

1 Grammar

$e ::=$	exprs.	$\tau ::=$	types
x	<i>variable</i>	X	<i>type variable</i>
v	<i>value</i>	$\{\bar{r}\}$	<i>effect set</i>
$e.\pi$	<i>operation call</i>	$\tau \rightarrow \tau$	<i>arrow</i>
$e e$	<i>application</i>	$\forall X <: \tau.\tau$	<i>universal type</i>
$e \tau$	<i>type application</i>		
$v ::=$	values	$\hat{\tau} ::=$	annotated types
r	<i>resource literal</i>	X	<i>type variable</i>
$\lambda x : \tau.e$	<i>abstraction</i>	$\{\bar{r}\}$	<i>resource set</i>
$\lambda X <: \tau.e$	<i>type polymorphism</i>	$\hat{\tau} \rightarrow_{\varepsilon} \hat{\tau}$	<i>annotated arrow</i>
		$\forall X <: \hat{\tau}.\hat{\tau} \text{ caps } \varepsilon$	<i>universal type</i>
		$\forall \phi \subseteq \varepsilon.\hat{\tau} \text{ caps } \varepsilon$	<i>universal effect set</i>
$\hat{e} ::=$	annotated exprs.	$\varepsilon ::=$	effects
x	<i>variable</i>	ϕ	<i>effect variable</i>
\hat{v}	<i>value</i>	$\{\bar{r}.\pi\}$	<i>effect set</i>
$\hat{e}.\pi$	<i>operation call</i>		
$\hat{e} \hat{e}$	<i>application</i>	$\Gamma ::=$	contexts
$e \hat{\tau}$	<i>type application</i>	\emptyset	<i>empty ctx.</i>
$e \varepsilon$	<i>effect application</i>	$\Gamma, x : \tau$	<i>var. binding</i>
$\text{import}(\varepsilon_s) x = \hat{e} \text{ in } e$	<i>import</i>	$\Gamma, X <: \tau$	<i>type var. binding</i>
$\hat{v} ::=$	annotated values	$\hat{\Gamma} ::=$	annotated contexts
r	<i>resource literal</i>	\emptyset	<i>empty ctx.</i>
$\lambda x : \hat{\tau}.\hat{e}$	<i>abstraction</i>	$\hat{\Gamma}, x : \hat{\tau}$	<i>var. binding</i>
$\lambda X <: \hat{\tau}.\hat{e}$	<i>type polymorphism</i>	$\hat{\Gamma}, X <: \hat{\tau}$	<i>type var. binding</i>
$\lambda \phi \subseteq \varepsilon.\hat{e}$	<i>effect polymorphism</i>	$\hat{\Gamma}, \phi \subseteq \varepsilon$	<i>effect var. binding</i>

2 Functions

Definition ($\text{annot} :: \tau \times \varepsilon \rightarrow \hat{\tau}$)

1. $\text{annot}(X, _) = X$
2. $\text{annot}(\{\bar{r}\}, _) = \{\bar{r}\}$
3. $\text{annot}(\tau_1 \rightarrow \tau_2, \varepsilon) = \text{annot}(\tau_1, \varepsilon) \rightarrow_{\varepsilon} \text{annot}(\tau_2, \varepsilon)$
4. $\text{annot}(\forall X <: \tau_1.\tau_2, \varepsilon) = \forall X <: \text{annot}(\tau_1, \varepsilon).\text{annot}(\tau_2, \varepsilon) \text{ caps } \varepsilon$

Definition ($\text{annot} :: e \times \varepsilon \rightarrow \hat{e}$)

1. $\text{annot}(x, _) = x$
2. $\text{annot}(r, _) = r$
3. $\text{annot}(\lambda x : \tau.e, \varepsilon) = \lambda x : \text{annot}(\tau, \varepsilon).\text{annot}(e, \varepsilon)$
4. $\text{annot}(e_1 e_2, \varepsilon) = \text{annot}(e_1, \varepsilon) \text{ annot}(e_2, \varepsilon)$
5. $\text{annot}(e.\pi, \varepsilon) = \text{annot}(e, \varepsilon).\pi$
6. $\text{annot}(\lambda X <: \tau_1.e, \varepsilon) = \lambda X <: \text{annot}(\tau_1, \varepsilon).\text{annot}(e, \varepsilon)$
7. $\text{annot}(e \tau, \varepsilon) = \text{annot}(e, \varepsilon) \text{ annot}(\tau, \varepsilon)$

Definition ($\text{annot} :: \Gamma \times \varepsilon \rightarrow \hat{\Gamma}$)

1. $\text{annot}(\emptyset, _) = \emptyset$
2. $\text{annot}((\Gamma, x : \tau), \varepsilon) = \text{annot}(\Gamma, \varepsilon), x : \text{annot}(\tau, \varepsilon)$
3. $\text{annot}((\Gamma, X <: \tau), \varepsilon) = \text{annot}(\Gamma, \varepsilon), X <: \text{annot}(\tau, \varepsilon)$

Definition ($\text{erase} :: \hat{\tau} \rightarrow \tau$)

1. $\text{erase}(X) = X$
2. $\text{erase}(\{\bar{r}\}) = \{\bar{r}\}$
3. $\text{erase}(\hat{\tau}_1 \rightarrow_{\varepsilon} \hat{\tau}_2) = \text{erase}(\hat{\tau}_1) \rightarrow \text{erase}(\hat{\tau}_2)$
4. $\text{erase}(\forall X <: \hat{\tau}_1. \hat{\tau}_2 \text{ caps } \varepsilon) = \forall X <: \text{erase}(\hat{\tau}_1). \text{erase}(\hat{\tau}_2)$

Definition ($\text{erase} :: \hat{e} \rightarrow e$)

1. $\text{erase}(x) = x$
2. $\text{erase}(r) = r$
3. $\text{erase}(\lambda x : \hat{\tau}. \hat{e}) = \lambda x : \text{erase}(\hat{\tau}). \text{erase}(\hat{e})$
4. $\text{erase}(\hat{e}_1 \ \hat{e}_2) = \text{erase}(\hat{e}_1) \text{erase}(\hat{e}_2)$
5. $\text{erase}(\hat{e}. \pi) = \text{erase}(\hat{e}). \pi$
6. $\text{erase}(\lambda X <: \hat{\tau}. \hat{e}) = \lambda X <: \text{erase}(\hat{\tau}). \text{erase}(\hat{e})$

Definition ($\text{erase} :: \hat{\Gamma} \rightarrow \Gamma$)

1. $\text{erase}(\emptyset) = \emptyset$
2. $\text{erase}(\hat{\Gamma}, x : \hat{\tau}) = \text{erase}(\hat{\Gamma}), x : \text{erase}(\hat{\tau})$
3. $\text{erase}(\hat{\Gamma}, X <: \hat{\tau}) = \text{erase}(\hat{\Gamma}), X <: \text{erase}(\hat{\tau})$

Definition ($\text{effects} :: \hat{\tau} \rightarrow \varepsilon$)

1. $\text{effects}(X) = \emptyset$
2. $\text{effects}(\{\bar{r}\}) = \{r. \pi \mid r \in \bar{r}, \pi \in \Pi\}$
3. $\text{effects}(\hat{\tau}_1 \rightarrow_{\varepsilon} \hat{\tau}_2) = \text{ho-effects}(\hat{\tau}_1) \cup \varepsilon \cup \text{effects}(\hat{\tau}_2)$
4. $\text{effects}(\forall X <: \hat{\tau}_1. \hat{\tau}_2 \text{ caps } \varepsilon_1) = \text{ho-effects}(\hat{\tau}_1) \cup \text{effects}([\hat{\tau}_1/X]\hat{\tau}_2) \cup \varepsilon_1$
5. $\text{effects}(\forall \phi \subseteq \varepsilon. \hat{\tau} \text{ caps } \varepsilon_1) = \text{effects}([\varepsilon/\phi]\hat{\tau}) \cup \varepsilon_1$

Definition ($\text{ho-effects} :: \hat{\tau} \rightarrow \varepsilon$)

1. $\text{ho-effects}(X) = \emptyset$
2. $\text{ho-effects}(\{\bar{r}\}) = \emptyset$
3. $\text{ho-effects}(\hat{\tau}_1 \rightarrow_{\varepsilon} \hat{\tau}_2) = \text{effects}(\hat{\tau}_1) \cup \text{ho-effects}(\hat{\tau}_2)$
4. $\text{ho-effects}(\forall X <: \hat{\tau}_1. \hat{\tau}_2 \text{ caps } \varepsilon) = \text{effects}(\hat{\tau}_1) \cup \text{ho-effects}([\hat{\tau}_1/X]\hat{\tau}_2)$
5. $\text{ho-effects}(\forall \phi \subseteq \varepsilon. \hat{\tau} \text{ caps } \varepsilon_1) = \varepsilon \cup \text{ho-effects}([\varepsilon/\phi]\hat{\tau})$

3 Static Rules

$\boxed{\Gamma \vdash e : \tau}$

$$\begin{array}{c}
\overline{\Gamma, x : \tau \vdash x : \tau} \text{ (T-VAR)} \quad \overline{\Gamma, r : \{\bar{r}\} \vdash r : \{\bar{r}\}} \text{ (T-RESOURCE)} \quad \frac{\Gamma \vdash e : \{\bar{r}\}}{\Gamma \vdash e. \pi : \mathbf{Unit}} \text{ (T-OPERCALL)} \\
\\
\frac{\Gamma, x : \tau_1 \vdash e : \tau_2}{\Gamma \vdash \lambda x : \tau_1. e : \tau_1 \rightarrow \tau_2} \text{ (T-ABS)} \quad \frac{\Gamma \vdash e_1 : \tau_2 \rightarrow \tau_3 \quad \Gamma \vdash e_2 : \tau_2}{\Gamma \vdash e_1 \ e_2 : \tau_3} \text{ (T-APP)} \\
\\
\frac{\Gamma, X <: \tau_1 \vdash e : \tau_2}{\Gamma \vdash \lambda X <: \tau_1. e : \forall X <: \tau_1. \tau_2} \text{ (T-POLYTYPEABS)} \quad \frac{\Gamma \vdash e : \forall X <: \tau_1. \tau_2 \quad \tau' <: \tau_1}{\Gamma \vdash e \ \tau' : [\tau'/X]\tau_2} \text{ (T-POLYTYPEAPP)}
\end{array}$$

$\boxed{\hat{\Gamma} \vdash \hat{e} : \hat{\tau} \text{ with } \varepsilon}$

$$\begin{array}{c}
\frac{}{\hat{I}, x : \tau \vdash x : \tau \text{ with } \emptyset} (\varepsilon\text{-VAR}) \quad \frac{}{\hat{I}, r : \{r\} \vdash r : \{r\} \text{ with } \emptyset} (\varepsilon\text{-RESOURCE}) \\
\\
\frac{\hat{I} \vdash \hat{e} : \{\bar{r}\} \text{ with } \varepsilon_1}{\hat{I} \vdash \hat{e}.\pi : \text{Unit with } \varepsilon_1 \cup \{r.\pi \mid r \in \bar{r}\}} (\varepsilon\text{-OPERCALL}) \quad \frac{\hat{I} \vdash e : \tau \text{ with } \varepsilon \quad \tau <: \tau' \quad \varepsilon \subseteq \varepsilon'}{\hat{I} \vdash e : \tau' \text{ with } \varepsilon'} (\varepsilon\text{-SUBSUME}) \\
\\
\frac{\hat{I}, x : \hat{\tau}_2 \vdash \hat{e} : \hat{\tau}_3 \text{ with } \varepsilon_3}{\hat{I} \vdash \lambda x : \tau_2. \hat{e} : \hat{\tau}_2 \rightarrow_{\varepsilon_3} \hat{\tau}_3 \text{ with } \emptyset} (\varepsilon\text{-ABS}) \quad \frac{\hat{I} \vdash \hat{e}_1 : \hat{\tau}_2 \rightarrow_{\varepsilon} \hat{\tau}_3 \text{ with } \varepsilon_1 \quad \hat{I} \vdash \hat{e}_2 : \hat{\tau}_2 \text{ with } \varepsilon_2}{\hat{I} \vdash \hat{e}_1 \hat{e}_2 : \hat{\tau}_3 \text{ with } \varepsilon_1 \cup \varepsilon_2 \cup \varepsilon} (\varepsilon\text{-APP}) \\
\\
\frac{\hat{I}, X <: \hat{\tau}_1 \vdash \hat{e} : \hat{\tau}_2 \text{ with } \varepsilon_1}{\hat{I} \vdash \lambda X <: \hat{\tau}_1. \hat{e} : \forall X <: \hat{\tau}_1. \hat{\tau}_2 \text{ caps } \varepsilon_1 \text{ with } \emptyset} (\varepsilon\text{-POLYTYPEABS}) \\
\\
\frac{\hat{I}, \phi \subseteq \varepsilon \vdash \hat{e} : \hat{\tau} \text{ with } \varepsilon_1}{\hat{I} \vdash \lambda \phi \subseteq \varepsilon. \hat{e} : \forall \phi \subseteq \varepsilon. \hat{\tau} \text{ caps } \varepsilon_1 \text{ with } \emptyset} (\varepsilon\text{-POLYFXABS}) \\
\\
\frac{\forall X <: \hat{\tau}_1. \hat{\tau}_2 \text{ caps } \varepsilon_1 \text{ with } \varepsilon_2 \quad \hat{\tau}' <: \hat{\tau}_1}{\hat{I} \vdash \hat{e} \hat{\tau}' : [\hat{\tau}'/X] \hat{\tau}_2 \text{ with } [\hat{\tau}'/X]_{\varepsilon_1} \cup \varepsilon_2} (\varepsilon\text{-POLYTYPEAPP}) \\
\\
\frac{\hat{I} \vdash \hat{e} : \forall \phi \subseteq \varepsilon. \hat{\tau} \text{ caps } \varepsilon_1 \text{ with } \varepsilon_2 \quad \varepsilon' \subseteq \varepsilon}{\hat{I} \vdash \hat{e} \varepsilon : [\varepsilon'/\phi] \hat{\tau} \text{ with } [\varepsilon'/\phi]_{\varepsilon_1} \cup \varepsilon_2} (\varepsilon\text{-POLYFXAPP}) \\
\\
\frac{\text{effects}(\hat{\tau}) \cup \text{ho-effects}(\text{annot}(\tau, \emptyset)) \subseteq \varepsilon \quad \hat{I} \vdash \hat{e} : \hat{\tau} \text{ with } \varepsilon_1 \quad \text{ho-safe}(\hat{\tau}, \varepsilon) \quad x : \text{erase}(\hat{\tau}) \vdash e : \tau}{\hat{I} \vdash \text{import}(\varepsilon) \ x = \hat{e} \text{ in } e : \text{annot}(\tau, \varepsilon) \text{ with } \varepsilon \cup \varepsilon_1} (\varepsilon\text{-IMPORT})
\end{array}$$

safe(τ, ε)

$$\begin{array}{c}
\frac{}{\text{safe}(\{\bar{r}\}, \varepsilon)} (\text{SAFE-RESOURCE}) \quad \frac{}{\text{safe}(\text{Unit}, \varepsilon)} (\text{SAFE-UNIT}) \\
\\
\frac{\varepsilon \subseteq \varepsilon' \quad \text{ho-safe}(\hat{\tau}_1, \varepsilon) \quad \text{safe}(\hat{\tau}_2, \varepsilon)}{\text{safe}(\hat{\tau}_1 \rightarrow_{\varepsilon'} \hat{\tau}_2, \varepsilon)} (\text{SAFE-ARROW}) \\
\\
\frac{\varepsilon_1 \subseteq \varepsilon \quad \text{safe}([\varepsilon_1/\phi] \hat{\tau}, \varepsilon)}{\text{safe}(\forall \phi \subseteq \varepsilon_1. \hat{\tau} \text{ caps } \varepsilon_2, \varepsilon)} (\text{SAFE-POLYFX}) \quad \frac{\text{ho-safe}(\hat{\tau}_1, \varepsilon) \quad \text{safe}([\hat{\tau}_1/X] \hat{\tau}_2, \varepsilon) \quad \varepsilon \subseteq \varepsilon_1}{\text{safe}(\forall X <: \hat{\tau}_1. \hat{\tau}_2 \text{ caps } \varepsilon_2, \varepsilon)} (\text{SAFE-POLYTYPE})
\end{array}$$

ho-safe($\hat{\tau}, \varepsilon$)

$$\begin{array}{c}
\frac{}{\text{ho-safe}(\{\bar{r}\}, \varepsilon)} (\text{HOSAFE-RESOURCE}) \quad \frac{}{\text{ho-safe}(\text{Unit}, \varepsilon)} (\text{HOSAFE-UNIT}) \\
\\
\frac{\text{safe}(\hat{\tau}_1, \varepsilon) \quad \text{ho-safe}(\hat{\tau}_2, \varepsilon)}{\text{ho-safe}(\hat{\tau}_1 \rightarrow_{\varepsilon'} \hat{\tau}_2, \varepsilon)} (\text{HOSAFE-ARROW}) \\
\\
\frac{\varepsilon_1 \subseteq \varepsilon \quad \text{safe}([\varepsilon_1/\phi] \hat{\tau}, \varepsilon)}{\text{ho-safe}(\forall \phi \subseteq \varepsilon_1. \hat{\tau} \text{ caps } \varepsilon_2, \varepsilon)} (\text{HOSAFE-POLYFX}) \quad \frac{\text{safe}(\hat{\tau}_1, \varepsilon) \quad \text{ho-safe}([\hat{\tau}_1/X] \hat{\tau}_2, \varepsilon)}{\text{ho-safe}(\forall X <: \hat{\tau}_1. \hat{\tau}_2 \text{ caps } \varepsilon_2, \varepsilon)} (\text{HOSAFE-POLYTYPE})
\end{array}$$

$\hat{I} \vdash \hat{\tau} <: \hat{\tau}$

$$\begin{array}{c}
\frac{}{\hat{I} \vdash \hat{\tau} <: \hat{\tau}} \text{ (S-REFLEXIVE)} \quad \frac{\hat{I} \vdash \hat{\tau}_1 <: \hat{\tau}_2 \quad \hat{I} \vdash \hat{\tau}_2 <: \hat{\tau}_3}{\hat{I} \vdash \hat{\tau}_1 <: \hat{\tau}_3} \text{ (S-TRANSITIVE)} \\
\\
\frac{r \in \overline{r_1} \implies r \in \overline{r_2}}{\hat{I} \vdash \{\overline{r_1}\} <: \{\overline{r_2}\}} \text{ (S-RESOURCESET)} \quad \frac{\hat{I} \vdash \hat{\tau}'_1 <: \hat{\tau}_1 \quad \hat{I} \vdash \hat{\tau}_2 <: \hat{\tau}'_2 \quad \varepsilon \subseteq \varepsilon'}{\hat{I} \vdash \tau_1 \rightarrow_\varepsilon \tau_2 <: \tau'_1 \rightarrow_{\varepsilon'} \tau'_2} \text{ (S-ARROW)} \\
\\
\frac{\hat{I} \vdash \hat{\tau}'_1 <: \hat{\tau}_1 \quad \hat{I}, Y <: \hat{\tau}'_1 \vdash \hat{\tau}_2 <: \hat{\tau}'_2}{\hat{I} \vdash (\forall X <: \hat{\tau}_1. \hat{\tau}_2) <: (\forall Y <: \hat{\tau}'_1. \hat{\tau}'_2)} \text{ (S-TYPEPOLY)} \quad \frac{}{\hat{I}, X <: \hat{\tau} \vdash X <: \hat{\tau}} \text{ (S-TYPEVAR)}
\end{array}$$

4 Dynamic Rules

$$\boxed{\hat{e} \longrightarrow \hat{e} \mid \varepsilon}$$

$$\begin{array}{c}
\frac{\hat{e}_1 \longrightarrow \hat{e}'_1 \mid \varepsilon}{\hat{e}_1 \hat{e}_2 \longrightarrow \hat{e}'_1 \hat{e}_2 \mid \varepsilon} \text{ (E-APP1)} \quad \frac{\hat{e}_2 \longrightarrow \hat{e}'_2 \mid \varepsilon}{\hat{v}_1 \hat{e}_2 \longrightarrow \hat{v}_1 \hat{e}'_2 \mid \varepsilon} \text{ (E-APP2)} \quad \frac{}{(\lambda x : \hat{\tau}. \hat{e}) \hat{v}_2 \longrightarrow [\hat{v}_2/x] \hat{e} \mid \emptyset} \text{ (E-APP3)} \\
\\
\frac{\hat{e} \rightarrow \hat{e}' \mid \varepsilon}{\hat{e}. \pi \longrightarrow \hat{e}'. \pi \mid \varepsilon} \text{ (E-OPERCALL1)} \quad \frac{r \in R \quad \pi \in \Pi}{r. \pi \longrightarrow \mathbf{unit} \mid \{r. \pi\}} \text{ (E-OPERCALL2)} \\
\\
\frac{\hat{e} \longrightarrow \hat{e}' \mid \varepsilon}{\hat{e} \hat{\tau} \longrightarrow \hat{e}' \hat{\tau} \mid \varepsilon} \text{ (E-POLYTYPEAPP1)} \quad \frac{}{(\lambda X. \hat{e}) \hat{\tau} \longrightarrow [\hat{\tau}/X] \hat{e} \mid \emptyset} \text{ (E-POLYTYPEAPP2)} \\
\\
\frac{\hat{e} \longrightarrow \hat{e}' \mid \varepsilon}{\hat{e} \hat{\tau} \longrightarrow \hat{e}' \hat{\tau} \mid \varepsilon} \text{ (E-POLYFXAPP1)} \quad \frac{}{(\lambda \phi. \hat{e}) \varepsilon \longrightarrow [\varepsilon/\phi] \hat{e} \mid \emptyset} \text{ (E-POLYFXAPP2)} \\
\\
\frac{\hat{e} \longrightarrow \hat{e}' \mid \varepsilon'}{\mathbf{import}(\varepsilon_s) \ x = \hat{e} \ \mathbf{in} \ e \longrightarrow \mathbf{import}(\varepsilon_s) \ x = \hat{e}' \ \mathbf{in} \ e \mid \varepsilon'} \text{ (E-IMPORT1)} \\
\\
\frac{}{\mathbf{import}(\varepsilon_s) \ x = \hat{e} \ \mathbf{in} \ e \longrightarrow [\hat{v}/x] \mathbf{annot}(e, \varepsilon_s) \mid \emptyset} \text{ (E-IMPORT2)}
\end{array}$$