



# Ensemble Methods

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# Ensemble Learning

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At a high level, Ensemble Learning is the aggregation of predictions of multiple weak learners with the goal of improving prediction performance.

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**bagging**: uses ensembles to reduce the variability of single ML models,

**boosting**: uses ensembles of ML models each capturing a specific subspace of predictor space.

# The story of the Netflix Prize

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Supervised learning task:

- Dataset of users and their ratings, (1,2,3,4 or 5 stars), of movies they have rated.
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The idea is that they can then recommend movies to those users if they predict they would rate them highly.

# The story of the Netflix Prize

Netflix would award \$1M for the first ML system that provided a 10% improvement to their existing system

# The story of the Netflix Prize

Existing system had  
a 0.9514 mean  
squared error



## Leaderboard

Team Name	Best Score	% Improvement
No Grand Prize candidates yet	-	-
<b><u>Grand Prize - RMSE &lt;= 0.8563</u></b>		
How low can he go?	0.9046	4.92
<a href="#">ML@UToronto A</a>	0.9046	4.92
<a href="#">ssorkin</a>	0.9089	4.47
<a href="#">wxyzconsulting.com</a>	0.9103	4.32
The Thought Gang	0.9113	4.21
<a href="#">NIPS Reject</a>	0.9118	4.16
<a href="#">simonfunk</a>	0.9145	3.88
<a href="#">Bozo_The_Clown</a>	0.9177	3.54

# The story of the Netflix Prize

Within three weeks,  
at least 40 teams had  
improved upon the  
existing Netflix  
system.

The top teams were  
showing  
improvement over  
5%.

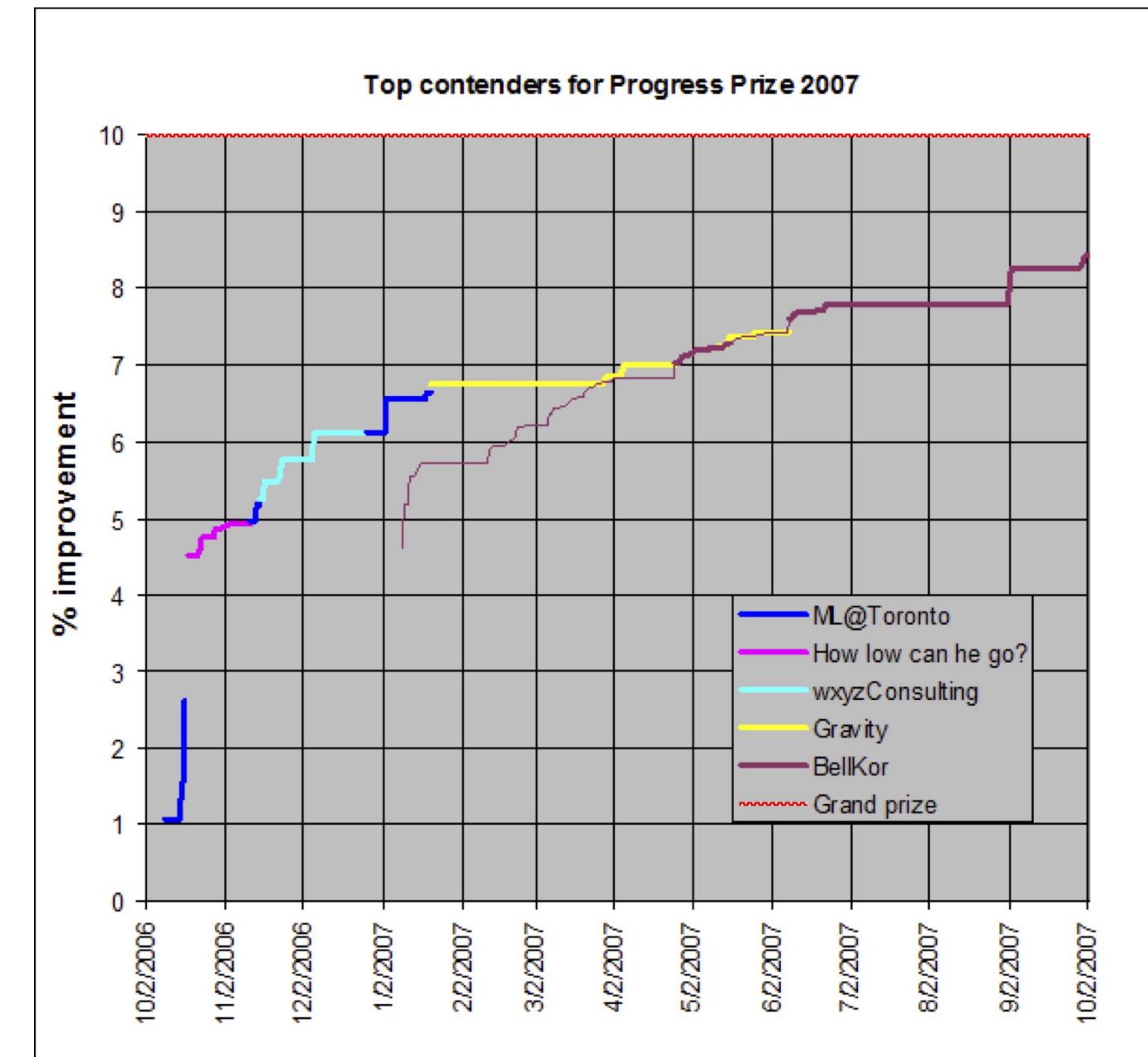


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# The story of the Netflix Prize

Progress soon slowed and teams were stuck at around 8-9% improvement over the existing system.



# The story of the Netflix Prize

Along the way, progress prizes were awarded based on the top team at each challenge anniversary.

The top teams were using ensemble methods.

No Progress Prize candidates yet			
<b>Progress Prize - RMSE &lt;= 0.8625</b>			
1	<a href="#">BellKor</a>	0.8705	8.50
<b>Progress Prize 2007 - RMSE = 0.8712 - Winning Team: KorBell</b>			
2	<a href="#">KorBell</a>	0.8712	8.43
3	<a href="#">When Gravity and Dinosaurs Unite</a>	0.8717	8.38
4	<a href="#">Gravity</a>	0.8743	8.10
5	<a href="#">basho</a>	0.8746	8.07
6	<a href="#">Dinosaur Planet</a>	0.8753	8.00
7	<a href="#">ML@UToronto A</a>	0.8787	7.64
8	<a href="#">Arek Paterek</a>	0.8789	7.62
9	<a href="#">NIPS Reject</a>	0.8808	7.42
10	<a href="#">Just a guy in a garage</a>	0.8834	7.15
11	<a href="#">Ensemble Experts</a>	0.8841	7.07
12	<a href="#">mathematical capital</a>	0.8844	7.04
13	<a href="#">HowLowCanHeGo2</a>	0.8847	7.01
14	<a href="#">The Thought Gang</a>	0.8849	6.99
15	<a href="#">Reel Ingenuity</a>	0.8855	6.93
16	<a href="#">strudeltamale</a>	0.8859	6.88
17	<a href="#">NIPS Submission</a>	0.8861	6.86
18	<a href="#">Three Blind Mice</a>	0.8869	6.78
19	<a href="#">TrainOnTest</a>	0.8869	6.78
20	<a href="#">Geoff Dean</a>	0.8869	6.78

# The story of the Netflix Prize

- **Arek Paterek:** "combine the results of many methods"
- **U of Toronto:** "when the predictions of multiple RBM and SVD models are linearly combined..."

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	.....	0.8870	6.78

# The story of the Netflix Prize

- **When Gravity and Dinosaurs Unite:** "Our common team blends the result of team Gravity and team Dinosaur Planet"
- **BellKor:** "Our final solution consists of blending 107 individual results"

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# The story of the Netflix Prize

Ultimately, the challenge was won when multiple top teams allied to combine their own ensemble models as ensembles.

## Leaderboard

Showing Test Score. [Click here to show quiz score](#)

Display top  leaders.

Rank	Team Name	Best Test Score	% Improvement	Best Submit Time
<b>Grand Prize - RMSE = 0.8567 - Winning Team: BellKor's Pragmatic Chaos</b>				
1	<a href="#">BellKor's Pragmatic Chaos</a>	0.8567	10.06	2009-07-26 18:18:28
2	<a href="#">The Ensemble</a>	0.8567	10.06	2009-07-26 18:38:22
3	<a href="#">Grand Prize Team</a>	0.8582	9.90	2009-07-10 21:24:40
4	<a href="#">Opera Solutions and Vandelay United</a>	0.8588	9.84	2009-07-10 01:12:31
5	<a href="#">Vandelay Industries !</a>	0.8591	9.81	2009-07-10 00:32:20
6	<a href="#">PragmaticTheory</a>	0.8594	9.77	2009-06-24 12:06:56
7	<a href="#">BellKor in BigChaos</a>	0.8601	9.70	2009-05-13 08:14:09
8	<a href="#">Dace</a>	0.8612	9.59	2009-07-24 17:18:43
9	<a href="#">Feeds2</a>	0.8622	9.48	2009-07-12 13:11:51
10	<a href="#">BigChaos</a>	0.8623	9.47	2009-04-07 12:33:59
11	<a href="#">Opera Solutions</a>	0.8623	9.47	2009-07-24 00:34:07
12	<a href="#">BellKor</a>	0.8624	9.46	2009-07-26 17:19:11
<b>Progress Prize 2008 - RMSE = 0.8627 - Winning Team: BellKor in BigChaos</b>				

# Intuition behind ensemble methods

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For instance, combining diverse independent opinions in human decision-making.

Averting risk by diversifying a stock portfolio.

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Second, it is generally difficult to establish precisely the type and complexity of model required for a specific learning task.

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A combination of models of diverse types and complexities can alleviate this challenge.

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Suppose we have completely independent classifiers, each having 70% accuracy on a specific task.

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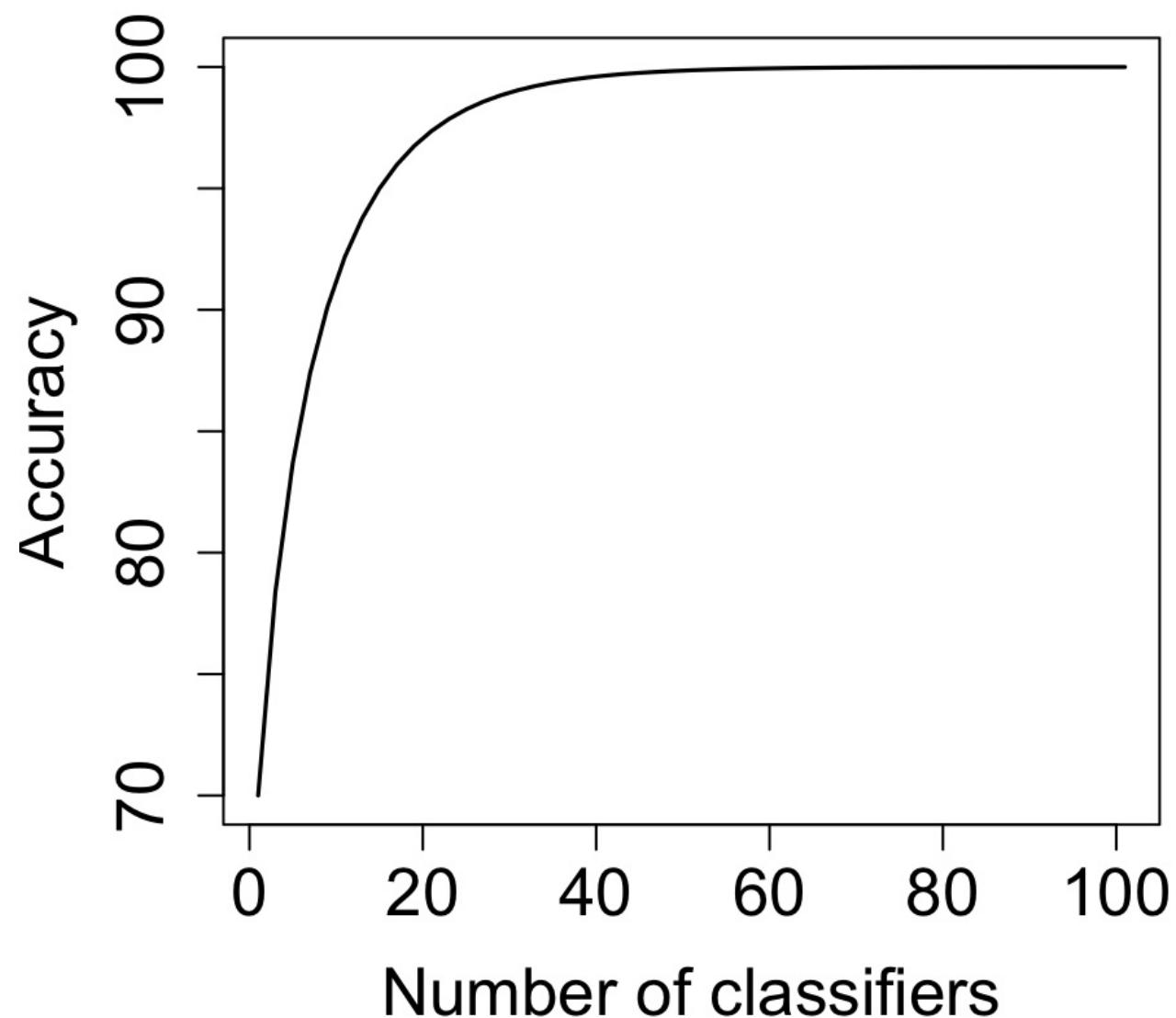
Suppose we have completely independent classifiers, each having 70% accuracy on a specific task.

Now, suppose we combine them using majority vote, where each classifier predicts a label for an instance, and we make a final prediction based on what the majority of classifiers predicted.

# Intuition behind ensemble methods

In this case, the accuracy of the ensemble increases as the number of classifiers increase.

In this case, we would reach 99% accuracy with about 40 classifiers.



# Ensemble Models

We will look at two strategies for building ensembles.

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Use different samples of instances and/or attributes to independently train diverse classifiers.

Then, aggregate the classifiers using majority vote (classification) or averaging (regression).

# Ensemble Models

Boosting: Build the ensemble sequentially, using the performance of the ensemble to train the next classifier in the ensemble.

# Bagging

The name Bagging comes from combining the bootstrap to generate different samples of instances and aggregation used to combine predictions.

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The name Bagging comes from combining the bootstrap to generate different samples of instances and aggregation used to combine predictions.

This method increases diversity of the weak learners using randomness in two ways

1. using bootstrap sampling to generate the training set for each weak learner
2. using random subsets of features to train each weak learner

# Bagging

The general bootstrap algorithm was designed to estimate variability of estimates when we do not have support for a specific data generating model.

# Bagging

The general bootstrap procedure is as follows for parameter  $\theta$  and estimator  $\hat{\theta}$  :

1. Select  $B$  independent bootstrap samples  $\{B_i\}_{i=1}^B$  each consisting of  $n$  data values drawing with replacement from training set  $\{x_1, x_2, \dots, x_n\}$ .
2. Evaluate each bootstrap replication to estimate  $\hat{\theta}_i$ , for  $i = 1, 2, \dots, B$ .
3. Estimate standard error of  $\hat{\theta}$  using the sample standard error of the estimates.

# Bagging

In the ensemble learning case,  $s$  is an ML training algorithm

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$\hat{\theta}_b$  is interpreted as the predictions made by the ML algorithm trained using bootstrap sample  $b$ .

In this case we are not necessarily interested in the inferential task.

Instead, we use aggregation of these multiple predictions to make final predictions.

# Bagging

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For some models, like linear regression models, we can show anatically that this strategy decreases variance without increasing bias.

# Stacking

A related idea is stacking (also known as blending)

This is what the Netflix teams did as they combined predictors near the end of the challenge.

# Stacking

The basic idea is to train a set of diverse predictors as done in bagging with some differences:

- If the set of predictors are very different types of models e.g., SVM, Decision Tree, KNN, train each on the entire set of training observations instead of bootstrapped samples
- Train a simple model (e.g., linear regression or logistic regression) using the predictions made by the predictors as features

# Boosting

Create an ensemble of classifiers sequentially.

At each iteration of the sequence we use a weighted version of the training instances so that instances that are incorrectly classified by the current set of classifiers are given higher weight.

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Here diversity is injected into the ensemble by re-weighting training instances.

# Boosting

Final prediction from the ensemble is also given by aggregation (majority decision or averaging)

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Predictions are weighted based on each classifiers accuracy.

# Adaboost

The Adaboost algorithm is probably the best known example of boosting.

The algorithm is as follows:

- Initialize weights: each instance gets the same weight

$$w_i = 1/N, i = 1, \dots, N$$

- Construct a classifier  $g_m$  using current weights (Decision Trees, SVM, linear/logistic regression). Compute the error of the new classifier

$$\epsilon_m = \frac{\sum_i w_i I{y_i \neq g_m(x_i)}}{\sum_i w_i}$$

# Adaboost

- Get the influence of the new classifier and update training instance weights

$$\alpha_m = \log\left(\frac{1 - \epsilon_i}{\epsilon_i}\right)$$

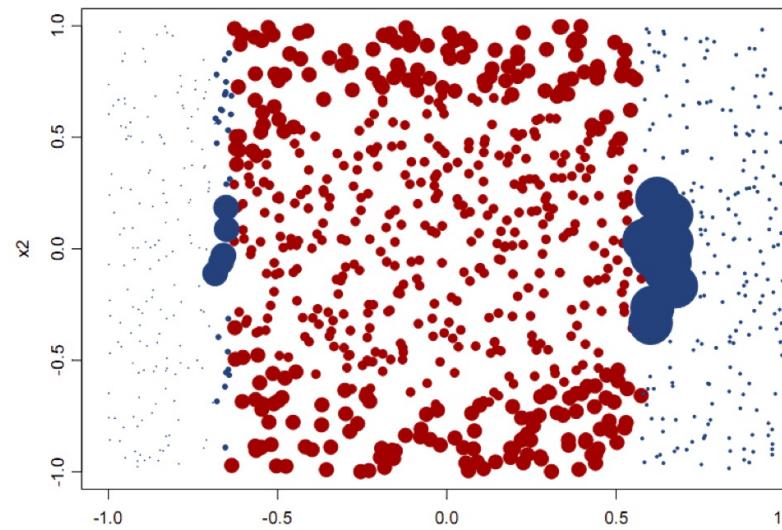
$$w_i \leftarrow w_i \exp \alpha_m I y_i \neq g_m(x_i)$$

- Goto step 2

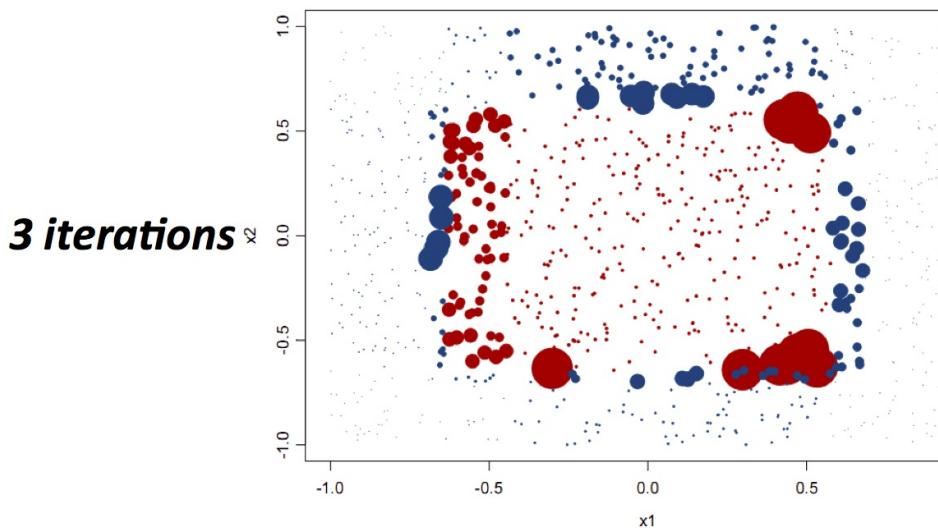
For final prediction, average predictions from each classifier with weight

$$\alpha_m$$

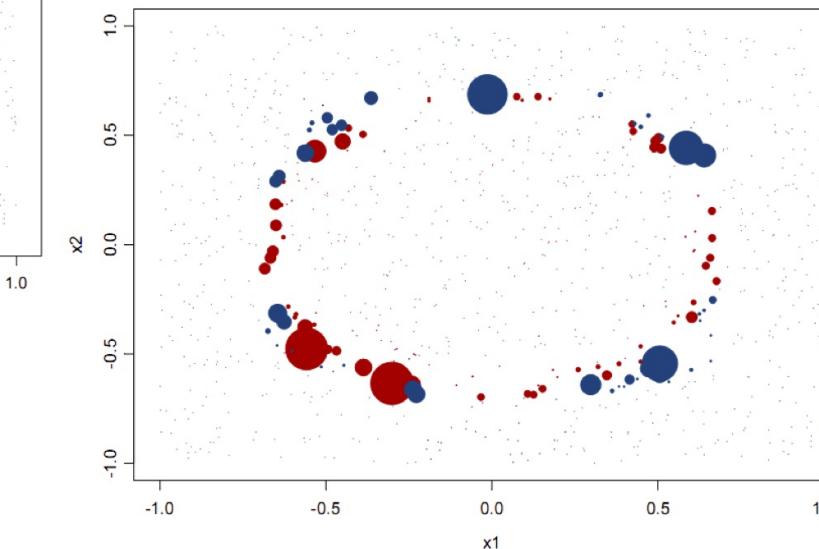
# Adaboost



**Classifications (colors) and  
Weights (size) after 1 iteration  
Of AdaBoost**



**3 iterations**



**20 iterations**

from Elder, John. From Trees to  
Forests and Rule Sets - A Unified  
Overview of Ensemble Methods. 2007.

# Adaboost

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Adaboost has the advantage of its simplicity

Like bagging, adaboost reduces the variability of individual weak learners.

Unlike bagging it is sensitive to noise and outliers.

# Summary

Ensemble methods seek to reduce variance of individual weak learners by aggregating their predictions.

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Bagging and boosting are alternative methods of training ensembles while providing diversity to each of the trained weak learners.