Lemma 1. *If* $\varepsilon \subseteq \texttt{effects}(\hat{\tau})$ *and* $\texttt{ho-safe}(\hat{\tau}, \varepsilon)$ *then* $\hat{\tau} <: \texttt{annot}(\texttt{erase}(\hat{\tau}), \varepsilon)$.

Counterexample. Let $\hat{\tau} = \texttt{Unit} \to_a \texttt{Unit}$ and $\varepsilon = \varnothing$. Note $\texttt{annot}(\texttt{erase}(\texttt{Unit} \to_a \texttt{Unit}), \varnothing) = \texttt{Unit} \to_\varnothing \texttt{Unit}$

$\texttt{effects}(\texttt{Unit} \to_a \texttt{Unit}) = \{a\} \supseteq \varepsilon = \varnothing$

$\texttt{ho-safe}(\texttt{Unit} \to_a \texttt{Unit}, \varnothing) = \texttt{safe}(\texttt{Unit}, \varnothing) \wedge \texttt{ho-safe}(\texttt{Unit}, \varnothing) = \texttt{True}$

The lemma applies, so $\texttt{Unit} \to_a \texttt{Unit} <: \texttt{Unit} \to_\varnothing \texttt{Unit}$. This implies that $\{a\} \subseteq \varnothing$.

Lemma 2. *If (1)* $\texttt{effects}(\hat{\tau}) \subseteq \varepsilon$ *and (2)* $\texttt{ho-safe}(\hat{\tau}, \varepsilon)$ *then* $\hat{\tau} <: \texttt{annot}(\texttt{erase}(\hat{\tau}), \varepsilon)$.

Counterexample. Let $\hat{\tau} = ((\texttt{Unit} \to_a \texttt{Unit}) \to_\varnothing \texttt{Unit}) \to_\varnothing \texttt{Unit}$ and $\varepsilon = \{a\}$.

Proof of (1).
$\texttt{effects}(((\texttt{Unit} \to_a \texttt{Unit}) \to_\varnothing \texttt{Unit}) \to_\varnothing \texttt{Unit})$
$= \texttt{ho-effects}((\texttt{Unit} \to_a \texttt{Unit}) \to_\varnothing \texttt{Unit})) \cup \texttt{effects}(\texttt{Unit})$
$= \texttt{effects}(\texttt{Unit} \to_a \texttt{Unit}) \cup \texttt{effects}(\texttt{Unit})$
$= \{a\}$, therefore (1) is true.

Proof of (2).
$\texttt{ho-safe}(((\texttt{Unit} \to_a \texttt{Unit}) \to_\varnothing \texttt{Unit}) \to_\varnothing \texttt{Unit}, \{a\})$
$= \texttt{safe}((\texttt{Unit} \to_a \texttt{Unit}) \to_\varnothing \texttt{Unit}, \{a\}) \wedge \texttt{ho-safe}(\texttt{Unit}, \{a\})$
$= \texttt{safe}((\texttt{Unit} \to_a \texttt{Unit}) \to_\varnothing \texttt{Unit}, \{a\})$
This is untrue as $\{a\} \nsubseteq \varnothing$, so this is not a valid counterexample.