1 Grammar

$e ::=$ **exprs.**
- $\mid\ x$ *variable*
- $\mid\ v$ *value*
- $\mid\ e.\pi$ *operation call*
- $\mid\ e\ e$ *application*
- $\mid\ e\ \tau$ *type application*

$v ::=$ **values**
- $\mid\ r$ *resource literal*
- $\mid\ \lambda x : \tau.e$ *abstraction*
- $\mid\ \lambda X <: \tau.e$ *type polymorphism*

$\hat{e} ::=$ **annotated exprs.**
- $\mid\ x$ *variable*
- $\mid\ \hat{v}$ *value*
- $\mid\ \hat{e}.\pi$ *operation call*
- $\mid\ \hat{e}\ \hat{e}$ *application*
- $\mid\ e\ \hat{\tau}$ *type application*
- $\mid\ e\ \varepsilon$ *effect application*
- $\mid\ \texttt{import}(\varepsilon_s)\ \overline{x = \hat{e}}\ \texttt{in}\ e$

$\hat{v} ::=$ **annotated values**
- $\mid\ r$ *resource literal*
- $\mid\ \lambda x : \hat{\tau}.\hat{e}$ *abstraction*
- $\mid\ \lambda X <: \hat{\tau}.\hat{e}$ *type polymorphism*
- $\mid\ \lambda \phi \subseteq \varepsilon.\hat{e}$ *effect polymorphism*

$\tau ::=$ **types**
- $\mid\ X$ *type variable*
- $\mid\ \{\bar{r}\}$ *effect set*
- $\mid\ \tau \to \tau$ *arrow*
- $\mid\ \forall X <: \tau.\tau$ *universal type*

$\hat{\tau} ::=$ **annotated types**
- $\mid\ X$ *type variable*
- $\mid\ \{\bar{r}\}$ *resource set*
- $\mid\ \hat{\tau} \to_\varepsilon \hat{\tau}$ *annotated arrow*
- $\mid\ \forall X <: \hat{\tau}.\hat{\tau}\ \texttt{caps}\ \varepsilon$ *universal type*
- $\mid\ \forall \phi \subseteq \varepsilon.\hat{\tau}\ \texttt{caps}\ \varepsilon$ *universal effect set*

$\varepsilon ::=$ **effects**
- $\mid\ \phi$ *effect variable*
- $\mid\ \{\overline{r.\pi}\}$ *effect set*

$\Gamma ::=$ **contexts**
- $\mid\ \varnothing$ *empty ctx.*
- $\mid\ \Gamma, x : \tau$ *var. binding*
- $\mid\ \Gamma, X <: \tau$ *type var. binding*

$\hat{\Gamma} ::=$ **annotated contexts**
- $\mid\ \varnothing$ *empty ctx.*
- $\mid\ \hat{\Gamma}, x : \hat{\tau}$ *var. binding*
- $\mid\ \hat{\Gamma}, X <: \hat{\tau}$ *type var. binding*
- $\mid\ \hat{\Gamma}, \phi \subseteq \varepsilon$ *effect var. binding*

2 Functions

Definition ($\texttt{annot} :: \tau \times \varepsilon \to \hat{\tau}$)

1. $\texttt{annot}(X, \_) = X$
2. $\texttt{annot}(\{\bar{r}\}, \_) = \{\bar{r}\}$
3. $\texttt{annot}(\tau_1 \to \tau_2, \varepsilon) = \texttt{annot}(\tau_1, \varepsilon) \to_\varepsilon \texttt{annot}(\tau_2, \varepsilon)$
4. $\texttt{annot}(\forall X <: \tau_1.\tau_2, \varepsilon) = \forall X <: \texttt{annot}(\tau_1, \varepsilon).\texttt{annot}(\tau_2, \varepsilon)\ \texttt{caps}\ \varepsilon$

Definition ($\texttt{annot} :: \texttt{e} \times \varepsilon \to \hat{\texttt{e}}$)

1. $\texttt{annot}(x, \_) = e$
2. $\texttt{annot}(r, \_) = r$
3. $\texttt{annot}(\lambda x : \tau.e, \varepsilon) = \lambda x : \texttt{annot}(\tau, \varepsilon).\texttt{annot}(e, \varepsilon)$
4. $\texttt{annot}(e_1\ e_2, \varepsilon) = \texttt{annot}(e_1)\ \texttt{annot}(e_2)$
5. $\texttt{annot}(e.\pi, \varepsilon) = \texttt{annot}(e, \varepsilon).\pi$
6. $\texttt{annot}(\lambda X <: \tau_1.e, \varepsilon) = \lambda X <: \texttt{annot}(\tau_1, \varepsilon).\texttt{annot}(e, \varepsilon)$
7. $\texttt{annot}(e\ \tau, \varepsilon) = \texttt{annot}(e, \varepsilon)\ \texttt{annot}(\tau, \varepsilon)$

Definition ($\texttt{annot} :: \Gamma \times \varepsilon \to \hat{\Gamma}$)

1. $\texttt{annot}(\varnothing, \_) = \varnothing$
2. $\texttt{annot}((\Gamma, x : \tau), \varepsilon) = \texttt{annot}(\Gamma, \varepsilon), x : \texttt{annot}(\tau, \varepsilon)$
3. $\texttt{annot}((\Gamma, X <: \tau), \varepsilon) = \texttt{annot}(\Gamma, \varepsilon), X <: \texttt{annot}(\tau, \varepsilon)$

Definition ($\texttt{erase} :: \hat{\boldsymbol{\tau}} \to \boldsymbol{\tau}$)

1. $\texttt{erase}(X) = X$
2. $\texttt{erase}(\{\bar{r}\}) = \{\bar{r}\}$
3. $\texttt{erase}(\hat{\tau}_1 \to_\varepsilon \hat{\tau}_2) = \texttt{erase}(\hat{\tau}_1) \to \texttt{erase}(\hat{\tau}_2)$
4. $\texttt{erase}(\forall X <: \hat{\tau}_1.\hat{\tau}_2 \texttt{ caps } \varepsilon) = \forall X <: \texttt{erase}(\hat{\tau}_1).\texttt{erase}(\hat{\tau}_2)$

Definition ($\texttt{erase} :: \hat{\mathsf{e}} \to \mathsf{e}$)

1. $\texttt{erase}(x) = x$
2. $\texttt{erase}(r) = r$
3. $\texttt{erase}(\lambda x : \hat{\tau}.\hat{e}) = \lambda x : \texttt{erase}(\hat{\tau}).\texttt{erase}(\hat{e})$
4. $\texttt{erase}(\hat{e}_1\ \hat{e}_2) = \texttt{erase}(\hat{e}_1)\texttt{erase}(\hat{e}_2)$
5. $\texttt{erase}(\hat{e}.\pi) = \texttt{erase}(\hat{e}).\pi$
6. $\texttt{erase}(\lambda X <: \hat{\tau}.\hat{e}) = \lambda X <: \texttt{erase}(\hat{\tau}).\texttt{erase}(\hat{e})$

Definition ($\texttt{erase} :: \hat{\Gamma} \to \Gamma$)

1. $\texttt{erase}(\varnothing) = \varnothing$
2. $\texttt{erase}(\hat{\Gamma}, x : \hat{\tau}) = \texttt{erase}(\hat{\Gamma}), x : \texttt{erase}(\hat{\tau})$
3. $\texttt{erase}(\hat{\Gamma}, X <: \hat{\tau}) = \texttt{erase}(\hat{\Gamma}), X <: \texttt{erase}(\hat{\tau})$

Definition ($\texttt{effects} :: \hat{\boldsymbol{\tau}} \to \boldsymbol{\varepsilon}$)

1. $\texttt{effects}(X) = \varnothing$
2. $\texttt{effects}(\{\bar{r}\}) = \{r.\pi \mid r \in \bar{r}, \pi \in \Pi\}$
3. $\texttt{effects}(\hat{\tau}_1 \to_\varepsilon \hat{\tau}_2) = \texttt{ho-effects}(\hat{\tau}_1) \cup \varepsilon \cup \texttt{effects}(\hat{\tau}_2)$
4. $\texttt{effects}(\forall X <: \hat{\tau}_1.\hat{\tau}_2 \texttt{ caps } \varepsilon_1) = \texttt{ho-effects}(\hat{\tau}_1) \cup \texttt{effects}([\hat{\tau}_1/X]\hat{\tau}_2) \cup \varepsilon_1$
5. $\texttt{effects}(\forall \phi \subseteq \varepsilon.\hat{\tau} \texttt{ caps } \varepsilon_1) = \texttt{effects}([\varepsilon/\phi]\hat{\tau}) \cup \varepsilon_1$

Definition ($\texttt{ho-effects} :: \hat{\boldsymbol{\tau}} \to \boldsymbol{\varepsilon}$)

1. $\texttt{ho-effects}(X) = \varnothing$
2. $\texttt{ho-effects}(\{\bar{r}\}) = \varnothing$
3. $\texttt{ho-effects}(\hat{\tau}_1 \to_\varepsilon \hat{\tau}_2) = \texttt{effects}(\hat{\tau}_1) \cup \texttt{ho-effects}(\hat{\tau}_2)$
4. $\texttt{ho-effects}(\forall X <: \hat{\tau}_1.\hat{\tau}_2 \texttt{ caps } \varepsilon) = \texttt{effects}(\hat{\tau}_1) \cup \texttt{ho-effects}([\hat{\tau}_1/X]\hat{\tau}_2)$
5. $\texttt{ho-effects}(\forall \phi \subseteq \varepsilon.\hat{\tau} \texttt{ caps } \varepsilon_1) = \varepsilon \cup \texttt{ho-effects}([\varepsilon/\phi]\hat{\tau})$

3 Static Rules

$\boxed{\Gamma \vdash e : \tau}$

$$\frac{}{\Gamma, x : \tau \vdash x : \tau}\ (\text{T-Var}) \qquad \frac{}{\Gamma, r : \{r\} \vdash r : \{r\}}\ (\text{T-Resource}) \qquad \frac{\Gamma \vdash e : \{\bar{r}\}}{\Gamma \vdash e.\pi : \texttt{Unit}}\ (\text{T-OperCall})$$

$$\frac{\Gamma, x : \tau_1 \vdash e : \tau_2}{\Gamma \vdash \lambda x : \tau_1.e : \tau_1 \to \tau_2}\ (\text{T-Abs}) \qquad \frac{\Gamma \vdash e_1 : \tau_2 \to \tau_3 \quad \Gamma \vdash e_2 : \tau_2}{\Gamma \vdash e_1\ e_2 : \tau_3}\ (\text{T-App})$$

$$\frac{\Gamma, X <: \tau_1 \vdash e : \tau_2}{\Gamma \vdash \lambda X <: \tau_1.e : \forall X <: \tau_1.\tau_2}\ (\text{T-PolyTypeAbs}) \qquad \frac{\Gamma \vdash e : \forall X <: \tau_1.\tau_2 \quad \tau' <: \tau_1}{\Gamma \vdash e\ \tau' : [\tau'/X]\tau_2}\ (\text{T-PolyTypeApp})$$

$\boxed{\hat{\Gamma} \vdash \hat{e} : \hat{\tau} \texttt{ with } \varepsilon}$

$$\frac{}{\hat{\Gamma}, x : \tau \vdash x : \tau \ \mathtt{with}\ \varnothing}\ (\varepsilon\text{-Var}) \qquad \frac{}{\hat{\Gamma}, r : \{r\} \vdash r : \{r\} \ \mathtt{with}\ \varnothing}\ (\varepsilon\text{-Resource})$$

$$\frac{\hat{\Gamma} \vdash \hat{e} : \{\bar{r}\} \ \mathtt{with}\ \varepsilon_1}{\hat{\Gamma} \vdash \hat{e}.\pi : \mathtt{Unit}\ \mathtt{with}\ \varepsilon_1 \cup \{r.\pi \mid r \in \bar{r}\}}\ (\varepsilon\text{-OperCall}) \qquad \frac{\hat{\Gamma} \vdash e : \tau \ \mathtt{with}\ \varepsilon \quad \tau <: \tau' \quad \varepsilon \subseteq \varepsilon'}{\hat{\Gamma} \vdash e : \tau' \ \mathtt{with}\ \varepsilon'}\ (\varepsilon\text{-Subsume})$$

$$\frac{\hat{\Gamma}, x : \hat{\tau}_2 \vdash \hat{e} : \hat{\tau}_3 \ \mathtt{with}\ \varepsilon_3}{\hat{\Gamma} \vdash \lambda x : \tau_2.\hat{e} : \hat{\tau}_2 \rightarrow_{\varepsilon_3} \hat{\tau}_3 \ \mathtt{with}\ \varnothing}\ (\varepsilon\text{-Abs}) \qquad \frac{\hat{\Gamma} \vdash \hat{e}_1 : \hat{\tau}_2 \rightarrow_{\varepsilon} \hat{\tau}_3 \ \mathtt{with}\ \varepsilon_1 \quad \hat{\Gamma} \vdash \hat{e}_2 : \hat{\tau}_2 \ \mathtt{with}\ \varepsilon_2}{\hat{\Gamma} \vdash \hat{e}_1 \hat{e}_2 : \hat{\tau}_3 \ \mathtt{with}\ \varepsilon_1 \cup \varepsilon_2 \cup \varepsilon}\ (\varepsilon\text{-App})$$

$$\frac{\hat{\Gamma}, X <: \hat{\tau}_1 \vdash \hat{e} : \hat{\tau}_2 \ \mathtt{with}\ \varepsilon_1}{\hat{\Gamma} \vdash \lambda X <: \hat{\tau}_1.\hat{e} : \forall X <: \hat{\tau}_1.\hat{\tau}_2 \ \mathtt{caps}\ \varepsilon_1 \ \mathtt{with}\ \varnothing}\ (\varepsilon\text{-PolyTypeAbs})$$

$$\frac{\hat{\Gamma}, \phi \subseteq \varepsilon \vdash \hat{e} : \hat{\tau} \ \mathtt{with}\ \varepsilon_1}{\hat{\Gamma} \vdash \lambda \phi \subseteq \varepsilon.\hat{e} : \forall \phi \subseteq \varepsilon.\hat{\tau} \ \mathtt{caps}\ \varepsilon_1 \ \mathtt{with}\ \varnothing}\ (\varepsilon\text{-PolyFxAbs})$$

$$\frac{\hat{\Gamma} \vdash \hat{e} : \forall X <: \hat{\tau}_1.\hat{\tau}_2 \ \mathtt{caps}\ \varepsilon_1 \ \mathtt{with}\ \varepsilon_2 \quad \hat{\tau}' <: \hat{\tau}_1}{\hat{\Gamma} \vdash \hat{e}\ \hat{\tau}' : [\hat{\tau}'/X]\hat{\tau}_2 \ \mathtt{with}\ [\hat{\tau}'/X]\varepsilon_1 \cup \varepsilon_2}\ (\varepsilon\text{-PolyTypeApp})$$

$$\frac{\hat{\Gamma} \vdash \hat{e} : \forall \phi \subseteq \varepsilon.\hat{\tau} \ \mathtt{caps}\ \varepsilon_1 \ \mathtt{with}\ \varepsilon_2 \quad \varepsilon' \subseteq \varepsilon}{\hat{\Gamma} \vdash \hat{e}\ \varepsilon' : [\varepsilon'/\phi]\hat{\tau} \ \mathtt{with}\ [\varepsilon'/\phi]\varepsilon_1 \cup \varepsilon_2}\ (\varepsilon\text{-PolyFxApp})$$

$$\frac{\begin{array}{c}\mathtt{effects}(\hat{\tau}_i) \cup \mathtt{ho\text{-}effects}(\mathtt{annot}(\tau, \varnothing)) \subseteq \varepsilon_s \\ \hat{\Gamma} \vdash \hat{e}_i : \hat{\tau}_i \ \mathtt{with}\ \varepsilon_i \qquad x_i : \mathtt{erase}(\hat{\tau}_i) \vdash e : \tau \qquad \mathtt{ho\text{-}safe}(\hat{\tau}_i, \varepsilon_s)\end{array}}{\hat{\Gamma} \vdash \mathtt{import}(\varepsilon_s)\ \overline{x = \hat{e}}\ \mathtt{in}\ e : \mathtt{annot}(\tau, \varepsilon_s) \ \mathtt{with}\ \varepsilon_s \cup \bigcup_i \varepsilon_i}\ (\varepsilon\text{-Import})$$

$\boxed{\mathtt{safe}(\tau, \varepsilon)}$

$$\frac{}{\mathtt{safe}(\{\bar{r}\}, \varepsilon)}\ (\text{Safe-Resource}) \qquad \frac{}{\mathtt{safe}(\mathtt{Unit}, \varepsilon)}\ (\text{Safe-Unit})$$

$$\frac{\varepsilon \subseteq \varepsilon' \quad \mathtt{ho\text{-}safe}(\hat{\tau}_1, \varepsilon) \quad \mathtt{safe}(\hat{\tau}_2, \varepsilon)}{\mathtt{safe}(\hat{\tau}_1 \rightarrow_{\varepsilon'} \hat{\tau}_2, \varepsilon)}\ (\text{Safe-Arrow})$$

$$\frac{\varepsilon_1 \subseteq \varepsilon \quad \mathtt{safe}([\varepsilon_1/\phi]\hat{\tau}, \varepsilon)}{\mathtt{safe}(\forall \phi \subseteq \varepsilon_1.\hat{\tau} \ \mathtt{caps}\ \varepsilon_2, \varepsilon)}\ (\text{Safe-PolyFx}) \qquad \frac{\mathtt{ho\text{-}safe}(\hat{\tau}_1, \varepsilon) \quad \mathtt{safe}([\hat{\tau}_1/X]\hat{\tau}_2, \varepsilon) \quad \varepsilon \subseteq \varepsilon_1}{\mathtt{safe}(\forall X <: \hat{\tau}_1.\hat{\tau}_2 \ \mathtt{caps}\ \varepsilon_2, \varepsilon)}\ (\text{Safe-PolyType})$$

$\boxed{\mathtt{ho\text{-}safe}(\hat{\tau}, \varepsilon)}$

$$\frac{}{\mathtt{ho\text{-}safe}(\{\bar{r}\}, \varepsilon)}\ (\text{HOSafe-Resource}) \qquad \frac{}{\mathtt{ho\text{-}safe}(\mathtt{Unit}, \varepsilon)}\ (\text{HOSafe-Unit})$$

$$\frac{\mathtt{safe}(\hat{\tau}_1, \varepsilon) \quad \mathtt{ho\text{-}safe}(\hat{\tau}_2, \varepsilon)}{\mathtt{ho\text{-}safe}(\hat{\tau}_1 \rightarrow_{\varepsilon'} \hat{\tau}_2, \varepsilon)}\ (\text{HOSafe-Arrow})$$

$$\frac{\varepsilon_1 \subseteq \varepsilon \quad \mathtt{safe}([\varepsilon_1/\phi]\hat{\tau}, \varepsilon)}{\mathtt{ho\text{-}safe}(\forall \phi \subseteq \varepsilon_1.\hat{\tau} \ \mathtt{caps}\ \varepsilon_2, \varepsilon)}\ (\text{HOSafe-PolyFx}) \qquad \frac{\mathtt{safe}(\hat{\tau}_1, \varepsilon) \quad \mathtt{ho\text{-}safe}([\hat{\tau}_1/X]\hat{\tau}_2, \varepsilon)}{\mathtt{ho\text{-}safe}(\forall X <: \hat{\tau}_1.\hat{\tau}_2 \ \mathtt{caps}\ \varepsilon_2, \varepsilon)}\ (\text{HOSafe-PolyType})$$

$\boxed{\hat{\Gamma} \vdash \hat{\tau} <: \hat{\tau}}$

$$\frac{}{\hat{\Gamma} \vdash \hat{\tau} <: \hat{\tau}} \text{ (S-REFLEXIVE)} \qquad \frac{\hat{\Gamma} \vdash \hat{\tau}_1 <: \hat{\tau}_2 \quad \hat{\Gamma} \vdash \hat{\tau}_2 <: \hat{\tau}_3}{\hat{\Gamma} \vdash \hat{\tau}_1 <: \hat{\tau}_3} \text{ (S-TRANSITIVE)}$$

$$\frac{r \in \overline{r_1} \implies r \in \overline{r_2}}{\hat{\Gamma} \vdash \{\overline{r_1}\} <: \{\overline{r_2}\}} \text{ (S-RESOURCESET)} \qquad \frac{\hat{\Gamma} \vdash \hat{\tau}_1' <: \hat{\tau}_1 \quad \hat{\Gamma} \vdash \hat{\tau}_2 <: \hat{\tau}_2' \quad \varepsilon \subseteq \varepsilon'}{\hat{\Gamma} \vdash \tau_1 \to_{\varepsilon} \tau_2 <: \tau_1' \to_{\varepsilon'} \tau_2'} \text{ (S-ARROW)}$$

$$\frac{\hat{\Gamma} \vdash \hat{\tau}_1' <: \hat{\tau}_1 \quad \hat{\Gamma}, Y <: \hat{\tau}_1' \vdash \hat{\tau}_2 <: \hat{\tau}_2'}{\hat{\Gamma} \vdash (\forall X <: \hat{\tau}_1 . \hat{\tau}_2) <: (\forall Y <: \hat{\tau}_1' . \hat{\tau}_2')} \text{ (S-TYPEPOLY)} \qquad \frac{}{\hat{\Gamma}, X <: \hat{\tau} \vdash X <: \hat{\tau}} \text{ (S-TYPEVAR)}$$

4 Dynamic Rules

$\boxed{\hat{e} \longrightarrow \hat{e} \mid \varepsilon}$

$$\frac{\hat{e}_1 \longrightarrow \hat{e}_1' \mid \varepsilon}{\hat{e}_1 \hat{e}_2 \longrightarrow \hat{e}_1' \hat{e}_2 \mid \varepsilon} \text{ (E-APP1)} \qquad \frac{\hat{e}_2 \longrightarrow \hat{e}_2' \mid \varepsilon}{\hat{v}_1 \hat{e}_2 \longrightarrow \hat{v}_1 \hat{e}_2' \mid \varepsilon} \text{ (E-APP2)} \qquad \frac{}{(\lambda x : \hat{\tau} . \hat{e}) \hat{v}_2 \longrightarrow [\hat{v}_2 / x] \hat{e} \mid \varnothing} \text{ (E-APP3)}$$

$$\frac{\hat{e} \to \hat{e}' \mid \varepsilon}{\hat{e}.\pi \longrightarrow \hat{e}'.\pi \mid \varepsilon} \text{ (E-OPERCALL1)} \qquad \frac{r \in R \quad \pi \in \Pi}{r.\pi \longrightarrow \texttt{unit} \mid \{r.\pi\}} \text{ (E-OPERCALL2)}$$

$$\frac{\hat{e} \longrightarrow \hat{e}' \mid \varepsilon}{\hat{e}\ \hat{\tau} \longrightarrow \hat{e}'\ \hat{\tau} \mid \varepsilon} \text{ (E-POLYTYPEAPP1)} \qquad \frac{}{(\lambda X <: \hat{\tau}_1 . \hat{e}) \hat{\tau} \longrightarrow [\hat{\tau} / X] \hat{e} \mid \varnothing} \text{ (E-POLYTYPEAPP2)}$$

$$\frac{\hat{e} \longrightarrow \hat{e}' \mid \varepsilon}{\hat{e}\ \hat{\tau} \longrightarrow \hat{e}'\ \hat{\tau} \mid \varepsilon} \text{ (E-POLYFXAPP1)} \qquad \frac{}{(\lambda \phi \subseteq \varepsilon_1 . \hat{e}) \varepsilon \longrightarrow [\varepsilon / \phi] \hat{e} \mid \varnothing} \text{ (E-POLYFXAPP2)}$$

$$\frac{\hat{e} \longrightarrow \hat{e}' \mid \varepsilon'}{\texttt{import}(\varepsilon_s)\ x = \hat{e}\ \texttt{in}\ e \longrightarrow \texttt{import}(\varepsilon_s)\ x = \hat{e}'\ \texttt{in}\ e \mid \varepsilon'} \text{ (E-IMPORT1)}$$

$$\frac{}{\texttt{import}(\varepsilon_s)\ x = \hat{e}\ \texttt{in}\ e \longrightarrow [\hat{v}/x] \texttt{annot}(e, \varepsilon_s) \mid \varnothing} \text{ (E-IMPORT2)}$$