1 Grammar

$e ::=$ **exprs.**
| x *variable*
| v *value*
| $e.\pi$ *operation call*
| $e\ e$ *application*
| $e\ \tau$ *type application*

$v ::=$ **values**
| r *resource literal*
| $\lambda x : \tau.e$ *abstraction*
| $\lambda X <: \tau.e$ *type polymorphism*

$\hat{e} ::=$ **annotated exprs.**
| x *variable*
| $\hat{v}$ *value*
| $\hat{e}.\pi$ *operation call*
| $\hat{e}\ \hat{e}$ *application*
| $e\ \hat{\tau}$ *type application*
| $e\ \varepsilon$ *effect application*
| $\texttt{import}(\varepsilon_s)\ \overline{x = \hat{e}}\ \texttt{in}\ e$

$\hat{v} ::=$ **annotated values**
| r *resource literal*
| $\lambda x : \hat{\tau}.\hat{e}$ *abstraction*
| $\lambda X <: \hat{\tau}.\hat{e}$ *type polymorphism*
| $\lambda \phi \subseteq \varepsilon.\hat{e}$ *effect polymorphism*

$\tau ::=$ **types**
| X *type variable*
| $\{\bar{r}\}$ *effect set*
| $\tau \to \tau$ *arrow*
| $\forall X <: \tau.\tau$ *universal type*

$\hat{\tau} ::=$ **annotated types**
| X *type variable*
| $\{\bar{r}\}$ *resource set*
| $\hat{\tau} \to_\varepsilon \hat{\tau}$ *annotated arrow*
| $\forall X <: \hat{\tau}.\hat{\tau}\ \texttt{caps}\ \varepsilon$ *universal type*
| $\forall \phi \subseteq \varepsilon.\hat{\tau}\ \texttt{caps}\ \varepsilon$ *universal effect set*

$\varepsilon ::=$ **effects**
| ϕ *effect variable*
| $\{\overline{r.\pi}\}$ *effect set*

$\Gamma ::=$ **contexts**
| $\varnothing$ *empty ctx.*
| $\Gamma, x : \tau$ *var. binding*
| $\Gamma, X <: \tau$ *type var. binding*

$\hat{\Gamma} ::=$ **annotated contexts**
| $\varnothing$ *empty ctx.*
| $\hat{\Gamma}, x : \hat{\tau}$ *var. binding*
| $\hat{\Gamma}, X <: \hat{\tau}$ *type var. binding*
| $\hat{\Gamma}, \phi \subseteq \varepsilon$ *effect var. binding*

2 Functions

Definition ($\texttt{annot} :: \tau \times \varepsilon \to \hat{\tau}$)

1. $\texttt{annot}(X, \_) = X$
2. $\texttt{annot}(\{\bar{r}\}, \_) = \{\bar{r}\}$
3. $\texttt{annot}(\tau_1 \to \tau_2, \varepsilon) = \texttt{annot}(\tau_1, \varepsilon) \to_\varepsilon \texttt{annot}(\tau_2, \varepsilon)$
4. $\texttt{annot}(\forall X <: \tau_1.\tau_2, \varepsilon) = \forall X <: \texttt{annot}(\tau_1, \varepsilon).\texttt{annot}(\tau_2, \varepsilon)\ \texttt{caps}\ \varepsilon$

Definition ($\texttt{annot} :: e \times \varepsilon \to \hat{e}$)

1. $\texttt{annot}(x, \_) = e$
2. $\texttt{annot}(r, \_) = r$
3. $\texttt{annot}(\lambda x : \tau.e, \varepsilon) = \lambda x : \texttt{annot}(\tau, \varepsilon).\texttt{annot}(e, \varepsilon)$
4. $\texttt{annot}(e_1\ e_2, \varepsilon) = \texttt{annot}(e_1)\ \texttt{annot}(e_2)$
5. $\texttt{annot}(e.\pi, \varepsilon) = \texttt{annot}(e, \varepsilon).\pi$
6. $\texttt{annot}(\lambda X <: \tau_1.e, \varepsilon) = \lambda X <: \texttt{annot}(\tau_1, \varepsilon).\texttt{annot}(e, \varepsilon)$
7. $\texttt{annot}(e\ \tau, \varepsilon) = \texttt{annot}(e, \varepsilon)\ \texttt{annot}(\tau, \varepsilon)$

Definition ($\texttt{annot} :: \Gamma \times \varepsilon \to \hat{\Gamma}$)

1. $\texttt{annot}(\varnothing, \_) = \varnothing$
2. $\texttt{annot}((\Gamma, x : \tau), \varepsilon) = \texttt{annot}(\Gamma, \varepsilon), x : \texttt{annot}(\tau, \varepsilon)$
3. $\texttt{annot}((\Gamma, X <: \tau), \varepsilon) = \texttt{annot}(\Gamma, \varepsilon), X <: \texttt{annot}(\tau, \varepsilon)$

Definition (erase :: $\hat{\tau} \to \tau$)

1. $\texttt{erase}(X) = X$
2. $\texttt{erase}(\{\bar{r}\}) = \{\bar{r}\}$
3. $\texttt{erase}(\hat{\tau}_1 \to_\varepsilon \hat{\tau}_2) = \texttt{erase}(\hat{\tau}_1) \to \texttt{erase}(\hat{\tau}_2)$
4. $\texttt{erase}(\forall X <: \hat{\tau}_1.\hat{\tau}_2 \texttt{ caps } \varepsilon) = \forall X <: \texttt{erase}(\hat{\tau}_1).\texttt{erase}(\hat{\tau}_2)$

Definition (erase :: $\hat{e} \to e$)

1. $\texttt{erase}(x) = x$
2. $\texttt{erase}(r) = r$
3. $\texttt{erase}(\lambda x : \hat{\tau}.\hat{e}) = \lambda x : \texttt{erase}(\hat{\tau}).\texttt{erase}(\hat{e})$
4. $\texttt{erase}(\hat{e}_1\ \hat{e}_2) = \texttt{erase}(\hat{e}_1)\texttt{erase}(\hat{e}_2)$
5. $\texttt{erase}(\hat{e}.\pi) = \texttt{erase}(\hat{e}).\pi$
6. $\texttt{erase}(\lambda X <: \hat{\tau}.\hat{e}) = \lambda X <: \texttt{erase}(\hat{\tau}).\texttt{erase}(\hat{e})$

Definition (erase :: $\hat{\Gamma} \to \Gamma$)

1. $\texttt{erase}(\varnothing) = \varnothing$
2. $\texttt{erase}(\hat{\Gamma}, x : \hat{\tau}) = \texttt{erase}(\hat{\Gamma}), x : \texttt{erase}(\hat{\tau})$
3. $\texttt{erase}(\hat{\Gamma}, X <: \hat{\tau}) = \texttt{erase}(\hat{\Gamma}), X <: \texttt{erase}(\hat{\tau})$

Definition (effects :: $\hat{\tau} \to \varepsilon$)

1. $\texttt{effects}(X) = \varnothing$
2. $\texttt{effects}(\{\bar{r}\}) = \{r.\pi \mid r \in \bar{r}, \pi \in \Pi\}$
3. $\texttt{effects}(\hat{\tau}_1 \to_\varepsilon \hat{\tau}_2) = \texttt{ho-effects}(\hat{\tau}_1) \cup \varepsilon \cup \texttt{effects}(\hat{\tau}_2)$
4. $\texttt{effects}(\forall X <: \hat{\tau}_1.\hat{\tau}_2 \texttt{ caps } \varepsilon_1) = \texttt{ho-effects}(\hat{\tau}_1) \cup \texttt{effects}([\hat{\tau}_1/X]\hat{\tau}_2) \cup \varepsilon_1$
5. $\texttt{effects}(\forall \phi \subseteq \varepsilon.\hat{\tau} \texttt{ caps } \varepsilon_1) = \texttt{effects}([\varepsilon/\phi]\hat{\tau}) \cup \varepsilon_1$

Defintion (effects :: $\hat{\tau} \times \overline{\hat{\tau}} \to \varepsilon$)

1. $\texttt{effects}(\hat{\tau}, \overline{\hat{\tau}}) = \texttt{effects}(\hat{\tau}) \cap \bigcup_i \texttt{effects}(\hat{\tau}_i)$, if $\hat{\tau}$ is polymorphic.
2. $\texttt{effects}(\hat{\tau}, \_) = \texttt{effects}(\hat{\tau})$, otherwise.

Note: the definitions given for non-polymorphic types could also be refined to give a more precise upper-bound on the effects they capture. This definition is also probably deficient when there is another polymorphic function in scope.

Definition (ho-effects :: $\hat{\tau} \to \varepsilon$)

1. $\texttt{ho-effects}(X) = \varnothing$
2. $\texttt{ho-effects}(\{\bar{r}\}) = \varnothing$
3. $\texttt{ho-effects}(\hat{\tau}_1 \to_\varepsilon \hat{\tau}_2) = \texttt{effects}(\hat{\tau}_1) \cup \texttt{ho-effects}(\hat{\tau}_2)$
4. $\texttt{ho-effects}(\forall X <: \hat{\tau}_1.\hat{\tau}_2 \texttt{ caps } \varepsilon) = \texttt{effects}(\hat{\tau}_1) \cup \texttt{ho-effects}([\hat{\tau}_1/X]\hat{\tau}_2)$
5. $\texttt{ho-effects}(\forall \phi \subseteq \varepsilon.\hat{\tau} \texttt{ caps } \varepsilon_1) = \varepsilon \cup \texttt{ho-effects}([\varepsilon/\phi]\hat{\tau})$

Definition (ho-effects :: $\hat{\tau} \times \overline{\hat{\tau}} \to \varepsilon$)

1. $\texttt{ho-effects}(\hat{\tau}, \overline{\hat{\tau}}) = \texttt{ho-effects}(\hat{\tau}) \cap \bigcup_i \texttt{ho-effects}(\hat{\tau}_i)$, if $\hat{\tau}$ is polymorphic.
2. $\texttt{ho-effects}(\hat{\tau}, \_) = \texttt{ho-effects}(\hat{\tau})$, otherwise.

Note: the definitions given for non-polymorphic types could also be refined to give a more precise upper-bound on the effects they capture. This definition is also probably deficient when there is another polymorphic function in scope.

3   Static Rules

$\boxed{\Gamma \vdash e : \tau}$

$$\frac{}{\Gamma, x : \tau \vdash x : \tau} \; \text{(T-Var)} \quad \frac{}{\Gamma, r : \{r\} \vdash r : \{r\}} \; \text{(T-Resource)} \quad \frac{\Gamma \vdash e : \{\bar{r}\}}{\Gamma \vdash e.\pi : \texttt{Unit}} \; \text{(T-OperCall)}$$

$$\frac{\Gamma, x : \tau_1 \vdash e : \tau_2}{\Gamma \vdash \lambda x : \tau_1.e : \tau_1 \to \tau_2} \; \text{(T-Abs)} \quad \frac{\Gamma \vdash e_1 : \tau_2 \to \tau_3 \quad \Gamma \vdash e_2 : \tau_2}{\Gamma \vdash e_1 \; e_2 : \tau_3} \; \text{(T-App)}$$

$$\frac{\Gamma, X <: \tau_1 \vdash e : \tau_2}{\Gamma \vdash \lambda X <: \tau_1.e : \forall X <: \tau_1.\tau_2} \; \text{(T-PolyTypeAbs)} \quad \frac{\Gamma \vdash e : \forall X <: \tau_1.\tau_2 \quad \tau' <: \tau_1}{\Gamma \vdash e \; \tau' : [\tau'/X]\tau_2} \; \text{(T-PolyTypeApp)}$$

$\boxed{\hat{\Gamma} \vdash \hat{e} : \hat{\tau} \; \texttt{with} \; \varepsilon}$

$$\frac{}{\hat{\Gamma}, x : \tau \vdash x : \tau \; \texttt{with} \; \varnothing} \; (\varepsilon\text{-Var}) \quad \frac{}{\hat{\Gamma}, r : \{r\} \vdash r : \{r\} \; \texttt{with} \; \varnothing} \; (\varepsilon\text{-Resource})$$

$$\frac{\hat{\Gamma} \vdash \hat{e} : \{\bar{r}\} \; \texttt{with} \; \varepsilon_1}{\hat{\Gamma} \vdash \hat{e}.\pi : \texttt{Unit} \; \texttt{with} \; \varepsilon_1 \cup \{r.\pi \mid r \in \bar{r}\}} \; (\varepsilon\text{-OperCall}) \quad \frac{\hat{\Gamma} \vdash e : \tau \; \texttt{with} \; \varepsilon \quad \hat{\Gamma} \vdash \tau <: \tau' \quad \varepsilon \subseteq \varepsilon'}{\hat{\Gamma} \vdash e : \tau' \; \texttt{with} \; \varepsilon'} \; (\varepsilon\text{-Subsume})$$

$$\frac{\hat{\Gamma}, x : \hat{\tau}_2 \vdash \hat{e} : \hat{\tau}_3 \; \texttt{with} \; \varepsilon_3}{\hat{\Gamma} \vdash \lambda x : \tau_2.\hat{e} : \hat{\tau}_2 \to_{\varepsilon_3} \hat{\tau}_3 \; \texttt{with} \; \varnothing} \; (\varepsilon\text{-Abs}) \quad \frac{\hat{\Gamma} \vdash \hat{e}_1 : \hat{\tau}_2 \to_{\varepsilon} \hat{\tau}_3 \; \texttt{with} \; \varepsilon_1 \quad \hat{\Gamma} \vdash \hat{e}_2 : \hat{\tau}_2 \; \texttt{with} \; \varepsilon_2}{\hat{\Gamma} \vdash \hat{e}_1\hat{e}_2 : \hat{\tau}_3 \; \texttt{with} \; \varepsilon_1 \cup \varepsilon_2 \cup \varepsilon} \; (\varepsilon\text{-App})$$

$$\frac{\hat{\Gamma}, X <: \hat{\tau}_1 \vdash \hat{e} : \hat{\tau}_2 \; \texttt{with} \; \varepsilon_1}{\hat{\Gamma} \vdash \lambda X <: \hat{\tau}_1.\hat{e} : \forall X <: \hat{\tau}_1.\hat{\tau}_2 \; \texttt{caps} \; \varepsilon_1 \; \texttt{with} \; \varnothing} \; (\varepsilon\text{-PolyTypeAbs})$$

$$\frac{\hat{\Gamma}, \phi \subseteq \varepsilon \vdash \hat{e} : \hat{\tau} \; \texttt{with} \; \varepsilon_1}{\hat{\Gamma} \vdash \lambda \phi \subseteq \varepsilon.\hat{e} : \forall \phi \subseteq \varepsilon.\hat{\tau} \; \texttt{caps} \; \varepsilon_1 \; \texttt{with} \; \varnothing} \; (\varepsilon\text{-PolyFxAbs})$$

$$\frac{\hat{\Gamma} \vdash \hat{e} : \forall X <: \hat{\tau}_1.\hat{\tau}_2 \; \texttt{caps} \; \varepsilon_1 \; \texttt{with} \; \varepsilon_2 \quad \hat{\Gamma} \vdash \hat{\tau}' <: \hat{\tau}_1}{\hat{\Gamma} \vdash \hat{e} \; \hat{\tau}' : [\hat{\tau}'/X]\hat{\tau}_2 \; \texttt{with} \; [\hat{\tau}'/X]\varepsilon_1 \cup \varepsilon_2} \; (\varepsilon\text{-PolyTypeApp})$$

$$\frac{\hat{\Gamma} \vdash \hat{e} : \forall \phi \subseteq \varepsilon.\hat{\tau} \; \texttt{caps} \; \varepsilon_1 \; \texttt{with} \; \varepsilon_2 \quad \varepsilon' \subseteq \varepsilon}{\hat{\Gamma} \vdash \hat{e} \; \varepsilon' : [\varepsilon'/\phi]\hat{\tau} \; \texttt{with} \; [\varepsilon'/\phi]\varepsilon_1 \cup \varepsilon_2} \; (\varepsilon\text{-PolyFxApp})$$

$$\frac{\texttt{effects}(\hat{\tau}_i, \bigcup_{j \neq i} \hat{\tau}_j) \cup \texttt{ho-effects}(\texttt{annot}(\tau, \varnothing)) \subseteq \varepsilon_s}{\hat{\Gamma} \vdash \hat{e}_i : \hat{\tau}_i \; \texttt{with} \; \varepsilon_i \quad x_i : \texttt{erase}(\hat{\tau}_i) \vdash e : \tau \quad \texttt{ho-safe}(\hat{\tau}_i, \varepsilon_s)}{\hat{\Gamma} \vdash \texttt{import}(\varepsilon_s) \; \overline{x = \hat{e}} \; \texttt{in} \; e : \texttt{annot}(\tau, \varepsilon_s) \; \texttt{with} \; \varepsilon_s \cup \bigcup_i \varepsilon_i} \; (\varepsilon\text{-Import})$$

$\boxed{\texttt{safe}(\tau, \varepsilon)}$

$$\frac{}{\texttt{safe}(\{\bar{r}\}, \varepsilon)} \; \text{(Safe-Resource)} \quad \frac{\varepsilon \subseteq \varepsilon' \quad \texttt{ho-safe}(\hat{\tau}_1, \varepsilon) \quad \texttt{safe}(\hat{\tau}_2, \varepsilon)}{\texttt{safe}(\hat{\tau}_1 \to_{\varepsilon'} \hat{\tau}_2, \varepsilon)} \; \text{(Safe-Arrow)}$$

$$\frac{\varepsilon_1 \subseteq \varepsilon \quad \texttt{safe}([\varepsilon_1/\phi]\hat{\tau}, \varepsilon)}{\texttt{safe}(\forall \phi \subseteq \varepsilon_1.\hat{\tau} \; \texttt{caps} \; \varepsilon_2, \varepsilon)} \; \text{(Safe-PolyFx)} \quad \frac{\texttt{ho-safe}(\hat{\tau}_1, \varepsilon) \quad \texttt{safe}([\hat{\tau}_1/X]\hat{\tau}_2, \varepsilon) \quad \varepsilon \subseteq \varepsilon_1}{\texttt{safe}(\forall X <: \hat{\tau}_1.\hat{\tau}_2 \; \texttt{caps} \; \varepsilon_2, \varepsilon)} \; \text{(Safe-PolyType)}$$

$\boxed{\texttt{ho-safe}(\hat{\tau}, \varepsilon)}$

$$\frac{}{\texttt{ho-safe}(\{\bar{r}\},\varepsilon)} \ (\textsc{HOSafe-Resource}) \qquad \frac{\texttt{safe}(\hat{\tau}_1,\varepsilon) \quad \texttt{ho-safe}(\hat{\tau}_2,\varepsilon)}{\texttt{ho-safe}(\hat{\tau}_1 \to_{\varepsilon'} \hat{\tau}_2,\varepsilon)} \ (\textsc{HOSafe-Arrow})$$

$$\frac{\varepsilon_1 \subseteq \varepsilon \quad \texttt{safe}([\varepsilon_1/\phi]\hat{\tau},\varepsilon)}{\texttt{ho-safe}(\forall \phi \subseteq \varepsilon_1.\hat{\tau} \texttt{ caps } \varepsilon_2,\varepsilon)} \ (\textsc{HOSafe-PolyFx}) \qquad \frac{\texttt{safe}(\hat{\tau}_1,\varepsilon) \quad \texttt{ho-safe}([\hat{\tau}_1/X]\hat{\tau}_2,\varepsilon)}{\texttt{ho-safe}(\forall X <: \hat{\tau}_1.\hat{\tau}_2 \texttt{ caps } \varepsilon_2,\varepsilon)} \ (\textsc{HOSafe-PolyType})$$

$$\boxed{\hat{\Gamma} \vdash \hat{\tau} <: \hat{\tau}}$$

$$\frac{}{\hat{\Gamma} \vdash \hat{\tau} <: \hat{\tau}} \ (\textsc{S-Reflexive}) \qquad \frac{\hat{\Gamma} \vdash \hat{\tau}_1 <: \hat{\tau}_2 \quad \hat{\Gamma} \vdash \hat{\tau}_2 <: \hat{\tau}_3}{\hat{\Gamma} \vdash \hat{\tau}_1 <: \hat{\tau}_3} \ (\textsc{S-Transitive})$$

$$\frac{r \in \overline{r_1} \implies r \in \overline{r_2}}{\hat{\Gamma} \vdash \{\overline{r_1}\} <: \{\overline{r_2}\}} \ (\textsc{S-ResourceSet}) \qquad \frac{\hat{\Gamma} \vdash \hat{\tau}'_1 <: \hat{\tau}_1 \quad \hat{\Gamma} \vdash \hat{\tau}_2 <: \hat{\tau}'_2 \quad \varepsilon \subseteq \varepsilon'}{\hat{\Gamma} \vdash \tau_1 \to_{\varepsilon} \tau_2 <: \tau'_1 \to_{\varepsilon'} \tau'_2} \ (\textsc{S-Arrow})$$

$$\frac{\hat{\Gamma} \vdash \hat{\tau}'_1 <: \hat{\tau}_1 \quad \hat{\Gamma}, Y <: \hat{\tau}'_1 \vdash \hat{\tau}_2 <: \hat{\tau}'_2}{\hat{\Gamma} \vdash (\forall X <: \hat{\tau}_1.\hat{\tau}_2) <: (\forall Y <: \hat{\tau}'_1.\hat{\tau}'_2)} \ (\textsc{S-TypePoly}) \qquad \frac{}{\hat{\Gamma}, X <: \hat{\tau} \vdash X <: \hat{\tau}} \ (\textsc{S-TypeVar})$$

4 Dynamic Rules

$$\boxed{\hat{e} \longrightarrow \hat{e} \mid \varepsilon}$$

$$\frac{\hat{e}_1 \longrightarrow \hat{e}'_1 \mid \varepsilon}{\hat{e}_1 \hat{e}_2 \longrightarrow \hat{e}'_1 \hat{e}_2 \mid \varepsilon} \ (\textsc{E-App1}) \qquad \frac{\hat{e}_2 \longrightarrow \hat{e}'_2 \mid \varepsilon}{\hat{v}_1 \hat{e}_2 \longrightarrow \hat{v}_1 \hat{e}'_2 \mid \varepsilon} \ (\textsc{E-App2}) \qquad \frac{}{(\lambda x : \hat{\tau}.\hat{e})\hat{v}_2 \longrightarrow [\hat{v}_2/x]\hat{e} \mid \varnothing} \ (\textsc{E-App3})$$

$$\frac{\hat{e} \to \hat{e}' \mid \varepsilon}{\hat{e}.\pi \longrightarrow \hat{e}'.\pi \mid \varepsilon} \ (\textsc{E-OperCall1}) \qquad \frac{r \in R \quad \pi \in \Pi}{r.\pi \longrightarrow \texttt{unit} \mid \{r.\pi\}} \ (\textsc{E-OperCall2})$$

$$\frac{\hat{e} \longrightarrow \hat{e}' \mid \varepsilon}{\hat{e} \ \hat{\tau} \longrightarrow \hat{e}' \ \hat{\tau} \mid \varepsilon} \ (\textsc{E-PolyTypeApp1}) \qquad \frac{}{(\lambda X <: \hat{\tau}_1.\hat{e})\hat{\tau} \longrightarrow [\hat{\tau}/X]\hat{e} \mid \varnothing} \ (\textsc{E-PolyTypeApp2})$$

$$\frac{\hat{e} \longrightarrow \hat{e}' \mid \varepsilon}{\hat{e} \ \hat{\tau} \longrightarrow \hat{e}' \ \hat{\tau} \mid \varepsilon} \ (\textsc{E-PolyFxApp1}) \qquad \frac{}{(\lambda \phi \subseteq \varepsilon_1.\hat{e})\varepsilon \longrightarrow [\varepsilon/\phi]\hat{e} \mid \varnothing} \ (\textsc{E-PolyFxApp2})$$

$$\frac{\hat{e} \longrightarrow \hat{e}' \mid \varepsilon'}{\texttt{import}(\varepsilon_s) \ x = \hat{e} \texttt{ in } e \longrightarrow \texttt{import}(\varepsilon_s) \ x = \hat{e}' \texttt{ in } e \mid \varepsilon'} \ (\textsc{E-Import1})$$

$$\frac{}{\texttt{import}(\varepsilon_s) \ x = \hat{e} \texttt{ in } e \longrightarrow [\hat{v}/x]\texttt{annot}(e,\varepsilon_s) \mid \varnothing} \ (\textsc{E-Import2})$$

5 Substitution Functions

Definition (sub :: $\hat{v} \times \hat{v} \to \hat{e}$)

1. $[\hat{v}/y]x = x$, if $x \neq y$
2. $[\hat{v}/y]y = \hat{v}$
3. $[\hat{v}/y]r = r$
4. $[\hat{v}/y](\lambda x : \hat{\tau}.\hat{e}) = \lambda x : \hat{\tau}.[\hat{v}/y]\hat{e}$, if $y \neq x$ and y does not occur free in $\hat{e}$
5. $[\hat{v}/y](\lambda X <: \hat{\tau}.\hat{e}) = \lambda X <: \hat{\tau}.[\hat{v}/y]\hat{e}$
6. $[\hat{v}/y](\lambda \phi \subseteq \varepsilon.\hat{e}) = \lambda \phi \subseteq \varepsilon.[\hat{v}/y]\hat{e}$
7. $[\hat{v}/y](\hat{e}.\pi) = ([\hat{v}/y]\hat{e}_1).\pi$
8. $[\hat{v}/y](\hat{e}_1 \ \hat{e}_2) = ([\hat{v}/y]\hat{e}_1) \ ([\hat{v}/y]\hat{e}_2)$
9. $[\hat{v}/y](\hat{e} \ \hat{\tau}) = [\hat{v}/y]\hat{e} \ \hat{\tau}$
10. $[\hat{v}/y](\hat{e} \ \varepsilon) = [\hat{v}/y]\hat{e} \ \hat{\varepsilon}$
11. $[\hat{v}/y](\texttt{import}(\varepsilon_s) \ \overline{x = \hat{e}} \texttt{ in } e) = \texttt{import}(\varepsilon_s) \ \overline{x = [\hat{v}/y]\hat{e}} \texttt{ in } e$

Definition (sub :: $\hat{\tau} \times \hat{v} \to \hat{e}$)

1. $[\hat{\tau}/Y]x = x$
2. $[\hat{\tau}/Y]r = r$
3. $[\hat{\tau}/Y](\lambda x : \hat{\tau}_1.\hat{e}) = \lambda x : [\hat{\tau}/Y]\hat{\tau}_1.[\hat{\tau}/Y]\hat{e}$
4. $[\hat{\tau}/Y](\lambda X <: \hat{\tau}_1.\hat{e}) = \lambda X <: [\hat{\tau}/Y]\hat{\tau}_1.[\hat{\tau}/Y]\hat{e}$, if $X \neq Y$ and Y does not occur free in $\hat{e}$
5. $[\hat{\tau}/Y](\lambda \phi \subseteq \varepsilon.\hat{e}) = \lambda \phi \subseteq \varepsilon.[\hat{\tau}/Y]\hat{e}$
6. $[\hat{\tau}/Y](\hat{e}.\pi) = ([\hat{\tau}/Y]\hat{e}_1).\pi$
7. $[\hat{\tau}/Y](\hat{e}_1 \ \hat{e}_2) = ([\hat{\tau}/Y]\hat{e}_1) \ ([\hat{\tau}/Y]\hat{e}_2)$
8. $[\hat{\tau}/Y](\hat{e} \ \hat{\tau}_1) = ([\hat{\tau}/Y]\hat{e}) \ ([\hat{\tau}/Y]\hat{\tau}_1)$
9. $[\hat{\tau}/Y](\hat{e} \ \varepsilon) = [\hat{\tau}/Y]\hat{e} \ \hat{\varepsilon}$
10. $[\hat{\tau}/Y](\texttt{import}(\varepsilon_s) \ \overline{x = \hat{e}} \ \texttt{in} \ e) = \texttt{import}(\varepsilon_s) \ \overline{x = [\hat{\tau}/Y]\hat{e}} \ \texttt{in} \ e$

Definition (sub :: $\hat{\tau} \times \hat{\tau} \to \hat{e}$)

1. $[\hat{\tau}/Y]Y = \hat{\tau}$
2. $[\hat{\tau}/Y]X = X$, if $X \neq Y$
3. $[\hat{\tau}/Y]\{\bar{r}\} = \{\bar{r}\}$
4. $[\hat{\tau}/Y](\hat{\tau}_1 \to_\varepsilon \hat{\tau}_2) = ([\hat{\tau}/Y]\hat{\tau}_1) \to_\varepsilon ([\hat{\tau}/Y]\hat{\tau}_2)$
5. $[\hat{\tau}/Y](\forall X <: \hat{\tau}_1.\hat{\tau}_2) = \forall X <: [\hat{\tau}/Y]\hat{\tau}_1.[\hat{\tau}/Y]\hat{\tau}_2$, if $X \neq Y$ and Y does not occur free in $\hat{\tau}_2$
6. $[\hat{\tau}/Y](\forall \phi \subseteq \varepsilon_1.\hat{e}) = \forall \phi \subseteq \varepsilon_1.[\hat{\tau}/Y]\hat{e}$

Definition (sub :: $\varepsilon \times \hat{v} \to \hat{e}$)

1. $[\varepsilon/\psi]\psi = \varepsilon$
2. $[\varepsilon/\psi]\phi = \phi$, if $\psi \neq \phi$
3. $[\varepsilon/\psi](\lambda x : \hat{\tau}_1.\hat{e}) = \lambda x : [\varepsilon/\psi]\hat{\tau}_1.[\varepsilon/\psi]\hat{e}$
4. $[\varepsilon/\psi](\lambda X <: \hat{\tau}_1.\hat{e}) = \lambda X <: [\varepsilon/\psi]\hat{\tau}_1.[\varepsilon/\psi]\hat{e}$
5. $[\varepsilon/\psi](\lambda \phi \subseteq \varepsilon_1.\hat{e}) = \lambda \phi \subseteq [\varepsilon/\psi]\varepsilon_1.[\varepsilon/\psi]\hat{e}$
6. $[\varepsilon/\psi](\hat{e}.\pi) = ([\varepsilon/\psi]\hat{e}_1).\pi$
7. $[\varepsilon/\psi](\hat{e}_1 \ \hat{e}_2) = ([\varepsilon/\psi]\hat{e}_1) \ ([\varepsilon/\psi]\hat{e}_2)$
8. $[\varepsilon/\psi](\hat{e} \ \hat{\tau}) = ([\varepsilon/\psi]\hat{e}) \ ([\varepsilon/\psi]\hat{\tau})$
9. $[\varepsilon/\psi](\hat{e} \ \varepsilon_1) = ([\varepsilon/\psi]\hat{e}) \ ([\varepsilon/\psi]\varepsilon_1)$
10. $[\varepsilon/\psi](\texttt{import}(\varepsilon_s) \ \overline{x = \hat{e}} \ \texttt{in} \ e) = \texttt{import}([\varepsilon/\psi]\varepsilon_s) \ \overline{x = [\varepsilon/\psi]\hat{e}} \ \texttt{in} \ e$

Definition (sub :: $\hat{\varepsilon} \times \hat{\tau} \to \hat{e}$)

1. $[\varepsilon/\psi]X = X$
2. $[\varepsilon/\psi]\{\bar{r}\} = \{\bar{r}\}$
3. $[\varepsilon/\psi](\hat{\tau}_1 \to_{\varepsilon_1} \hat{\tau}_2) = ([\varepsilon/\psi]\hat{\tau}_1) \to_{[\varepsilon/\psi]\varepsilon_1} ([\varepsilon/\psi]\hat{\tau}_2)$
4. $[\varepsilon/\psi](\forall X <: \hat{\tau}_1.\hat{\tau}_2) = \forall X <: [\varepsilon/\psi]\hat{\tau}_1.[\varepsilon/\psi]\hat{\tau}_2$
5. $[\varepsilon/\psi](\forall \phi \subseteq \varepsilon_1.\hat{e}) = \forall \phi \subseteq [\varepsilon/\psi]\varepsilon_1.[\varepsilon/\psi]\hat{e}$, if $\psi \neq \phi$ and ψ does not occur free in $\hat{e}$

Definition (sub :: $\varepsilon \times \varepsilon \to \hat{e}$)

1. $[\varepsilon/\psi]\psi = \varepsilon$
2. $[\varepsilon/\psi]\phi = \phi$, if $\phi \neq \psi$
3. $[\varepsilon/\psi]\{\overline{r.\pi}\} = \{\overline{r.\pi}\}$