

1 Grammar

$e ::=$	exprs.	$\tau ::=$	types
x	<i>variable</i>	X	<i>type variable</i>
v	<i>value</i>	$\{\bar{r}\}$	<i>effect set</i>
$e e$	<i>application</i>	$\tau \rightarrow \tau$	<i>arrow</i>
$e.\pi$	<i>operation call</i>	$\forall X.\tau$	<i>universal type</i>
$\lambda X.e$	<i>type polymorphism</i>	$\forall \phi.\tau$	<i>universal effect set</i>
$e \tau$	<i>type application</i>		
$v ::=$	values	$\hat{\tau} ::=$	annotated types
r	<i>resource literal</i>	t	<i>type variable</i>
$\lambda x : \tau.e$	<i>abstraction</i>	$\{\bar{r}\}$	<i>resource set</i>
		$\hat{\tau} \rightarrow_{\varepsilon} \hat{\tau}$	<i>annotated arrow</i>
		$\forall X.\hat{\tau}$	<i>universal type</i>
		$\forall \phi.\hat{\tau}$	<i>universal effect set</i>
$\hat{e} ::=$	annotated exprs.	$\varepsilon ::=$	effects
x	<i>variable</i>	ϕ	<i>effect variable</i>
\hat{v}	<i>value</i>	$\{\bar{r}.\pi\}$	<i>effect set</i>
$\hat{e} \hat{e}$	<i>application</i>		
$\hat{e}.\pi$	<i>operation call</i>	$\Gamma ::=$	contexts
$\lambda X.\hat{e}$	<i>type polymorphism</i>	\emptyset	<i>empty ctx.</i>
$\lambda \phi.\hat{e}$	<i>effect polymorphism</i>	$\Gamma, x : \tau$	<i>var. binding</i>
$e \tau$	<i>type application</i>	Γ, X	<i>type var. binding</i>
$e \varepsilon$	<i>effect application</i>		
$\text{import}(\varepsilon_s) x = \hat{e} \text{ in } e$	<i>import</i>	$\hat{\Gamma} ::=$	annotated contexts
		\emptyset	<i>empty ctx.</i>
$\hat{v} ::=$	annotated values	$\hat{\Gamma}, x : \hat{\tau}$	<i>var. binding</i>
r	<i>resource literal</i>	$\hat{\Gamma}, X$	<i>type var. binding</i>
$\lambda x : \hat{\tau}.\hat{e}$	<i>abstraction</i>	$\hat{\Gamma}, \phi$	<i>effect var. binding</i>

2 Functions

Definition ($\text{annot} :: \tau \times \varepsilon \rightarrow \hat{\tau}$)

1. $\text{annot}(X, _) = X$
2. $\text{annot}(\{\bar{r}\}, _) = \{\bar{r}\}$
3. $\text{annot}(\tau_1 \rightarrow \tau_2, \varepsilon) = \text{annot}(\tau_1, \varepsilon) \rightarrow_{\varepsilon} \text{annot}(\tau_2, \varepsilon)$
4. $\text{annot}(\forall X.\tau, \varepsilon) = \forall X.\text{annot}(\tau, \varepsilon)$
5. $\text{annot}(\forall \phi.\tau, \varepsilon) = \forall \phi.\text{annot}(\tau, \varepsilon)$

Definition ($\text{annot} :: e \times \varepsilon \rightarrow \hat{e}$)

1. $\text{annot}(x, _) = x$
2. $\text{annot}(r, _) = r$
3. $\text{annot}(\lambda x : \tau.e, \varepsilon) = \lambda x : \text{annot}(\tau, \varepsilon).\text{annot}(e, \varepsilon)$
4. $\text{annot}(e_1 e_2, \varepsilon) = \text{annot}(e_1, \varepsilon) \text{ annot}(e_2, \varepsilon)$
5. $\text{annot}(e.\pi, \varepsilon) = \text{annot}(e, \varepsilon).\pi$
6. $\text{annot}(\lambda X.e, \varepsilon) = \lambda X.\text{annot}(e, \varepsilon)$
7. $\text{annot}(e \tau, \varepsilon) = \text{annot}(e, \varepsilon) \text{ annot}(\tau, \varepsilon)$

Definition ($\text{annot} :: \Gamma \times \varepsilon \rightarrow \hat{\Gamma}$)

1. $\text{annot}(\emptyset, _) = \emptyset$
2. $\text{annot}((\Gamma, x : \tau), \varepsilon) = \text{annot}(\Gamma, \varepsilon), x : \text{annot}(\tau, \varepsilon)$
3. $\text{annot}((\Gamma, X), \varepsilon) = \text{annot}(\Gamma, \varepsilon), X$

Definition ($\text{erase} :: \hat{\tau} \rightarrow \tau$)

1. $\text{erase}(X) = X$
2. $\text{erase}(\{\bar{r}\}) = \{\bar{r}\}$
3. $\text{erase}(\hat{\tau}_1 \rightarrow_{\varepsilon} \hat{\tau}_2) = \text{erase}(\hat{\tau}_1) \rightarrow \text{erase}(\hat{\tau}_2)$
4. $\text{erase}(\forall X. \hat{\tau}) = \forall X. \text{erase}(\hat{\tau})$

Definition ($\text{erase} :: \hat{e} \rightarrow e$)

1. $\text{erase}(x) = x$
2. $\text{erase}(r) = r$
3. $\text{erase}(\lambda x : \hat{\tau}. \hat{e}) = \lambda x : \text{erase}(\hat{\tau}). \text{erase}(\hat{e})$
4. $\text{erase}(\hat{e}_1 \hat{e}_2) = \text{erase}(\hat{e}_1) \text{erase}(\hat{e}_2)$
5. $\text{erase}(\hat{e}. \pi) = \text{erase}(\hat{e}). \pi$
6. $\text{erase}(\lambda X. \hat{e}) = \lambda X. \text{erase}(\hat{e})$

Definition ($\text{erase} :: \hat{\Gamma} \rightarrow \Gamma$)

1. $\text{erase}(\emptyset) = \emptyset$
2. $\text{erase}(\hat{\Gamma}, x : \hat{\tau}) = \text{erase}(\hat{\Gamma}), x : \text{erase}(\hat{\tau})$
3. $\text{erase}(\hat{\Gamma}, X) = \text{erase}(\hat{\Gamma}), X$

Definition ($\text{effects} :: \hat{\tau} \rightarrow \varepsilon$)

1. $\text{effects}(X) = \emptyset$
2. $\text{effects}(\{\bar{r}\}) = \{r. \pi \mid r \in \bar{r}, \pi \in \Pi\}$
3. $\text{effects}(\hat{\tau}_1 \rightarrow_{\varepsilon} \hat{\tau}_2) = \text{ho-effects}(\hat{\tau}_1) \cup \varepsilon \cup \text{effects}(\hat{\tau}_2)$

Definition ($\text{ho-effects} :: \hat{\tau} \rightarrow \varepsilon$)

1. $\text{ho-effects}(t) = \emptyset$
2. $\text{ho-effects}(\{\bar{r}\}) = \emptyset$
3. $\text{ho-effects}(\hat{\tau}_1 \rightarrow_{\varepsilon} \hat{\tau}_2) = \text{effects}(\hat{\tau}_1) \cup \text{ho-effects}(\hat{\tau}_2)$

Definition ($\text{substitution} :: \hat{e} \times \hat{v} \times \hat{v} \rightarrow \hat{e}$)

The notation $[\hat{v}/x]\hat{e}$ is short-hand for $\text{substitution}(\hat{e}, \hat{v}, x)$. This function is partial, because the third input must be a variable. We adopt the usual renaming conventions to avoid accidental capture.

1. $[\hat{v}/y]x = \hat{v}$, if $x = y$
2. $[\hat{v}/y]x = x$, if $x \neq y$
3. $[\hat{v}/y](\lambda x : \hat{\tau}. \hat{e}) = \lambda x : \hat{\tau}. [\hat{v}/y]\hat{e}$, if $y \neq x$ and y does not occur free in \hat{e}
4. $[\hat{v}/y](\hat{e}_1 \hat{e}_2) = ([\hat{v}/y]\hat{e}_1)([\hat{v}/y]\hat{e}_2)$
5. $[\hat{v}/y](\hat{e}. \pi) = ([\hat{v}/y]\hat{e}). \pi$
6. $[\hat{v}/y](\lambda X. \hat{e}) = \lambda X. [\hat{v}/y]\hat{e}$
7. $[\hat{v}/y](\lambda \phi. \hat{e}) = \lambda \phi. [\hat{v}/y]\hat{e}$
8. $[\hat{v}/y](\hat{e} \hat{\tau}) = [\hat{v}/y]\hat{e} \hat{\tau}$
9. $[\hat{v}/y](\hat{e} \varepsilon) = [\hat{v}/y]\hat{e} \varepsilon$
10. $[\hat{v}/y](\text{import}(\varepsilon_s) x = \hat{e} \text{ in } e) = \text{import}(\varepsilon_s) x = [\hat{v}/y]\hat{e} \text{ in } e$

When performing multiple substitutions the notation $[\hat{v}_1/x_1, \hat{v}_2/x_2]\hat{e}$ is used as shorthand for $[\hat{v}_2/x_2]([\hat{v}_1/x_1]\hat{e})$ (note the order of the variables has been flipped; the substitutions occur as they are written, left-to-right).

Definition (substitution :: $\hat{\tau} \times \hat{\tau} \times \hat{\tau} \rightarrow \hat{\tau}$)

1. $[\hat{\tau}/Y]X = \hat{\tau}$, if $X = Y$
2. $[\hat{\tau}/Y]X = X$, if $X \neq Y$
3. $[\hat{\tau}/Y]\{\bar{r}\} = \{\bar{r}\}$
4. $[\hat{\tau}/Y](\hat{\tau}_1 \rightarrow_{\varepsilon} \hat{\tau}_2) = [\hat{\tau}/Y]\hat{\tau}_1 \rightarrow_{\varepsilon} [\hat{\tau}/Y]\hat{\tau}_2$
5. $[\hat{\tau}/Y](\forall X.\hat{\tau}_1) = \forall X.[\hat{\tau}/Y]\tau_1$, if $Y \neq X$ and Y does not occur free in τ_1
6. $[\hat{\tau}/Y](\forall \phi.\hat{\tau}_1) = \forall \phi.[\hat{\tau}/Y]\tau_1$

Definition (substitution :: $\varepsilon \times \varepsilon \times \varepsilon \rightarrow \hat{\tau}$)

1. $[\varepsilon/\phi]\Phi = \varepsilon$, if $\phi = \Phi$
2. $[\varepsilon/\phi]\Phi = \Phi$, if $\phi \neq \Phi$
3. $[\varepsilon/\phi]\{\bar{r}.\bar{\pi}\} = \{\bar{r}.\bar{\pi}\}$

Definition (substitution :: $\hat{\tau} \times \varepsilon \times \varepsilon \rightarrow \hat{\tau}$)

1. $[\varepsilon/\phi]X = X$
2. $[\varepsilon/\phi]\{\bar{r}\} = \{\bar{r}\}$
3. $[\varepsilon/\phi](\hat{\tau}_1 \rightarrow'_{\varepsilon} \hat{\tau}_2) = [\varepsilon/\phi]\hat{\tau}_1 \rightarrow_{[\varepsilon/\phi]\varepsilon'} [\varepsilon/\phi]\hat{\tau}_2$
4. $[\varepsilon/\phi](\forall X.\hat{\tau}) = \forall X.[\varepsilon/\phi]\hat{\tau}$
5. $[\varepsilon/\phi](\forall \Phi.\hat{\tau}) = \forall \Phi.[\varepsilon/\phi]\hat{\tau}$, if $\phi \neq \Phi$ and ϕ does not occur free in $\hat{\tau}$

3 Static Rules

$$\boxed{\Gamma \vdash e : \tau}$$

$$\begin{array}{c} \frac{}{\Gamma, x : \tau \vdash x : \tau} \text{ (T-VAR)} \quad \frac{}{\Gamma, r : \{r\} \vdash r : \{r\}} \text{ (T-RESOURCE)} \quad \frac{\Gamma \vdash e : \{\bar{r}\}}{\Gamma \vdash e.\pi : \mathbf{Unit}} \text{ (T-OPERCALL)} \\[10pt] \frac{\Gamma, x : \tau_1 \vdash e : \tau_2}{\Gamma \vdash \lambda x : \tau_1. e : \tau_1 \rightarrow \tau_2} \text{ (T-ABS)} \quad \frac{\Gamma \vdash e_1 : \tau_2 \rightarrow \tau_3 \quad \Gamma \vdash e_2 : \tau_2}{\Gamma \vdash e_1 e_2 : \tau_3} \text{ (T-APP)} \\[10pt] \frac{\Gamma, X \vdash e : \tau}{\Gamma \vdash \lambda X. e : \tau} \text{ (T-POLYTYPEABS)} \quad \frac{\Gamma \vdash e : \forall X. \tau}{\Gamma \vdash e : \tau' : [\tau'/X]\tau} \text{ (T-POLYTYPEAPP)} \end{array}$$

$$\boxed{\hat{\Gamma} \vdash \hat{e} : \hat{\tau} \text{ with } \varepsilon}$$

$$\begin{array}{c} \frac{}{\hat{\Gamma}, x : \tau \vdash x : \tau \text{ with } \emptyset} (\varepsilon\text{-VAR}) \quad \frac{}{\hat{\Gamma}, r : \{r\} \vdash r : \{r\} \text{ with } \emptyset} (\varepsilon\text{-RESOURCE}) \\[10pt] \frac{\hat{\Gamma} \vdash \hat{e} : \{\bar{r}\}}{\hat{\Gamma} \vdash \hat{e}.\pi : \mathbf{Unit} \text{ with } \{r.\pi \mid r \in \bar{r}\}} (\varepsilon\text{-OPERCALL}) \quad \frac{\hat{\Gamma} \vdash e : \tau \text{ with } \varepsilon \quad \tau <: \tau' \quad \varepsilon \subseteq \varepsilon'}{\hat{\Gamma} \vdash e : \tau' \text{ with } \varepsilon'} (\varepsilon\text{-SUBSUME}) \\[10pt] \frac{\hat{\Gamma}, x : \hat{\tau}_2 \vdash \hat{e} : \hat{\tau}_3 \text{ with } \varepsilon_3}{\hat{\Gamma} \vdash \lambda x : \tau_2. \hat{e} : \hat{\tau}_2 \rightarrow_{\varepsilon_3} \hat{\tau}_3 \text{ with } \emptyset} (\varepsilon\text{-ABS}) \quad \frac{\hat{\Gamma} \vdash \hat{e}_1 : \hat{\tau}_2 \rightarrow_{\varepsilon} \hat{\tau}_3 \text{ with } \varepsilon_1 \quad \hat{\Gamma} \vdash \hat{e}_2 : \hat{\tau}_2 \text{ with } \varepsilon_2}{\hat{\Gamma} \vdash \hat{e}_1 \hat{e}_2 : \hat{\tau}_3 \text{ with } \varepsilon_1 \cup \varepsilon_2 \cup \varepsilon} (\varepsilon\text{-APP}) \\[10pt] \frac{\hat{\Gamma}, X \vdash \hat{e} : \hat{\tau} \text{ with } \varepsilon}{\hat{\Gamma} \vdash \lambda X. \hat{e} : \hat{\tau} \text{ with } \emptyset} (\varepsilon\text{-POLYTYPEABS}) \quad \frac{\hat{\Gamma} \vdash e : \forall X. \hat{\tau} \text{ with } \varepsilon_1}{\hat{\Gamma} \vdash e : \tau' : [\hat{\tau}'/X]\hat{\tau} \text{ with } \varepsilon_1 \cup \mathbf{effects}(\hat{\tau}')} (\varepsilon\text{-POLYTYPEAPP}) \\[10pt] \frac{\mathbf{effects}(\hat{\tau}) \cup \mathbf{ho-effects}(\mathbf{annot}(\tau, \emptyset)) \subseteq \varepsilon \quad \hat{\Gamma} \vdash \hat{e} : \hat{\tau} \text{ with } \varepsilon_1 \quad \mathbf{ho-safe}(\hat{\tau}, \varepsilon) \quad x : \mathbf{erase}(\hat{\tau}) \vdash e : \tau}{\hat{\Gamma} \vdash \mathbf{import}(\varepsilon) \ x = \hat{e} \text{ in } e : \mathbf{annot}(\tau, \varepsilon) \text{ with } \varepsilon \cup \varepsilon_1} (\varepsilon\text{-IMPORT}) \end{array}$$

$\text{safe}(\tau, \varepsilon)$

$$\frac{}{\text{safe}(\{\bar{r}\}, \varepsilon)} \text{ (SAFE-RESOURCE)} \quad \frac{}{\text{safe}(\text{Unit}, \varepsilon)} \text{ (SAFE-UNIT)}$$

$$\frac{\varepsilon \subseteq \varepsilon' \quad \text{ho-safe}(\hat{\tau}_1, \varepsilon) \quad \text{safe}(\hat{\tau}_2, \varepsilon)}{\text{safe}(\hat{\tau}_1 \rightarrow_{\varepsilon'} \hat{\tau}_2, \varepsilon)} \text{ (SAFE-ARROW)}$$

$\text{ho-safe}(\hat{\tau}, \varepsilon)$

$$\frac{}{\text{ho-safe}(\{\bar{r}\}, \varepsilon)} \text{ (HOSAFE-RESOURCE)} \quad \frac{}{\text{ho-safe}(\text{Unit}, \varepsilon)} \text{ (HOSAFE-UNIT)}$$

$$\frac{\text{safe}(\hat{\tau}_1, \varepsilon) \quad \text{ho-safe}(\hat{\tau}_2, \varepsilon)}{\text{ho-safe}(\hat{\tau}_1 \rightarrow_{\varepsilon'} \hat{\tau}_2, \varepsilon)} \text{ (HOSAFE-ARROW)}$$

$\hat{\tau} <: \hat{\tau}$

$$\frac{\varepsilon \subseteq \varepsilon' \quad \hat{\tau}_2 <: \hat{\tau}'_2 \quad \hat{\tau}'_1 <: \hat{\tau}_1}{\hat{\tau}_1 \rightarrow_{\varepsilon} \hat{\tau}_2 <: \hat{\tau}'_1 \rightarrow_{\varepsilon'} \hat{\tau}'_2} \text{ (S-EFFECTS)} \quad \frac{r \in \bar{r}_2 \implies r \in \bar{r}_1}{\{\bar{r}_2\} <: \{\bar{r}_1\}} \text{ (S-RESOURCESET)}$$

4 Dynamic Rules

$\hat{e} \longrightarrow \hat{e} \mid \varepsilon$

$$\frac{\hat{e}_1 \longrightarrow \hat{e}'_1 \mid \varepsilon}{\hat{e}_1 \hat{e}_2 \longrightarrow \hat{e}'_1 \hat{e}_2 \mid \varepsilon} \text{ (E-APP1)} \quad \frac{\hat{e}_2 \longrightarrow \hat{e}'_2 \mid \varepsilon}{\hat{v}_1 \hat{e}_2 \longrightarrow \hat{v}_1 \hat{e}'_2 \mid \varepsilon} \text{ (E-APP2)} \quad \frac{}{(\lambda x : \hat{\tau}. \hat{e}) \hat{v}_2 \longrightarrow [\hat{v}_2/x] \hat{e} \mid \emptyset} \text{ (E-APP3)}$$

$$\frac{\hat{e} \rightarrow \hat{e}' \mid \varepsilon}{\hat{e}. \pi \longrightarrow \hat{e}'. \pi \mid \varepsilon} \text{ (E-OPERCALL1)} \quad \frac{r \in R \quad \pi \in \Pi}{r. \pi \longrightarrow \text{unit} \mid \{r. \pi\}} \text{ (E-OPERCALL2)}$$

$$\frac{\hat{e} \longrightarrow \hat{e}' \mid \varepsilon}{\hat{e} \hat{\tau} \longrightarrow \hat{e}' \hat{\tau} \mid \varepsilon} \text{ (E-POLYTYPEAPP1)} \quad \frac{}{(\lambda X. \hat{e}) \hat{\tau} \longrightarrow [\hat{\tau}/X] \hat{e} \mid \emptyset} \text{ (E-POLYTYPEAPP2)}$$

$$\frac{\hat{e} \longrightarrow \hat{e}' \mid \varepsilon}{\hat{e} \hat{\tau} \longrightarrow \hat{e}' \hat{\tau} \mid \varepsilon} \text{ (E-POLYFXAPP1)} \quad \frac{}{(\lambda \phi. \hat{e}) \varepsilon \longrightarrow [\varepsilon/\phi] \hat{e} \mid \emptyset} \text{ (E-POLYFXAPP2)}$$

$$\frac{\hat{e} \longrightarrow \hat{e}' \mid \varepsilon'}{\text{import}(\varepsilon_s) x = \hat{e} \text{ in } e \longrightarrow \text{import}(\varepsilon_s) x = \hat{e}' \text{ in } e \mid \varepsilon'} \text{ (E-IMPORT1)}$$

$$\frac{}{\text{import}(\varepsilon_s) x = \hat{e} \text{ in } e \longrightarrow [\hat{v}/x] \text{annot}(e, \varepsilon_s) \mid \emptyset} \text{ (E-IMPORT2)}$$

5 Encodings

5.1 \perp

The bottom type is defined as $\perp \stackrel{\text{def}}{=} \emptyset$, which is the literal for an empty set of resources.

$$\frac{}{\Gamma \vdash \perp : \emptyset} \text{ (T-}\perp\text{)} \quad \frac{}{\Gamma \vdash \perp : \emptyset \text{ with } \emptyset} \text{ (}\varepsilon\text{-}\perp\text{)}$$

5.2 unit, Unit

Define $\mathbf{unit} = \lambda x : \emptyset. x$, i.e. the function which takes an empty set of resources and returns it. We shall refer to its type, which is $\emptyset \rightarrow_{\emptyset} \emptyset$, as \mathbf{Unit} . It has various properties befitting \mathbf{unit} .

1. \mathbf{unit} cannot be invoked as \emptyset is uninhabited.
2. \mathbf{unit} is a value.
3. The only term with type \mathbf{Unit} is \mathbf{unit} .
4. $\vdash \mathbf{unit} : \mathbf{Unit}$ by using ε -ABS and ε -VAR.
5. $\mathbf{effects}(\mathbf{Unit}) = \mathbf{ho-effects}(\mathbf{Unit}) = \emptyset$
6. $\mathbf{safe}(\mathbf{Unit}, \varepsilon)$ and $\mathbf{ho-safe}(\mathbf{Unit}, \varepsilon)$

$$\frac{}{\Gamma \vdash \mathbf{unit} : \mathbf{Unit}} \text{ (T-UNIT)} \qquad \frac{}{\Gamma \vdash \mathbf{unit} : \mathbf{Unit} \text{ with } \emptyset} \text{ (\varepsilon-UNIT)}$$