1 Grammar

e ::=		**exprs.**
|	x	*variable*
|	v	*value*
|	$e.\pi$	*operation call*
|	$e\ e$	*application*
|	$e\ \tau$	*type application*

v ::=		**values**
|	r	*resource literal*
|	$\lambda x : \tau.e$	*abstraction*
|	$\lambda X.e$	*type polymorphism*

$\hat{e}$ ::=		**annotated exprs.**
|	x	*variable*
|	$\hat{v}$	*value*
|	$\hat{e}.\pi$	*operation call*
|	$\hat{e}\ \hat{e}$	*application*
|	$e\ \tau$	*type application*
|	$e\ \varepsilon$	*effect application*
|	$\texttt{import}(\varepsilon_s)\ x = \hat{e}\ \texttt{in}\ e$	*import*

$\hat{v}$ ::=		**annotated values**
|	r	*resource literal*
|	$\lambda x : \hat{\tau}.\hat{e}$	*abstraction*
|	$\lambda X.\hat{e}$	*type polymorphism*
|	$\lambda \phi.\hat{e}$	*effect polymorphism*

τ ::=		**types**
|	X	*type variable*
|	$\{\bar{r}\}$	*effect set*
|	$\tau \to \tau$	*arrow*
|	$\forall X.\tau$	*universal type*
|	$\forall \phi.\tau$	*universal effect set*

$\hat{\tau}$ ::=		**annotated types**
|	t	*type variable*
|	$\{\bar{r}\}$	*resource set*
|	$\hat{\tau} \to_\varepsilon \hat{\tau}$	*annotated arrow*
|	$\forall X.\hat{\tau}$	*universal type*
|	$\forall \phi.\hat{\tau}$	*universal effect set*

ε ::=		**effects**
|	ϕ	*effect variable*
|	$\{\overline{r.\pi}\}$	*effect set*

Γ ::=		**contexts**
|	$\varnothing$	*empty ctx.*
|	$\Gamma, x : \tau$	*var. binding*
|	Γ, X	*type var. binding*

$\hat{\Gamma}$ ::=		**annotated contexts**
|	$\varnothing$	*empty ctx.*
|	$\hat{\Gamma}, x : \hat{\tau}$	*var. binding*
|	$\hat{\Gamma}, X$	*type var. binding*
|	$\hat{\Gamma}, \phi$	*effect var. binding*

2 Functions

Definition ($\texttt{annot} :: \boldsymbol{\tau} \times \boldsymbol{\varepsilon} \to \boldsymbol{\hat{\tau}}$)

1. $\texttt{annot}(X, \_) = X$
2. $\texttt{annot}(\{\bar{r}\}, \_) = \{\bar{r}\}$
3. $\texttt{annot}(\tau_1 \to \tau_2, \varepsilon) = \texttt{annot}(\tau_1, \varepsilon) \to_\varepsilon \texttt{annot}(\tau_2, \varepsilon)$
4. $\texttt{annot}(\forall X.\tau, \varepsilon) = \forall X.\texttt{annot}(\tau, \varepsilon)$
5. $\texttt{annot}(\forall \phi.\tau, \varepsilon) = \forall \phi.\texttt{annot}(\tau, \varepsilon)$

Definition ($\texttt{annot} :: \texttt{e} \times \boldsymbol{\varepsilon} \to \hat{\texttt{e}}$)

1. $\texttt{annot}(x, \_) = e$
2. $\texttt{annot}(r, \_) = r$
3. $\texttt{annot}(\lambda x : \tau.e, \varepsilon) = \lambda x : \texttt{annot}(\tau, \varepsilon).\texttt{annot}(e, \varepsilon)$
4. $\texttt{annot}(e_1\ e_2, \varepsilon) = \texttt{annot}(e_1)\ \texttt{annot}(e_2)$
5. $\texttt{annot}(e.\pi, \varepsilon) = \texttt{annot}(e, \varepsilon).\pi$
6. $\texttt{annot}(\lambda X.e, \varepsilon) = \lambda X.\texttt{annot}(e, \varepsilon)$
7. $\texttt{annot}(e\ \tau, \varepsilon) = \texttt{annot}(e, \varepsilon)\ \texttt{annot}(\tau, \varepsilon)$

Definition ($\texttt{annot} :: \Gamma \times \boldsymbol{\varepsilon} \to \hat{\Gamma}$)

1. $\texttt{annot}(\varnothing, \_) = \varnothing$
2. $\texttt{annot}((\Gamma, x : \tau), \varepsilon) = \texttt{annot}(\Gamma, \varepsilon), x : \texttt{annot}(\tau, \varepsilon)$
3. $\texttt{annot}((\Gamma, X), \varepsilon) = \texttt{annot}(\Gamma, \varepsilon), X$

Definition (erase :: $\hat{\tau} \to \tau$)

1. $\texttt{erase}(X) = X$
2. $\texttt{erase}(\{\bar{r}\}) = \{\bar{r}\}$
3. $\texttt{erase}(\hat{\tau}_1 \to_\varepsilon \hat{\tau}_2) = \texttt{erase}(\hat{\tau}_1) \to \texttt{erase}(\hat{\tau}_2)$
4. $\texttt{erase}(\forall X.\hat{\tau}) = \forall X.\texttt{erase}(\hat{\tau})$

Definition (erase :: $\hat{\mathsf{e}} \to \mathsf{e}$)

1. $\texttt{erase}(x) = x$
2. $\texttt{erase}(r) = r$
3. $\texttt{erase}(\lambda x : \hat{\tau}.\hat{e}) = \lambda x : \texttt{erase}(\hat{\tau}).\texttt{erase}(\hat{e})$
4. $\texttt{erase}(\hat{e}_1\ \hat{e}_2) = \texttt{erase}(\hat{e}_1)\texttt{erase}(\hat{e}_2)$
5. $\texttt{erase}(\hat{e}.\pi) = \texttt{erase}(\hat{e}).\pi$
6. $\texttt{erase}(\lambda X.\hat{e}) = \lambda X.\texttt{erase}(\hat{e})$

Definition (erase :: $\hat{\Gamma} \to \Gamma$)

1. $\texttt{erase}(\varnothing) = \varnothing$
2. $\texttt{erase}(\hat{\Gamma}, x : \hat{\tau}) = \texttt{erase}(\hat{\Gamma}), x : \texttt{erase}(\hat{\tau})$
3. $\texttt{erase}(\hat{\Gamma}, X) = \texttt{erase}(\hat{\Gamma}), X$

Definition (effects :: $\hat{\tau} \to \varepsilon$)

1. $\texttt{effects}(X) = \varnothing$
2. $\texttt{effects}(\{\bar{r}\}) = \{r.\pi \mid r \in \bar{r}, \pi \in \mathit{\Pi}\}$
3. $\texttt{effects}(\hat{\tau}_1 \to_\varepsilon \hat{\tau}_2) = \texttt{ho-effects}(\hat{\tau}_1) \cup \varepsilon \cup \texttt{effects}(\hat{\tau}_2)$

Definition (ho-effects :: $\hat{\tau} \to \varepsilon$)

1. $\texttt{ho-effects}(t) = \varnothing$
2. $\texttt{ho-effects}(\{\bar{r}\}) = \varnothing$
3. $\texttt{ho-effects}(\hat{\tau}_1 \to_\varepsilon \hat{\tau}_2) = \texttt{effects}(\hat{\tau}_1) \cup \texttt{ho-effects}(\hat{\tau}_2)$

Definition (substitution :: $\hat{\mathsf{e}} \times \hat{\mathsf{v}} \times \hat{\mathsf{v}} \to \hat{\mathsf{e}}$)

The notation $[\hat{v}/x]\hat{e}$ is short-hand for $\texttt{substitution}(\hat{e}, \hat{v}, x)$. This function is partial, because the third input must be a variable. We adopt the usual renaming conventions to avoid accidental capture.

1. $[\hat{v}/y]x = \hat{v}$, if $x = y$
2. $[\hat{v}/y]x = x$, if $x \neq y$
3. $[\hat{v}/y](\lambda x : \hat{\tau}.\hat{e}) = \lambda x : \hat{\tau}.[\hat{v}/y]\hat{e}$, if $y \neq x$ and y does not occur free in $\hat{e}$
4. $[\hat{v}/y](\hat{e}_1\ \hat{e}_2) = ([\hat{v}/y]\hat{e}_1)([\hat{v}/y]\hat{e}_2)$
5. $[\hat{v}/y](\hat{e}.\pi) = ([\hat{v}/y]\hat{e}).\pi$
6. $[\hat{v}/y](\lambda X.\hat{e}) = \lambda X.[\hat{v}/y]\hat{e}$
7. $[\hat{v}/y](\lambda \phi.\hat{e}) = \lambda \phi.[\hat{v}/y]\hat{e}$
8. $[\hat{v}/y](\hat{e}\ \hat{\tau}) = [\hat{v}/y]\hat{e}\ \hat{\tau}$
9. $[\hat{v}/y](\hat{e}\ \varepsilon) = [\hat{v}/y]\hat{e}\ \varepsilon$
10. $[\hat{v}/y](\texttt{import}(\varepsilon_s)\ x = \hat{e}\ \texttt{in}\ e) = \texttt{import}(\varepsilon_s)\ x = [\hat{v}/y]\hat{e}\ \texttt{in}\ e$

When performing multiple substitutions the notation $[\hat{v}_1/x_1, \hat{v}_2/x_2]\hat{e}$ is used as shorthand for $[\hat{v}_2/x_2]([\hat{v}_1/x_1]\hat{e})$ (note the order of the variables has been flipped; the substitutions occur as they are written, left-to-right).

Definition (`substitution` :: $\hat{\boldsymbol{\tau}} \times \hat{\boldsymbol{\tau}} \times \hat{\boldsymbol{\tau}} \to \hat{\boldsymbol{\tau}}$)

1. $[\hat{\tau}/Y]X = \hat{\tau}$, if $X = Y$
2. $[\hat{\tau}/Y]X = X$, if $X \neq Y$
3. $[\hat{\tau}/Y]\{\bar{r}\} = \{\bar{r}\}$
4. $[\hat{\tau}/Y](\hat{\tau}_1 \to_\varepsilon \hat{\tau}_2) = [\hat{\tau}/Y]\hat{\tau}_1 \to_\varepsilon [\hat{\tau}/Y]\hat{\tau}_2$
5. $[\hat{\tau}/Y](\forall X.\hat{\tau}_1) = \forall X.[\hat{\tau}/Y]\tau_1$, if $Y \neq X$ and Y does not occur free in τ_1
6. $[\hat{\tau}/Y](\forall \phi.\hat{\tau}_1) = \forall \phi.[\hat{\tau}/Y]\tau_1$

Definition (`substitution` :: $\boldsymbol{\varepsilon} \times \boldsymbol{\varepsilon} \times \boldsymbol{\varepsilon} \to \hat{\boldsymbol{\tau}}$)

1. $[\varepsilon/\phi]\Phi = \varepsilon$, if $\phi = \Phi$
2. $[\varepsilon/\phi]\Phi = \Phi$, if $\phi \neq \Phi$
3. $[\varepsilon/\phi]\{\overline{r.\pi}\} = \{\overline{r.\pi}\}$

Definition (`substitution` :: $\hat{\boldsymbol{\tau}} \times \boldsymbol{\varepsilon} \times \boldsymbol{\varepsilon} \to \hat{\boldsymbol{\tau}}$)

1. $[\varepsilon/\phi]X = X$
2. $[\varepsilon/\phi]\{\bar{r}\} = \{\bar{r}\}$
3. $[\varepsilon/\phi](\hat{\tau}_1 \to'_\varepsilon \hat{\tau}_2) = [\varepsilon/\phi]\hat{\tau}_1 \to_{[\varepsilon/\phi]\varepsilon'} [\varepsilon/\phi]\hat{\tau}_2$
4. $[\varepsilon/\phi](\forall X.\hat{\tau}) = \forall X.[\varepsilon/\phi]\hat{\tau}$
5. $[\varepsilon/\phi](\forall \Phi.\hat{\tau}) = \forall \Phi.[\varepsilon/\phi]$, if $\phi \neq \Phi$ and ϕ does not occur free in $\hat{\tau}$

3 Static Rules

$\boxed{\Gamma \vdash e : \tau}$

$$\frac{}{\Gamma, x : \tau \vdash x : \tau}\ (\text{T-Var}) \qquad \frac{}{\Gamma, r : \{r\} \vdash r : \{r\}}\ (\text{T-Resource}) \qquad \frac{\Gamma \vdash e : \{\bar{r}\}}{\Gamma \vdash e.\pi : \texttt{Unit}}\ (\text{T-OperCall})$$

$$\frac{\Gamma, x : \tau_1 \vdash e : \tau_2}{\Gamma \vdash \lambda x : \tau_1.e : \tau_1 \to \tau_2}\ (\text{T-Abs}) \qquad \frac{\Gamma \vdash e_1 : \tau_2 \to \tau_3 \quad \Gamma \vdash e_2 : \tau_2}{\Gamma \vdash e_1\ e_2 : \tau_3}\ (\text{T-App})$$

$$\frac{\Gamma, X \vdash e : \tau}{\Gamma \vdash \lambda X.e : \tau}\ (\text{T-PolyTypeAbs}) \qquad \frac{\Gamma \vdash e : \forall X.\tau}{\Gamma \vdash e\ \tau' : [\tau'/X]\tau}\ (\text{T-PolyTypeApp})$$

$\boxed{\hat{\Gamma} \vdash \hat{e} : \hat{\tau} \texttt{ with } \varepsilon}$

$$\frac{}{\hat{\Gamma}, x : \tau \vdash x : \tau \texttt{ with } \varnothing}\ (\varepsilon\text{-Var}) \qquad \frac{}{\hat{\Gamma}, r : \{r\} \vdash r : \{r\} \texttt{ with } \varnothing}\ (\varepsilon\text{-Resource})$$

$$\frac{\hat{\Gamma} \vdash \hat{e} : \{\bar{r}\}}{\hat{\Gamma} \vdash \hat{e}.\pi : \texttt{Unit with } \{r.\pi \mid r \in \bar{r}\}}\ (\varepsilon\text{-OperCall}) \qquad \frac{\hat{\Gamma} \vdash e : \tau \texttt{ with } \varepsilon \quad \tau <: \tau' \quad \varepsilon \subseteq \varepsilon'}{\hat{\Gamma} \vdash e : \tau' \texttt{ with } \varepsilon'}\ (\varepsilon\text{-Subsume})$$

$$\frac{\hat{\Gamma}, x : \hat{\tau}_2 \vdash \hat{e} : \hat{\tau}_3 \texttt{ with } \varepsilon_3}{\hat{\Gamma} \vdash \lambda x : \tau_2.\hat{e} : \hat{\tau}_2 \to_{\varepsilon_3} \hat{\tau}_3 \texttt{ with } \varnothing}\ (\varepsilon\text{-Abs}) \qquad \frac{\hat{\Gamma} \vdash \hat{e}_1 : \hat{\tau}_2 \to_\varepsilon \hat{\tau}_3 \texttt{ with } \varepsilon_1 \quad \hat{\Gamma} \vdash \hat{e}_2 : \hat{\tau}_2 \texttt{ with } \varepsilon_2}{\hat{\Gamma} \vdash \hat{e}_1 \hat{e}_2 : \hat{\tau}_3 \texttt{ with } \varepsilon_1 \cup \varepsilon_2 \cup \varepsilon}\ (\varepsilon\text{-App})$$

$$\frac{\hat{\Gamma}, X \vdash \hat{e} : \hat{\tau} \texttt{ with } \varepsilon}{\hat{\Gamma} \vdash \lambda X.\hat{e} : \hat{\tau} \texttt{ with } \varnothing}\ (\varepsilon\text{-PolyTypeAbs}) \qquad \frac{\hat{\Gamma} \vdash \hat{e} : \forall X.\hat{\tau} \texttt{ with } \varepsilon_1}{\hat{\Gamma} \vdash \hat{e}\ \hat{\tau}' : [\hat{\tau}'/X]\hat{\tau} \texttt{ with } \varepsilon_1 \cup \texttt{effects}(\hat{\tau}')}\ (\varepsilon\text{-PolyTypeApp})$$

$$\frac{\hat{\Gamma} \vdash \hat{e} : \hat{\tau} \texttt{ with } \varnothing}{\hat{\Gamma} \vdash \lambda\phi.\hat{e} : \forall\phi.\hat{\tau} \texttt{ with } \varnothing}\ (\varepsilon\text{-PolyFxAbs}) \qquad \frac{\hat{\Gamma} \vdash \hat{e} : \forall X.\phi \texttt{ with } \varepsilon_1}{\hat{\Gamma} \vdash \hat{e}\ \varepsilon : [\phi/\varepsilon]\hat{\tau} \texttt{ with } \varepsilon_1 \cup \varepsilon}\ (\varepsilon\text{-PolyFxApp})$$

$$\frac{\begin{array}{c}\texttt{effects}(\hat{\tau}) \cup \texttt{ho-effects}(\texttt{annot}(\tau, \varnothing)) \subseteq \varepsilon \\ \hat{\Gamma} \vdash \hat{e} : \hat{\tau} \texttt{ with } \varepsilon_1 \qquad \texttt{ho-safe}(\hat{\tau}, \varepsilon) \qquad x : \texttt{erase}(\hat{\tau}) \vdash e : \tau\end{array}}{\hat{\Gamma} \vdash \texttt{import}(\varepsilon)\ x = \hat{e} \texttt{ in } e : \texttt{annot}(\tau, \varepsilon) \texttt{ with } \varepsilon \cup \varepsilon_1}\ (\varepsilon\text{-Import})$$

$\boxed{\texttt{safe}(\tau, \varepsilon)}$

$$\frac{}{\texttt{safe}(\{\bar{r}\}, \varepsilon)}\ (\textsc{Safe-Resource}) \qquad \frac{}{\texttt{safe}(\texttt{Unit}, \varepsilon)}\ (\textsc{Safe-Unit})$$

$$\frac{\varepsilon \subseteq \varepsilon' \quad \texttt{ho-safe}(\hat{\tau}_1, \varepsilon) \quad \texttt{safe}(\hat{\tau}_2, \varepsilon)}{\texttt{safe}(\hat{\tau}_1 \to_{\varepsilon'} \hat{\tau}_2, \varepsilon)}\ (\textsc{Safe-Arrow})$$

$\boxed{\texttt{ho-safe}(\hat{\tau}, \varepsilon)}$

$$\frac{}{\texttt{ho-safe}(\{\bar{r}\}, \varepsilon)}\ (\textsc{HOSafe-Resource}) \qquad \frac{}{\texttt{ho-safe}(\texttt{Unit}, \varepsilon)}\ (\textsc{HOSafe-Unit})$$

$$\frac{\texttt{safe}(\hat{\tau}_1, \varepsilon) \quad \texttt{ho-safe}(\hat{\tau}_2, \varepsilon)}{\texttt{ho-safe}(\hat{\tau}_1 \to_{\varepsilon'} \hat{\tau}_2, \varepsilon)}\ (\textsc{HOSafe-Arrow})$$

$\boxed{\hat{\tau} <: \hat{\tau}}$

$$\frac{\varepsilon \subseteq \varepsilon' \quad \hat{\tau}_2 <: \hat{\tau}_2' \quad \hat{\tau}_1' <: \hat{\tau}_1}{\hat{\tau}_1 \to_{\varepsilon} \hat{\tau}_2 <: \hat{\tau}_1' \to_{\varepsilon'} \hat{\tau}_2'}\ (\textsc{S-Effects}) \qquad \frac{r \in \bar{r}_2 \implies r \in \bar{r}_1}{\{\bar{r}_2\} <: \{\bar{r}_1\}}\ (\textsc{S-ResourceSet})$$

4 Dynamic Rules

$\boxed{\hat{e} \longrightarrow \hat{e} \mid \varepsilon}$

$$\frac{\hat{e}_1 \longrightarrow \hat{e}_1' \mid \varepsilon}{\hat{e}_1 \hat{e}_2 \longrightarrow \hat{e}_1' \hat{e}_2 \mid \varepsilon}\ (\textsc{E-App1}) \qquad \frac{\hat{e}_2 \longrightarrow \hat{e}_2' \mid \varepsilon}{\hat{v}_1 \hat{e}_2 \longrightarrow \hat{v}_1 \hat{e}_2' \mid \varepsilon}\ (\textsc{E-App2}) \qquad \frac{}{(\lambda x : \hat{\tau}.\hat{e})\hat{v}_2 \longrightarrow [\hat{v}_2/x]\hat{e} \mid \varnothing}\ (\textsc{E-App3})$$

$$\frac{\hat{e} \to \hat{e}' \mid \varepsilon}{\hat{e}.\pi \longrightarrow \hat{e}'.\pi \mid \varepsilon}\ (\textsc{E-OperCall1}) \qquad \frac{r \in R \quad \pi \in \Pi}{r.\pi \longrightarrow \texttt{unit} \mid \{r.\pi\}}\ (\textsc{E-OperCall2})$$

$$\frac{\hat{e} \longrightarrow \hat{e}' \mid \varepsilon}{\hat{e}\ \hat{\tau} \longrightarrow \hat{e}'\ \hat{\tau} \mid \varepsilon}\ (\textsc{E-PolyTypeApp1}) \qquad \frac{}{(\lambda X.\hat{e})\hat{\tau} \longrightarrow [\hat{\tau}/X]\hat{e} \mid \varnothing}\ (\textsc{E-PolyTypeApp2})$$

$$\frac{\hat{e} \longrightarrow \hat{e}' \mid \varepsilon}{\hat{e}\ \hat{\tau} \longrightarrow \hat{e}'\ \hat{\tau} \mid \varepsilon}\ (\textsc{E-PolyFxApp1}) \qquad \frac{}{(\lambda \phi.\hat{e})\varepsilon \longrightarrow [\varepsilon/\phi]\hat{e} \mid \varnothing}\ (\textsc{E-PolyFxApp2})$$

$$\frac{\hat{e} \longrightarrow \hat{e}' \mid \varepsilon'}{\texttt{import}(\varepsilon_s)\ x = \hat{e}\ \texttt{in}\ e \longrightarrow \texttt{import}(\varepsilon_s)\ x = \hat{e}'\ \texttt{in}\ e \mid \varepsilon'}\ (\textsc{E-Import1})$$

$$\frac{}{\texttt{import}(\varepsilon_s)\ x = \hat{e}\ \texttt{in}\ e \longrightarrow [\hat{v}/x]\texttt{annot}(e, \varepsilon_s) \mid \varnothing}\ (\textsc{E-Import2})$$

5 Encodings

5.1 $\bot$

The bottom type is defined as $\bot \stackrel{\text{def}}{=} \varnothing$, which is the literal for an empty set of resources.

$$\frac{}{\Gamma \vdash \bot : \varnothing}\ (\text{T-}\bot) \qquad \frac{}{\Gamma \vdash \bot : \varnothing\ \texttt{with}\ \varnothing}\ (\varepsilon\text{-}\bot)$$

5.2 `unit`, `Unit`

Define $\texttt{unit} = \lambda\texttt{x} : \varnothing.\texttt{x}$, i.e. the function which takes an empty set of resources and returns it. We shall refer to its type, which is $\varnothing \to_{\varnothing} \varnothing$, as `Unit`. It has various properties befitting `unit`.

1. `unit` cannot be invoked as $\varnothing$ is uninhabited.
2. `unit` is a value.
3. The only term with type `Unit` is `unit`.
4. $\vdash \texttt{unit} : \texttt{Unit}$ by using ε-Abs and ε-Var.
5. $\texttt{effects}(\texttt{Unit}) = \texttt{ho-effects}(\texttt{Unit}) = \varnothing$
6. $\texttt{safe}(\texttt{Unit}, \varepsilon)$ and $\texttt{ho-safe}(\texttt{Unit}, \varepsilon)$

$$\frac{}{\Gamma \vdash \texttt{unit} : \texttt{Unit}} \;(\text{T-Unit}) \qquad \frac{}{\Gamma \vdash \texttt{unit} : \texttt{Unit with } \varnothing} \;(\varepsilon\text{-Unit})$$