1 Grammar

$e ::=$		**exprs.**
$\mid$	x	*variable*
$\mid$	v	*value*
$\mid$	$e\ e$	*application*
$\mid$	$e.\pi$	*operation call*
$\mid$	$\forall t : \tau.e$	*type polymorphism*
$v ::=$		**values**
$\mid$	r	*resource literal*
$\mid$	$\lambda x : \tau.e$	*abstraction*
$\hat{e} ::=$		**annotated exprs.**
$\mid$	x	*variable*
$\mid$	$\hat{v}$	*value*
$\mid$	$\hat{e}\ \hat{e}$	*application*
$\mid$	$\hat{e}.\pi$	*operation call*
$\mid$	$\forall t : \hat{\tau}.\hat{e}$	*type polymorphism*
$\mid$	$\forall \epsilon : \varepsilon.\hat{e}$	*effect polymorphism*
$\mid$	$\texttt{import}(\varepsilon_s)\ x = \hat{e}\ \texttt{in}\ e$	*import*
$\hat{v} ::=$		**annotated values**
$\mid$	r	*resource literal*
$\mid$	$\lambda x : \hat{\tau}.\hat{e}$	*abstraction*

$\tau ::=$		**types**
$\mid$	t	*type variable*
$\mid$	$\{\bar{r}\}$	*effect set*
$\mid$	$\tau \to \tau$	*arrow*
$\hat{\tau} ::=$		**annotated types**
$\mid$	t	*type variable*
$\mid$	$\{\bar{r}\}$	*resource set*
$\mid$	$\hat{\tau} \to_\varepsilon \hat{\tau}$	*annotated arrow*
$\varepsilon ::=$		**effects**
$\mid$	ϵ	*effect variable*
$\mid$	$\{\overline{r.\pi}\}$	*effect set*
$\Gamma ::=$		**contexts**
$\mid$	$\varnothing$	*empty ctx.*
$\mid$	$\Gamma, x : \tau$	*var. binding*
$\mid$	$\Gamma, t : \tau$	*type var. binding*
$\hat{\Gamma} ::=$		**annotated contexts**
$\mid$	$\varnothing$	*empty ctx.*
$\mid$	$\hat{\Gamma}, x : \hat{\tau}$	*var. binding*
$\mid$	$\hat{\Gamma}, \epsilon : \varepsilon$	*effect var. binding*
$\mid$	$\hat{\Gamma}, t : \hat{\tau}$	*type var. binding*

2 Functions

Definition (annot :: $\tau \times \varepsilon \to \hat{\tau}$)

1. $\texttt{annot}(t, \_) = t$
2. $\texttt{annot}(\{\bar{r}\}, \_) = \{\bar{r}\}$
3. $\texttt{annot}(\tau_1 \to \tau_2, \varepsilon) = \texttt{annot}(\tau_1, \varepsilon) \to_\varepsilon \texttt{annot}(\tau_2, \varepsilon)$

Definition (annot :: e $\times\ \varepsilon \to$ ê)

1. $\texttt{annot}(x, \_) = e$
2. $\texttt{annot}(r, \_) = r$
3. $\texttt{annot}(\lambda x : \tau.e, \varepsilon) = \lambda x : \texttt{annot}(\tau, \varepsilon).\texttt{annot}(e, \varepsilon)$
4. $\texttt{annot}(e_1\ e_2, \varepsilon) = \texttt{annot}(e_1)\texttt{annot}(e_2)$
5. $\texttt{annot}(e.\pi, \varepsilon) = \texttt{annot}(e).\pi$
6. $\texttt{annot}(\forall t : \tau.e, \varepsilon) = \forall t : \texttt{annot}(\tau, \varepsilon).\texttt{annot}(e, \varepsilon)$
7. $\texttt{annot}(\forall \epsilon : \varepsilon'.e, \varepsilon) = \forall \epsilon : \varepsilon'.\texttt{annot}(e, \varepsilon)$

Definition (annot :: $\Gamma \times \varepsilon \to \hat{\Gamma}$)

1. $\texttt{annot}(\varnothing, \_) = \varnothing$
2. $\texttt{annot}((\Gamma, x : \tau), \varepsilon) = \texttt{annot}(\Gamma, \varepsilon), x : \texttt{annot}(\tau, \varepsilon)$
3. $\texttt{annot}((\Gamma, t : \tau), \varepsilon) = \texttt{annot}(\Gamma, \varepsilon), x : \texttt{annot}(\tau, \varepsilon)$

Definition (erase :: $\hat{\tau} \to \tau$)

1. $\texttt{erase}(t) = e$
2. $\texttt{erase}(\{\bar{r}\}, \_) = \{\bar{r}\}$
3. $\texttt{erase}(\hat{\tau}_1 \to_\varepsilon \hat{\tau}_2) = \texttt{erase}(\hat{\tau}_1) \to \texttt{erase}(\hat{\tau}_2)$

Definition ($\texttt{erase} :: \hat{\texttt{e}} \rightarrow \texttt{e}$)

1. $\texttt{erase}(x) = x$
2. $\texttt{erase}(r) = r$
3. $\texttt{erase}(\lambda x : \hat{\tau}.\hat{e}) = \lambda x : \texttt{erase}(\hat{\tau}).\texttt{erase}(\hat{e})$
4. $\texttt{erase}(e_1\ e_2) = \texttt{erase}(e_1)\texttt{erase}(e_2)$
5. $\texttt{erase}(e.\pi) = \texttt{erase}(e).\pi$
6. $\texttt{erase}(\forall t : \hat{\tau}.\hat{e}) = \forall t : \texttt{erase}(\hat{\tau}).\texttt{erase}(\hat{e})$
7. $\texttt{erase}(\forall \epsilon : \varepsilon.\hat{e}) = \forall \epsilon : \varepsilon.\texttt{erase}(\hat{e})$

Definition ($\texttt{erase} :: \hat{\Gamma} \rightarrow \Gamma$)

1. $\texttt{erase}(\varnothing) = \varnothing$
2. $\texttt{erase}(\hat{\Gamma}, x : \hat{\tau}) = \texttt{erase}(\hat{\Gamma}), x : \texttt{erase}(\hat{\tau})$
3. $\texttt{erase}(\Gamma, t : \hat{\tau}) = \texttt{erase}(\Gamma), x : \texttt{erase}(\hat{\tau})$
4. $\texttt{erase}(\Gamma, \epsilon : \varepsilon) = \texttt{erase}(\Gamma)$

Definition ($\texttt{effects} :: \hat{\tau} \rightarrow \varepsilon$)

1. $\texttt{effects}(t) = \varnothing$
2. $\texttt{effects}(\{\bar{r}\}) = \{r.\pi \mid r \in \bar{r}, \pi \in \Pi\}$
3. $\texttt{effects}(\hat{\tau}_1 \rightarrow_{\varepsilon} \hat{\tau}_2) = \texttt{ho-effects}(\hat{\tau}_1) \cup \varepsilon \cup \texttt{effects}(\hat{\tau}_2)$

Definition ($\texttt{ho-effects} :: \hat{\tau} \rightarrow \varepsilon$)

1. $\texttt{ho-effects}(t) = \varnothing$
2. $\texttt{ho-effects}(\{\bar{r}\}) = \varnothing$
3. $\texttt{ho-effects}(\hat{\tau}_1 \rightarrow_{\varepsilon} \hat{\tau}_2) = \texttt{effects}(\hat{\tau}_1) \cup \texttt{ho-effects}(\hat{\tau}_2)$

Definition ($\texttt{substitution} :: \hat{\texttt{e}} \times \hat{\texttt{v}} \times \hat{\texttt{v}} \rightarrow \hat{\texttt{e}}$)

The notation $[\hat{v}/x]\hat{e}$ is short-hand for $\texttt{substitution}(\hat{e}, \hat{v}, x)$. This function is partial, because the third input must be a variable. We adopt the usual renaming conventions to avoid accidental capture.

1. $[\hat{v}/y]x = \hat{v}$, if $x = y$
2. $[\hat{v}/y]x = x$, if $x \neq y$
3. $[\hat{v}/y](\lambda x : \hat{\tau}.\hat{e}) = \lambda x : \hat{\tau}.[\hat{v}/y]\hat{e}$, if $y \neq x$ and y does not occur free in $\hat{e}$
4. $[\hat{v}/y](\hat{e}_1\ \hat{e}_2) = ([\hat{v}/y]\hat{e}_1)([\hat{v}/y]\hat{e}_2)$
5. $[\hat{v}/y](\hat{e}.\pi) = ([\hat{v}/y]\hat{e}).\pi$
6. $[\hat{v}/y](\forall t : \hat{\tau}.\hat{e}) = \forall t : \hat{\tau}.[\hat{v}/y]\hat{e}$, if $y \neq t$ and y does not occur free in $\hat{e}$
7. $[\hat{v}/y](\forall \epsilon : \varepsilon.\hat{e}) = \forall \epsilon : \varepsilon.[\hat{v}/y]\hat{e}$, if $y \neq \epsilon$ and y does not occur free in $\hat{e}$
8. $[\hat{v}/y](\texttt{import}(\varepsilon_s)\ x = \hat{e}\ \texttt{in}\ e) = \texttt{import}(\varepsilon_s)\ x = [\hat{v}/y]\hat{e}\ \texttt{in}\ e$

When performing multiple substitutions the notation $[\hat{v}_1/x_1, \hat{v}_2/x_2]\hat{e}$ is used as shorthand for $[\hat{v}_2/x_2]([\hat{v}_1/x_1]\hat{e})$ (note the order of the variables has been flipped; the substitutions occur as they are written, left-to-right).

3 Static Rules

$\boxed{\Gamma \vdash e : \tau}$

$$\frac{}{\Gamma, x : \tau \vdash x : \tau}\ (\text{T-Var}) \qquad \frac{}{\Gamma, r : \{r\} \vdash r : \{r\}}\ (\text{T-Resource}) \qquad \frac{\Gamma, x : \tau_1 \vdash e : \tau_2}{\Gamma \vdash \lambda x : \tau_1.e : \tau_1 \rightarrow \tau_2}\ (\text{T-Abs})$$

$$\frac{\Gamma \vdash e_1 : \tau_2 \rightarrow \tau_3 \quad \Gamma \vdash e_2 : \tau_2}{\Gamma \vdash e_1\ e_2 : \tau_3}\ (\text{T-App}) \qquad \frac{\Gamma \vdash e : \{\bar{r}\}}{\Gamma \vdash e.\pi : \texttt{Unit}}\ (\text{T-OperCall})$$

$\boxed{\hat{\Gamma} \vdash \hat{e} : \hat{\tau} \texttt{ with } \varepsilon}$

$$\frac{}{\hat{\Gamma}, x : \tau \vdash x : \tau \texttt{ with } \varnothing} \ (\varepsilon\text{-Var}) \qquad \frac{}{\hat{\Gamma}, r : \{r\} \vdash r : \{r\} \texttt{ with } \varnothing} \ (\varepsilon\text{-Resource})$$

$$\frac{\hat{\Gamma}, x : \hat{\tau}_2 \vdash \hat{e} : \hat{\tau}_3 \texttt{ with } \varepsilon_3}{\hat{\Gamma} \vdash \lambda x : \tau_2.\hat{e} : \hat{\tau}_2 \rightarrow_{\varepsilon_3} \hat{\tau}_3 \texttt{ with } \varnothing} \ (\varepsilon\text{-Abs})$$

$$\frac{\hat{\Gamma} \vdash \hat{e}_1 : \hat{\tau}_2 \rightarrow_{\varepsilon} \hat{\tau}_3 \texttt{ with } \varepsilon_1 \quad \hat{\Gamma} \vdash \hat{e}_2 : \hat{\tau}_2 \texttt{ with } \varepsilon_2}{\hat{\Gamma} \vdash \hat{e}_1 \hat{e}_2 : \hat{\tau}_3 \texttt{ with } \varepsilon_1 \cup \varepsilon_2 \cup \varepsilon} \ (\varepsilon\text{-App})$$

$$\frac{\hat{\Gamma} \vdash \hat{e} : \{\bar{r}\}}{\hat{\Gamma} \vdash \hat{e}.\pi : \texttt{Unit with } \{r.\pi \mid r \in \bar{r}\}} \ (\varepsilon\text{-OperCall}) \qquad \frac{\hat{\Gamma} \vdash e : \tau \texttt{ with } \varepsilon \quad \tau <: \tau' \quad \varepsilon \subseteq \varepsilon'}{\hat{\Gamma} \vdash e : \tau' \texttt{ with } \varepsilon'} \ (\varepsilon\text{-Subsume})$$

$$\frac{\begin{array}{c}\texttt{effects}(\hat{\tau}) \cup \texttt{ho-effects}(\texttt{annot}(\tau, \varnothing)) \subseteq \varepsilon \\ \hat{\Gamma} \vdash \hat{e} : \hat{\tau} \texttt{ with } \varepsilon_1 \qquad \texttt{ho-safe}(\hat{\tau}, \varepsilon) \qquad x : \texttt{erase}(\hat{\tau}) \vdash e : \tau\end{array}}{\hat{\Gamma} \vdash \texttt{import}(\varepsilon)\ x = \hat{e} \texttt{ in } e : \texttt{annot}(\tau, \varepsilon) \texttt{ with } \varepsilon \cup \varepsilon_1} \ (\varepsilon\text{-Import})$$

$\boxed{\texttt{safe}(\tau, \varepsilon)}$

$$\frac{}{\texttt{safe}(\{\bar{r}\}, \varepsilon)} \ (\text{Safe-Resource}) \qquad \frac{}{\texttt{safe}(\texttt{Unit}, \varepsilon)} \ (\text{Safe-Unit})$$

$$\frac{\varepsilon \subseteq \varepsilon' \quad \texttt{ho-safe}(\hat{\tau}_1, \varepsilon) \quad \texttt{safe}(\hat{\tau}_2, \varepsilon)}{\texttt{safe}(\hat{\tau}_1 \rightarrow_{\varepsilon'} \hat{\tau}_2, \varepsilon)} \ (\text{Safe-Arrow})$$

$\boxed{\texttt{ho-safe}(\hat{\tau}, \varepsilon)}$

$$\frac{}{\texttt{ho-safe}(\{\bar{r}\}, \varepsilon)} \ (\text{HOSafe-Resource}) \qquad \frac{}{\texttt{ho-safe}(\texttt{Unit}, \varepsilon)} \ (\text{HOSafe-Unit})$$

$$\frac{\texttt{safe}(\hat{\tau}_1, \varepsilon) \quad \texttt{ho-safe}(\hat{\tau}_2, \varepsilon)}{\texttt{ho-safe}(\hat{\tau}_1 \rightarrow_{\varepsilon'} \hat{\tau}_2, \varepsilon)} \ (\text{HOSafe-Arrow})$$

$\boxed{\hat{\tau} <: \hat{\tau}}$

$$\frac{\varepsilon \subseteq \varepsilon' \quad \hat{\tau}_2 <: \hat{\tau}_2' \quad \hat{\tau}_1' <: \hat{\tau}_1}{\hat{\tau}_1 \rightarrow_{\varepsilon} \hat{\tau}_2 <: \hat{\tau}_1' \rightarrow_{\varepsilon'} \hat{\tau}_2'} \ (\text{S-Effects}) \qquad \frac{r \in \bar{r}_2 \implies r \in \bar{r}_1}{\{\bar{r}_2\} <: \{\bar{r}_1\}} \ (\text{S-ResourceSet})$$

4 Dynamic Rules

$\boxed{\hat{e} \longrightarrow \hat{e} \mid \varepsilon}$

$$\frac{\hat{e}_1 \longrightarrow \hat{e}_1' \mid \varepsilon}{\hat{e}_1\hat{e}_2 \longrightarrow \hat{e}_1'\hat{e}_2 \mid \varepsilon} \text{ (E-APP1)} \qquad \frac{\hat{e}_2 \longrightarrow \hat{e}_2' \mid \varepsilon}{\hat{v}_1\hat{e}_2 \longrightarrow \hat{v}_1\hat{e}_2' \mid \varepsilon} \text{ (E-APP2)} \qquad \frac{}{(\lambda x : \hat{\tau}.\hat{e})\hat{v}_2 \longrightarrow [\hat{v}_2/x]\hat{e} \mid \varnothing} \text{ (E-APP3)}$$

$$\frac{\hat{e} \to \hat{e}' \mid \varepsilon}{\hat{e}.\pi \longrightarrow \hat{e}'.\pi \mid \varepsilon} \text{ (E-OPERCALL1)} \qquad \frac{r \in R \quad \pi \in \Pi}{r.\pi \longrightarrow \texttt{unit} \mid \{r.\pi\}} \text{ (E-OPERCALL2)}$$

$$\frac{}{\forall t : \tau.e \longrightarrow [\tau/t]e} \text{ (E-TYPEPOLY)} \qquad \frac{}{\forall \epsilon : \varepsilon.e \longrightarrow [\varepsilon/\epsilon]e} \text{ (E-FXPOLY)}$$

$$\frac{\hat{e} \longrightarrow \hat{e}' \mid \varepsilon'}{\texttt{import}(\varepsilon_s)\ x = \hat{e}\ \texttt{in}\ e \longrightarrow \texttt{import}(\varepsilon_s)\ x = \hat{e}'\ \texttt{in}\ e \mid \varepsilon'} \text{ (E-IMPORT1)}$$

$$\frac{}{\texttt{import}(\varepsilon_s)\ x = \hat{e}\ \texttt{in}\ e \longrightarrow [\hat{v}/x]\texttt{annot}(e, \varepsilon_s) \mid \varnothing} \text{ (E-IMPORT2)}$$

5 Encodings

5.1 $\perp$

The bottom type is defined as $\perp \stackrel{\text{def}}{=} \varnothing$, which is the literal for an empty set of resources.

$$\frac{}{\Gamma \vdash \perp : \varnothing} \text{ (T-}\perp\text{)} \qquad \frac{}{\Gamma \vdash \perp : \varnothing\ \texttt{with}\ \varnothing} \text{ (}\varepsilon\text{-}\perp\text{)}$$

5.2 `unit`, `Unit`

Define `unit` $= \lambda\texttt{x} : \varnothing.\texttt{x}$, i.e. the function which takes an empty set of resources and returns it. We shall refer to its type, which is $\varnothing \to_{\varnothing} \varnothing$, as `Unit`. It has various properties befitting `unit`.

1. `unit` cannot be invoked as $\varnothing$ is uninhabited.
2. `unit` is a value.
3. The only term with type `Unit` is `unit`.
4. $\vdash$ `unit` : `Unit` by using ε-ABS and ε-VAR.
5. `effects`(`Unit`) = `ho-effects`(`Unit`) = $\varnothing$
6. `safe`(`Unit`, ε) and `ho-safe`(`Unit`, ε)

$$\frac{}{\Gamma \vdash \texttt{unit} : \texttt{Unit}} \text{ (T-UNIT)} \qquad \frac{}{\Gamma \vdash \texttt{unit} : \texttt{Unit with } \varnothing} \text{ (}\varepsilon\text{-UNIT)}$$