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1 Stanford Encyclopaedia of Philosophy

1.1 Extensionality

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Definition. (Extension) The thing to which a reference points. For a term, this is exactly the referent; for the predicate P, it is the set of things for which P holds; for the sentence, it is the truth-value.

Definition. (Intension) The sense or meaning of a reference provided by a semantics; the aspect of the semantics determining its extension.

Definition. (Logic) A formal language with a semantics, a semantics being a theory which provides rigorous definitions of truth, validity, and logical consequence inside the language. A logic is *extensional* if the truth-value of every sentence is determined entirely by its form; otherwise it is *intentional*. Extensional logics obey the *substitution principle*, while intensional logics do not (because truth appeals to something higher than form).

Propositional/predicate logic are extensional. Suppose the following sentences are true:

1. "All John's dogs are mammals": $\forall x(Dx \to Mx)$.
2. "All John's pets are mammals: $\forall x(Px \to Mx)$.

Because both are true, they have the same extension. Therefore, we may replace one with the other inside a third sentence, and truth will be preserved. Once we add modalities, this no longer holds.

1. "Necessarily, all of John's dogs are mammals": $\Box(\forall x(Dx \to Mx))$
2. "Necessarily, all of John's pets are mammals": $\Box(\forall x(Px \to Mx))$

The first sentence is true, but the second is not. This shows modal logic is intensional, and that the substitution principle does not hold.

1.2 Possible Worlds

To regain extensionality, possible world semantics was developed, which describes the semantics of modal sentences in terms of first-order logic (which is extensional). Modal operators are interpreted as quantifiers over possible worlds.

- $\Box\phi$ is true if and only if ϕ is true in every possible world.
- $\Diamond\phi$ is true if and only if ϕ is true in some possible world.