1 Grammar

$$
\begin{aligned}
e ::= \ & x && \textit{exprs.} \\
| \ & r \\
| \ & \lambda x : \tau.e \\
| \ & e \ e \\
| \ & e.\pi
\end{aligned}
$$

$$
\begin{aligned}
\hat{e} ::= \ & x && \textit{labelled exprs.} \\
| \ & r \\
| \ & \lambda x : \hat{\tau}.\hat{e} \\
| \ & \hat{e} \ \hat{e} \\
| \ & \hat{e}.\pi \\
| \ & \texttt{import}(\varepsilon) \ x = \hat{e} \ \texttt{in} \ e
\end{aligned}
$$

$$
\begin{aligned}
v ::= \ & r && \textit{values.} \\
| \ & \lambda x : \tau.e
\end{aligned}
$$

$$
\begin{aligned}
\hat{v} ::= \ & r && \textit{labelled values} \\
| \ & \lambda x : \hat{\tau}.\hat{e}
\end{aligned}
$$

$$
\begin{aligned}
\varepsilon ::= \ & \{\overline{r.\pi}\} && \textit{effects}
\end{aligned}
$$

$$
\begin{aligned}
\tau ::= \ & \{\bar{r}\} && \textit{types} \\
| \ & \tau \to \tau
\end{aligned}
$$

$$
\begin{aligned}
\hat{\tau} ::= \ & \{\bar{r}\} && \textit{labelled types} \\
| \ & \hat{\tau} \to_{\varepsilon} \hat{\tau}
\end{aligned}
$$

$$
\begin{aligned}
\Gamma ::= \ & \varnothing && \textit{type ctx.} \\
| \ & \Gamma, x : \tau
\end{aligned}
$$

$$
\begin{aligned}
\hat{\Gamma} ::= \ & \varnothing && \textit{labelled type ctx.} \\
| \ & \hat{\Gamma}, x : \hat{\tau}
\end{aligned}
$$

2 Functions

Definition (annot :: $\tau \times \varepsilon \to \hat{\tau}$)

1. $\texttt{annot}(\{\bar{r}\}, \_) = \{\bar{r}\}$
2. $\texttt{annot}(\tau_1 \to \tau_2, \varepsilon) = \texttt{annot}(\tau_1, \varepsilon) \to_{\varepsilon} \texttt{annot}(\tau_2, \varepsilon)$

Definition (annot :: e $\times \varepsilon \to$ ê)

1. $\texttt{annot}(x, \_) = e$
2. $\texttt{annot}(r, \_) = r$
3. $\texttt{annot}(e_1 e_2, \varepsilon) = \texttt{annot}(e_1)\texttt{annot}(e_2)$
4. $\texttt{annot}(e.\pi, \varepsilon) = \texttt{annot}(e).\pi$
5. $\texttt{annot}(\lambda x : \tau.e, \varepsilon) = \lambda x : \texttt{annot}(\tau, \varepsilon).\texttt{annot}(e, \varepsilon)$

Definition (annot :: $\Gamma \times \varepsilon \to \hat{\Gamma}$)

1. $\texttt{annot}(\varnothing, \_) = \varnothing$
2. $\texttt{annot}((\Gamma, x : \tau), \varepsilon) = \texttt{annot}(\Gamma, \varepsilon), x : \texttt{annot}(\tau, \varepsilon)$

Definition (erase :: $\hat{\tau} \to \tau$)

1. $\texttt{erase}(\{\bar{r}\}, \_) = \{\bar{r}\}$
2. $\texttt{erase}(\hat{\tau}_1 \to_{\varepsilon} \hat{\tau}_2) = \texttt{erase}(\hat{\tau}_1) \to \texttt{erase}(\hat{\tau}_2)$

Definition (erase :: ê $\to$ e)

1. $\texttt{erase}(x) = x$
2. $\texttt{erase}(r) = r$
3. $\texttt{erase}(e_1 e_2) = \texttt{erase}(e_1)\texttt{erase}(e_2)$
4. $\texttt{erase}(e.\pi) = \texttt{erase}(e).\pi$
5. $\texttt{erase}(\lambda x : \hat{\tau}.\hat{e}) = \lambda x : \texttt{erase}(\hat{\tau}).\texttt{erase}(\hat{e})$

Definition (effects :: $\tau \to \varepsilon$)

1. $\texttt{effects}(\{\bar{r}\}) = \{r.\pi \mid r \in \bar{r}, \pi \in \Pi\}$
2. $\texttt{effects}(\hat{\tau}_1 \to_\varepsilon \hat{\tau}_2) = \texttt{ho-effects}(\hat{\tau}_1) \cup \varepsilon \cup \texttt{effects}(\hat{\tau}_2)$

This function computes those effects:

- Directly invoked by a function of type $\hat{\tau}$.
- Captured by functions created/returned by $\hat{\tau}$.

Definition (ho-effects :: $\tau \to \varepsilon$)

1. $\texttt{ho-effects}(\{\bar{r}\}) = \varnothing$
2. $\texttt{ho-effects}(\hat{\tau}_1 \to_\varepsilon \hat{\tau}_2) = \texttt{effects}(\hat{\tau}_1) \cup \texttt{ho-effects}(\hat{\tau}_2)$

This function computes those effects:

- Captured by functions passed into $\hat{\tau}$.

Examples

Suppose a is a base type that captures no effects.

Consider $\hat{\tau} = (a \to_b (a \to_c a)) \to_d (a \to_e a)$.
   $\texttt{effects}(\hat{\tau}) = \{d, e\}$
   $\texttt{ho-effects}(\hat{\tau}) = \{b, c\}$

Consider $\hat{\tau} = ((a \to_b a) \to_c (a \to_d a)) \to_e ((a \to_f a) \to_g (a \to_h a))$
   $\texttt{effects}(\hat{\tau}) = \{e, b, g, h\}$
   $\texttt{ho-effects}(\hat{\tau}) = \{c, d, f\}$

3 Static Rules

$$\boxed{\Gamma \vdash e : \tau}$$

$$\frac{}{\Gamma, x : \tau \vdash x : \tau} \ (\text{T-Var}) \qquad \frac{}{\Gamma, r : \{r\} \vdash r : \{r\}} \ (\text{T-Resource}) \qquad \frac{\Gamma, x : \tau_1 \vdash e : \tau_2}{\Gamma \vdash \lambda x : \tau_1.e : \tau_1 \to \tau_2} \ (\text{T-Abs})$$

$$\frac{\Gamma \vdash e_1 : \tau_2 \to \tau_3 \quad \Gamma \vdash e_2 : \tau_2}{\Gamma \vdash e_1\ e_2 : \tau_3} \ (\text{T-App}) \qquad \frac{\Gamma \vdash e : \{\bar{r}\} \quad \forall r \in \bar{r} \mid r \in R \quad \pi \in \Pi}{\Gamma \vdash e.\pi : \texttt{Unit}} \ (\text{T-OperCall})$$

$$\boxed{\hat{\Gamma} \vdash \hat{e} : \hat{\tau} \ \texttt{with} \ \varepsilon}$$

$$\frac{}{\hat{\Gamma}, x : \tau \vdash x : \tau \ \texttt{with} \ \varnothing} \ (\varepsilon\text{-Var}) \qquad \frac{}{\hat{\Gamma}, r : \{r\} \vdash r : \{r\} \ \texttt{with} \ \varnothing} \ (\varepsilon\text{-Resource})$$

$$\frac{\hat{\Gamma}, x : \hat{\tau}_2 \vdash \hat{e} : \hat{\tau}_3 \ \texttt{with} \ \varepsilon_3}{\hat{\Gamma} \vdash \lambda x : \tau_2.\hat{e} : \hat{\tau}_2 \to_\varepsilon \hat{\tau}_3 \ \texttt{with} \ \varnothing} \ (\varepsilon\text{-Abs}) \frac{\hat{\Gamma} \vdash \hat{e}_1 : \hat{\tau}_2 \to_\varepsilon \hat{\tau}_3 \ \texttt{with} \ \varepsilon_1 \quad \hat{\Gamma} \vdash \hat{e}_2 : \hat{\tau}_2 \ \texttt{with} \ \varepsilon_2}{\hat{\Gamma} \vdash \hat{e}_1\hat{e}_2 : \hat{\tau}_3 \ \texttt{with} \ \varepsilon_1 \cup \varepsilon_2 \cup \varepsilon} \ (\varepsilon\text{-App})$$

$$\frac{\hat{\Gamma} \vdash \hat{e} : \{\bar{r}\} \quad \forall r \in \bar{r} \mid r \in R \quad \pi \in \Pi}{\hat{\Gamma} \vdash \hat{e}.\pi : \texttt{Unit} \ \texttt{with} \ \{\bar{r}.\pi\}} \ (\varepsilon\text{-OperCall})$$

$$\frac{\begin{array}{c} \hat{\Gamma} \vdash \hat{e} : \hat{\tau} \ \texttt{with} \ \varepsilon_1 \qquad \varepsilon = \texttt{effects}(\hat{\tau}) \\ \texttt{ho-safe}(\hat{\tau}, \varepsilon) \qquad x : \texttt{erase}(\hat{\tau}) \vdash e : \tau \end{array}}{\hat{\Gamma} \vdash \texttt{import}(\varepsilon)\ x = \hat{e} \ \texttt{in} \ e : \texttt{annot}(\tau, \varepsilon) \ \texttt{with} \ \varepsilon \cup \varepsilon_1} \ (\varepsilon\text{-Module})$$

$\boxed{\texttt{safe}(\tau, \varepsilon)}$

$$\frac{}{\texttt{safe}(\{\bar{r}\}, \varepsilon)} \text{ (SAFE-RESOURCE)} \qquad \frac{}{\texttt{safe}(\texttt{Unit}, \varepsilon)} \text{ (SAFE-UNIT)}$$

$$\frac{\varepsilon \subseteq \varepsilon_2 \quad \texttt{ho-safe}(\hat{\tau}_1, \varepsilon) \quad \texttt{safe}(\hat{\tau}_2, \varepsilon)}{\texttt{safe}(\hat{\tau}_1 \rightarrow_{\varepsilon_2} \hat{\tau}_2, \varepsilon)} \text{ (SAFE-ARROW)}$$

$\boxed{\texttt{ho-safe}(\hat{\tau}, \varepsilon)}$

$$\frac{}{\texttt{ho-safe}(\{\bar{r}\}, \varepsilon)} \text{ (HOSAFE-RESOURCE)} \qquad \frac{}{\texttt{ho-safe}(\texttt{Unit}, \varepsilon)} \text{ (HOSAFE-UNIT)}$$

$$\frac{\texttt{safe}(\hat{\tau}_1, \varepsilon) \quad \texttt{ho-safe}(\hat{\tau}_2, \varepsilon)}{\texttt{ho-safe}(\hat{\tau}_1 \rightarrow_{\varepsilon_2} \hat{\tau}_2, \varepsilon)} \text{ (HOSAFE-ARROW)}$$

$\boxed{\hat{\tau} <: \hat{\tau}}$

$$\frac{\varepsilon \subseteq \varepsilon' \quad \hat{\tau}_2 <: \hat{\tau}_2' \quad \hat{\tau}_1' <: \hat{\tau}_1}{\hat{\tau}_1 \rightarrow_{\varepsilon} \hat{\tau}_2 <: \hat{\tau}_1' \rightarrow_{\varepsilon'} \hat{\tau}_2'} \text{ (S-EFFECTS)}$$

4 Dynamic Rules

$\boxed{\hat{e} \longrightarrow \hat{e} \mid \varepsilon}$

$$\frac{\hat{e}_1 \longrightarrow \hat{e}_1' \mid \varepsilon}{\hat{e}_1 \hat{e}_2 \longrightarrow \hat{e}_1' \hat{e}_2 \mid \varepsilon} \text{ (E-APP1)} \qquad \frac{\hat{e}_2 \longrightarrow \hat{e}_2' \mid \varepsilon}{\hat{v}_1 \hat{e}_2 \longrightarrow \hat{v}_1 \hat{e}_2' \mid \varepsilon} \text{ (E-APP2)} \qquad \frac{}{(\lambda x : \hat{\tau}.\hat{e})\hat{v}_2 \longrightarrow [\hat{v}_2/x]\hat{e} \mid \varnothing} \text{ (E-APP3)}$$

$$\frac{\hat{e} \rightarrow \hat{e}' \mid \varepsilon}{\hat{e}.\pi \longrightarrow \hat{e}'.\pi \mid \varepsilon} \text{ (E-OPERCALL1)} \qquad \frac{r \in R \quad \pi \in \Pi}{r.\pi \longrightarrow \texttt{unit} \mid \{r.\pi\}} \text{ (E-OPERCALL2)}$$

$$\frac{\hat{e} \longrightarrow \hat{e}' \mid \varepsilon'}{\texttt{import}(\varepsilon)\ x = \hat{e}\ \texttt{in}\ e \longrightarrow \texttt{import}(\varepsilon)\ x = \hat{e}'\ \texttt{in}\ e \mid \varepsilon'} \text{ (E-MODULE1)}$$

$$\frac{}{\texttt{import}(\varepsilon)\ x = \hat{v}\ \texttt{in}\ e \longrightarrow [\hat{v}/x]\texttt{annot}(e, \varepsilon) \mid \varnothing} \text{ (E-MODULE2)}$$

5 Encodings

5.1 $\perp$

We can define the bottom type as $\perp = \{\}$, because there is no empty-set literal.

5.2 unit, Unit

Define $\texttt{unit} = \lambda \texttt{x} : \{\}.\texttt{x}$, i.e. the function which takes an empty set of resources and returns it. We shall refer to its type, which is $\{\} \rightarrow_{\varnothing} \{\}$, as Unit. It has various properties befitting unit.

1. unit cannot be invoked, as $\{\}$ is uninhabited.
2. unit is a value.
3. The only term with type Unit is unit.
4. $\vdash \texttt{unit} : \texttt{Unit}$, by using ε-ABS and ε-VAR.
5. $\texttt{effects}(\texttt{Unit}) = \texttt{ho-effects}(\texttt{Unit}) = \varnothing$
6. $\texttt{safe}(\texttt{Unit}, \varepsilon)$ and $\texttt{ho-safe}(\texttt{Unit}, \varepsilon)$

6   Proofs

Theorem 1 (Progress). *If $\hat{\Gamma} \vdash \hat{e}_A : \hat{\tau}_A$ with ε_A then $\hat{e}_A$ is a value or $\hat{e}_A \longrightarrow \hat{e}_B \mid \varepsilon$.*

Proof. By induction on $\hat{\Gamma} \vdash \hat{e}_A : \hat{\tau}_A$ with ε_A.

Case: ε-RESOURCE, ε-UNIT, ε-ABS   Then $\hat{e}_A$ is a value.

Case: ε-APP   Then $\hat{e}_A = \hat{e}_1\ \hat{e}_2$. We consider the cases in which $\hat{e}_1$ and $\hat{e}_2$ are values.
   If $\hat{e}_1$ is not a value then by inductive assumption there is a reduction $\hat{e}_1 \longrightarrow \hat{e}_1' \mid \varepsilon$. Then $\hat{e}_1\ \hat{e}_2$ reduces by the rule E-APP1, giving $\hat{e}_1\ \hat{e}_2 \longrightarrow \hat{e}_1'\ \hat{e}_2 \mid \varepsilon$.
   If $\hat{e}_2$ is not a value then WLOG $\hat{e}_1$ is a value. By inductive assumption $\hat{e}_2 \longrightarrow \hat{e}_2' \mid \varepsilon$. Then $\hat{v}_1\ \hat{e}_2$ reduces by the rule E-APP2, giving $\hat{v}_1\ \hat{e}_2 \longrightarrow \hat{v}_1\ \hat{e}_2' \mid \varepsilon$.
   If $\hat{e}_1$ and $\hat{e}_2$ are both values then by canonical forms $\hat{e}_1 = \hat{v}_1 = \lambda x : \tau_2.e$. Then $\hat{v}_1\ \hat{v}_2$ reduces by the rule E-APP3, giving $\hat{v}_1\ \hat{v}_2 \longrightarrow [\hat{v}_2/x]\hat{e} \mid \varnothing$.

Case: ε-OPERCALL   Then $\hat{e}_A = \hat{e}_1.\pi$. We consider whether $\hat{e}_1$ is a value.
   If $\hat{e}_1$ is not a value then by inductive assumption there is a reduction $\hat{e}_1 \longrightarrow \hat{e}_1' \mid \varepsilon$. Then $\hat{e}_1.\pi$ reduces by the rule E-OPERCALL1, giving $\hat{e}_1.\pi \longrightarrow \hat{e}_1'.\pi \mid \varepsilon$.
   If $\hat{e}_1$ is a value then $\hat{e}_1 = r$ by canonical forms. By the assumption that $r.\pi$ is closed under Γ, we know $r \in R$ and $\pi \in \Pi$. Then $\hat{e}_1.\pi$ reduces by the rule E-OPERCALL2, giving $r.\pi \longrightarrow \mathtt{unit} \mid \varepsilon$.

Case: ε-MODULE   Then $e_A = \mathtt{import}(\varepsilon)\ x = \hat{e}$ in e. If $\hat{e}$ is an expression then it can be reduced, so $\hat{e} \longrightarrow \hat{e}' \mid \varepsilon'$, and so by E-MODULE1 we get $\mathtt{import}(\varepsilon)\ x = \hat{e}$ in $e \longrightarrow \mathtt{import}(\varepsilon)\ x = \hat{e}'$ in $e \mid \varepsilon'$. Otherwise $\hat{e} = \hat{v}$ is a value. Then by E-MODULE2 we get $\mathtt{import}(\varepsilon)\ x = \hat{v} \longrightarrow [\hat{v}/x]\mathtt{annot}(e, \varepsilon) \mid \varnothing$.

Lemma 1 (Substitution). *If $\hat{\Gamma}, x : \hat{\tau}' \vdash e : \hat{\tau}$ with ε and $\hat{\Gamma} \vdash v : \hat{\tau}'$ with $\varnothing$ then $\hat{\Gamma} \vdash [v/x]e : \hat{\tau}$ with ε.*

Lemma 2. *If $\varepsilon' \subseteq \varepsilon$ then $\mathtt{safe}(\hat{\tau}, \varepsilon) \implies \mathtt{safe}(\hat{\tau}, \varepsilon')$ and $\mathtt{ho\text{-}safe}(\hat{\tau}, \varepsilon) \implies \mathtt{ho\text{-}safe}(\hat{\tau}, \varepsilon')$.*

Proof. If $\hat{\tau} = \{\bar{r}\}$ then the result holds immediately. Otherwise $\hat{\tau} = \hat{\tau}_1 \rightarrow_{\varepsilon_2} \hat{\tau}_2$. We proceed by simultaneous induction on derivations of $\mathtt{safe}(\hat{\tau}, \varepsilon)$ and $\mathtt{ho\text{-}safe}(\hat{\tau}, \varepsilon)$.

$\mathtt{safe}(\hat{\tau}_1 \rightarrow_{\varepsilon_2} \hat{\tau}_2, \varepsilon)$ By definition this means $\varepsilon \subseteq \varepsilon_2$ and $\mathtt{ho\text{-}safe}(\hat{\tau}_1, \varepsilon)$ and $\mathtt{safe}(\hat{\tau}_2, \varepsilon)$. By inductive assumption, $\mathtt{ho\text{-}safe}(\hat{\tau}_1, \varepsilon')$ and $\mathtt{safe}(\hat{\tau}_2, \varepsilon')$. Also $\varepsilon' \subseteq \varepsilon \subseteq \varepsilon_2$. Therefore $\mathtt{safe}(\hat{\tau}_1 \rightarrow_{\varepsilon_2} \hat{\tau}_2, \varepsilon')$.

$\mathtt{ho\text{-}safe}(\hat{\tau}_1 \rightarrow_{\varepsilon_2} \hat{\tau}_2, \varepsilon)$ By definition this means $\mathtt{safe}(\hat{\tau}_1, \varepsilon)$ and $\mathtt{ho\text{-}safe}(\hat{\tau}_2, \varepsilon)$. By inductive assumption, $\mathtt{safe}(\hat{\tau}_1, \varepsilon')$ and $\mathtt{ho\text{-}safe}(\hat{\tau}_2, \varepsilon')$. Therefore $\mathtt{ho\text{-}safe}(\hat{\tau}_1 \rightarrow_{\varepsilon_2} \hat{\tau}_2, \varepsilon')$.

Lemma 3. *If $\varepsilon \subseteq \mathtt{effects}(\hat{\tau})$ and $\mathtt{ho\text{-}safe}(\hat{\tau}, \varepsilon)$ then $\hat{\tau} <: \mathtt{annot}(\mathtt{erase}(\hat{\tau}), \varepsilon)$.*

Lemma 4. *If $\varepsilon \subseteq \mathtt{ho\text{-}effects}(\hat{\tau})$ and $\mathtt{safe}(\hat{\tau}, \varepsilon)$ then $\mathtt{annot}(\mathtt{erase}(\hat{\tau}), \varepsilon) <: \hat{\tau}$.*

Proof. By simultaneous induction on derivations.

Case: Unit, $\{\bar{r}\}$ Then $\mathtt{annot}(\mathtt{erase}(\hat{\tau}), \varepsilon) = \hat{\tau}$.

Case: $\hat{\tau} = \hat{\tau}_1 \rightarrow_{\varepsilon'} \hat{\tau}_2$, $\varepsilon \subseteq \mathtt{effects}(\hat{\tau})$, $\mathtt{ho\text{-}safe}(\hat{\tau}, \varepsilon)$
$\mathtt{effects}(\hat{\tau}_1 \rightarrow_{\varepsilon'} \hat{\tau}_2) = \mathtt{ho\text{-}effects}(\hat{\tau}_1) \cup \varepsilon \cup \mathtt{effects}(\hat{\tau}_2)$.
$\mathtt{ho\text{-}safe}(\hat{\tau}_1 \rightarrow_{\varepsilon'} \hat{\tau}_2, \varepsilon) = \mathtt{safe}(\hat{\tau}_1, \varepsilon) \wedge \mathtt{ho\text{-}safe}(\hat{\tau}_2, \varepsilon)$.

Case: $\hat{\tau} = \hat{\tau}_1 \rightarrow_{\varepsilon'} \hat{\tau}_2$, $\varepsilon \subseteq \mathtt{ho\text{-}effects}(\hat{\tau})$, $\mathtt{safe}(\hat{\tau}, \varepsilon)$
$\mathtt{ho\text{-}effects}(\hat{\tau}_1 \rightarrow_{\varepsilon'} \hat{\tau}_2) = \mathtt{effects}(\hat{\tau}_1) \cup \mathtt{ho\text{-}effects}(\hat{\tau}_2)$
$\mathtt{safe}(\hat{\tau}_1 \rightarrow_{\varepsilon'} \hat{\tau}_2) = \varepsilon \subseteq \varepsilon' \wedge \mathtt{ho\text{-}safe}(\hat{\tau}_1, \varepsilon) \wedge \mathtt{safe}(\hat{\tau}_2, \varepsilon)$

Theorem 2 (Preservation). *If $\hat{\Gamma} \vdash \hat{e}_A : \hat{\tau}_A$ `with` ε_A and $e_A \longrightarrow e_B \mid \varepsilon$, then $\hat{\Gamma} \vdash e_B : \tau_B$ `with` ε_B, where $e_B <: e_B$ and $\varepsilon \cup \varepsilon_B \subseteq \varepsilon_A$.*

Proof. By induction on $\hat{\Gamma} \vdash \hat{e}_A : \tau_A$ `with` ε_A, and then on $e_A \longrightarrow e_B \mid \varepsilon$.

$\boxed{\varepsilon\text{-VAR, }\varepsilon\text{-RESOURCE, }\varepsilon\text{-UNIT, }\varepsilon\text{-ABS}}$ Then e_A cannot be reduced and so the theorem statement vacuously holds.

$\boxed{\varepsilon\text{-APP}}$ Then $e_A = \hat{e}_1 \hat{e}_2$ and $\hat{e}_1 : \hat{\tau}_2 \rightarrow_\varepsilon \hat{\tau}_3$ `with` ε_1 and $\hat{\Gamma} \vdash \hat{e}_2 : \hat{\tau}_2$ `with` ε_2. If the reduction rule used was E-APP1 or E-APP2, then the result follows by applying the inductive hypothesis to $\hat{e}_1$ and $\hat{e}_2$ respectively.

Otherwise the rule used was E-APP3. Then $(\lambda x : \hat{\tau}_2.\hat{e})\hat{v}_2 \longrightarrow [\hat{v}_2/x]\hat{e} \mid \varnothing$. By inversion on the typing rule for $\lambda x : \hat{\tau}_2.\hat{e}$ we know $\Gamma, x : \hat{\tau}_2 \vdash \hat{e} : \hat{\tau}_3$ `with` ε_3. By canonical forms, $\varepsilon_2 = \varnothing$ because $\hat{e}_2 = \hat{v}_2$ is a value. Then by the substitution lemma, $\hat{\Gamma} \vdash [\hat{v}_2/x]\hat{e} : \hat{\tau}_3$ `with` ε_3. By canonical forms, $\varepsilon_1 = \varepsilon_2 = \varnothing = \varepsilon$. Therefore $\varepsilon_A = \varepsilon_3 = \varepsilon_B \cup \varnothing$.

$\boxed{\varepsilon\text{-OperCall}}$ Then $e_A = e_1.\pi$ and $\hat{\Gamma} \vdash e_1 : \{\bar{r}\}$ `with` ε_1. If the reduction rule used was E-OPERCALL1 then the result follows by applying the inductive hypothesis to $\hat{e}_1$.

Otherwise the reduction rule used was E-OPERCALL2 and $v_1.\pi \longrightarrow$ `unit` $\mid \{r.\pi\}$. By canonical forms, $\hat{\Gamma} \vdash v_1 :$ `unit` `with` $\{r.\pi\}$. Also, $\hat{\Gamma} \vdash$ `unit` `: Unit with` $\varnothing$. Then $\tau_B = \tau_A$. Also, $\varepsilon \cup \varepsilon_B = \{r.\pi\} = \varepsilon_A$.

$\boxed{\varepsilon\text{-Module}}$ Then $e_A =$ `import`(ε) $x = \hat{e}$ `in` e. If the reduction rule used was E-MODULECALL1 then the result follows by applying the inductive hypothesis to $\hat{e}$.

Otherwise the following are true:

1. $e_A =$ `import`(ε) $x = \hat{v}$ `in` e
2. $\hat{\Gamma} \vdash e_A :$ `annot`(τ, ε) `with` $\varepsilon \cup \varepsilon_1$
3. `import`(ε) $x = \hat{v}$ `in` $e \longrightarrow [\hat{v}/x]$`annot`$(e, \varepsilon) \mid \varnothing$
4. $\hat{\Gamma} \vdash \hat{v} : \hat{\tau}$ `with` $\varnothing$
5. $\varepsilon =$ `effects`$(\hat{\tau})$
6. `ho-safe`$(\hat{\tau}, \varepsilon)$
7. $x :$ `erase`$(\hat{\tau}) \vdash e : \tau$

Show that $\hat{\Gamma}, x : \hat{\tau} \vdash$ `annot`$(e, \varepsilon) :$ `annot`(τ, ε) `with` ε.

Then by substitution lemma, $\hat{\Gamma} \vdash [\hat{v}/x]$`annot`$(e, \varepsilon) :$ `annot`(τ, ε) `with` ε.