1 Grammar

$$
\begin{array}{lll}
e ::= & & \textbf{exprs.} \\
\mid & x & \textit{variable} \\
\mid & v & \textit{value} \\
\mid & e.\pi & \textit{operation call} \\
\mid & e\ e & \textit{application} \\
\mid & e\ \tau & \textit{type application}
\end{array}
$$

$$
\begin{array}{lll}
v ::= & & \textbf{values} \\
\mid & r & \textit{resource literal} \\
\mid & \lambda x : \tau.e & \textit{abstraction} \\
\mid & \lambda X <: \tau.e & \textit{type polymorphism}
\end{array}
$$

$$
\begin{array}{lll}
\hat{e} ::= & & \textbf{annotated exprs.} \\
\mid & x & \textit{variable} \\
\mid & \hat{v} & \textit{value} \\
\mid & \hat{e}.\pi & \textit{operation call} \\
\mid & \hat{e}\ \hat{e} & \textit{application} \\
\mid & e\ \tau & \textit{type application} \\
\mid & e\ \varepsilon & \textit{effect application} \\
\mid & \texttt{import}(\varepsilon_s)\ x = \hat{e}\ \texttt{in}\ e & \textit{import}
\end{array}
$$

$$
\begin{array}{lll}
\hat{v} ::= & & \textbf{annotated values} \\
\mid & r & \textit{resource literal} \\
\mid & \lambda x : \hat{\tau}.\hat{e} & \textit{abstraction} \\
\mid & \lambda X <: \tau.\hat{e} & \textit{type polymorphism} \\
\mid & \lambda \phi \subseteq \varepsilon.\hat{e} & \textit{effect polymorphism}
\end{array}
$$

$$
\begin{array}{lll}
\tau ::= & & \textbf{types} \\
\mid & X & \textit{type variable} \\
\mid & \{\bar{r}\} & \textit{effect set} \\
\mid & \tau \to \tau & \textit{arrow} \\
\mid & \forall X <: \tau.\tau & \textit{universal type}
\end{array}
$$

$$
\begin{array}{lll}
\hat{\tau} ::= & & \textbf{annotated types} \\
\mid & t & \textit{type variable} \\
\mid & \{\bar{r}\} & \textit{resource set} \\
\mid & \hat{\tau} \to_{\varepsilon} \hat{\tau} & \textit{annotated arrow} \\
\mid & \forall X <: \hat{\tau}.\hat{\tau} & \textit{universal type} \\
\mid & \forall \phi \subseteq \varepsilon.\hat{\tau} & \textit{universal effect set}
\end{array}
$$

$$
\begin{array}{lll}
\varepsilon ::= & & \textbf{effects} \\
\mid & \phi & \textit{effect variable} \\
\mid & \{\overline{r.\pi}\} & \textit{effect set}
\end{array}
$$

$$
\begin{array}{lll}
\Gamma ::= & & \textbf{contexts} \\
\mid & \varnothing & \textit{empty ctx.} \\
\mid & \Gamma, x : \tau & \textit{var. binding} \\
\mid & \Gamma, X <: \tau & \textit{type var. binding}
\end{array}
$$

$$
\begin{array}{lll}
\hat{\Gamma} ::= & & \textbf{annotated contexts} \\
\mid & \varnothing & \textit{empty ctx.} \\
\mid & \hat{\Gamma}, x : \hat{\tau} & \textit{var. binding} \\
\mid & \hat{\Gamma}, X <: \hat{\tau} & \textit{type var. binding} \\
\mid & \hat{\Gamma}, \phi \subseteq \varepsilon & \textit{effect var. binding}
\end{array}
$$

2 Functions

Definition ($\texttt{annot} :: \tau \times \varepsilon \to \hat{\tau}$)

1. $\texttt{annot}(X, \_) = X$
2. $\texttt{annot}(\{\bar{r}\}, \_) = \{\bar{r}\}$
3. $\texttt{annot}(\tau_1 \to \tau_2, \varepsilon) = \texttt{annot}(\tau_1, \varepsilon) \to_{\varepsilon} \texttt{annot}(\tau_2, \varepsilon)$
4. $\texttt{annot}(\forall X <: \tau_1.\tau_2, \varepsilon) = \forall X <: \texttt{annot}(\tau_1, \varepsilon).\texttt{annot}(\tau_2, \varepsilon)$
5. $\texttt{annot}(\forall \phi \subseteq \varepsilon.\tau, \varepsilon) = \forall \phi \subseteq \varepsilon.\texttt{annot}(\tau, \varepsilon)$

Definition ($\texttt{annot} :: \texttt{e} \times \varepsilon \to \hat{\texttt{e}}$)

1. $\texttt{annot}(x, \_) = e$
2. $\texttt{annot}(r, \_) = r$
3. $\texttt{annot}(\lambda x : \tau.e, \varepsilon) = \lambda x : \texttt{annot}(\tau, \varepsilon).\texttt{annot}(e, \varepsilon)$
4. $\texttt{annot}(e_1\ e_2, \varepsilon) = \texttt{annot}(e_1)\ \texttt{annot}(e_2)$
5. $\texttt{annot}(e.\pi, \varepsilon) = \texttt{annot}(e, \varepsilon).\pi$
6. $\texttt{annot}(\lambda X <: \tau_1.e, \varepsilon) = \lambda X <: \texttt{annot}(\tau_1, \varepsilon).\texttt{annot}(e, \varepsilon)$
7. $\texttt{annot}(e\ \tau, \varepsilon) = \texttt{annot}(e, \varepsilon)\ \texttt{annot}(\tau, \varepsilon)$

Definition ($\texttt{annot} :: \Gamma \times \varepsilon \to \hat{\Gamma}$)

1. $\texttt{annot}(\varnothing, \_) = \varnothing$
2. $\texttt{annot}((\Gamma, x : \tau), \varepsilon) = \texttt{annot}(\Gamma, \varepsilon), x : \texttt{annot}(\tau, \varepsilon)$
3. $\texttt{annot}((\Gamma, X <: \tau), \varepsilon) = \texttt{annot}(\Gamma, \varepsilon), X <: \texttt{annot}(\tau, \varepsilon)$

Definition ($\texttt{erase} :: \hat{\boldsymbol{\tau}} \rightarrow \boldsymbol{\tau}$)

1. $\texttt{erase}(X) = X$
2. $\texttt{erase}(\{\bar{r}\}) = \{\bar{r}\}$
3. $\texttt{erase}(\hat{\tau}_1 \rightarrow_\varepsilon \hat{\tau}_2) = \texttt{erase}(\hat{\tau}_1) \rightarrow \texttt{erase}(\hat{\tau}_2)$
4. $\texttt{erase}(\forall X <: \hat{\tau}_1.\hat{\tau}_2) = \forall X <: \texttt{erase}(\hat{\tau}_1).\texttt{erase}(\hat{\tau}_2)$

Definition ($\texttt{erase} :: \hat{\texttt{e}} \rightarrow \texttt{e}$)

1. $\texttt{erase}(x) = x$
2. $\texttt{erase}(r) = r$
3. $\texttt{erase}(\lambda x : \hat{\tau}.\hat{e}) = \lambda x : \texttt{erase}(\hat{\tau}).\texttt{erase}(\hat{e})$
4. $\texttt{erase}(\hat{e}_1\ \hat{e}_2) = \texttt{erase}(\hat{e}_1)\texttt{erase}(\hat{e}_2)$
5. $\texttt{erase}(\hat{e}.\pi) = \texttt{erase}(\hat{e}).\pi$
6. $\texttt{erase}(\lambda X <: \hat{\tau}.\hat{e}) = \lambda X <: \texttt{erase}(\hat{\tau}).\texttt{erase}(\hat{e})$

Definition ($\texttt{erase} :: \hat{\Gamma} \rightarrow \Gamma$)

1. $\texttt{erase}(\varnothing) = \varnothing$
2. $\texttt{erase}(\hat{\varGamma}, x : \hat{\tau}) = \texttt{erase}(\hat{\varGamma}), x : \texttt{erase}(\hat{\tau})$
3. $\texttt{erase}(\hat{\varGamma}, X <: \hat{\tau}) = \texttt{erase}(\hat{\varGamma}), X <: \texttt{erase}(\hat{\tau})$

Definition ($\texttt{effects} :: \hat{\boldsymbol{\tau}} \rightarrow \boldsymbol{\varepsilon}$)

1. $\texttt{effects}(X) = \varnothing$
2. $\texttt{effects}(\{\bar{r}\}) = \{r.\pi \mid r \in \bar{r}, \pi \in \varPi\}$
3. $\texttt{effects}(\hat{\tau}_1 \rightarrow_\varepsilon \hat{\tau}_2) = \texttt{ho-effects}(\hat{\tau}_1) \cup \varepsilon \cup \texttt{effects}(\hat{\tau}_2)$
4. $\texttt{effects}(\forall \hat{\tau}_1.\hat{\tau}_2) = \texttt{ho-effects}(\hat{\tau}_1) \cup \texttt{effects}(\hat{\tau}_2)$
5. $\texttt{effects}(\forall \phi \subseteq \varepsilon.\hat{\tau}) = \texttt{effects}(\hat{\tau})$

Definition ($\texttt{ho-effects} :: \hat{\boldsymbol{\tau}} \rightarrow \boldsymbol{\varepsilon}$)

1. $\texttt{ho-effects}(t) = \varnothing$
2. $\texttt{ho-effects}(\{\bar{r}\}) = \varnothing$
3. $\texttt{ho-effects}(\hat{\tau}_1 \rightarrow_\varepsilon \hat{\tau}_2) = \texttt{effects}(\hat{\tau}_1) \cup \texttt{ho-effects}(\hat{\tau}_2)$
4. $\texttt{ho-effects}(\forall \hat{\tau}_1.\hat{\tau}_2) = \texttt{effects}(\hat{\tau}_1) \cup \texttt{ho-effects}(\hat{\tau}_2)$
5. $\texttt{ho-effects}(\forall \phi \subseteq \varepsilon.\hat{\tau}) = \varepsilon \cup \texttt{ho-effects}(\hat{\tau})$

Definition ($\texttt{substitution} :: \hat{\texttt{e}} \times \hat{\texttt{v}} \times \hat{\texttt{v}} \rightarrow \hat{\texttt{e}}$)

The notation $[\hat{v}/x]\hat{e}$ is short-hand for $\texttt{substitution}(\hat{e}, \hat{v}, x)$. This function is partial, because the third input must be a variable. We adopt the usual renaming conventions to avoid accidental capture.

1. $[\hat{v}/y]x = \hat{v}$, if $x = y$
2. $[\hat{v}/y]x = x$, if $x \neq y$
3. $[\hat{v}/y](\lambda x : \hat{\tau}.\hat{e}) = \lambda x : \hat{\tau}.[\hat{v}/y]\hat{e}$, if $y \neq x$ and y does not occur free in $\hat{e}$
4. $[\hat{v}/y](\hat{e}_1\ \hat{e}_2) = ([\hat{v}/y]\hat{e}_1)([\hat{v}/y]\hat{e}_2)$
5. $[\hat{v}/y](\hat{e}.\pi) = ([\hat{v}/y]\hat{e}).\pi$
6. $[\hat{v}/y](\lambda X.\hat{e}) = \lambda X.[\hat{v}/y]\hat{e}$
7. $[\hat{v}/y](\lambda \phi.\hat{e}) = \lambda \phi.[\hat{v}/y]\hat{e}$
8. $[\hat{v}/y](\hat{e}\ \hat{\tau}) = [\hat{v}/y]\hat{e}\ \hat{\tau}$
9. $[\hat{v}/y](\hat{e}\ \varepsilon) = [\hat{v}/y]\hat{e}\ \varepsilon$
10. $[\hat{v}/y](\texttt{import}(\varepsilon_s)\ x = \hat{e}\ \texttt{in}\ e) = \texttt{import}(\varepsilon_s)\ x = [\hat{v}/y]\hat{e}\ \texttt{in}\ e$

When performing multiple substitutions the notation $[\hat{v}_1/x_1, \hat{v}_2/x_2]\hat{e}$ is used as shorthand for $[\hat{v}_2/x_2]([\hat{v}_1/x_1]\hat{e})$ (note the order of the variables has been flipped; the substitutions occur as they are written, left-to-right).

Definition ($\texttt{substitution} :: \hat{\boldsymbol{\tau}} \times \hat{\boldsymbol{\tau}} \times \hat{\boldsymbol{\tau}} \to \hat{\boldsymbol{\tau}}$)

1. $[\hat{\tau}/Y]X = \hat{\tau}$, if $X = Y$
2. $[\hat{\tau}/Y]X = X$, if $X \neq Y$
3. $[\hat{\tau}/Y]\{\bar{r}\} = \{\bar{r}\}$
4. $[\hat{\tau}/Y](\hat{\tau}_1 \to_\varepsilon \hat{\tau}_2) = [\hat{\tau}/Y]\hat{\tau}_1 \to_{[\hat{\tau}/Y]\varepsilon} [\hat{\tau}/Y]\hat{\tau}_2$
5. $[\hat{\tau}/Y](\forall X.\hat{\tau}_1) = \forall X.[\hat{\tau}/Y]\tau_1$, if $Y \neq X$ and Y does not occur free in τ_1
6. $[\hat{\tau}/Y](\forall \phi.\hat{\tau}_1) = \forall \phi.[\hat{\tau}/Y]\tau_1$

Definition ($\texttt{substitution} :: \boldsymbol{\varepsilon} \times \boldsymbol{\varepsilon} \times \boldsymbol{\varepsilon} \to \hat{\boldsymbol{\tau}}$)

1. $[\varepsilon/\phi]\Phi = \varepsilon$, if $\phi = \Phi$
2. $[\varepsilon/\phi]\Phi = \Phi$, if $\phi \neq \Phi$
3. $[\varepsilon/\phi]\{\overline{r.\pi}\} = \{\overline{r.\pi}\}$

Definition ($\texttt{substitution} :: \hat{\boldsymbol{\tau}} \times \boldsymbol{\varepsilon} \times \boldsymbol{\varepsilon} \to \hat{\boldsymbol{\tau}}$)

1. $[\varepsilon/\phi]X = X$
2. $[\varepsilon/\phi]\{\bar{r}\} = \{\bar{r}\}$
3. $[\varepsilon/\phi](\hat{\tau}_1 \to'_\varepsilon \hat{\tau}_2) = [\varepsilon/\phi]\hat{\tau}_1 \to_{[\varepsilon/\phi]\varepsilon'} [\varepsilon/\phi]\hat{\tau}_2$
4. $[\varepsilon/\phi](\forall X.\hat{\tau}) = \forall X.[\varepsilon/\phi]\hat{\tau}$
5. $[\varepsilon/\phi](\forall \Phi.\hat{\tau}) = \forall \Phi.[\varepsilon/\phi]$, if $\phi \neq \Phi$ and ϕ does not occur free in $\hat{\tau}$

3 Static Rules

$\boxed{\Gamma \vdash e : \tau}$

$$\frac{}{\Gamma, x : \tau \vdash x : \tau}\ \text{(T-VAR)} \qquad \frac{}{\Gamma, r : \{r\} \vdash r : \{r\}}\ \text{(T-RESOURCE)} \qquad \frac{\Gamma \vdash e : \{\bar{r}\}}{\Gamma \vdash e.\pi : \texttt{Unit}}\ \text{(T-OPERCALL)}$$

$$\frac{\Gamma, x : \tau_1 \vdash e : \tau_2}{\Gamma \vdash \lambda x : \tau_1.e : \tau_1 \to \tau_2}\ \text{(T-ABS)} \qquad \frac{\Gamma \vdash e_1 : \tau_2 \to \tau_3 \quad \Gamma \vdash e_2 : \tau_2}{\Gamma \vdash e_1\ e_2 : \tau_3}\ \text{(T-APP)}$$

$$\frac{\Gamma, X <: \tau_1 \vdash e : \tau_2}{\Gamma \vdash \lambda X <: \tau_1.e : \forall X <: \tau_1.\tau_2}\ \text{(T-POLYTYPEABS)} \qquad \frac{\Gamma \vdash e : \forall X <: \tau_1.\tau_2 \quad \tau' <: \tau_1}{\Gamma \vdash e\ \tau' : [\tau'/X]\tau_2}\ \text{(T-POLYTYPEAPP)}$$

$\boxed{\hat{\Gamma} \vdash \hat{e} : \hat{\tau}\ \texttt{with}\ \varepsilon}$

$$\frac{}{\hat{\Gamma}, x : \tau \vdash x : \tau\ \texttt{with}\ \varnothing}\ (\varepsilon\text{-VAR}) \qquad \frac{}{\hat{\Gamma}, r : \{r\} \vdash r : \{r\}\ \texttt{with}\ \varnothing}\ (\varepsilon\text{-RESOURCE})$$

$$\frac{\hat{\Gamma} \vdash \hat{e} : \{\bar{r}\}\ \texttt{with}\ \varepsilon_1}{\hat{\Gamma} \vdash \hat{e}.\pi : \texttt{Unit}\ \texttt{with}\ \varepsilon_1 \cup \{r.\pi \mid r \in \bar{r}\}}\ (\varepsilon\text{-OPERCALL}) \frac{\hat{\Gamma} \vdash e : \tau\ \texttt{with}\ \varepsilon \quad \tau <: \tau' \quad \varepsilon \subseteq \varepsilon'}{\hat{\Gamma} \vdash e : \tau'\ \texttt{with}\ \varepsilon'}\ (\varepsilon\text{-SUBSUME})$$

$$\frac{\hat{\Gamma}, x : \hat{\tau}_2 \vdash \hat{e} : \hat{\tau}_3\ \texttt{with}\ \varepsilon_3}{\hat{\Gamma} \vdash \lambda x : \tau_2.\hat{e} : \hat{\tau}_2 \to_{\varepsilon_3} \hat{\tau}_3\ \texttt{with}\ \varnothing}\ (\varepsilon\text{-ABS}) \quad \frac{\hat{\Gamma} \vdash \hat{e}_1 : \hat{\tau}_2 \to_\varepsilon \hat{\tau}_3\ \texttt{with}\ \varepsilon_1 \quad \hat{\Gamma} \vdash \hat{e}_2 : \hat{\tau}_2\ \texttt{with}\ \varepsilon_2}{\hat{\Gamma} \vdash \hat{e}_1\hat{e}_2 : \hat{\tau}_3\ \texttt{with}\ \varepsilon_1 \cup \varepsilon_2 \cup \varepsilon}\ (\varepsilon\text{-APP})$$

$$\frac{\hat{\Gamma} \vdash \hat{e} : \forall X <: \hat{\tau}_1.\hat{\tau}_2\ \texttt{caps}\ \varepsilon\ \texttt{with}\ \varepsilon_1 \quad \hat{\tau}' <: \hat{\tau}_1}{\hat{\Gamma} \vdash \hat{e}\ \hat{\tau}' : [\hat{\tau}'/X]\hat{\tau}_2\ \texttt{with}\ [\hat{\tau}'/X]\varepsilon \cup \varepsilon_1}\ (\varepsilon\text{-POLYTYPEAPP})$$

$$\frac{\hat{\Gamma} \vdash \hat{e} : \forall \phi \subseteq \varepsilon.\hat{\tau}\ \texttt{caps}\ \varepsilon_1\ \texttt{with}\ \varepsilon_2 \quad \varepsilon' \subseteq \varepsilon}{\hat{\Gamma} \vdash \hat{e}\ \varepsilon : [\varepsilon'/\phi]\hat{\tau}\ \texttt{with}\ [\varepsilon'/\phi]\varepsilon_1 \cup \varepsilon_2}\ (\varepsilon\text{-POLYFXAPP})$$

$$\frac{\begin{array}{c}\texttt{effects}(\hat{\tau}) \cup \texttt{ho-effects}(\texttt{annot}(\tau, \varnothing)) \subseteq \varepsilon \\ \hat{\Gamma} \vdash \hat{e} : \hat{\tau}\ \texttt{with}\ \varepsilon_1 \qquad \texttt{ho-safe}(\hat{\tau}, \varepsilon) \qquad x : \texttt{erase}(\hat{\tau}) \vdash e : \tau\end{array}}{\hat{\Gamma} \vdash \texttt{import}(\varepsilon)\ x = \hat{e}\ \texttt{in}\ e : \texttt{annot}(\tau, \varepsilon)\ \texttt{with}\ \varepsilon \cup \varepsilon_1}\ (\varepsilon\text{-IMPORT})$$

$\boxed{\hat{\Gamma} \vdash \hat{e} : \hat{\tau} \; \texttt{caps} \; \varepsilon \; \texttt{with} \; \varepsilon}$

$$\frac{\hat{\Gamma}, X <: \hat{\tau}_1 \vdash \hat{e} : \hat{\tau}_2 \; \texttt{with} \; \varepsilon}{\hat{\Gamma} \vdash \lambda X <: \hat{\tau}_1 . \hat{e} : \forall X <: \hat{\tau}_1 . \hat{\tau}_2 \; \texttt{caps} \; \varepsilon \; \texttt{with} \; \varnothing} \; (\varepsilon\text{-PolyTypeAbs})$$

$$\frac{\hat{\Gamma}, \phi \subseteq \varepsilon \vdash \hat{e} : \hat{\tau} \; \texttt{with} \; \varepsilon_1}{\hat{\Gamma} \vdash \lambda \phi \subseteq \varepsilon . \hat{e} : \forall \phi \subseteq \varepsilon . \hat{\tau} \; \texttt{caps} \; \varepsilon_1 \; \texttt{with} \; \varnothing} \; (\varepsilon\text{-PolyFxAbs})$$

$\boxed{\texttt{safe}(\tau, \varepsilon)}$

$$\frac{}{\texttt{safe}(\{\bar{r}\}, \varepsilon)} \; (\text{Safe-Resource}) \qquad \frac{}{\texttt{safe}(\texttt{Unit}, \varepsilon)} \; (\text{Safe-Unit})$$

$$\frac{\varepsilon \subseteq \varepsilon' \quad \texttt{ho-safe}(\hat{\tau}_1, \varepsilon) \quad \texttt{safe}(\hat{\tau}_2, \varepsilon)}{\texttt{safe}(\hat{\tau}_1 \rightarrow_{\varepsilon'} \hat{\tau}_2, \varepsilon)} \; (\text{Safe-Arrow})$$

$\boxed{\texttt{ho-safe}(\hat{\tau}, \varepsilon)}$

$$\frac{}{\texttt{ho-safe}(\{\bar{r}\}, \varepsilon)} \; (\text{HOSafe-Resource}) \qquad \frac{}{\texttt{ho-safe}(\texttt{Unit}, \varepsilon)} \; (\text{HOSafe-Unit})$$

$$\frac{\texttt{safe}(\hat{\tau}_1, \varepsilon) \quad \texttt{ho-safe}(\hat{\tau}_2, \varepsilon)}{\texttt{ho-safe}(\hat{\tau}_1 \rightarrow_{\varepsilon'} \hat{\tau}_2, \varepsilon)} \; (\text{HOSafe-Arrow})$$

$\boxed{\hat{\Gamma} \vdash \hat{\tau} <: \hat{\tau}}$

$$\frac{}{\hat{\Gamma} \vdash \hat{\tau} <: \hat{\tau}} \; (\text{S-Reflexive}) \qquad \frac{\hat{\Gamma} \vdash \hat{\tau}_1 <: \hat{\tau}_2 \quad \hat{\Gamma} \vdash \hat{\tau}_2 <: \hat{\tau}_3}{\hat{\Gamma} \vdash \hat{\tau}_1 <: \hat{\tau}_3} \; (\text{S-Transitive})$$

$$\frac{r \in \overline{r_1} \implies r \in \overline{r_2}}{\hat{\Gamma} \vdash \{\overline{r_1}\} <: \{\overline{r_2}\}} \; (\text{S-ResourceSet}) \qquad \frac{\hat{\Gamma} \vdash \hat{\tau}'_1 <: \hat{\tau}_1 \quad \hat{\Gamma} \vdash \hat{\tau}_2 <: \hat{\tau}'_2 \quad \varepsilon \subseteq \varepsilon'}{\hat{\Gamma} \vdash \tau_1 \rightarrow_{\varepsilon} \tau_2 <: \tau'_1 \rightarrow_{\varepsilon'} \tau'_2} \; (\text{S-Arrow})$$

$$\frac{\hat{\Gamma} \vdash \hat{\tau}'_1 <: \hat{\tau}_1 \quad \hat{\Gamma}, Y <: \hat{\tau}'_1 \vdash \hat{\tau}_2 <: \hat{\tau}'_2}{\hat{\Gamma} \vdash (\forall X <: \hat{\tau}_1 . \hat{\tau}_2) <: (\forall Y <: \hat{\tau}'_1 . \hat{\tau}'_2)} \; (\text{S-TypePoly}) \qquad \frac{}{\hat{\Gamma}, X <: \hat{\tau} \vdash X <: \hat{\tau}} \; (\text{S-TypeVar})$$

4 Dynamic Rules

$\boxed{\hat{e} \longrightarrow \hat{e} \mid \varepsilon}$

$$\frac{\hat{e}_1 \longrightarrow \hat{e}_1' \mid \varepsilon}{\hat{e}_1 \hat{e}_2 \longrightarrow \hat{e}_1' \hat{e}_2 \mid \varepsilon} \text{ (E-App1)} \qquad \frac{\hat{e}_2 \longrightarrow \hat{e}_2' \mid \varepsilon}{\hat{v}_1 \hat{e}_2 \longrightarrow \hat{v}_1 \hat{e}_2' \mid \varepsilon} \text{ (E-App2)} \qquad \frac{}{(\lambda x : \hat{\tau}.\hat{e})\hat{v}_2 \longrightarrow [\hat{v}_2/x]\hat{e} \mid \varnothing} \text{ (E-App3)}$$

$$\frac{\hat{e} \rightarrow \hat{e}' \mid \varepsilon}{\hat{e}.\pi \longrightarrow \hat{e}'.\pi \mid \varepsilon} \text{ (E-OperCall1)} \qquad \frac{r \in R \quad \pi \in \Pi}{r.\pi \longrightarrow \texttt{unit} \mid \{r.\pi\}} \text{ (E-OperCall2)}$$

$$\frac{\hat{e} \longrightarrow \hat{e}' \mid \varepsilon}{\hat{e}\ \hat{\tau} \longrightarrow \hat{e}'\ \hat{\tau} \mid \varepsilon} \text{ (E-PolyTypeApp1)} \qquad \frac{}{(\lambda X.\hat{e})\hat{\tau} \longrightarrow [\hat{\tau}/X]\hat{e} \mid \varnothing} \text{ (E-PolyTypeApp2)}$$

$$\frac{\hat{e} \longrightarrow \hat{e}' \mid \varepsilon}{\hat{e}\ \hat{\tau} \longrightarrow \hat{e}'\ \hat{\tau} \mid \varepsilon} \text{ (E-PolyFxApp1)} \qquad \frac{}{(\lambda \phi.\hat{e})\varepsilon \longrightarrow [\varepsilon/\phi]\hat{e} \mid \varnothing} \text{ (E-PolyFxApp2)}$$

$$\frac{\hat{e} \longrightarrow \hat{e}' \mid \varepsilon'}{\texttt{import}(\varepsilon_s)\ x = \hat{e}\ \texttt{in}\ e \longrightarrow \texttt{import}(\varepsilon_s)\ x = \hat{e}'\ \texttt{in}\ e \mid \varepsilon'} \text{ (E-Import1)}$$

$$\frac{}{\texttt{import}(\varepsilon_s)\ x = \hat{e}\ \texttt{in}\ e \longrightarrow [\hat{v}/x]\texttt{annot}(e, \varepsilon_s) \mid \varnothing} \text{ (E-Import2)}$$

5 Encodings

5.1 $\perp$

The bottom type is defined as $\perp \stackrel{\text{def}}{=} \varnothing$, which is the literal for an empty set of resources.

$$\frac{}{\Gamma \vdash \perp : \varnothing} \text{ (T-}\perp\text{)} \qquad \frac{}{\Gamma \vdash \perp : \varnothing\ \texttt{with}\ \varnothing} \text{ (}\varepsilon\text{-}\perp\text{)}$$

5.2 `unit`, `Unit`

Define $\texttt{unit} = \lambda \texttt{x} : \varnothing.\texttt{x}$, i.e. the function which takes an empty set of resources and returns it. We shall refer to its type, which is $\varnothing \rightarrow_\varnothing \varnothing$, as `Unit`. It has various properties befitting `unit`.

1. `unit` cannot be invoked as $\varnothing$ is uninhabited.
2. `unit` is a value.
3. The only term with type `Unit` is `unit`.
4. $\vdash \texttt{unit} : \texttt{Unit}$ by using ε-Abs and ε-Var.
5. $\texttt{effects}(\texttt{Unit}) = \texttt{ho-effects}(\texttt{Unit}) = \varnothing$
6. $\texttt{safe}(\texttt{Unit}, \varepsilon)$ and $\texttt{ho-safe}(\texttt{Unit}, \varepsilon)$

$$\frac{}{\Gamma \vdash \texttt{unit} : \texttt{Unit}} \text{ (T-Unit)} \qquad \frac{}{\Gamma \vdash \texttt{unit} : \texttt{Unit with } \varnothing} \text{ (}\varepsilon\text{-Unit)}$$