1 Grammar

$$
\begin{array}{lll}
e ::= & & \textbf{exprs.} \\
\mid & x & \textit{variable} \\
\mid & v & \textit{value} \\
\mid & e.\pi & \textit{operation call} \\
\mid & e\ e & \textit{application} \\
\mid & e\ \tau & \textit{type application}
\end{array}
$$

$$
\begin{array}{lll}
v ::= & & \textbf{values} \\
\mid & r & \textit{resource literal} \\
\mid & \lambda x : \tau.e & \textit{abstraction} \\
\mid & \lambda X <: \tau.e & \textit{type polymorphism}
\end{array}
$$

$$
\begin{array}{lll}
\hat{e} ::= & & \textbf{annotated exprs.} \\
\mid & x & \textit{variable} \\
\mid & \hat{v} & \textit{value} \\
\mid & \hat{e}.\pi & \textit{operation call} \\
\mid & \hat{e}\ \hat{e} & \textit{application} \\
\mid & e\ \hat{\tau} & \textit{type application} \\
\mid & e\ \varepsilon & \textit{effect application} \\
\mid & \texttt{import}(\varepsilon_s)\ \overline{x = \hat{e}}\ \texttt{in}\ e &
\end{array}
$$

$$
\begin{array}{lll}
\hat{v} ::= & & \textbf{annotated values} \\
\mid & r & \textit{resource literal} \\
\mid & \lambda x : \hat{\tau}.\hat{e} & \textit{abstraction} \\
\mid & \lambda X <: \hat{\tau}.\hat{e} & \textit{type polymorphism} \\
\mid & \lambda \phi \subseteq \varepsilon.\hat{e} & \textit{effect polymorphism}
\end{array}
$$

$$
\begin{array}{lll}
\tau ::= & & \textbf{types} \\
\mid & X & \textit{type variable} \\
\mid & \{\bar{r}\} & \textit{effect set} \\
\mid & \tau \to \tau & \textit{arrow} \\
\mid & \forall X <: \tau.\tau & \textit{universal type}
\end{array}
$$

$$
\begin{array}{lll}
\hat{\tau} ::= & & \textbf{annotated types} \\
\mid & X & \textit{type variable} \\
\mid & \{\bar{r}\} & \textit{resource set} \\
\mid & \hat{\tau} \to_{\varepsilon} \hat{\tau} & \textit{annotated arrow} \\
\mid & \forall X <: \hat{\tau}.\hat{\tau}\ \texttt{caps}\ \varepsilon & \textit{universal type} \\
\mid & \forall \phi \subseteq \varepsilon.\hat{\tau}\ \texttt{caps}\ \varepsilon & \textit{universal effect set}
\end{array}
$$

$$
\begin{array}{lll}
\varepsilon ::= & & \textbf{effects} \\
\mid & \phi & \textit{effect variable} \\
\mid & \{\overline{r.\pi}\} & \textit{effect set}
\end{array}
$$

$$
\begin{array}{lll}
\Gamma ::= & & \textbf{contexts} \\
\mid & \varnothing & \textit{empty ctx.} \\
\mid & \Gamma, x : \tau & \textit{var. binding} \\
\mid & \Gamma, X <: \tau & \textit{type var. binding}
\end{array}
$$

$$
\begin{array}{lll}
\hat{\Gamma} ::= & & \textbf{annotated contexts} \\
\mid & \varnothing & \textit{empty ctx.} \\
\mid & \hat{\Gamma}, x : \hat{\tau} & \textit{var. binding} \\
\mid & \hat{\Gamma}, X <: \hat{\tau} & \textit{type var. binding} \\
\mid & \hat{\Gamma}, \phi \subseteq \varepsilon & \textit{effect var. binding}
\end{array}
$$

2 Functions

Definition ($\texttt{annot} :: \tau \times \varepsilon \to \hat{\tau}$)

1. $\texttt{annot}(X, \_) = X$
2. $\texttt{annot}(\{\bar{r}\}, \_) = \{\bar{r}\}$
3. $\texttt{annot}(\tau_1 \to \tau_2, \varepsilon) = \texttt{annot}(\tau_1, \varepsilon) \to_{\varepsilon} \texttt{annot}(\tau_2, \varepsilon)$
4. $\texttt{annot}(\forall X <: \tau_1.\tau_2, \varepsilon) = \forall X <: \texttt{annot}(\tau_1, \varepsilon).\texttt{annot}(\tau_2, \varepsilon)\ \texttt{caps}\ \varepsilon$

Definition ($\texttt{annot} :: \texttt{e} \times \varepsilon \to \hat{\texttt{e}}$)

1. $\texttt{annot}(x, \_) = e$
2. $\texttt{annot}(r, \_) = r$
3. $\texttt{annot}(\lambda x : \tau.e, \varepsilon) = \lambda x : \texttt{annot}(\tau, \varepsilon).\texttt{annot}(e, \varepsilon)$
4. $\texttt{annot}(e_1\ e_2, \varepsilon) = \texttt{annot}(e_1)\ \texttt{annot}(e_2)$
5. $\texttt{annot}(e.\pi, \varepsilon) = \texttt{annot}(e, \varepsilon).\pi$
6. $\texttt{annot}(\lambda X <: \tau_1.e, \varepsilon) = \lambda X <: \texttt{annot}(\tau_1, \varepsilon).\texttt{annot}(e, \varepsilon)$
7. $\texttt{annot}(e\ \tau, \varepsilon) = \texttt{annot}(e, \varepsilon)\ \texttt{annot}(\tau, \varepsilon)$

Definition ($\texttt{annot} :: \Gamma \times \varepsilon \to \hat{\Gamma}$)

1. $\texttt{annot}(\varnothing, \_) = \varnothing$
2. $\texttt{annot}((\Gamma, x : \tau), \varepsilon) = \texttt{annot}(\Gamma, \varepsilon), x : \texttt{annot}(\tau, \varepsilon)$
3. $\texttt{annot}((\Gamma, X <: \tau), \varepsilon) = \texttt{annot}(\Gamma, \varepsilon), X <: \texttt{annot}(\tau, \varepsilon)$

Definition (erase :: $\hat{\boldsymbol{\tau}} \to \boldsymbol{\tau}$)

1. $\texttt{erase}(X) = X$
2. $\texttt{erase}(\{\bar{r}\}) = \{\bar{r}\}$
3. $\texttt{erase}(\hat{\tau}_1 \to_\varepsilon \hat{\tau}_2) = \texttt{erase}(\hat{\tau}_1) \to \texttt{erase}(\hat{\tau}_2)$
4. $\texttt{erase}(\forall X <: \hat{\tau}_1.\hat{\tau}_2 \ \texttt{caps}\ \varepsilon) = \forall X <: \texttt{erase}(\hat{\tau}_1).\texttt{erase}(\hat{\tau}_2)$

Definition (erase :: $\hat{\mathtt{e}} \to \mathtt{e}$)

1. $\texttt{erase}(x) = x$
2. $\texttt{erase}(r) = r$
3. $\texttt{erase}(\lambda x : \hat{\tau}.\hat{e}) = \lambda x : \texttt{erase}(\hat{\tau}).\texttt{erase}(\hat{e})$
4. $\texttt{erase}(\hat{e}_1\ \hat{e}_2) = \texttt{erase}(\hat{e}_1)\texttt{erase}(\hat{e}_2)$
5. $\texttt{erase}(\hat{e}.\pi) = \texttt{erase}(\hat{e}).\pi$
6. $\texttt{erase}(\lambda X <: \hat{\tau}.\hat{e}) = \lambda X <: \texttt{erase}(\hat{\tau}).\texttt{erase}(\hat{e})$

Definition (erase :: $\hat{\Gamma} \to \Gamma$)

1. $\texttt{erase}(\varnothing) = \varnothing$
2. $\texttt{erase}(\hat{\mathit{\Gamma}}, x : \hat{\tau}) = \texttt{erase}(\hat{\mathit{\Gamma}}), x : \texttt{erase}(\hat{\tau})$
3. $\texttt{erase}(\hat{\mathit{\Gamma}}, X <: \hat{\tau}) = \texttt{erase}(\hat{\mathit{\Gamma}}), X <: \texttt{erase}(\hat{\tau})$

Definition (effects :: $\hat{\boldsymbol{\tau}} \to \boldsymbol{\varepsilon}$)

1. $\texttt{effects}(X) = \varnothing$
2. $\texttt{effects}(\{\bar{r}\}) = \{r.\pi \mid r \in \bar{r}, \pi \in \mathit{\Pi}\}$
3. $\texttt{effects}(\hat{\tau}_1 \to_\varepsilon \hat{\tau}_2) = \texttt{ho-effects}(\hat{\tau}_1) \cup \varepsilon \cup \texttt{effects}(\hat{\tau}_2)$
4. $\texttt{effects}(\forall X <: \hat{\tau}_1.\hat{\tau}_2 \ \texttt{caps}\ \varepsilon_1) = \texttt{ho-effects}(\hat{\tau}_1) \cup \texttt{effects}([\hat{\tau}_1/X]\hat{\tau}_2) \cup \varepsilon_1$
5. $\texttt{effects}(\forall \phi \subseteq \varepsilon.\hat{\tau} \ \texttt{caps}\ \varepsilon_1) = \texttt{effects}([\varepsilon/\phi]\hat{\tau}) \cup \varepsilon_1$

Defintion (effects :: $\hat{\boldsymbol{\tau}} \times \overline{\hat{\boldsymbol{\tau}}} \to \boldsymbol{\varepsilon}$)

1. $\texttt{effects}(\hat{\tau}, \overline{\hat{\tau}}) = \texttt{effects}(\hat{\tau}) \cap \bigcup_i \texttt{effects}(\hat{\tau}_i)$, if $\hat{\tau}$ is polymorphic.
2. $\texttt{effects}(\hat{\tau}, \_) = \texttt{effects}(\hat{\tau})$, otherwise.

Note: the definitions given for non-polymorphic types could also be refined to give a more precise upper-bound on the effects they capture. This definition is also probably deficient when there is another polymorphic function in scope.

Definition (ho-effects :: $\hat{\boldsymbol{\tau}} \to \boldsymbol{\varepsilon}$)

1. $\texttt{ho-effects}(X) = \varnothing$
2. $\texttt{ho-effects}(\{\bar{r}\}) = \varnothing$
3. $\texttt{ho-effects}(\hat{\tau}_1 \to_\varepsilon \hat{\tau}_2) = \texttt{effects}(\hat{\tau}_1) \cup \texttt{ho-effects}(\hat{\tau}_2)$
4. $\texttt{ho-effects}(\forall X <: \hat{\tau}_1.\hat{\tau}_2 \ \texttt{caps}\ \varepsilon) = \texttt{effects}(\hat{\tau}_1) \cup \texttt{ho-effects}([\hat{\tau}_1/X]\hat{\tau}_2)$
5. $\texttt{ho-effects}(\forall \phi \subseteq \varepsilon.\hat{\tau} \ \texttt{caps}\ \varepsilon_1) = \varepsilon \cup \texttt{ho-effects}([\varepsilon/\phi]\hat{\tau})$

Definition (ho-effects :: $\hat{\boldsymbol{\tau}} \times \overline{\hat{\boldsymbol{\tau}}} \to \boldsymbol{\varepsilon}$)

1. $\texttt{ho-effects}(\hat{\tau}, \overline{\hat{\tau}}) = \texttt{ho-effects}(\hat{\tau}) \cap \bigcup_i \texttt{ho-effects}(\hat{\tau}_i)$, if $\hat{\tau}$ is polymorphic.
2. $\texttt{ho-effects}(\hat{\tau}, \_) = \texttt{ho-effects}(\hat{\tau})$, otherwise.

Note: the definitions given for non-polymorphic types could also be refined to give a more precise upper-bound on the effects they capture. This definition is also probably deficient when there is another polymorphic function in scope.

3 Static Rules

$\boxed{\Gamma \vdash e : \tau}$

$$\frac{}{\Gamma, x : \tau \vdash x : \tau}\ (\text{T-Var}) \qquad \frac{}{\Gamma, r : \{r\} \vdash r : \{r\}}\ (\text{T-Resource}) \qquad \frac{\Gamma \vdash e : \{\bar{r}\}}{\Gamma \vdash e.\pi : \mathtt{Unit}}\ (\text{T-OperCall})$$

$$\frac{\Gamma, x : \tau_1 \vdash e : \tau_2}{\Gamma \vdash \lambda x : \tau_1.e : \tau_1 \to \tau_2}\ (\text{T-Abs}) \qquad \frac{\Gamma \vdash e_1 : \tau_2 \to \tau_3 \quad \Gamma \vdash e_2 : \tau_2}{\Gamma \vdash e_1\ e_2 : \tau_3}\ (\text{T-App})$$

$$\frac{\Gamma, X <: \tau_1 \vdash e : \tau_2}{\Gamma \vdash \lambda X <: \tau_1.e : \forall X <: \tau_1.\tau_2}\ (\text{T-PolyTypeAbs}) \qquad \frac{\Gamma \vdash e : \forall X <: \tau_1.\tau_2 \quad \tau' <: \tau_1}{\Gamma \vdash e\ \tau' : [\tau'/X]\tau_2}\ (\text{T-PolyTypeApp})$$

$\boxed{\hat{\Gamma} \vdash \hat{e} : \hat{\tau}\ \mathtt{with}\ \varepsilon}$

$$\frac{}{\hat{\Gamma}, x : \tau \vdash x : \tau\ \mathtt{with}\ \varnothing}\ (\varepsilon\text{-Var}) \qquad \frac{}{\hat{\Gamma}, r : \{r\} \vdash r : \{r\}\ \mathtt{with}\ \varnothing}\ (\varepsilon\text{-Resource})$$

$$\frac{\hat{\Gamma} \vdash \hat{e} : \{\bar{r}\}\ \mathtt{with}\ \varepsilon_1}{\hat{\Gamma} \vdash \hat{e}.\pi : \mathtt{Unit}\ \mathtt{with}\ \varepsilon_1 \cup \{r.\pi \mid r \in \bar{r}\}}\ (\varepsilon\text{-OperCall}) \qquad \frac{\hat{\Gamma} \vdash e : \hat{\tau}\ \mathtt{with}\ \varepsilon \quad \hat{\Gamma} \vdash \hat{\tau} <: \hat{\tau}' \quad \hat{\Gamma} \vdash \varepsilon \subseteq \varepsilon'}{\hat{\Gamma} \vdash e : \hat{\tau}'\ \mathtt{with}\ \varepsilon'}\ (\varepsilon\text{-Subsume})$$

$$\frac{\hat{\Gamma}, x : \hat{\tau}_2 \vdash \hat{e} : \hat{\tau}_3\ \mathtt{with}\ \varepsilon_3}{\hat{\Gamma} \vdash \lambda x : \tau_2.\hat{e} : \hat{\tau}_2 \to_{\varepsilon_3} \hat{\tau}_3\ \mathtt{with}\ \varnothing}\ (\varepsilon\text{-Abs}) \qquad \frac{\hat{\Gamma} \vdash \hat{e}_1 : \hat{\tau}_2 \to_{\varepsilon} \hat{\tau}_3\ \mathtt{with}\ \varepsilon_1 \quad \hat{\Gamma} \vdash \hat{e}_2 : \hat{\tau}_2\ \mathtt{with}\ \varepsilon_2}{\hat{\Gamma} \vdash \hat{e}_1 \hat{e}_2 : \hat{\tau}_3\ \mathtt{with}\ \varepsilon_1 \cup \varepsilon_2 \cup \varepsilon}\ (\varepsilon\text{-App})$$

$$\frac{\hat{\Gamma}, X <: \hat{\tau}_1 \vdash \hat{e} : \hat{\tau}_2\ \mathtt{with}\ \varepsilon_1}{\hat{\Gamma} \vdash \lambda X <: \hat{\tau}_1.\hat{e} : \forall X <: \hat{\tau}_1.\hat{\tau}_2\ \mathtt{caps}\ \varepsilon_1\ \mathtt{with}\ \varnothing}\ (\varepsilon\text{-PolyTypeAbs})$$

$$\frac{\hat{\Gamma}, \phi \subseteq \varepsilon \vdash \hat{e} : \hat{\tau}\ \mathtt{with}\ \varepsilon_1}{\hat{\Gamma} \vdash \lambda \phi \subseteq \varepsilon.\hat{e} : \forall \phi \subseteq \varepsilon.\hat{\tau}\ \mathtt{caps}\ \varepsilon_1\ \mathtt{with}\ \varnothing}\ (\varepsilon\text{-PolyFxAbs})$$

$$\frac{\hat{\Gamma} \vdash \hat{e} : \forall X <: \hat{\tau}_1.\hat{\tau}_2\ \mathtt{caps}\ \varepsilon_1\ \mathtt{with}\ \varepsilon_2 \quad \hat{\Gamma} \vdash \hat{\tau}' <: \hat{\tau}_1}{\hat{\Gamma} \vdash \hat{e}\ \hat{\tau}' : [\hat{\tau}'/X]\hat{\tau}_2\ \mathtt{with}\ \varepsilon_1 \cup \varepsilon_2}\ (\varepsilon\text{-PolyTypeApp})$$

$$\frac{\hat{\Gamma} \vdash \hat{e} : \forall \phi \subseteq \varepsilon.\hat{\tau}\ \mathtt{caps}\ \varepsilon_1\ \mathtt{with}\ \varepsilon_2 \quad \varepsilon' \subseteq \varepsilon}{\hat{\Gamma} \vdash \hat{e}\ \varepsilon' : [\varepsilon'/\phi]\hat{\tau}\ \mathtt{with}\ [\varepsilon'/\phi]\varepsilon_1 \cup \varepsilon_2}\ (\varepsilon\text{-PolyFxApp})$$

$$\frac{\begin{array}{c}\mathtt{effects}(\hat{\tau}_i, \bigcup_{j \neq i} \hat{\tau}_j) \cup \mathtt{ho\text{-}effects}(\mathtt{annot}(\tau, \varnothing)) \subseteq \varepsilon_s \\ \hat{\Gamma} \vdash \hat{e}_i : \hat{\tau}_i\ \mathtt{with}\ \varepsilon_i \qquad x_i : \mathtt{erase}(\hat{\tau}_i) \vdash e : \tau \qquad \mathtt{ho\text{-}safe}(\hat{\tau}_i, \varepsilon_s)\end{array}}{\hat{\Gamma} \vdash \mathtt{import}(\varepsilon_s)\ \overline{x = \hat{e}}\ \mathtt{in}\ e : \mathtt{annot}(\tau, \varepsilon_s)\ \mathtt{with}\ \varepsilon_s \cup \bigcup_i \varepsilon_i}\ (\varepsilon\text{-Import})$$

$\boxed{\mathtt{safe}(\tau, \varepsilon)}$

$$\frac{}{\mathtt{safe}(\{\bar{r}\}, \varepsilon)}\ (\text{Safe-Resource}) \qquad \frac{\varepsilon \subseteq \varepsilon' \quad \mathtt{ho\text{-}safe}(\hat{\tau}_1, \varepsilon) \quad \mathtt{safe}(\hat{\tau}_2, \varepsilon)}{\mathtt{safe}(\hat{\tau}_1 \to_{\varepsilon'} \hat{\tau}_2, \varepsilon)}\ (\text{Safe-Arrow})$$

$$\frac{\varepsilon_1 \subseteq \varepsilon \quad \mathtt{safe}([\varepsilon_1/\phi]\hat{\tau}, \varepsilon)}{\mathtt{safe}(\forall \phi \subseteq \varepsilon_1.\hat{\tau}\ \mathtt{caps}\ \varepsilon_2, \varepsilon)}\ (\text{Safe-PolyFx}) \qquad \frac{\mathtt{ho\text{-}safe}(\hat{\tau}_1, \varepsilon) \quad \mathtt{safe}([\hat{\tau}_1/X]\hat{\tau}_2, \varepsilon) \quad \varepsilon \subseteq \varepsilon_1}{\mathtt{safe}(\forall X <: \hat{\tau}_1.\hat{\tau}_2\ \mathtt{caps}\ \varepsilon_2, \varepsilon)}\ (\text{Safe-PolyType})$$

$\boxed{\mathtt{ho\text{-}safe}(\hat{\tau}, \varepsilon)}$

$$\frac{}{\texttt{ho-safe}(\{\bar{r}\}, \varepsilon)} \text{ (HOSafe-Resource)} \quad \frac{\texttt{safe}(\hat{\tau}_1, \varepsilon) \quad \texttt{ho-safe}(\hat{\tau}_2, \varepsilon)}{\texttt{ho-safe}(\hat{\tau}_1 \rightarrow_{\varepsilon'} \hat{\tau}_2, \varepsilon)} \text{ (HOSafe-Arrow)}$$

$$\frac{\varepsilon_1 \subseteq \varepsilon \quad \texttt{safe}([\varepsilon_1/\phi]\hat{\tau}, \varepsilon)}{\texttt{ho-safe}(\forall \phi \subseteq \varepsilon_1.\hat{\tau} \ \texttt{caps} \ \varepsilon_2, \varepsilon)} \text{ (HOSafe-PolyFx)} \quad \frac{\texttt{safe}(\hat{\tau}_1, \varepsilon) \quad \texttt{ho-safe}([\hat{\tau}_1/X]\hat{\tau}_2, \varepsilon)}{\texttt{ho-safe}(\forall X <: \hat{\tau}_1.\hat{\tau}_2 \ \texttt{caps} \ \varepsilon_2, \varepsilon)} \text{ (HOSafe-PolyType)}$$

$\boxed{\hat{\Gamma} \vdash \hat{\tau} <: \hat{\tau}}$

$$\frac{}{\hat{\Gamma} \vdash \hat{\tau} <: \hat{\tau}} \text{ (S-Reflexive)} \quad \frac{\hat{\Gamma} \vdash \hat{\tau}_1 <: \hat{\tau}_2 \quad \hat{\Gamma} \vdash \hat{\tau}_2 <: \hat{\tau}_3}{\hat{\Gamma} \vdash \hat{\tau}_1 <: \hat{\tau}_3} \text{ (S-Transitive)}$$

$$\frac{r \in \overline{r_1} \implies r \in \overline{r_2}}{\hat{\Gamma} \vdash \{\overline{r_1}\} <: \{\overline{r_2}\}} \text{ (S-ResourceSet)} \quad \frac{\hat{\Gamma} \vdash \hat{\tau}_1' <: \hat{\tau}_1 \quad \hat{\Gamma} \vdash \hat{\tau}_2 <: \hat{\tau}_2' \quad \varepsilon \subseteq \varepsilon'}{\hat{\Gamma} \vdash \hat{\tau}_1 \rightarrow_{\varepsilon} \hat{\tau}_2 <: \hat{\tau}_1' \rightarrow_{\varepsilon'} \hat{\tau}_2'} \text{ (S-Arrow)}$$

$$\frac{\hat{\Gamma} \vdash \hat{\tau}_1' <: \hat{\tau}_1 \quad \hat{\Gamma}, Y <: \hat{\tau}_1' \vdash \hat{\tau}_2 <: \hat{\tau}_2'}{\hat{\Gamma} \vdash (\forall X <: \hat{\tau}_1.\hat{\tau}_2) <: (\forall Y <: \hat{\tau}_1'.\hat{\tau}_2')} \text{ (S-PolyType)} \quad \frac{\hat{\Gamma} \vdash \varepsilon' \subseteq \varepsilon \quad \hat{\Gamma}, \Phi <: \varepsilon' \vdash \hat{\tau}_1 <: \hat{\tau}_1'}{\hat{\Gamma} \vdash (\forall \phi \subseteq \varepsilon.\hat{\tau}_1) <: (\forall \Phi \subseteq \varepsilon'.\hat{\tau}_1')} \text{ (S-PolyFx)}$$

$$\frac{}{\hat{\Gamma}, X <: \hat{\tau} \vdash X <: \hat{\tau}} \text{ (S-TypeVar)}$$

$\boxed{\hat{\Gamma} \vdash \varepsilon \subseteq \varepsilon}$

$$\frac{r.\pi \in \overline{r.\pi_1} \implies r.\pi \in \overline{r.\pi_2}}{\hat{\Gamma} \vdash \{\overline{r.\pi_1}\} \subseteq \{\overline{r.\pi_2}\}} \text{ (S-FxSet)} \quad \frac{}{\hat{\Gamma}, \phi \subseteq \varepsilon \vdash \phi \subseteq \varepsilon} \text{ (S-FxVar)}$$

4 Dynamic Rules

$\boxed{\hat{e} \longrightarrow \hat{e} \mid \varepsilon}$

$$\frac{\hat{e}_1 \longrightarrow \hat{e}_1' \mid \varepsilon}{\hat{e}_1\hat{e}_2 \longrightarrow \hat{e}_1'\hat{e}_2 \mid \varepsilon} \text{ (E-App1)} \quad \frac{\hat{e}_2 \longrightarrow \hat{e}_2' \mid \varepsilon}{\hat{v}_1\hat{e}_2 \longrightarrow \hat{v}_1\hat{e}_2' \mid \varepsilon} \text{ (E-App2)} \quad \frac{}{(\lambda x : \hat{\tau}.\hat{e})\hat{v}_2 \longrightarrow [\hat{v}_2/x]\hat{e} \mid \varnothing} \text{ (E-App3)}$$

$$\frac{\hat{e} \rightarrow \hat{e}' \mid \varepsilon}{\hat{e}.\pi \longrightarrow \hat{e}'.\pi \mid \varepsilon} \text{ (E-OperCall1)} \quad \frac{r \in R \quad \pi \in \Pi}{r.\pi \longrightarrow \texttt{unit} \mid \{r.\pi\}} \text{ (E-OperCall2)}$$

$$\frac{\hat{e} \longrightarrow \hat{e}' \mid \varepsilon}{\hat{e} \ \hat{\tau} \longrightarrow \hat{e}' \ \hat{\tau} \mid \varepsilon} \text{ (E-PolyTypeApp1)} \quad \frac{}{(\lambda X <: \hat{\tau}_1.\hat{e})\hat{\tau} \longrightarrow [\hat{\tau}/X]\hat{e} \mid \varnothing} \text{ (E-PolyTypeApp2)}$$

$$\frac{\hat{e} \longrightarrow \hat{e}' \mid \varepsilon}{\hat{e} \ \hat{\tau} \longrightarrow \hat{e}' \ \hat{\tau} \mid \varepsilon} \text{ (E-PolyFxApp1)} \quad \frac{}{(\lambda \phi \subseteq \varepsilon_1.\hat{e})\varepsilon \longrightarrow [\varepsilon/\phi]\hat{e} \mid \varnothing} \text{ (E-PolyFxApp2)}$$

$$\frac{\hat{e} \longrightarrow \hat{e}' \mid \varepsilon'}{\texttt{import}(\varepsilon_s) \ x = \hat{e} \ \texttt{in} \ e \longrightarrow \texttt{import}(\varepsilon_s) \ x = \hat{e}' \ \texttt{in} \ e \mid \varepsilon'} \text{ (E-Import1)}$$

$$\frac{}{\texttt{import}(\varepsilon_s) \ x = \hat{e} \ \texttt{in} \ e \longrightarrow [\hat{v}/x]\texttt{annot}(e, \varepsilon_s) \mid \varnothing} \text{ (E-Import2)}$$

5 Substitution Functions

Definition (sub :: $\hat{v} \times \hat{v} \rightarrow \hat{e}$)

1. $[\hat{v}/y]x = x$, if $x \neq y$

2. $[\hat{v}/y]y = \hat{v}$
3. $[\hat{v}/y]r = r$
4. $[\hat{v}/y](\lambda x : \hat{\tau}.\hat{e}) = \lambda x : \hat{\tau}.[\hat{v}/y]\hat{e}$, if $y \neq x$ and y does not occur free in $\hat{e}$
5. $[\hat{v}/y](\lambda X <: \hat{\tau}.\hat{e}) = \lambda X <: \hat{\tau}.[\hat{v}/y]\hat{e}$
6. $[\hat{v}/y](\lambda \phi \subseteq \varepsilon.\hat{e}) = \lambda \phi \subseteq \varepsilon.[\hat{v}/y]\hat{e}$
7. $[\hat{v}/y](\hat{e}.\pi) = ([\hat{v}/y]\hat{e}_1).\pi$
8. $[\hat{v}/y](\hat{e}_1\ \hat{e}_2) = ([\hat{v}/y]\hat{e}_1)\ ([\hat{v}/y]\hat{e}_2)$
9. $[\hat{v}/y](\hat{e}\ \hat{\tau}) = [\hat{v}/y]\hat{e}\ \hat{\tau}$
10. $[\hat{v}/y](\hat{e}\ \varepsilon) = [\hat{v}/y]\hat{e}\ \hat{\varepsilon}$
11. $[\hat{v}/y](\texttt{import}(\varepsilon_s)\ \overline{x = \hat{e}}\ \texttt{in}\ e) = \texttt{import}(\varepsilon_s)\ \overline{x = [\hat{v}/y]\hat{e}}\ \texttt{in}\ e$

Definition (sub :: $\hat{\tau} \times \hat{v} \to \hat{e}$)

1. $[\hat{\tau}/Y]x = x$
2. $[\hat{\tau}/Y]r = r$
3. $[\hat{\tau}/Y](\lambda x : \hat{\tau}_1.\hat{e}) = \lambda x : [\hat{\tau}/Y]\hat{\tau}_1.[\hat{\tau}/Y]\hat{e}$
4. $[\hat{\tau}/Y](\lambda X <: \hat{\tau}_1.\hat{e}) = \lambda X <: [\hat{\tau}/Y]\hat{\tau}_1.[\hat{\tau}/Y]\hat{e}$, if $X \neq Y$ and Y does not occur free in $\hat{e}$
5. $[\hat{\tau}/Y](\lambda \phi \subseteq \varepsilon.\hat{e}) = \lambda \phi \subseteq \varepsilon.[\hat{\tau}/Y]\hat{e}$
6. $[\hat{\tau}/Y](\hat{e}.\pi) = ([\hat{\tau}/Y]\hat{e}_1).\pi$
7. $[\hat{\tau}/Y](\hat{e}_1\ \hat{e}_2) = ([\hat{\tau}/Y]\hat{e}_1)\ ([\hat{\tau}/Y]\hat{e}_2)$
8. $[\hat{\tau}/Y](\hat{e}\ \hat{\tau}_1) = ([\hat{\tau}/Y]\hat{e})\ ([\hat{\tau}/Y]\hat{\tau}_1)$
9. $[\hat{\tau}/Y](\hat{e}\ \varepsilon) = [\hat{\tau}/Y]\hat{e}\ \hat{\varepsilon}$
10. $[\hat{\tau}/Y](\texttt{import}(\varepsilon_s)\ \overline{x = \hat{e}}\ \texttt{in}\ e) = \texttt{import}(\varepsilon_s)\ \overline{x = [\hat{\tau}/Y]\hat{e}}\ \texttt{in}\ e$

Definition (sub :: $\hat{\tau} \times \hat{\tau} \to \hat{e}$)

1. $[\hat{\tau}/Y]Y = \hat{\tau}$
2. $[\hat{\tau}/Y]X = X$, if $X \neq Y$
3. $[\hat{\tau}/Y]\{\bar{r}\} = \{\bar{r}\}$
4. $[\hat{\tau}/Y](\hat{\tau}_1 \to_\varepsilon \hat{\tau}_2) = ([\hat{\tau}/Y]\hat{\tau}_1) \to_\varepsilon ([\hat{\tau}/Y]\hat{\tau}_2)$
5. $[\hat{\tau}/Y](\forall X <: \hat{\tau}_1.\hat{\tau}_2) = \forall X <: [\hat{\tau}/Y]\hat{\tau}_1.[\hat{\tau}/Y]\hat{\tau}_2$, if $X \neq Y$ and Y does not occur free in $\hat{\tau}_2$
6. $[\hat{\tau}/Y](\forall \phi \subseteq \varepsilon_1.\hat{e}) = \forall \phi \subseteq \varepsilon_1.[\hat{\tau}/Y]\hat{e}$

Definition (sub :: $\varepsilon \times \hat{v} \to \hat{e}$)

1. $[\varepsilon/\psi]\psi = \varepsilon$
2. $[\varepsilon/\psi]\phi = \phi$, if $\psi \neq \phi$
3. $[\varepsilon/\psi](\lambda x : \hat{\tau}_1.\hat{e}) = \lambda x : [\varepsilon/\psi]\hat{\tau}_1.[\varepsilon/\psi]\hat{e}$
4. $[\varepsilon/\psi](\lambda X <: \hat{\tau}_1.\hat{e}) = \lambda X <: [\varepsilon/\psi]\hat{\tau}_1.[\varepsilon/\psi]\hat{e}$
5. $[\varepsilon/\psi](\lambda \phi \subseteq \varepsilon_1.\hat{e}) = \lambda \phi \subseteq [\varepsilon/\psi]\varepsilon_1.[\varepsilon/\psi]\hat{e}$
6. $[\varepsilon/\psi](\hat{e}.\pi) = ([\varepsilon/\psi]\hat{e}_1).\pi$
7. $[\varepsilon/\psi](\hat{e}_1\ \hat{e}_2) = ([\varepsilon/\psi]\hat{e}_1)\ ([\varepsilon/\psi]\hat{e}_2)$
8. $[\varepsilon/\psi](\hat{e}\ \hat{\tau}) = ([\varepsilon/\psi]\hat{e})\ ([\varepsilon/\psi]\hat{\tau})$
9. $[\varepsilon/\psi](\hat{e}\ \varepsilon_1) = ([\varepsilon/\psi]\hat{e})\ ([\varepsilon/\psi]\varepsilon_1)$
10. $[\varepsilon/\psi](\texttt{import}(\varepsilon_s)\ \overline{x = \hat{e}}\ \texttt{in}\ e) = \texttt{import}([\varepsilon/\psi]\varepsilon_s)\ \overline{x = [\varepsilon/\psi]\hat{e}}\ \texttt{in}\ e$

Definition (sub :: $\hat{\varepsilon} \times \hat{\tau} \to \hat{e}$)

1. $[\varepsilon/\psi]X = X$
2. $[\varepsilon/\psi]\{\bar{r}\} = \{\bar{r}\}$
3. $[\varepsilon/\psi](\hat{\tau}_1 \to_{\varepsilon_1} \hat{\tau}_2) = ([\varepsilon/\psi]\hat{\tau}_1) \to_{[\varepsilon/\psi]\varepsilon_1} ([\varepsilon/\psi]\hat{\tau}_2)$
4. $[\varepsilon/\psi](\forall X <: \hat{\tau}_1.\hat{\tau}_2) = \forall X <: [\varepsilon/\psi]\hat{\tau}_1.[\varepsilon/\psi]\hat{\tau}_2$
5. $[\varepsilon/\psi](\forall \phi \subseteq \varepsilon_1.\hat{e}) = \forall \phi \subseteq [\varepsilon/\psi]\varepsilon_1.[\varepsilon/\psi]\hat{e}$, if $\psi \neq \phi$ and ψ does not occur free in $\hat{e}$

Definition (sub :: $\varepsilon \times \varepsilon \to \hat{e}$)

1. $[\varepsilon/\psi]\psi = \varepsilon$
2. $[\varepsilon/\psi]\phi = \phi$, if $\phi \neq \psi$
3. $[\varepsilon/\psi]\{\overline{r.\pi}\} = \{\overline{r.\pi}\}$