

# Transformations Exercise

CMSC320

Consider data for variable  $\mathbf{x} = x_1, x_2, \dots, x_n$ . We use  $\bar{x}$  to denote the sample mean of  $\mathbf{x}$ , and  $s_x$  is the sample standard deviation of  $\mathbf{x}$ .

## Part I

For each of the following three transformations derive (a) the sample mean  $\bar{z}$ , and (b) the sample standard deviation  $s_z$ .

1. Centering

$$z_i = (x_i - \bar{x})$$

2. Scaling

$$z_i = \frac{x_i}{s_x}$$

3. Centering and scaling (standardizing)

$$z_i = \frac{(x_i - \bar{x})}{s_x}$$

## Part II

4. Consider transformation  $z_i = \log x_i$ . Show that the sample mean  $\bar{z}$  equals the logarithm of the geometric sample mean of the original data  $x_i$ .

*Note:* The sample mean we use most commonly is the *arithmetic* mean ( $\bar{x} = \frac{1}{n} \sum_i x_i$ ). For strictly positive data, especially where there is skew, the *geometric* mean is a better summary of central trend. It is defined as:

$$\text{gm}(\mathbf{x}) = \left( \prod_i x_i \right)^{1/n}$$

So, your problem is to show that  $\bar{z} = \log \text{gm}(\mathbf{x})$ .