

## 1 Grammar

$e ::=$	<b>exprs.</b>	$\tau ::=$	<b>types</b>
$x$	<i>variable</i>	$X$	<i>type variable</i>
$v$	<i>value</i>	$\{\bar{r}\}$	<i>effect set</i>
$e.\pi$	<i>operation call</i>	$\tau \rightarrow \tau$	<i>arrow</i>
$e e$	<i>application</i>	$\forall X <: \tau.\tau$	<i>universal type</i>
$e \tau$	<i>type application</i>		
$v ::=$	<b>values</b>	$\hat{\tau} ::=$	<b>annotated types</b>
$r$	<i>resource literal</i>	$X$	<i>type variable</i>
$\lambda x : \tau.e$	<i>abstraction</i>	$\{\bar{r}\}$	<i>resource set</i>
$\lambda X <: \tau.e$	<i>type polymorphism</i>	$\hat{\tau} \rightarrow_{\varepsilon} \hat{\tau}$	<i>annotated arrow</i>
		$\forall X <: \hat{\tau}.\hat{\tau} \text{ caps } \varepsilon$	<i>universal type</i>
		$\forall \phi \subseteq \varepsilon.\hat{\tau} \text{ caps } \varepsilon$	<i>universal effect set</i>
$\hat{e} ::=$	<b>annotated exprs.</b>	$\varepsilon ::=$	<b>effects</b>
$x$	<i>variable</i>	$\phi$	<i>effect variable</i>
$\hat{v}$	<i>value</i>	$\{\bar{r}.\pi\}$	<i>effect set</i>
$\hat{e}.\pi$	<i>operation call</i>		
$\hat{e} \hat{e}$	<i>application</i>	$\Gamma ::=$	<b>contexts</b>
$e \hat{\tau}$	<i>type application</i>	$\emptyset$	<i>empty ctx.</i>
$e \varepsilon$	<i>effect application</i>	$\Gamma, x : \tau$	<i>var. binding</i>
$\text{import}(\varepsilon_s) \overline{x = \hat{e}} \text{ in } e$		$\Gamma, X <: \tau$	<i>type var. binding</i>
$\hat{v} ::=$	<b>annotated values</b>	$\hat{\Gamma} ::=$	<b>annotated contexts</b>
$r$	<i>resource literal</i>	$\emptyset$	<i>empty ctx.</i>
$\lambda x : \hat{\tau}.\hat{e}$	<i>abstraction</i>	$\hat{\Gamma}, x : \hat{\tau}$	<i>var. binding</i>
$\lambda X <: \hat{\tau}.\hat{e}$	<i>type polymorphism</i>	$\hat{\Gamma}, X <: \hat{\tau}$	<i>type var. binding</i>
$\lambda \phi \subseteq \varepsilon.\hat{e}$	<i>effect polymorphism</i>	$\hat{\Gamma}, \phi \subseteq \varepsilon$	<i>effect var. binding</i>

## 2 Functions

**Definition** ( $\text{annot} :: \tau \times \varepsilon \rightarrow \hat{\tau}$ )

1.  $\text{annot}(X, \_) = X$
2.  $\text{annot}(\{\bar{r}\}, \_) = \{\bar{r}\}$
3.  $\text{annot}(\tau_1 \rightarrow \tau_2, \varepsilon) = \text{annot}(\tau_1, \varepsilon) \rightarrow_{\varepsilon} \text{annot}(\tau_2, \varepsilon)$
4.  $\text{annot}(\forall X <: \tau_1.\tau_2, \varepsilon) = \forall X <: \text{annot}(\tau_1, \varepsilon).\text{annot}(\tau_2, \varepsilon) \text{ caps } \varepsilon$

**Definition** ( $\text{annot} :: e \times \varepsilon \rightarrow \hat{e}$ )

1.  $\text{annot}(x, \_) = x$
2.  $\text{annot}(r, \_) = r$
3.  $\text{annot}(\lambda x : \tau.e, \varepsilon) = \lambda x : \text{annot}(\tau, \varepsilon).\text{annot}(e, \varepsilon)$
4.  $\text{annot}(e_1 e_2, \varepsilon) = \text{annot}(e_1, \varepsilon) \text{ annot}(e_2, \varepsilon)$
5.  $\text{annot}(e.\pi, \varepsilon) = \text{annot}(e, \varepsilon).\pi$
6.  $\text{annot}(\lambda X <: \tau_1.e, \varepsilon) = \lambda X <: \text{annot}(\tau_1, \varepsilon).\text{annot}(e, \varepsilon)$
7.  $\text{annot}(e \tau, \varepsilon) = \text{annot}(e, \varepsilon) \text{ annot}(\tau, \varepsilon)$

**Definition** ( $\text{annot} :: \Gamma \times \varepsilon \rightarrow \hat{\Gamma}$ )

1.  $\text{annot}(\emptyset, \_) = \emptyset$
2.  $\text{annot}((\Gamma, x : \tau), \varepsilon) = \text{annot}(\Gamma, \varepsilon), x : \text{annot}(\tau, \varepsilon)$
3.  $\text{annot}((\Gamma, X <: \tau), \varepsilon) = \text{annot}(\Gamma, \varepsilon), X <: \text{annot}(\tau, \varepsilon)$

**Definition** ( $\text{erase} :: \hat{\tau} \rightarrow \tau$ )

1.  $\text{erase}(X) = X$
2.  $\text{erase}(\{\bar{r}\}) = \{\bar{r}\}$
3.  $\text{erase}(\hat{\tau}_1 \rightarrow_{\varepsilon} \hat{\tau}_2) = \text{erase}(\hat{\tau}_1) \rightarrow \text{erase}(\hat{\tau}_2)$
4.  $\text{erase}(\forall X <: \hat{\tau}_1. \hat{\tau}_2 \text{ caps } \varepsilon) = \forall X <: \text{erase}(\hat{\tau}_1). \text{erase}(\hat{\tau}_2)$

**Definition** ( $\text{erase} :: \hat{e} \rightarrow e$ )

1.  $\text{erase}(x) = x$
2.  $\text{erase}(r) = r$
3.  $\text{erase}(\lambda x : \hat{\tau}. \hat{e}) = \lambda x : \text{erase}(\hat{\tau}). \text{erase}(\hat{e})$
4.  $\text{erase}(\hat{e}_1 \hat{e}_2) = \text{erase}(\hat{e}_1) \text{erase}(\hat{e}_2)$
5.  $\text{erase}(\hat{e}. \pi) = \text{erase}(\hat{e}). \pi$
6.  $\text{erase}(\lambda X <: \hat{\tau}. \hat{e}) = \lambda X <: \text{erase}(\hat{\tau}). \text{erase}(\hat{e})$

**Definition** ( $\text{erase} :: \hat{\Gamma} \rightarrow \Gamma$ )

1.  $\text{erase}(\emptyset) = \emptyset$
2.  $\text{erase}(\hat{\Gamma}, x : \hat{\tau}) = \text{erase}(\hat{\Gamma}), x : \text{erase}(\hat{\tau})$
3.  $\text{erase}(\hat{\Gamma}, X <: \hat{\tau}) = \text{erase}(\hat{\Gamma}), X <: \text{erase}(\hat{\tau})$

**Definition** ( $\text{effects} :: \hat{\tau} \rightarrow \varepsilon$ )

1.  $\text{effects}(\{\bar{r}\}) = \{r. \pi \mid r \in \bar{r}, \pi \in \Pi\}$
2.  $\text{effects}(\hat{\tau}_1 \rightarrow_{\varepsilon} \hat{\tau}_2) = \text{ho-effects}(\hat{\tau}_1) \cup \varepsilon \cup \text{effects}(\hat{\tau}_2)$
3.  $\text{effects}(\forall X <: \hat{\tau}_1. \hat{\tau}_2 \text{ caps } \varepsilon_2) = \varepsilon_2 \cup \text{effects}([\text{annot}(\text{erase}(\hat{\tau}_1), \emptyset)/X] \hat{\tau}_2)$
4.  $\text{effects}(\forall \Phi \subseteq \varepsilon_1. \hat{\tau}_2 \text{ caps } \varepsilon_2) = \varepsilon_2 \cup \text{effects}([\emptyset/\Phi] \hat{\tau}_2)$

**Definition** ( $\text{ho-effects} :: \hat{\tau} \rightarrow \varepsilon$ )

1.  $\text{ho-effects}(\{\bar{r}\}) = \emptyset$
2.  $\text{ho-effects}(\hat{\tau}_1 \rightarrow_{\varepsilon} \hat{\tau}_2) = \text{effects}(\hat{\tau}_1) \cup \text{ho-effects}(\hat{\tau}_2)$
3.  $\text{ho-effects}(\forall X <: \hat{\tau}_1. \hat{\tau}_2 \text{ caps } \varepsilon_2) = \text{effects}(\hat{\tau}_1) \cup \text{ho-effects}([\hat{\tau}_1/X] \hat{\tau}_2)$
4.  $\text{ho-effects}(\forall \Phi \subseteq \varepsilon_1. \hat{\tau}_2 \text{ caps } \varepsilon_2) = \varepsilon_1 \cup \text{ho-effects}([\emptyset/\Phi] \hat{\tau}_2)$

### 3 Static Rules

$$\boxed{\Gamma \vdash e : \tau}$$

$$\begin{array}{c} \frac{}{\Gamma, x : \tau \vdash x : \tau} \text{ (T-VAR)} \quad \frac{}{\Gamma, r : \{r\} \vdash r : \{r\}} \text{ (T-RESOURCE)} \quad \frac{\Gamma \vdash e : \{\bar{r}\}}{\Gamma \vdash e. \pi : \mathbf{Unit}} \text{ (T-OPERCALL)} \\[10pt] \frac{\Gamma, x : \tau_1 \vdash e : \tau_2}{\Gamma \vdash \lambda x : \tau_1. e : \tau_1 \rightarrow \tau_2} \text{ (T-ABS)} \quad \frac{\Gamma \vdash e_1 : \tau_2 \rightarrow \tau_3 \quad \Gamma \vdash e_2 : \tau_2}{\Gamma \vdash e_1 \ e_2 : \tau_3} \text{ (T-APP)} \\[10pt] \frac{\Gamma, X <: \tau_1 \vdash e : \tau_2}{\Gamma \vdash \lambda X <: \tau_1. e : \forall X <: \tau_1. \tau_2} \text{ (T-POLYTYPEABS)} \quad \frac{\Gamma \vdash e : \forall X <: \tau_1. \tau_2 \quad \tau' <: \tau_1}{\Gamma \vdash e \ \tau' : [\tau'/X] \tau_2} \text{ (T-POLYTYPEAPP)} \end{array}$$

$$\boxed{\hat{\Gamma} \vdash \hat{e} : \hat{\tau} \text{ with } \varepsilon}$$

$$\begin{array}{c}
\frac{}{\hat{I}, x : \tau \vdash x : \tau \text{ with } \emptyset} (\varepsilon\text{-VAR}) \quad \frac{}{\hat{I}, r : \{r\} \vdash r : \{r\} \text{ with } \emptyset} (\varepsilon\text{-RESOURCE}) \\
\\
\frac{\hat{I} \vdash \hat{e} : \{\bar{r}\} \text{ with } \varepsilon_1}{\hat{I} \vdash \hat{e}.\pi : \mathbf{Unit} \text{ with } \varepsilon_1 \cup \{r.\pi \mid r \in \bar{r}\}} (\varepsilon\text{-OPERCALL}) \quad \frac{\hat{I} \vdash e : \hat{\tau} \text{ with } \varepsilon \quad \hat{I} \vdash \hat{\tau} <: \hat{\tau}' \quad \hat{I} \vdash \varepsilon \subseteq \varepsilon'}{\hat{I} \vdash e : \hat{\tau}' \text{ with } \varepsilon'} (\varepsilon\text{-SUBSUME}) \\
\\
\frac{\hat{I}, x : \hat{\tau}_2 \vdash \hat{e} : \hat{\tau}_3 \text{ with } \varepsilon_3}{\hat{I} \vdash \lambda x : \tau_2. \hat{e} : \hat{\tau}_2 \rightarrow_{\varepsilon_3} \hat{\tau}_3 \text{ with } \emptyset} (\varepsilon\text{-ABS}) \quad \frac{\hat{I} \vdash \hat{e}_1 : \hat{\tau}_2 \rightarrow_{\varepsilon} \hat{\tau}_3 \text{ with } \varepsilon_1 \quad \hat{I} \vdash \hat{e}_2 : \hat{\tau}_2 \text{ with } \varepsilon_2}{\hat{I} \vdash \hat{e}_1 \hat{e}_2 : \hat{\tau}_3 \text{ with } \varepsilon_1 \cup \varepsilon_2 \cup \varepsilon} (\varepsilon\text{-APP}) \\
\\
\frac{\hat{I}, X <: \hat{\tau}_1 \vdash \hat{e} : \hat{\tau}_2 \text{ with } \varepsilon_1}{\hat{I} \vdash \lambda X <: \hat{\tau}_1. \hat{e} : \forall X <: \hat{\tau}_1. \hat{\tau}_2 \text{ caps } \varepsilon_1 \text{ with } \emptyset} (\varepsilon\text{-POLYTYPEABS}) \\
\\
\frac{\hat{I}, \phi \subseteq \varepsilon \vdash \hat{e} : \hat{\tau} \text{ with } \varepsilon_1}{\hat{I} \vdash \lambda \phi \subseteq \varepsilon. \hat{e} : \forall \phi \subseteq \varepsilon. \hat{\tau} \text{ caps } \varepsilon_1 \text{ with } \emptyset} (\varepsilon\text{-POLYFXABS}) \\
\\
\frac{\hat{I} \vdash \hat{e} : \forall X <: \hat{\tau}_1. \hat{\tau}_2 \text{ caps } \varepsilon_1 \text{ with } \varepsilon_2 \quad \hat{I} \vdash \hat{\tau}' <: \hat{\tau}_1}{\hat{I} \vdash \hat{e} \hat{\tau}' : [\hat{\tau}'/X] \hat{\tau}_2 \text{ with } \varepsilon_1 \cup \varepsilon_2} (\varepsilon\text{-POLYTYPEAPP}) \\
\\
\frac{\hat{I} \vdash \hat{e} : \forall \phi \subseteq \varepsilon. \hat{\tau} \text{ caps } \varepsilon_1 \text{ with } \varepsilon_2 \quad \varepsilon' \subseteq \varepsilon}{\hat{I} \vdash \hat{e} \varepsilon' : [\varepsilon'/\phi] \hat{\tau} \text{ with } [\varepsilon'/\phi] \varepsilon_1 \cup \varepsilon_2} (\varepsilon\text{-POLYFXAPP}) \\
\\
\frac{\text{effects}(\hat{\tau}_i) \cup \text{ho-effects}(\text{annot}(\tau, \emptyset)) \subseteq \varepsilon_s \quad \hat{I} \vdash \hat{e}_i : \hat{\tau}_i \text{ with } \varepsilon_i \quad x_i : \text{erase}(\hat{\tau}_i) \vdash e : \tau \quad \text{ho-safe}(\hat{\tau}_i, \varepsilon_s)}{\hat{I} \vdash \text{import}(\varepsilon_s) \bar{x} = \hat{e} \text{ in } e : \text{annot}(\tau, \varepsilon_s) \text{ with } \varepsilon_s \cup \bigcup_i \varepsilon_i} (\varepsilon\text{-IMPORT})
\end{array}$$

$\text{safe}(\tau, \varepsilon)$

$$\begin{array}{c}
\frac{}{\text{safe}(\{\bar{r}\}, \varepsilon)} (\text{SAFE-RESOURCE}) \quad \frac{\varepsilon \subseteq \varepsilon' \quad \text{ho-safe}(\hat{\tau}_1, \varepsilon) \quad \text{safe}(\hat{\tau}_2, \varepsilon)}{\text{safe}(\hat{\tau}_1 \rightarrow_{\varepsilon'} \hat{\tau}_2, \varepsilon)} (\text{SAFE-ARROW}) \\
\\
\frac{\varepsilon_2 \subseteq \varepsilon \quad \text{safe}([\emptyset/\Phi] \hat{\tau}_2, \varepsilon)}{\text{safe}(\forall \Phi \subseteq \varepsilon_1. \hat{\tau}_2 \text{ caps } \varepsilon_2, \varepsilon)} (\text{SAFE-POLYFX}) \\
\\
\frac{\varepsilon_2 \subseteq \varepsilon \quad \text{ho-safe}(\hat{\tau}_1, \varepsilon) \quad \text{safe}([\text{annot}(\text{erase}(\hat{\tau}_1), \emptyset)/X] \hat{\tau}_2, \varepsilon)}{\text{safe}(\forall X <: \hat{\tau}_1. \hat{\tau}_2 \text{ caps } \varepsilon_2, \varepsilon)} (\text{SAFE-POLYTYPE})
\end{array}$$

$\text{ho-safe}(\hat{\tau}, \varepsilon)$

$$\begin{array}{c}
\frac{}{\text{ho-safe}(\{\bar{r}\}, \varepsilon)} (\text{HOSAFE-RESOURCE}) \quad \frac{\text{safe}(\hat{\tau}_1, \varepsilon) \quad \text{ho-safe}(\hat{\tau}_2, \varepsilon)}{\text{ho-safe}(\hat{\tau}_1 \rightarrow_{\varepsilon'} \hat{\tau}_2, \varepsilon)} (\text{HOSAFE-ARROW}) \\
\\
\frac{\varepsilon \subseteq \varepsilon_1 \quad \text{ho-safe}([\emptyset/\Phi] \hat{\tau}_2, \varepsilon)}{\text{ho-safe}(\forall \Phi \subseteq \varepsilon_1. \hat{\tau}_2 \text{ caps } \varepsilon_2, \varepsilon)} (\text{HOSAFE-POLYFX}) \\
\\
\frac{\text{safe}(\hat{\tau}_1, \varepsilon) \quad \text{ho-safe}([\text{annot}(\text{erase}(\hat{\tau}_1), \emptyset)/X] \hat{\tau}_2, \varepsilon)}{\text{ho-safe}(\forall X <: \hat{\tau}_1. \hat{\tau}_2 \text{ caps } \varepsilon_2, \varepsilon)} (\text{HOSAFE-POLYTYPE})
\end{array}$$

$\hat{I} \vdash \hat{\tau} <: \hat{\tau}$

$$\begin{array}{c}
\frac{}{\hat{I} \vdash \hat{\tau} <: \hat{\tau}} \text{ (S-REFLEXIVE)} \quad \frac{}{\hat{I}, X <: \hat{\tau} \vdash X <: \hat{\tau}} \text{ (S-TYPEVAR)} \quad \frac{r \in \overline{r_1} \implies r \in \overline{r_2}}{\hat{I} \vdash \{\overline{r_1}\} <: \{\overline{r_2}\}} \text{ (S-RESOURCESET)} \\
\\
\frac{\hat{I} \vdash \hat{\tau}_1 <: \hat{\tau}_2 \quad \hat{I} \vdash \hat{\tau}_2 <: \hat{\tau}_3}{\hat{I} \vdash \hat{\tau}_1 <: \hat{\tau}_3} \text{ (S-TRANSITIVE)} \quad \frac{\hat{I} \vdash \hat{\tau}'_1 <: \hat{\tau}_1 \quad \hat{I} \vdash \hat{\tau}_2 <: \hat{\tau}'_2 \quad \varepsilon \subseteq \varepsilon'}{\hat{I} \vdash \hat{\tau}_1 \rightarrow_{\varepsilon} \hat{\tau}_2 <: \hat{\tau}'_1 \rightarrow_{\varepsilon'} \hat{\tau}'_2} \text{ (S-ARROW)} \\
\\
\frac{\hat{I} \vdash \hat{\tau}'_1 <: \hat{\tau}_1 \quad \hat{I}, Y <: \hat{\tau}'_1 \vdash \hat{\tau}_2 <: \hat{\tau}'_2 \quad \hat{I}, Y <: \hat{\tau}'_1 \vdash \varepsilon_3 \subseteq \varepsilon'_3}{\hat{I} \vdash (\forall X <: \hat{\tau}_1. \hat{\tau}_2 \text{ caps } \varepsilon_3) <: (\forall Y <: \hat{\tau}'_1. \hat{\tau}_2 \text{ caps } \varepsilon'_3)} \text{ (S-POLYTYPE)} \\
\\
\frac{\hat{I} \vdash \varepsilon' \subseteq \varepsilon \quad \hat{I}, \Phi <: \varepsilon' \vdash \hat{\tau}_1 <: \hat{\tau}'_1 \quad \hat{I}, \Phi \subseteq \varepsilon' \vdash \varepsilon_3 \subseteq \varepsilon'_3}{\hat{I} \vdash (\forall \phi \subseteq \varepsilon. \hat{\tau}_1 \text{ caps } \varepsilon_3) <: (\forall \Phi \subseteq \varepsilon'. \hat{\tau}'_1 \text{ caps } \varepsilon'_3)} \text{ (S-POLYFX)} \\
\\
\boxed{\hat{I} \vdash \varepsilon \subseteq \varepsilon} \\
\\
\frac{r.\pi \in \overline{r.\pi_1} \implies r.\pi \in \overline{r.\pi_2}}{\hat{I} \vdash \{\overline{r.\pi_1}\} \subseteq \{\overline{r.\pi_2}\}} \text{ (S-FXSET)} \quad \frac{}{\hat{I}, \phi \subseteq \varepsilon \vdash \phi \subseteq \varepsilon} \text{ (S-FXVAR)} \\
\\
\frac{}{\hat{I} \vdash \varepsilon \subseteq \varepsilon} \text{ (S-REFLEX)} \quad \frac{\hat{I} \vdash \varepsilon_1 \subseteq \varepsilon_2 \quad \hat{I} \vdash \varepsilon_2 \subseteq \varepsilon_3}{\hat{I} \vdash \varepsilon_1 \subseteq \varepsilon_3} \text{ (S-TRANS)}
\end{array}$$

## 4 Dynamic Rules

$$\boxed{\hat{e} \longrightarrow \hat{e} \mid \varepsilon}$$

$$\begin{array}{c}
\frac{\hat{e}_1 \longrightarrow \hat{e}'_1 \mid \varepsilon}{\hat{e}_1 \hat{e}_2 \longrightarrow \hat{e}'_1 \hat{e}_2 \mid \varepsilon} \text{ (E-APP1)} \quad \frac{\hat{e}_2 \longrightarrow \hat{e}'_2 \mid \varepsilon}{\hat{v}_1 \hat{e}_2 \longrightarrow \hat{v}_1 \hat{e}'_2 \mid \varepsilon} \text{ (E-APP2)} \quad \frac{}{(\lambda x : \hat{\tau}. \hat{e}) \hat{v}_2 \longrightarrow [\hat{v}_2/x] \hat{e} \mid \emptyset} \text{ (E-APP3)} \\
\\
\frac{\hat{e} \rightarrow \hat{e}' \mid \varepsilon}{\hat{e}.\pi \longrightarrow \hat{e}'.\pi \mid \varepsilon} \text{ (E-OPERCALL1)} \quad \frac{r \in R \quad \pi \in \Pi}{r.\pi \longrightarrow \mathbf{unit} \mid \{r.\pi\}} \text{ (E-OPERCALL2)} \\
\\
\frac{\hat{e} \longrightarrow \hat{e}' \mid \varepsilon}{\hat{e} \hat{\tau} \longrightarrow \hat{e}' \hat{\tau} \mid \varepsilon} \text{ (E-POLYTYPEAPP1)} \quad \frac{}{(\lambda X <: \hat{\tau}_1. \hat{e}) \hat{\tau} \longrightarrow [\hat{\tau}/X] \hat{e} \mid \emptyset} \text{ (E-POLYTYPEAPP2)} \\
\\
\frac{\hat{e} \longrightarrow \hat{e}' \mid \varepsilon}{\hat{e} \hat{\tau} \longrightarrow \hat{e}' \hat{\tau} \mid \varepsilon} \text{ (E-POLYFXAPP1)} \quad \frac{}{(\lambda \phi \subseteq \varepsilon_1. \hat{e}) \varepsilon \longrightarrow [\varepsilon/\phi] \hat{e} \mid \emptyset} \text{ (E-POLYFXAPP2)} \\
\\
\frac{\hat{e} \longrightarrow \hat{e}' \mid \varepsilon'}{\mathbf{import}(\varepsilon_s) \ x = \hat{e} \ \mathbf{in} \ e \longrightarrow \mathbf{import}(\varepsilon_s) \ x = \hat{e}' \ \mathbf{in} \ e \mid \varepsilon'} \text{ (E-IMPORT1)} \\
\\
\frac{}{\mathbf{import}(\varepsilon_s) \ x = \hat{e} \ \mathbf{in} \ e \longrightarrow [\hat{v}/x] \mathbf{annot}(e, \varepsilon_s) \mid \emptyset} \text{ (E-IMPORT2)}
\end{array}$$

## 5 Substitution Functions

**Definition** ( $\text{sub} :: \hat{v} \times \hat{v} \rightarrow \hat{e}$ )

1.  $[\hat{v}/y]x = x$ , if  $x \neq y$
2.  $[\hat{v}/y]y = \hat{v}$
3.  $[\hat{v}/y]r = r$
4.  $[\hat{v}/y](\lambda x : \hat{\tau}. \hat{e}) = \lambda x : \hat{\tau}. [\hat{v}/y]\hat{e}$ , if  $y \neq x$  and  $y$  does not occur free in  $\hat{e}$

5.  $[\hat{v}/y](\lambda X <: \hat{\tau}. \hat{e}) = \lambda X <: \hat{\tau}. [\hat{v}/y]\hat{e}$
6.  $[\hat{v}/y](\lambda \phi \subseteq \varepsilon. \hat{e}) = \lambda \phi \subseteq \varepsilon. [\hat{v}/y]\hat{e}$
7.  $[\hat{v}/y](\hat{e}. \pi) = ([\hat{v}/y]\hat{e}_1). \pi$
8.  $[\hat{v}/y](\hat{e}_1 \hat{e}_2) = ([\hat{v}/y]\hat{e}_1) ([\hat{v}/y]\hat{e}_2)$
9.  $[\hat{v}/y](\hat{e} \hat{\tau}) = [\hat{v}/y]\hat{e} \hat{\tau}$
10.  $[\hat{v}/y](\hat{e} \varepsilon) = [\hat{v}/y]\hat{e} \hat{\varepsilon}$
11.  $[\hat{v}/y](\text{import}(\varepsilon_s) \overline{x = \hat{e} \text{ in } e}) = \text{import}(\varepsilon_s) \overline{x = [\hat{v}/y]\hat{e} \text{ in } e}$

**Definition (sub ::  $\hat{\tau} \times \hat{v} \rightarrow \hat{e}$ )**

1.  $[\hat{\tau}/Y]x = x$
2.  $[\hat{\tau}/Y]r = r$
3.  $[\hat{\tau}/Y](\lambda x : \hat{\tau}_1. \hat{e}) = \lambda x : [\hat{\tau}/Y]\hat{\tau}_1. [\hat{\tau}/Y]\hat{e}$
4.  $[\hat{\tau}/Y](\lambda X <: \hat{\tau}_1. \hat{e}) = \lambda X <: [\hat{\tau}/Y]\hat{\tau}_1. [\hat{\tau}/Y]\hat{e}$ , if  $X \neq Y$  and  $Y$  does not occur free in  $\hat{e}$
5.  $[\hat{\tau}/Y](\lambda \phi \subseteq \varepsilon. \hat{e}) = \lambda \phi \subseteq \varepsilon. [\hat{\tau}/Y]\hat{e}$
6.  $[\hat{\tau}/Y](\hat{e}. \pi) = ([\hat{\tau}/Y]\hat{e}_1). \pi$
7.  $[\hat{\tau}/Y](\hat{e}_1 \hat{e}_2) = ([\hat{\tau}/Y]\hat{e}_1) ([\hat{\tau}/Y]\hat{e}_2)$
8.  $[\hat{\tau}/Y](\hat{e} \hat{\tau}_1) = ([\hat{\tau}/Y]\hat{e}) ([\hat{\tau}/Y]\hat{\tau}_1)$
9.  $[\hat{\tau}/Y](\hat{e} \varepsilon) = [\hat{\tau}/Y]\hat{e} \hat{\varepsilon}$
10.  $[\hat{\tau}/Y](\text{import}(\varepsilon_s) \overline{x = \hat{e} \text{ in } e}) = \text{import}(\varepsilon_s) \overline{x = [\hat{\tau}/Y]\hat{e} \text{ in } e}$

**Definition (sub ::  $\hat{\tau} \times \hat{\tau} \rightarrow \hat{e}$ )**

1.  $[\hat{\tau}/Y]Y = \hat{\tau}$
2.  $[\hat{\tau}/Y]X = X$ , if  $X \neq Y$
3.  $[\hat{\tau}/Y]\{\bar{r}\} = \{\bar{r}\}$
4.  $[\hat{\tau}/Y](\hat{\tau}_1 \rightarrow_{\varepsilon} \hat{\tau}_2) = ([\hat{\tau}/Y]\hat{\tau}_1) \rightarrow_{\varepsilon} ([\hat{\tau}/Y]\hat{\tau}_2)$
5.  $[\hat{\tau}/Y](\forall X <: \hat{\tau}_1. \hat{\tau}_2) = \forall X <: [\hat{\tau}/Y]\hat{\tau}_1. [\hat{\tau}/Y]\hat{\tau}_2$ , if  $X \neq Y$  and  $Y$  does not occur free in  $\hat{\tau}_2$
6.  $[\hat{\tau}/Y](\forall \phi \subseteq \varepsilon_1. \hat{e}) = \forall \phi \subseteq \varepsilon_1. [\hat{\tau}/Y]\hat{e}$

**Definition (sub ::  $\varepsilon \times \hat{v} \rightarrow \hat{e}$ )**

1.  $[\varepsilon/\psi]\psi = \varepsilon$
2.  $[\varepsilon/\psi]\phi = \phi$ , if  $\psi \neq \phi$
3.  $[\varepsilon/\psi](\lambda x : \hat{\tau}_1. \hat{e}) = \lambda x : [\varepsilon/\psi]\hat{\tau}_1. [\varepsilon/\psi]\hat{e}$
4.  $[\varepsilon/\psi](\lambda X <: \hat{\tau}_1. \hat{e}) = \lambda X <: [\varepsilon/\psi]\hat{\tau}_1. [\varepsilon/\psi]\hat{e}$
5.  $[\varepsilon/\psi](\lambda \phi \subseteq \varepsilon_1. \hat{e}) = \lambda \phi \subseteq [\varepsilon/\psi]\varepsilon_1. [\varepsilon/\psi]\hat{e}$
6.  $[\varepsilon/\psi](\hat{e}. \pi) = ([\varepsilon/\psi]\hat{e}_1). \pi$
7.  $[\varepsilon/\psi](\hat{e}_1 \hat{e}_2) = ([\varepsilon/\psi]\hat{e}_1) ([\varepsilon/\psi]\hat{e}_2)$
8.  $[\varepsilon/\psi](\hat{e} \hat{\tau}) = ([\varepsilon/\psi]\hat{e}) ([\varepsilon/\psi]\hat{\tau})$
9.  $[\varepsilon/\psi](\hat{e} \varepsilon_1) = ([\varepsilon/\psi]\hat{e}) ([\varepsilon/\psi]\varepsilon_1)$
10.  $[\varepsilon/\psi](\text{import}(\varepsilon_s) \overline{x = \hat{e} \text{ in } e}) = \text{import}([\varepsilon/\psi]\varepsilon_s) \overline{x = [\varepsilon/\psi]\hat{e} \text{ in } e}$

**Definition (sub ::  $\hat{e} \times \hat{\tau} \rightarrow \hat{e}$ )**

1.  $[\varepsilon/\psi]X = X$
2.  $[\varepsilon/\psi]\{\bar{r}\} = \{\bar{r}\}$
3.  $[\varepsilon/\psi](\hat{\tau}_1 \rightarrow_{\varepsilon_1} \hat{\tau}_2) = ([\varepsilon/\psi]\hat{\tau}_1) \rightarrow_{[\varepsilon/\psi]\varepsilon_1} ([\varepsilon/\psi]\hat{\tau}_2)$
4.  $[\varepsilon/\psi](\forall X <: \hat{\tau}_1. \hat{\tau}_2) = \forall X <: [\varepsilon/\psi]\hat{\tau}_1. [\varepsilon/\psi]\hat{\tau}_2$
5.  $[\varepsilon/\psi](\forall \phi \subseteq \varepsilon_1. \hat{e}) = \forall \phi \subseteq [\varepsilon/\psi]\varepsilon_1. [\varepsilon/\psi]\hat{e}$ , if  $\psi \neq \phi$  and  $\psi$  does not occur free in  $\hat{e}$

**Definition (sub ::  $\varepsilon \times \varepsilon \rightarrow \hat{e}$ )**

1.  $[\varepsilon/\psi]\psi = \varepsilon$
2.  $[\varepsilon/\psi]\phi = \phi$ , if  $\phi \neq \psi$
3.  $[\varepsilon/\psi]\{\bar{r}. \bar{\pi}\} = \{\bar{r}. \bar{\pi}\}$