Literature Survey

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April 22, 2022

1 Baseline

Our project's target is not necessarily to produce an algorithm that improves performance, but instead to provide a tool that can be used to help developers make better informed decisions when choosing an architecture to run a circuit on. As a result, our more fundamental sources detail approaches for mapping circuits to target architectures and estimating circuit fidelity.

1.1 $t\,|ket\rangle$: A Retargetable Compiler for NISQ Devices

URL: `https://arxiv.org/abs/2003.10611`

This is the white paper for $t\,|ket\rangle$, a language-agnostic compiler for NISQ devices. The authors provide a synopsis of the compiler detailing its design, circuit representation, optimizations, qubit mapping techniques (including noise aware placement), and benchmarking results for compilation and noise aware placement. At 43 pages, the paper provides a great overview of a quantum compiler design. An additional paper, On the qubit routing problem, provides a more detailed description of the heuristic method used by $t\,|ket\rangle$ to determine qubit mapping.

We plan on utilizing $t\,|ket\rangle$ and its techniques as a way to reduce the engineering burden required for this project by taking advantage of its efficient qubit mapping and circuit generation techniques to generate quantum circuits for candidate machines. Our tool can then evaluate the fidelity of these circuits to determine an optimal machine choice for the proposed circuit.

The authors of the paper discuss some of the improvements that could be made to $t\,|ket\rangle$, being mostly improvements to its optimization techniques and depth management. The most relevant weakness to $t\,|ket\rangle$ that we can determine at the time of writing is the naive noise model the system employs. This could result in sub-optimal qubit placement if we are unable to properly communicate the machine characterization data to the compiler. Further examination and revision of the noise model in $t\,|ket\rangle$ could be another way in which we could fine tune the tool we are developing.

1.2 Error rates in quantum circuits

URL: `https://arxiv.org/pdf/1511.00727.pdf`

While we are strongly leaning towards utilizing $t\,|ket\rangle$ as our primary framework, this paper details an initial circuit fidelity metric we could use when calculating the value of a circuit/machine pairing.

Diamond distance, the worst-case error of a quantum circuit, seems like a promising metric to report when evaluating a circuit. However, this work explains that the diamond distance is difficult to efficiently estimate and may be overly pessimistic compared to the actual circuit error rate. Instead, the author details an efficient approach to estimate upper and lower bounds for the diamond distance and shows that the average circuit error rate is proportional to the diamond distance. Calculating these bounds could be one approach for our project when reporting predicted error rates.

There are some potential difficulties with this approach. Since we are calculating the bounds of the potential error, we will need to determine a way to effectively communicate this information to the developer in a form that is meaningful. Additionally, if the range between the bounds is too large we may not be able to determine any advantage to running the given quantum circuit on one machine as opposed to another. Lastly, we may determine that this metric is not what we want to utilize for our tool. As a backup, the paper Quantum Fidelity measures for mixed states details a wide variety of fidelity metrics alongside many referenced papers detailing techniques that could be used as alternatives.

2 Circuit/Noise Simulation

2.1 Modelling and Simulating the Noisy Behaviour of Near-term Quantum Computers

URL: `https://journals.aps.org/pra/abstract/10.1103/PhysRevA.104.062432`

This paper proposes a method for simulating a quantum program by combining three error groups, which are the main sources of noise in quantum computing, and using a calibrated error rate. It is pretty recent work, and it seems to work well. This will also serve as a reference for a simulation algorithm. Having knowledge of quantum noise simulation models could be beneficial if we decide to incorporate a system for using simulated noise in the absence of characterization data for a machine.

2.2 Modeling quantum noise for efficient testing of fault-tolerant circuits

URL: `https://journals.aps.org/pra/abstract/10.1103/PhysRevA.87.012324`

This paper proposes a method to efficiently test the quantum circuit with noise model through Monte Carlo simulation. As we have seen in our previous assignments, noise simulation can greatly increase the computational complexity of a quantum simulation. This paper could help us figure out how to simulate the noise model in reasonable time.

2.3 Stochastic Quantum Circuit Simulation Using Decision Diagrams

URL: `https://ieeexplore.ieee.org/abstract/document/9474135`

This paper proposes a way to reduce resources for quantum circuit simulation, using decision diagrams. Most simulation methods simplify the problem a lot, but this method reflects various errors that can occur in real quantum computing. It might be helpful to implement our simulation.

2.4 Noise-Adaptive Compiler Mappings for Noisy Intermediate-Scale Quantum Computers

URL: `https://dl.acm.org/doi/abs/10.1145/3297858.3304075`

This paper proposes and evaluates various back end compiler algorithms that can be applied to NISQ with diverse hardware characteristics, from LLVM IR to a quantum program that can be executed on an actual IBM machine. In addition, this paper evaluates various methods through the implemented framework. This will help us figure out how to deal with multiple layers in NISQ.

2.5 Simulating Noisy Quantum Circuits with Matrix Product Density Operators

URL: `https://arxiv.org/abs/2004.02388`

The paper provides a method to simulate random quantum circuits in 1D with Matrix Product Density Operators (MPDO). It allows to test circuits for various noise models such as dephasing, depolarizing, and amplitude damping. This is a follow-up research based on Matrix Product Operators (MPO). MPO extend the error model to introduce random noise in an more concise way, and the author claims that MPDO reflects clearer physical picture of noise and quantum entanglement than MPO.

3 Qubit Mapping/Routing

3.1 Tackling the Qubit Mapping Problem for NISQ-Era Quantum Devices

URL: `https://dl.acm.org/doi/abs/10.1145/3297858.3304023`

This paper proposes a SWAP-based heuristic search algorithm as a solution to the qubit mapping problem. The heuristic aims to find a solution to qubit mapping problem with the objectives: 1. flexibility, 2. fidelity, 3. parallelism and 4. scalability. The preprocessing finds a distance matrix indicates the number of SWAPs required, and form a DAG represents the execution constraints. It claims that the method is applicable to NISQ devices with arbitrary connections between qubits.

3.2 On the qubit routing problem

URL: `https://arxiv.org/abs/1902.08091`

Architecture-agnostic methodology for mapping quantum circuits to realistic quantum computing devices. This is the method used in $t\,|ket\rangle$. Goal is to globally optimize the circuit. Four steps to algorithm: Slice Circuit: Circuit is divided into time steps (slices) comprising gates that act on disjoint sets of qubits. Determine initial qubit placement: Represent the problem as a graph and iterate over time steps, using qubits as vertices and edges as interactions. Each vertex can have at most two edges. A sub-graph with high average degree and low diameter is used as initial mapping. On a Hamiltonian connected architecture this usually means the first two time steps require no swaps (gates on edges removed when breaking rings are an exception). Route across time steps: Iterate over time steps. Any unmapped qubits are placed on nodes closest to their partner nodes. Gates that can be executed with the current mapping are output to the circuit. If any gates remain, swaps are required. Final cleanup: If SWAP operations are not supported they are replaced with hardware appropriate gates. Any redundant gates are pruned.

The authors find the approach used by $t\,|ket\rangle$ to be considerably faster and more scalable to circuit size than other compiler techniques while still achieving equivalent or better results for circuit depth and total gate count.

The paper additionally describes some architecture graphs seen in quantum machines such as ring, cyclic butterfly, and square grid graphs.

3.3 Qubit allocation

URL: `https://dl.acm.org/doi/abs/10.1145/3168822`

Describes an optimal algorithm for the qubit allocation problem, however this approach does not scale well so the authors provide a heuristic approach as well. Details the Qubit Allocation Problem (similar to the Register Assignment Problem) and Swap Minimization Problems (Token Swapping Problem) as being NP-complete.

- **The Qubit Allocation Problem**: Given a coupling graph, a list of control relations between qubits, and an integer K, and list of allowed quantum transformations with their costs, determine if we can produce a mapping which satisfies the coupling graph and whose cost does not exceed K.

The author's algorithm utilizes memoization and dynamic programming techniques to optimize the computation, however the algorithm suffers from an exponential runtime and is impractical for larger graphs. The authors offer a heuristic algorithm instead that achieves results near the optimal but at more practical run times.

- In the initial mapping, preference is given to qubits that appear frequently in the list of qubit interactions. Swaps are utilized if qubit interactions cannot be satisfied by the current mapping and follow the shortest path required to satisfy the interaction.

Compared to the work by Cowtan et al, the heuristic in this paper appears to approach the problem from a functional perspective. This work tackles the problem by giving functions that create an initial mapping and that return a circuit solution for a given initial mapping and creating swaps when conflicts occur in the interaction set. The approach by $t\,|ket\rangle$ in comparison appears to be more iterative and centers on generating the circuit over the course of the time steps.

4 Variational Algorithms, Fidelity Calculation

4.1 Noise resilience of variational quantum compiling

URL: `https://iopscience.iop.org/article/10.1088/1367-2630/ab784c/meta`

Details the noise resilience of variational hybrid quantum-classical algorithms (VHQCA) used for quantum compiling. A VHQCA uses a classical machine for optimizing a cost function while evaluating the function on a quantum machine. This allows the algorithm to take advantage of the computational power of the quantum machine and the error resilience of the classical system. Recent work has been done on utilizing VHQCA for variational quantum compiling (VQC) but there is concern about the target unitary generated from the compiler being impacted by noise. However, the authors demonstrate that VQC is resilient to measurement noise and resistant to gate noise in portions of the cost evaluation circuit.

While the work is not directly related to our project, this paper illustrates some of the cutting edge approaches to quantum computing and provides references to additional sources that cover VHQCA which may be useful if we consider using quantum circuits in our work.

4.2 Variational Quantum Fidelity Estimation

URL: `https://quantum-journal.org/papers/q-2020-03-26-248/`

Proposes an algorithm for estimating lower and upper bounds for quantum state fidelity by utilizing VHQCA techniques. The authors demonstrate that this method typically outperforms the preexisting quantum algorithm which computes fixed sub/super fidelity bounds (SSFB). This work demonstrates a potential use case of VHQCA relevant to our project.

4.3 Quantum fidelity measures for mixed states

URL: `https://iopscience.iop.org/article/10.1088/1361-6633/ab1ca4/meta`

The authors discuss the challenges of calculating fidelity for mixed quantum states. A side effect of this is that the paper details a variety of metrics for measuring quantum state fidelity and the properties of these functions. This work serves our purposes as a literature survey of quantum fidelity measurement techniques and properties and can serve as an excellent resource for expanding what metrics we collect. With 153 references in the work, we can explore more specific details regarding fidelity measurement types if needed.

4.4 Cryptographic distinguishability measures for quantum-mechanical states

URL: `https://ieeexplore.ieee.org/document/761271`

Surveys four different measurements for determining distinguishability of quantum-mechanical states:

1. Probability of identification error
2. Kolmogorov distance

3. Bhattacharyya Coefficient
4. Shannon Distinguishability

While this paper is focused on examine these metrics from a cryptographer's point of view, understanding these metrics and what it means for quantum states to be "close" or "far apart" can help us better understand how to estimate the fidelity of a circuit.

5 Characterization/Error Rate Data Collection

5.1 Practical Characterization of Quantum Devices without Tomography

URL: `https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.107.210404`

Tomography is the reconstruction of a quantum state using measurements of an ensemble of identical quantum states. Resources needed for quantum tomography grow exponentially with device size. The authors are able to estimate the fidelity of an experimental outcome in much less time. The paper proposes two methods. One is to select a theoretical description that is the best match for the experimental outcome. The other is to estimate fidelity between experimental outcome and theoretical target. The runtime to estimate the fidelity can change depending on the class of expected state and operations. This is perhaps something we can exploit - we may be able to dynamically change the way we estimate the fidelity of a program depending on the expected state space or operations.

5.2 Scalable and Robust Randomized Benchmarking of Quantum Processes

URL: `https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.106.180504`

This letter proposes an efficient, reliable estimate of the average error-rate for a set of operations. It seems that randomization is a popular way to characterize quantum processes. This protocol requires $O(n^2)$ quantum gates and $O(n^4)$ classical preprocessing. This protocol applies only to Clifford gates, and not all unitary operations. They chose to restrict themselves to Clifford gates to allow for a formal proof of scalability.

While we are not necessarily interested in running our own benchmarks on quantum hardware, and are planning to rely on characterization data that is collected for us, understanding the complexity in collecting the data can help us understand what challenges lie ahead.

5.3 Direct Fidelity Estimation from Few Pauli Measurements

URL: `https://journals.aps.org/prl/pdf/10.1103/PhysRevLett.106.230501`

This paper assumes that the target state is pure. The authors provide a simple method for estimating the fidelity between the actual state produced and the target state. The number of measurements required is constant, regardless of the number of qubits for Clifford circuits. This paper also uses randomization. So this paper seems quite similar to the previous one.

This paper's solution randomly samples Pauli gates weighted by their importance. It seems that if the target state is a stabilizer state, the runtime to certify that the quantum system produces the target state is lower. This seems to be common among other characterization algorithms.

5.4 Randomized Benchmarking of Quantum Gates

URL: `https://arxiv.org/abs/0707.0963`

The paper suggests a randomized benchmarking method to estimates of errors without relying on accurate state preparation an measurements, which are need for standard process tomography. The randomized benchmarking method apply random sequence of gates of varying lengths to a standard initial state. Each sequence ends with a randomized measurement that checks whether the final state is correct. This randomized benchmarking provides an overall average fidelity for the noise in gates. Paper basically provides iteration for single-qubit, but if the set of computational gates is expanded to include multiqubit gates, it can be applied to two or more qubits. The author implemented the method only for trapped atomic ion qubits, but the idea seems applicable to our simulation.

5.5 Scalable Benchmarks for Gate-Based Quantum Computers

URL: `https://arxiv.org/abs/2104.10698`

The paper provides a testing framework measure the performance of universal quantum devices without intuitive metric or understanding on hardware. The measure is based on six structured tests, and allows immediate comparison between devices. The list of structured circuits are 1. Bell Test, 2. Schödinger's Microscope, 3. Mandelbrot, 4. Line Drawing, 5. Quantum Matrix Inversion, 6. Platonic Fractals. The author also explains how these benchmarks efficiently scale to larger devices. This paper is helpful for our goal by suggesting what 'performance' should be considered in quantum computing.