training Data : D

$(x, y_D) \in D$

ture label: y.

$f(x;D)$ Model trained on D, Model outcom for input x

Expected. Prediction $\bar{f}(x) = E_D[f(x;D)]$

Variance due to different training Data. $E_D[(f(x;D) - \bar{f}(x))^2]$

Noise $\varepsilon^2 = E_D[(y_D - y)^2]$

$y_D - y = \varepsilon$

$\text{bias}^2(x) = (\bar{f}(x) - y)^2$

$E_D(y_D - y) = 0$

Expect. Test Error

$$E_D[(f(x;D) - y_D)^2]$$

$$= E_D[(f(x;D) - \bar{f}(x) + \bar{f}(x) - y_D)^2]$$

$$= \underbrace{E_D[(f(x;D) - \bar{f}(x))^2]}_{\text{variance}} + E_D[(\bar{f}(x) - y_D)^2]$$

$$\Downarrow$$

$$E_D[(\bar{f}(x) - y + y - y_D)^2]$$

$$= E_D[(\bar{f}(x) - y)^2] + E_D[(y - y_D)^2]$$

$$= \underbrace{(\bar{f}(x) - y)^2}_{\Downarrow\ \text{Bias}^2} + \underbrace{E_D[(y - y_D)^2]}_{\Downarrow\ \varepsilon^2}$$