat{\tau} \text{ caps } \varepsilon_1) = \varepsilon \cup \text{ho-effects}([\varepsilon/\phi]\hat{\tau})$

Definition ($\text{ho-effects} :: \hat{\tau} \times \bar{\tau} \rightarrow \varepsilon$)

1. $\text{ho-effects}(\hat{\tau}, \bar{\tau}) = \text{ho-effects}(\hat{\tau}) \cap \bigcup_i \text{ho-effects}(\hat{\tau}_i)$, if $\hat{\tau}$ is polymorphic.
2. $\text{ho-effects}(\hat{\tau}, _) = \text{ho-effects}(\hat{\tau})$, otherwise.

Note: the definitions given for non-polymorphic types could also be refined to give a more precise upper-bound on the effects they capture. This definition is also probably deficient when there is another polymorphic function in scope.

3 Static Rules

$$\boxed{\Gamma \vdash e : \tau}$$

$$\begin{array}{c} \frac{}{\Gamma, x : \tau \vdash x : \tau} \text{ (T-VAR)} \quad \frac{}{\Gamma, r : \{r\} \vdash r : \{r\}} \text{ (T-RESOURCE)} \quad \frac{\Gamma \vdash e : \{\bar{r}\}}{\Gamma \vdash e.\pi : \mathbf{Unit}} \text{ (T-OPERCALL)} \\[10pt] \frac{\Gamma, x : \tau_1 \vdash e : \tau_2}{\Gamma \vdash \lambda x : \tau_1. e : \tau_1 \rightarrow \tau_2} \text{ (T-ABS)} \quad \frac{\Gamma \vdash e_1 : \tau_2 \rightarrow \tau_3 \quad \Gamma \vdash e_2 : \tau_2}{\Gamma \vdash e_1 e_2 : \tau_3} \text{ (T-APP)} \\[10pt] \frac{\Gamma, X <: \tau_1 \vdash e : \tau_2}{\Gamma \vdash \lambda X <: \tau_1. e : \forall X <: \tau_1. \tau_2} \text{ (T-POLYTYPEABS)} \quad \frac{\Gamma \vdash e : \forall X <: \tau_1. \tau_2 \quad \tau' <: \tau_1}{\Gamma \vdash e \tau' : [\tau'/X]\tau_2} \text{ (T-POLYTYPEAPP)} \end{array}$$

$$\boxed{\hat{\Gamma} \vdash \hat{e} : \hat{\tau} \text{ with } \varepsilon}$$

$$\begin{array}{c} \frac{}{\hat{\Gamma}, x : \tau \vdash x : \tau \text{ with } \emptyset} \text{ (\varepsilon-VAR)} \quad \frac{}{\hat{\Gamma}, r : \{r\} \vdash r : \{r\} \text{ with } \emptyset} \text{ (\varepsilon-RESOURCE)} \\[10pt] \frac{\hat{\Gamma} \vdash \hat{e} : \{\bar{r}\} \text{ with } \varepsilon_1}{\hat{\Gamma} \vdash \hat{e}.\pi : \mathbf{Unit} \text{ with } \varepsilon_1 \cup \{r.\pi \mid r \in \bar{r}\}} \text{ (\varepsilon-OPERCALL)} \quad \frac{\hat{\Gamma} \vdash e : \hat{\tau} \text{ with } \varepsilon \quad \hat{\Gamma} \vdash \hat{\tau} <: \hat{\tau}' \quad \hat{\Gamma} \vdash \varepsilon \subseteq \varepsilon'}{\hat{\Gamma} \vdash e : \hat{\tau}' \text{ with } \varepsilon'} \text{ (\varepsilon-SUBSUME)} \\[10pt] \frac{\hat{\Gamma}, x : \hat{\tau}_2 \vdash \hat{e} : \hat{\tau}_3 \text{ with } \varepsilon_3}{\hat{\Gamma} \vdash \lambda x : \tau_2. \hat{e} : \hat{\tau}_2 \rightarrow_{\varepsilon_3} \hat{\tau}_3 \text{ with } \emptyset} \text{ (\varepsilon-ABS)} \quad \frac{\hat{\Gamma} \vdash \hat{e}_1 : \hat{\tau}_2 \rightarrow_{\varepsilon} \hat{\tau}_3 \text{ with } \varepsilon_1 \quad \hat{\Gamma} \vdash \hat{e}_2 : \hat{\tau}_2 \text{ with } \varepsilon_2}{\hat{\Gamma} \vdash \hat{e}_1 \hat{e}_2 : \hat{\tau}_3 \text{ with } \varepsilon_1 \cup \varepsilon_2 \cup \varepsilon} \text{ (\varepsilon-APP)} \\[10pt] \frac{\hat{\Gamma}, X <: \hat{\tau}_1 \vdash \hat{e} : \hat{\tau}_2 \text{ with } \varepsilon_1}{\hat{\Gamma} \vdash \lambda X <: \hat{\tau}_1. \hat{e} : \forall X <: \hat{\tau}_1. \hat{\tau}_2 \text{ caps } \varepsilon_1 \text{ with } \emptyset} \text{ (\varepsilon-POLYTYPEABS)} \\[10pt] \frac{\hat{\Gamma}, \phi \subseteq \varepsilon \vdash \hat{e} : \hat{\tau} \text{ with } \varepsilon_1}{\hat{\Gamma} \vdash \lambda \phi \subseteq \varepsilon. \hat{e} : \forall \phi \subseteq \varepsilon. \hat{\tau} \text{ caps } \varepsilon_1 \text{ with } \emptyset} \text{ (\varepsilon-POLYFXABS)} \\[10pt] \frac{\hat{\Gamma} \vdash \hat{e} : \forall X <: \hat{\tau}_1. \hat{\tau}_2 \text{ caps } \varepsilon_1 \text{ with } \varepsilon_2 \quad \hat{\Gamma} \vdash \hat{\tau}' <: \hat{\tau}_1}{\hat{\Gamma} \vdash \hat{e} \hat{\tau}' : [\hat{\tau}'/X]\hat{\tau}_2 \text{ with } [\hat{\tau}'/X]\varepsilon_1 \cup \varepsilon_2} \text{ (\varepsilon-POLYTYPEAPP)} \\[10pt] \frac{\hat{\Gamma} \vdash \hat{e} : \forall \phi \subseteq \varepsilon. \hat{\tau} \text{ caps } \varepsilon_1 \text{ with } \varepsilon_2 \quad \varepsilon' \subseteq \varepsilon}{\hat{\Gamma} \vdash \hat{e} \varepsilon' : [\varepsilon'/\phi]\hat{\tau} \text{ with } [\varepsilon'/\phi]\varepsilon_1 \cup \varepsilon_2} \text{ (\varepsilon-POLYFXAPP)} \\[10pt] \frac{\text{effects}(\hat{\tau}_i, \bigcup_{j \neq i} \hat{\tau}_j) \cup \text{ho-effects}(\text{annot}(\tau, \emptyset)) \subseteq \varepsilon_s \quad \hat{\Gamma} \vdash \hat{e}_i : \hat{\tau}_i \text{ with } \varepsilon_i \quad x_i : \text{erase}(\hat{\tau}_i) \vdash e : \tau \quad \text{ho-safe}(\hat{\tau}_i, \varepsilon_s)}{\hat{\Gamma} \vdash \text{import}(\varepsilon_s) \overline{x = \hat{e}} \text{ in } e : \text{annot}(\tau, \varepsilon_s) \text{ with } \varepsilon_s \cup \bigcup_i \varepsilon_i} \text{ (\varepsilon-IMPORT)} \end{array}$$

$$\boxed{\text{safe}(\tau, \varepsilon)}$$

$$\begin{array}{c} \frac{}{\text{safe}(\{\bar{r}\}, \varepsilon)} \text{ (SAFE-RESOURCE)} \quad \frac{\varepsilon \subseteq \varepsilon' \quad \text{ho-safe}(\hat{\tau}_1, \varepsilon) \quad \text{safe}(\hat{\tau}_2, \varepsilon)}{\text{safe}(\hat{\tau}_1 \rightarrow_{\varepsilon'} \hat{\tau}_2, \varepsilon)} \text{ (SAFE-ARROW)} \\[10pt] \frac{\varepsilon_1 \subseteq \varepsilon \quad \text{safe}([\varepsilon_1/\phi]\hat{\tau}, \varepsilon)}{\text{safe}(\forall \phi \subseteq \varepsilon_1. \hat{\tau} \text{ caps } \varepsilon_2, \varepsilon)} \text{ (SAFE-POLYFX)} \quad \frac{\text{ho-safe}(\hat{\tau}_1, \varepsilon) \quad \text{safe}([\hat{\tau}_1/X]\hat{\tau}_2, \varepsilon) \quad \varepsilon \subseteq \varepsilon_1}{\text{safe}(\forall X <: \hat{\tau}_1. \hat{\tau}_2 \text{ caps } \varepsilon_2, \varepsilon)} \text{ (SAFE-POLYTYPE)} \end{array}$$

$$\boxed{\text{ho-safe}(\hat{\tau}, \varepsilon)}$$

$$\begin{array}{c}
\frac{}{\text{ho-safe}(\{\bar{r}\}, \varepsilon)} \text{ (HOSAFE-RESOURCE)} \quad \frac{\text{safe}(\hat{\tau}_1, \varepsilon) \quad \text{ho-safe}(\hat{\tau}_2, \varepsilon)}{\text{ho-safe}(\hat{\tau}_1 \rightarrow_{\varepsilon'} \hat{\tau}_2, \varepsilon)} \text{ (HOSAFE-ARROW)} \\
\\
\frac{\varepsilon_1 \subseteq \varepsilon \quad \text{safe}([\varepsilon_1/\phi]\hat{\tau}, \varepsilon)}{\text{ho-safe}(\forall \phi \subseteq \varepsilon_1. \hat{\tau} \text{ caps } \varepsilon_2, \varepsilon)} \text{ (HOSAFE-POLYFX)} \quad \frac{\text{safe}(\hat{\tau}_1, \varepsilon) \quad \text{ho-safe}([\hat{\tau}_1/X]\hat{\tau}_2, \varepsilon)}{\text{ho-safe}(\forall X <: \hat{\tau}_1. \hat{\tau}_2 \text{ caps } \varepsilon_2, \varepsilon)} \text{ (HOSAFE-POLYTYPE)}
\end{array}$$

$$\boxed{\hat{I} \vdash \hat{\tau} <: \hat{\tau}}$$

$$\begin{array}{c}
\frac{}{\hat{I} \vdash \hat{\tau} <: \hat{\tau}} \text{ (S-REFLEXIVE)} \quad \frac{\hat{I} \vdash \hat{\tau}_1 <: \hat{\tau}_2 \quad \hat{I} \vdash \hat{\tau}_2 <: \hat{\tau}_3}{\hat{I} \vdash \hat{\tau}_1 <: \hat{\tau}_3} \text{ (S-TRANSITIVE)} \\
\\
\frac{r \in \bar{r}_1 \implies r \in \bar{r}_2}{\hat{I} \vdash \{\bar{r}_1\} <: \{\bar{r}_2\}} \text{ (S-RESOURCESET)} \quad \frac{\hat{I} \vdash \hat{\tau}'_1 <: \hat{\tau}_1 \quad \hat{I} \vdash \hat{\tau}_2 <: \hat{\tau}'_2 \quad \varepsilon \subseteq \varepsilon'}{\hat{I} \vdash \hat{\tau}_1 \rightarrow_{\varepsilon} \hat{\tau}_2 <: \hat{\tau}'_1 \rightarrow_{\varepsilon'} \hat{\tau}'_2} \text{ (S-ARROW)} \\
\\
\frac{\hat{I} \vdash \hat{\tau}'_1 <: \hat{\tau}_1 \quad \hat{I}, Y <: \hat{\tau}'_1 \vdash \hat{\tau}_2 <: \hat{\tau}'_2}{\hat{I} \vdash (\forall X <: \hat{\tau}_1. \hat{\tau}_2) <: (\forall Y <: \hat{\tau}'_1. \hat{\tau}'_2)} \text{ (S-TYPEPOLY)} \quad \frac{}{\hat{I}, X <: \hat{\tau} \vdash X <: \hat{\tau}} \text{ (S-TYPEVAR)}
\end{array}$$

$$\boxed{\hat{I} \vdash \varepsilon \subseteq \varepsilon}$$

$$\frac{r.\pi \in \varepsilon_1 \implies r.\pi \in \varepsilon_2}{\hat{I} \vdash \varepsilon_1 \subseteq \varepsilon_2} \text{ (S-FXSET)} \quad \frac{}{\hat{I}, \phi \subseteq \varepsilon \vdash \phi \subseteq \varepsilon} \text{ (S-FXVAR)}$$

4 Dynamic Rules

$$\boxed{\hat{e} \longrightarrow \hat{e} \mid \varepsilon}$$

$$\begin{array}{c}
\frac{\hat{e}_1 \longrightarrow \hat{e}'_1 \mid \varepsilon}{\hat{e}_1 \hat{e}_2 \longrightarrow \hat{e}'_1 \hat{e}_2 \mid \varepsilon} \text{ (E-APP1)} \quad \frac{\hat{e}_2 \longrightarrow \hat{e}'_2 \mid \varepsilon}{\hat{v}_1 \hat{e}_2 \longrightarrow \hat{v}_1 \hat{e}'_2 \mid \varepsilon} \text{ (E-APP2)} \quad \frac{}{(\lambda x : \hat{\tau}. \hat{e}) \hat{v}_2 \longrightarrow [\hat{v}_2/x] \hat{e} \mid \emptyset} \text{ (E-APP3)} \\
\\
\frac{\hat{e} \longrightarrow \hat{e}' \mid \varepsilon}{\hat{e}.\pi \longrightarrow \hat{e}'.\pi \mid \varepsilon} \text{ (E-OPERCALL1)} \quad \frac{r \in R \quad \pi \in \Pi}{r.\pi \longrightarrow \text{unit} \mid \{r.\pi\}} \text{ (E-OPERCALL2)} \\
\\
\frac{\hat{e} \longrightarrow \hat{e}' \mid \varepsilon}{\hat{e} \hat{\tau} \longrightarrow \hat{e}' \hat{\tau} \mid \varepsilon} \text{ (E-POLYTYPEAPP1)} \quad \frac{}{(\lambda X <: \hat{\tau}_1. \hat{e}) \hat{\tau} \longrightarrow [\hat{\tau}/X] \hat{e} \mid \emptyset} \text{ (E-POLYTYPEAPP2)} \\
\\
\frac{\hat{e} \longrightarrow \hat{e}' \mid \varepsilon}{\hat{e} \hat{\tau} \longrightarrow \hat{e}' \hat{\tau} \mid \varepsilon} \text{ (E-POLYFXAPP1)} \quad \frac{}{(\lambda \phi \subseteq \varepsilon_1. \hat{e}) \varepsilon \longrightarrow [\varepsilon/\phi] \hat{e} \mid \emptyset} \text{ (E-POLYFXAPP2)} \\
\\
\frac{\hat{e} \longrightarrow \hat{e}' \mid \varepsilon'}{\text{import}(\varepsilon_s) \ x = \hat{e} \text{ in } e \longrightarrow \text{import}(\varepsilon_s) \ x = \hat{e}' \text{ in } e \mid \varepsilon'} \text{ (E-IMPORT1)} \\
\\
\frac{}{\text{import}(\varepsilon_s) \ x = \hat{e} \text{ in } e \longrightarrow [\hat{v}/x] \text{annot}(e, \varepsilon_s) \mid \emptyset} \text{ (E-IMPORT2)}
\end{array}$$

5 Substitution Functions

Definition ($\text{sub} :: \hat{v} \times \hat{v} \rightarrow \hat{e}$)

1. $[\hat{v}/y]x = x$, if $x \neq y$
2. $[\hat{v}/y]y = \hat{v}$
3. $[\hat{v}/y]r = r$
4. $[\hat{v}/y](\lambda x : \hat{\tau}. \hat{e}) = \lambda x : \hat{\tau}. [\hat{v}/y]\hat{e}$, if $y \neq x$ and y does not occur free in \hat{e}

5. $[\hat{v}/y](\lambda X <: \hat{\tau}. \hat{e}) = \lambda X <: \hat{\tau}. [\hat{v}/y]\hat{e}$
6. $[\hat{v}/y](\lambda \phi \subseteq \varepsilon. \hat{e}) = \lambda \phi \subseteq \varepsilon. [\hat{v}/y]\hat{e}$
7. $[\hat{v}/y](\hat{e}. \pi) = ([\hat{v}/y]\hat{e}_1). \pi$
8. $[\hat{v}/y](\hat{e}_1 \hat{e}_2) = ([\hat{v}/y]\hat{e}_1) ([\hat{v}/y]\hat{e}_2)$
9. $[\hat{v}/y](\hat{e} \hat{\tau}) = [\hat{v}/y]\hat{e} \hat{\tau}$
10. $[\hat{v}/y](\hat{e} \varepsilon) = [\hat{v}/y]\hat{e} \hat{\varepsilon}$
11. $[\hat{v}/y](\text{import}(\varepsilon_s) \overline{x = \hat{e} \text{ in } e}) = \text{import}(\varepsilon_s) \overline{x = [\hat{v}/y]\hat{e} \text{ in } e}$

Definition (sub :: $\hat{\tau} \times \hat{v} \rightarrow \hat{e}$)

1. $[\hat{\tau}/Y]x = x$
2. $[\hat{\tau}/Y]r = r$
3. $[\hat{\tau}/Y](\lambda x : \hat{\tau}_1. \hat{e}) = \lambda x : [\hat{\tau}/Y]\hat{\tau}_1. [\hat{\tau}/Y]\hat{e}$
4. $[\hat{\tau}/Y](\lambda X <: \hat{\tau}_1. \hat{e}) = \lambda X <: [\hat{\tau}/Y]\hat{\tau}_1. [\hat{\tau}/Y]\hat{e}$, if $X \neq Y$ and Y does not occur free in \hat{e}
5. $[\hat{\tau}/Y](\lambda \phi \subseteq \varepsilon. \hat{e}) = \lambda \phi \subseteq \varepsilon. [\hat{\tau}/Y]\hat{e}$
6. $[\hat{\tau}/Y](\hat{e}. \pi) = ([\hat{\tau}/Y]\hat{e}_1). \pi$
7. $[\hat{\tau}/Y](\hat{e}_1 \hat{e}_2) = ([\hat{\tau}/Y]\hat{e}_1) ([\hat{\tau}/Y]\hat{e}_2)$
8. $[\hat{\tau}/Y](\hat{e} \hat{\tau}_1) = ([\hat{\tau}/Y]\hat{e}) ([\hat{\tau}/Y]\hat{\tau}_1)$
9. $[\hat{\tau}/Y](\hat{e} \varepsilon) = [\hat{\tau}/Y]\hat{e} \hat{\varepsilon}$
10. $[\hat{\tau}/Y](\text{import}(\varepsilon_s) \overline{x = \hat{e} \text{ in } e}) = \text{import}(\varepsilon_s) \overline{x = [\hat{\tau}/Y]\hat{e} \text{ in } e}$

Definition (sub :: $\hat{\tau} \times \hat{\tau} \rightarrow \hat{e}$)

1. $[\hat{\tau}/Y]Y = \hat{\tau}$
2. $[\hat{\tau}/Y]X = X$, if $X \neq Y$
3. $[\hat{\tau}/Y]\{\bar{r}\} = \{\bar{r}\}$
4. $[\hat{\tau}/Y](\hat{\tau}_1 \rightarrow_{\varepsilon} \hat{\tau}_2) = ([\hat{\tau}/Y]\hat{\tau}_1) \rightarrow_{\varepsilon} ([\hat{\tau}/Y]\hat{\tau}_2)$
5. $[\hat{\tau}/Y](\forall X <: \hat{\tau}_1. \hat{\tau}_2) = \forall X <: [\hat{\tau}/Y]\hat{\tau}_1. [\hat{\tau}/Y]\hat{\tau}_2$, if $X \neq Y$ and Y does not occur free in $\hat{\tau}_2$
6. $[\hat{\tau}/Y](\forall \phi \subseteq \varepsilon_1. \hat{e}) = \forall \phi \subseteq \varepsilon_1. [\hat{\tau}/Y]\hat{e}$

Definition (sub :: $\varepsilon \times \hat{v} \rightarrow \hat{e}$)

1. $[\varepsilon/\psi]\psi = \varepsilon$
2. $[\varepsilon/\psi]\phi = \phi$, if $\psi \neq \phi$
3. $[\varepsilon/\psi](\lambda x : \hat{\tau}_1. \hat{e}) = \lambda x : [\varepsilon/\psi]\hat{\tau}_1. [\varepsilon/\psi]\hat{e}$
4. $[\varepsilon/\psi](\lambda X <: \hat{\tau}_1. \hat{e}) = \lambda X <: [\varepsilon/\psi]\hat{\tau}_1. [\varepsilon/\psi]\hat{e}$
5. $[\varepsilon/\psi](\lambda \phi \subseteq \varepsilon_1. \hat{e}) = \lambda \phi \subseteq [\varepsilon/\psi]\varepsilon_1. [\varepsilon/\psi]\hat{e}$
6. $[\varepsilon/\psi](\hat{e}. \pi) = ([\varepsilon/\psi]\hat{e}_1). \pi$
7. $[\varepsilon/\psi](\hat{e}_1 \hat{e}_2) = ([\varepsilon/\psi]\hat{e}_1) ([\varepsilon/\psi]\hat{e}_2)$
8. $[\varepsilon/\psi](\hat{e} \hat{\tau}) = ([\varepsilon/\psi]\hat{e}) ([\varepsilon/\psi]\hat{\tau})$
9. $[\varepsilon/\psi](\hat{e} \varepsilon_1) = ([\varepsilon/\psi]\hat{e}) ([\varepsilon/\psi]\varepsilon_1)$
10. $[\varepsilon/\psi](\text{import}(\varepsilon_s) \overline{x = \hat{e} \text{ in } e}) = \text{import}([\varepsilon/\psi]\varepsilon_s) \overline{x = [\varepsilon/\psi]\hat{e} \text{ in } e}$

Definition (sub :: $\hat{e} \times \hat{\tau} \rightarrow \hat{e}$)

1. $[\varepsilon/\psi]X = X$
2. $[\varepsilon/\psi]\{\bar{r}\} = \{\bar{r}\}$
3. $[\varepsilon/\psi](\hat{\tau}_1 \rightarrow_{\varepsilon_1} \hat{\tau}_2) = ([\varepsilon/\psi]\hat{\tau}_1) \rightarrow_{[\varepsilon/\psi]\varepsilon_1} ([\varepsilon/\psi]\hat{\tau}_2)$
4. $[\varepsilon/\psi](\forall X <: \hat{\tau}_1. \hat{\tau}_2) = \forall X <: [\varepsilon/\psi]\hat{\tau}_1. [\varepsilon/\psi]\hat{\tau}_2$
5. $[\varepsilon/\psi](\forall \phi \subseteq \varepsilon_1. \hat{e}) = \forall \phi \subseteq [\varepsilon/\psi]\varepsilon_1. [\varepsilon/\psi]\hat{e}$, if $\psi \neq \phi$ and ψ does not occur free in \hat{e}

Definition (sub :: $\varepsilon \times \varepsilon \rightarrow \hat{e}$)

1. $[\varepsilon/\psi]\psi = \varepsilon$
2. $[\varepsilon/\psi]\phi = \phi$, if $\phi \neq \psi$
3. $[\varepsilon/\psi]\{\bar{r}. \bar{\pi}\} = \{\bar{r}. \bar{\pi}\}$