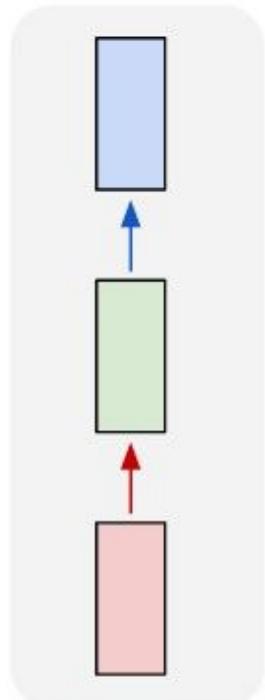


Lecture 10: Recurrent Neural Networks

“Vanilla” Neural Network

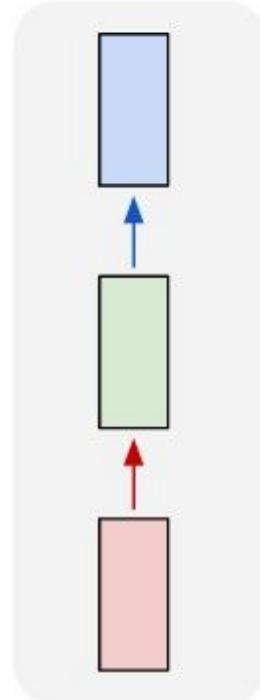
one to one



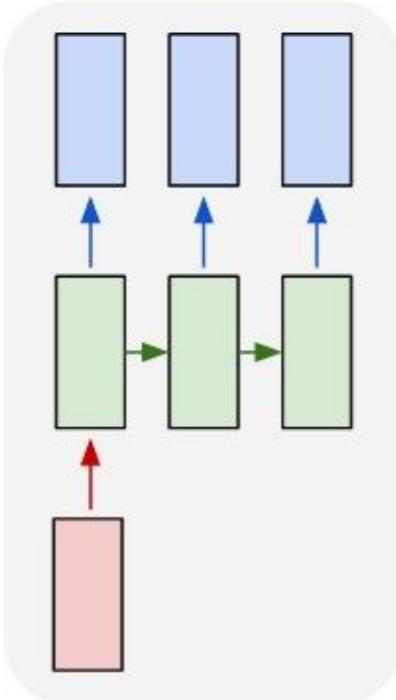
Vanilla Neural Networks

Recurrent Neural Networks: Process Sequences

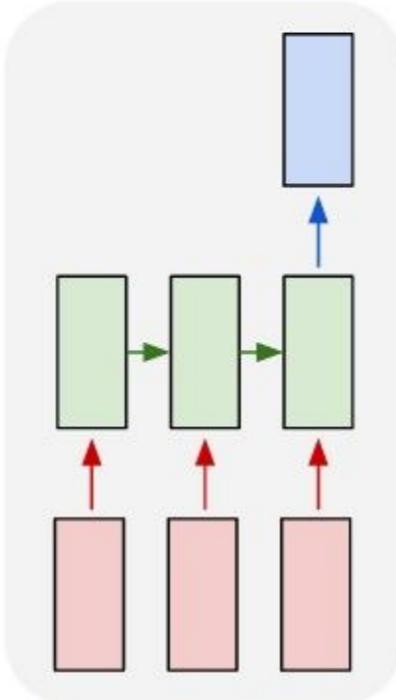
one to one



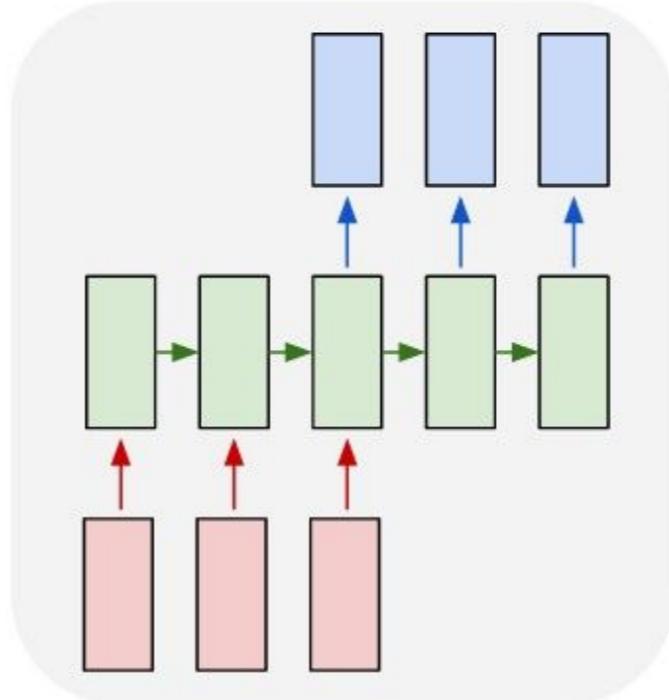
one to many



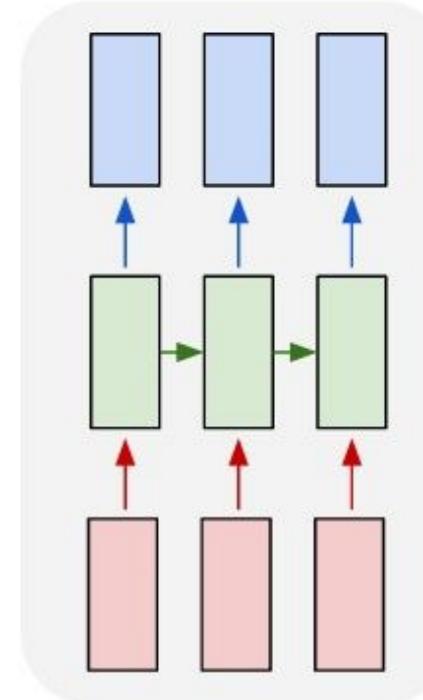
many to one



many to many



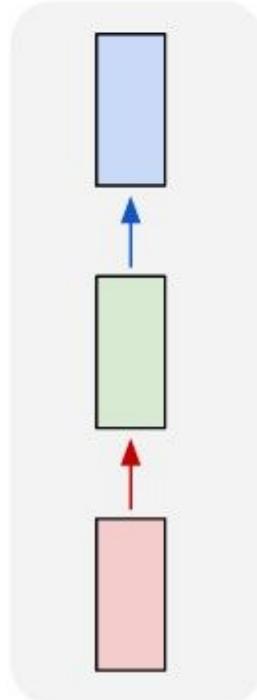
many to many



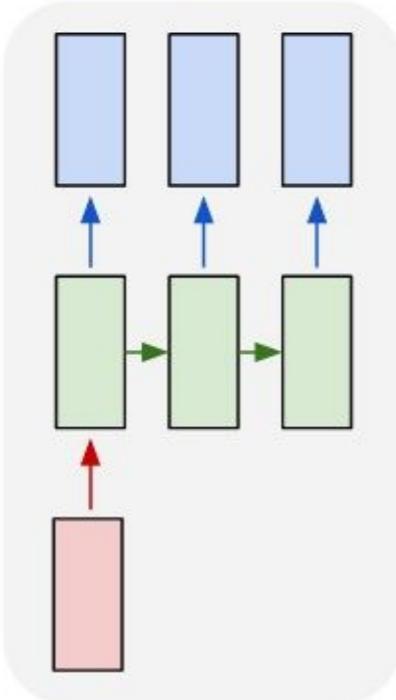
e.g. **Image Captioning**
image -> sequence of words

Recurrent Neural Networks: Process Sequences

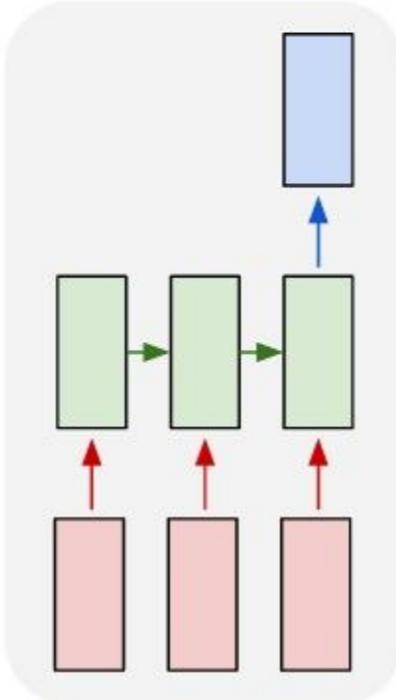
one to one



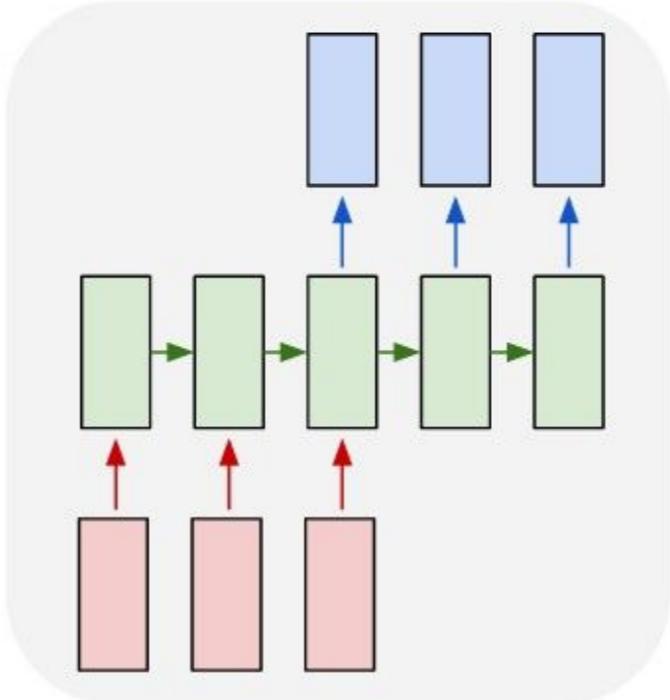
one to many



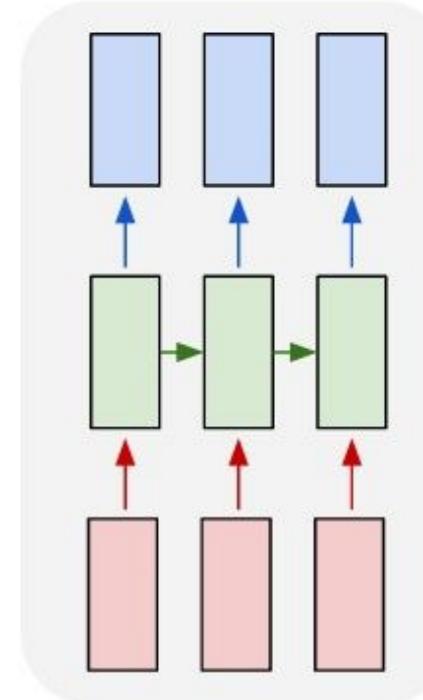
many to one



many to many



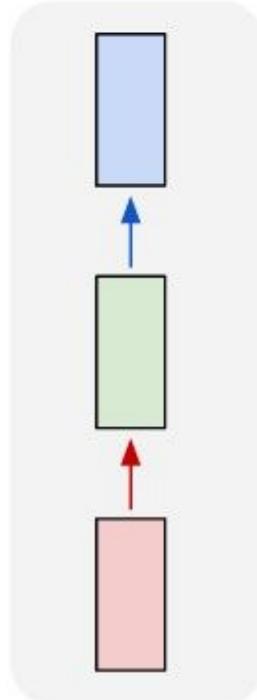
many to many



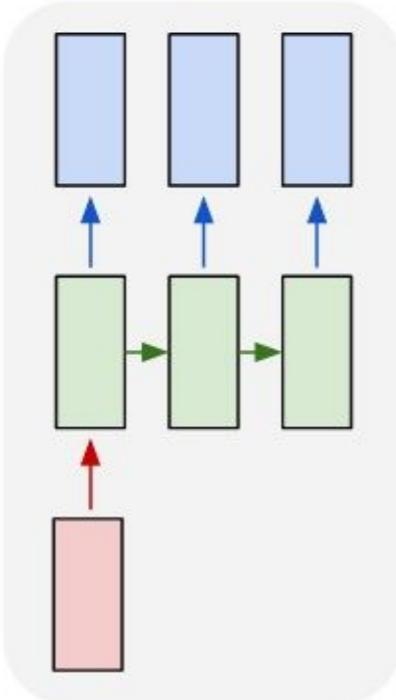
e.g. **Sentiment Classification**
sequence of words -> sentiment

Recurrent Neural Networks: Process Sequences

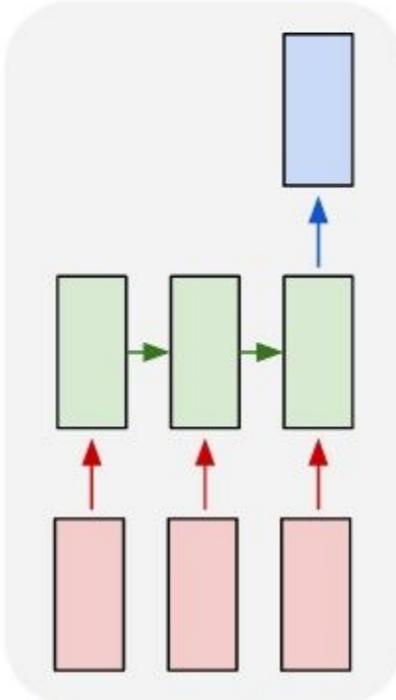
one to one



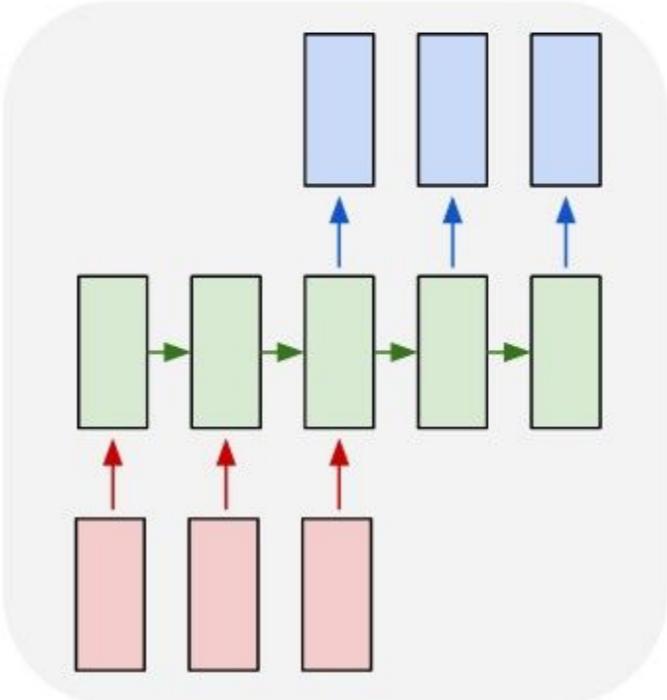
one to many



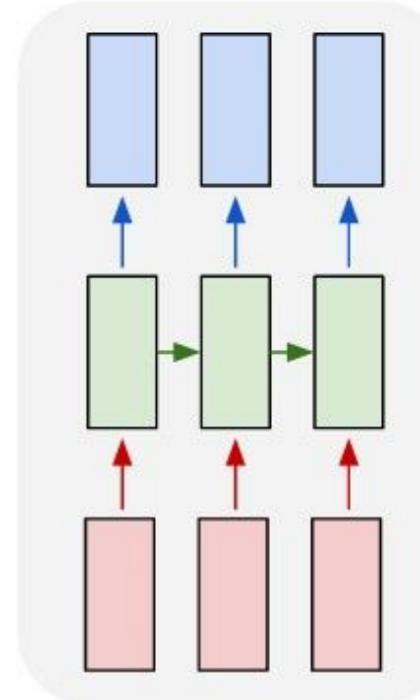
many to one



many to many



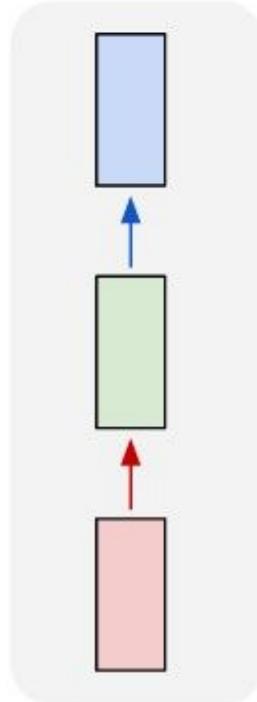
many to many



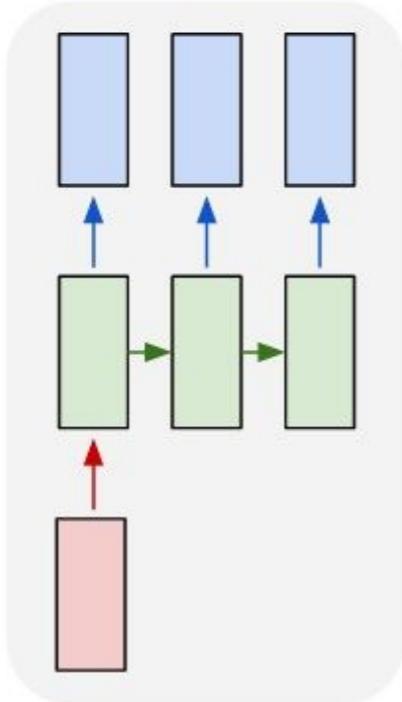
↑
e.g. **Machine Translation**
seq of words -> seq of words

Recurrent Neural Networks: Process Sequences

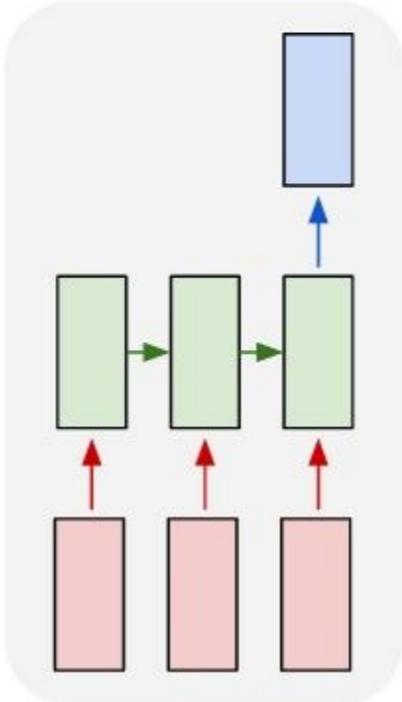
one to one



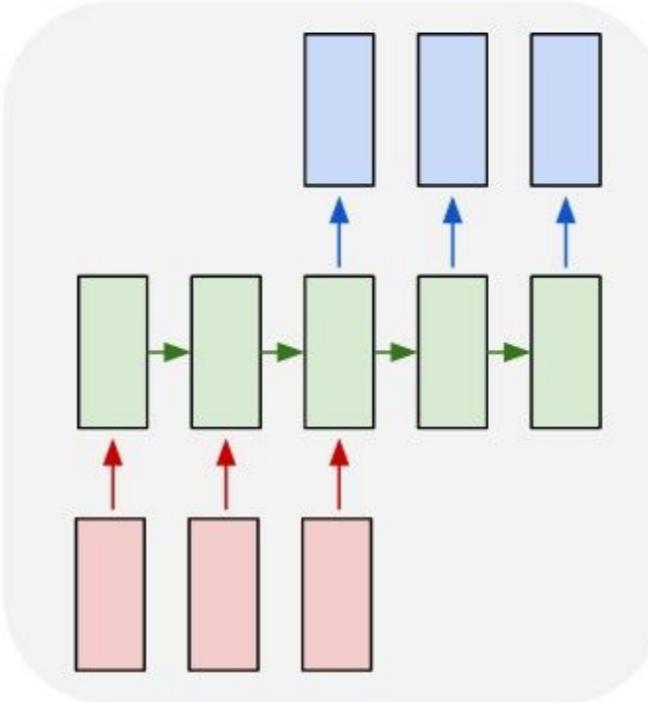
one to many



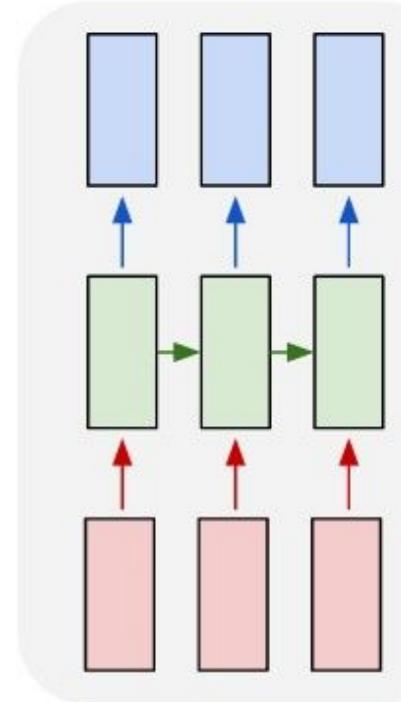
many to one



many to many

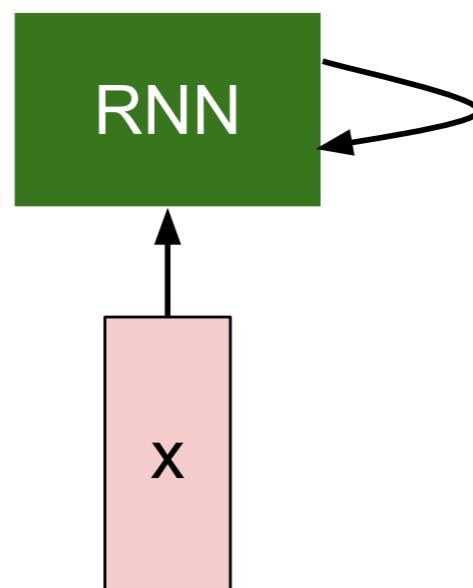


many to many

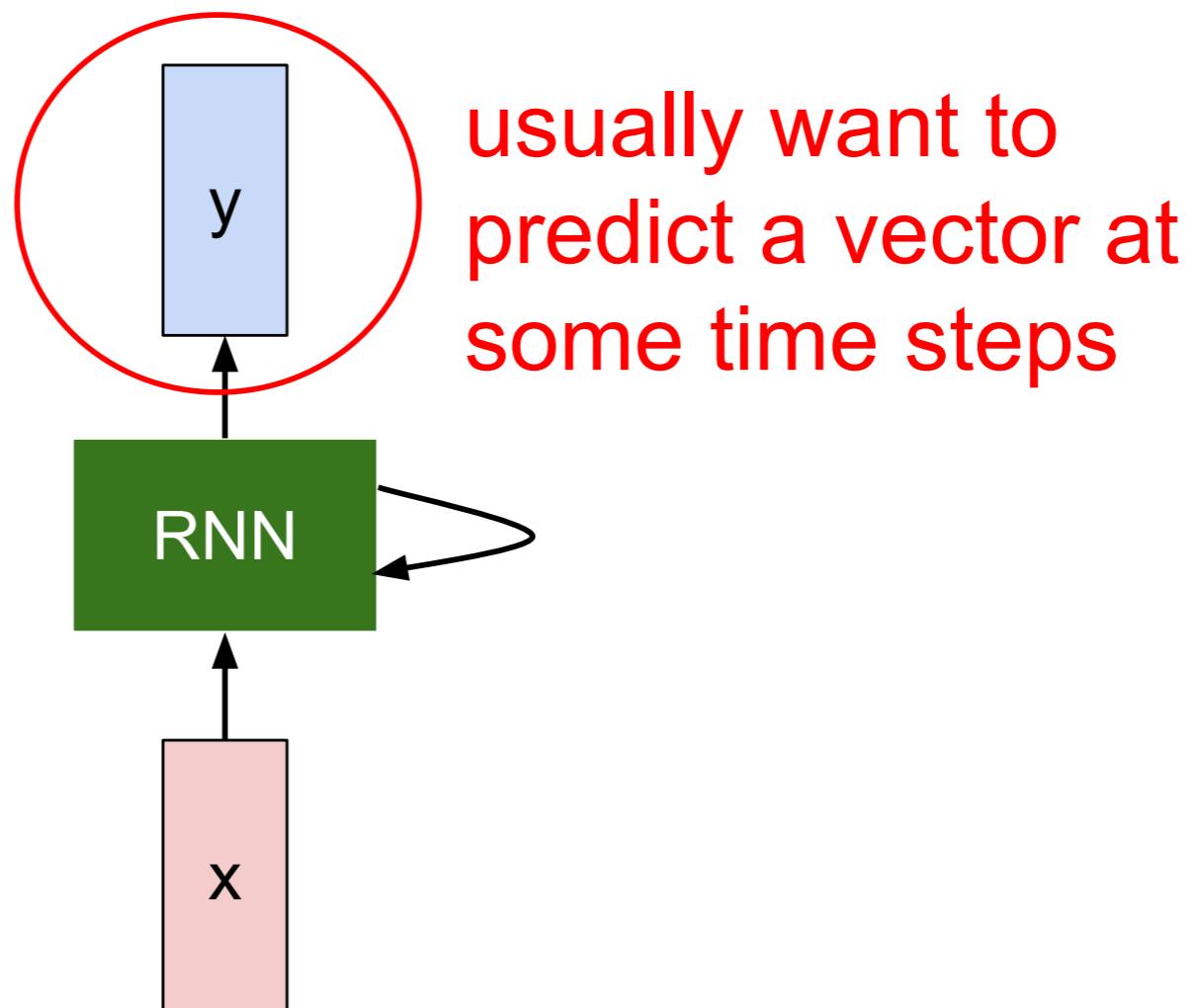


e.g. **Video classification on frame level**

Recurrent Neural Network



Recurrent Neural Network

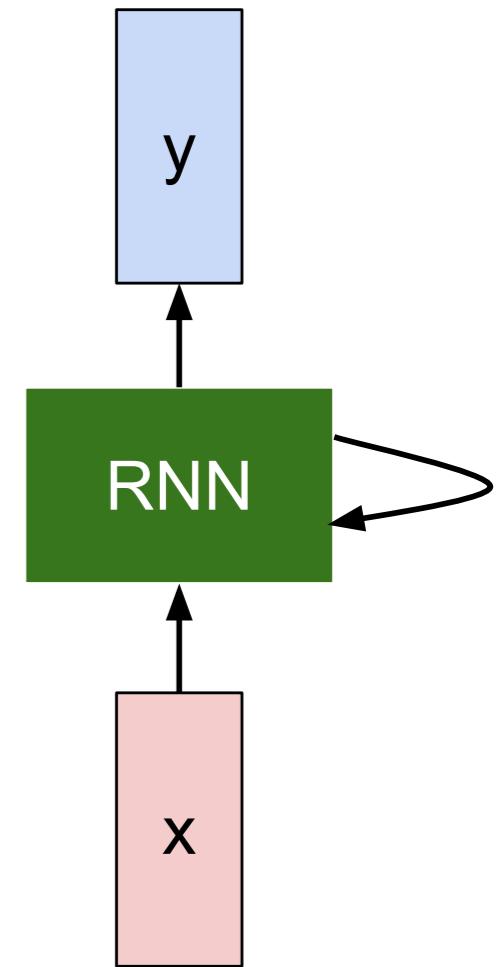


Recurrent Neural Network

We can process a sequence of vectors \mathbf{x} by applying a **recurrence formula** at every time step:

$$h_t = f_W(h_{t-1}, x_t)$$

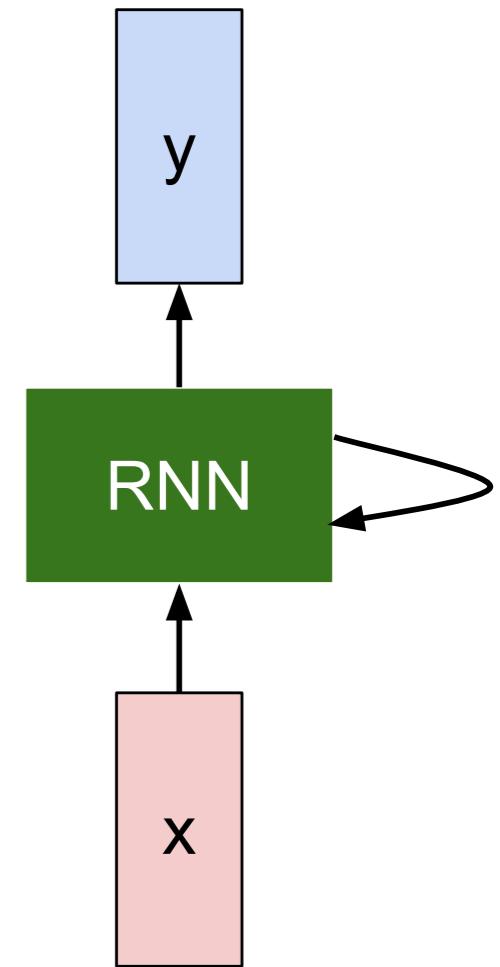
new state old state input vector at
some function with parameters W some time step



Recurrent Neural Network

We can process a sequence of vectors \mathbf{x} by applying a **recurrence formula** at every time step:

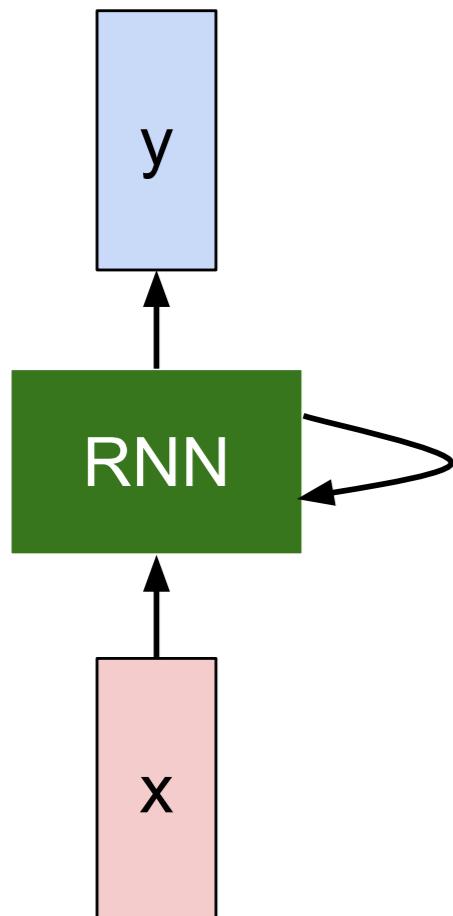
$$h_t = f_W(h_{t-1}, x_t)$$



Notice: the same function and the same set of parameters are used at every time step.

(Vanilla) Recurrent Neural Network

The state consists of a single “*hidden*” vector \mathbf{h} :

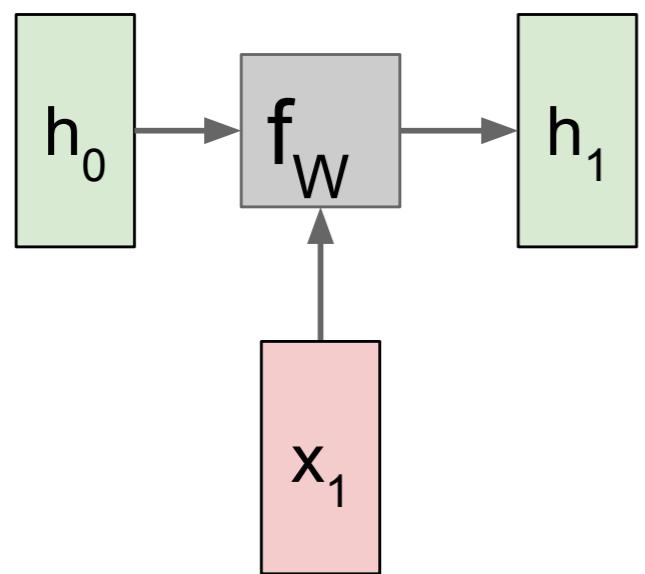


$$\mathbf{h}_t = f_W(\mathbf{h}_{t-1}, \mathbf{x}_t)$$

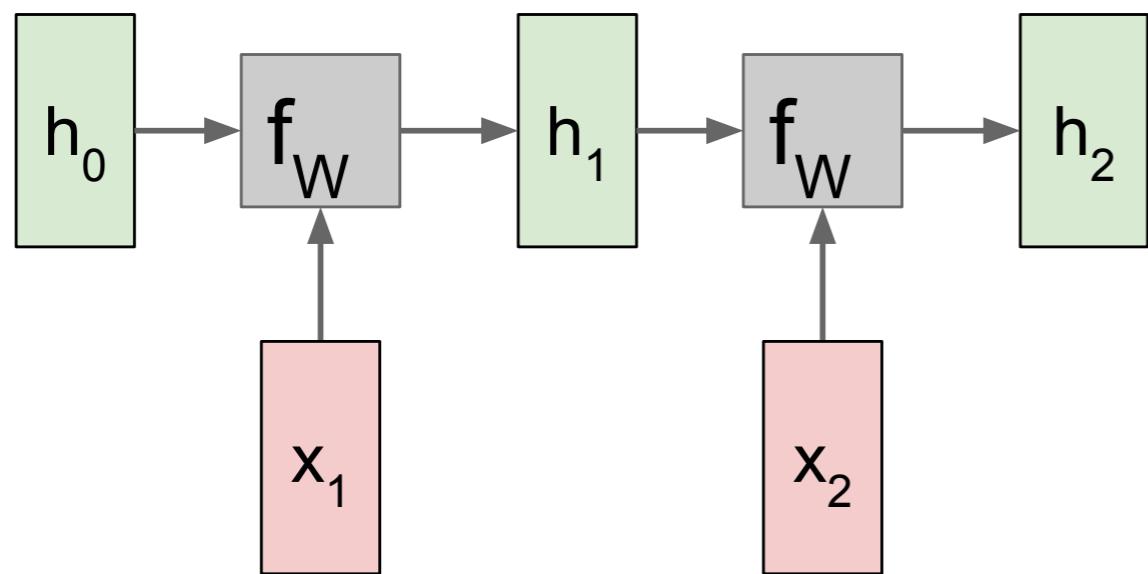
$$\mathbf{h}_t = \tanh(W_{hh}\mathbf{h}_{t-1} + W_{xh}\mathbf{x}_t)$$

$$y_t = W_{hy}\mathbf{h}_t$$

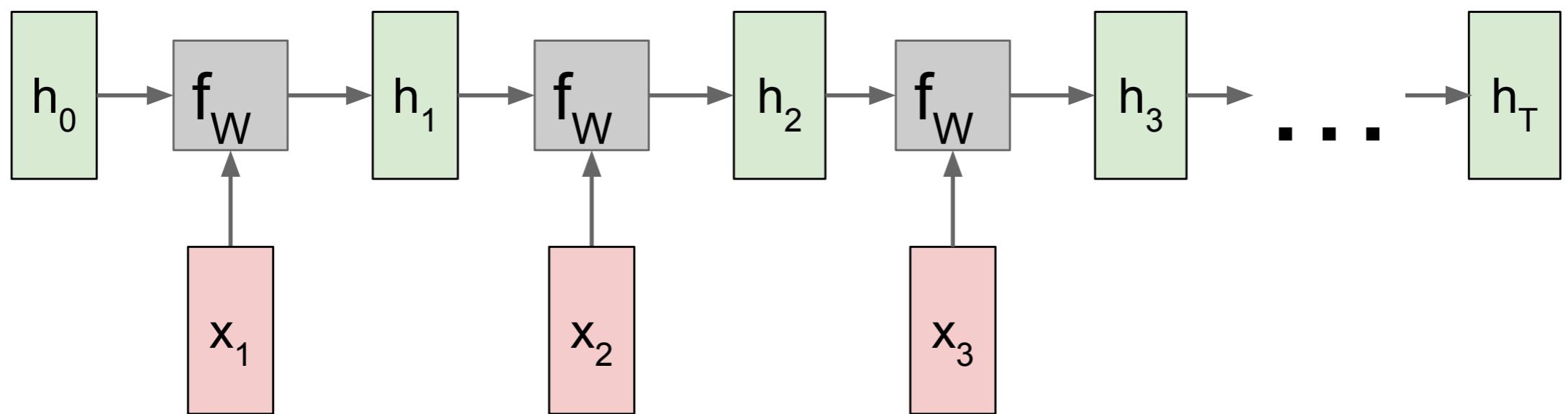
RNN: Computational Graph



RNN: Computational Graph

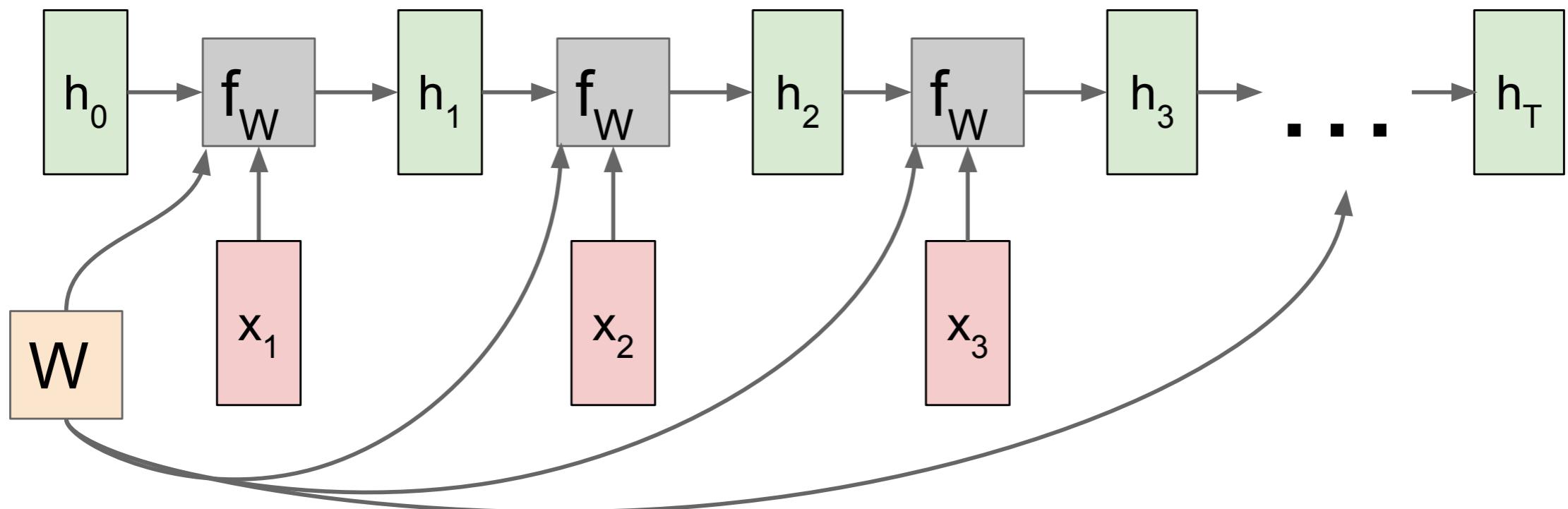


RNN: Computational Graph

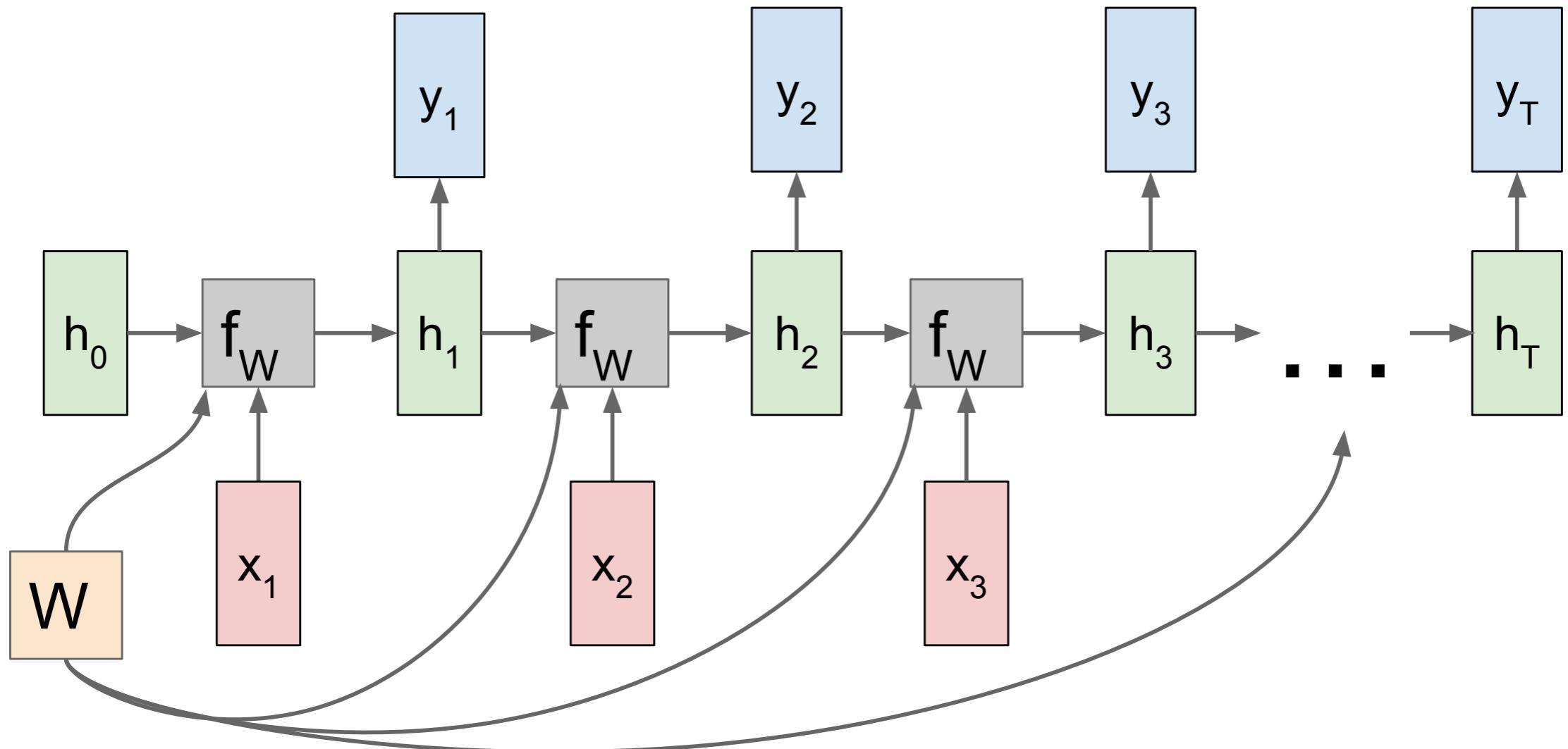


RNN: Computational Graph

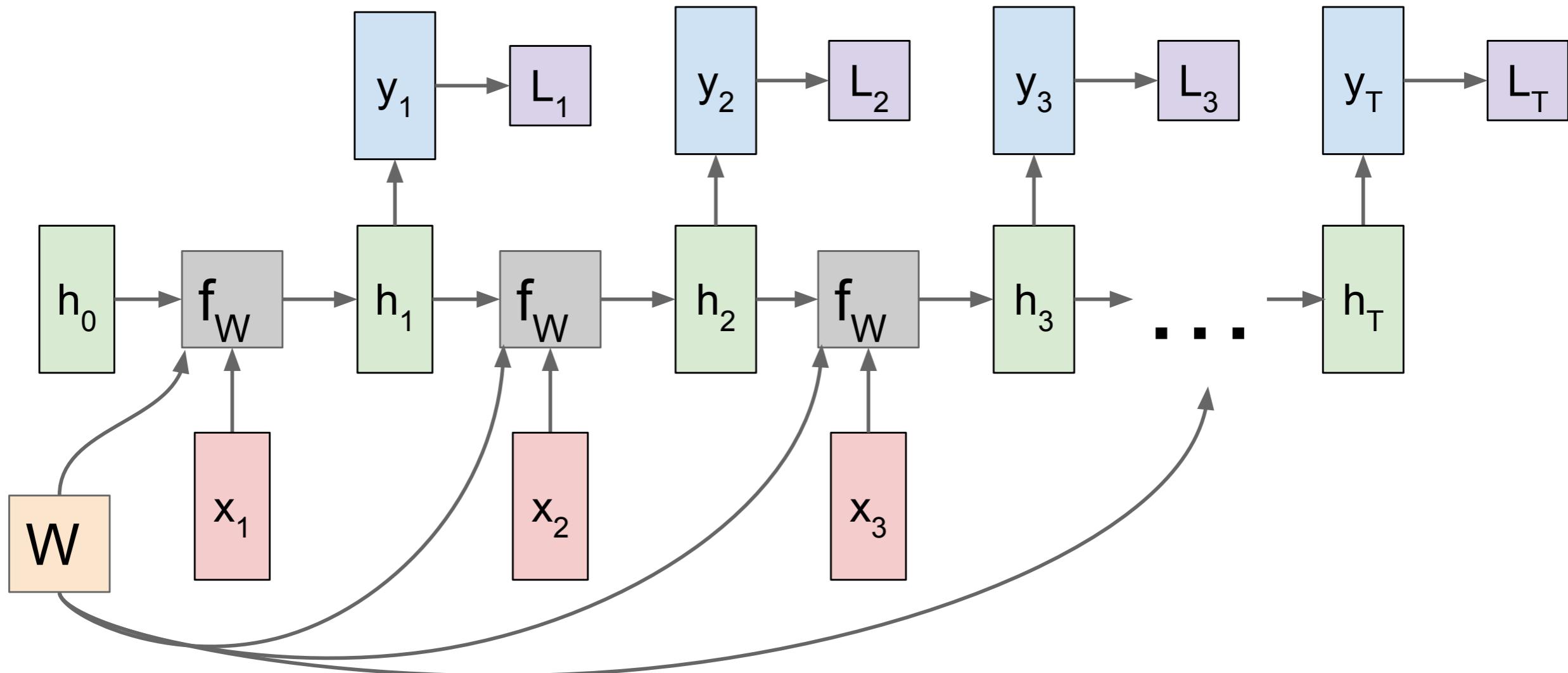
Re-use the same weight matrix at every time-step



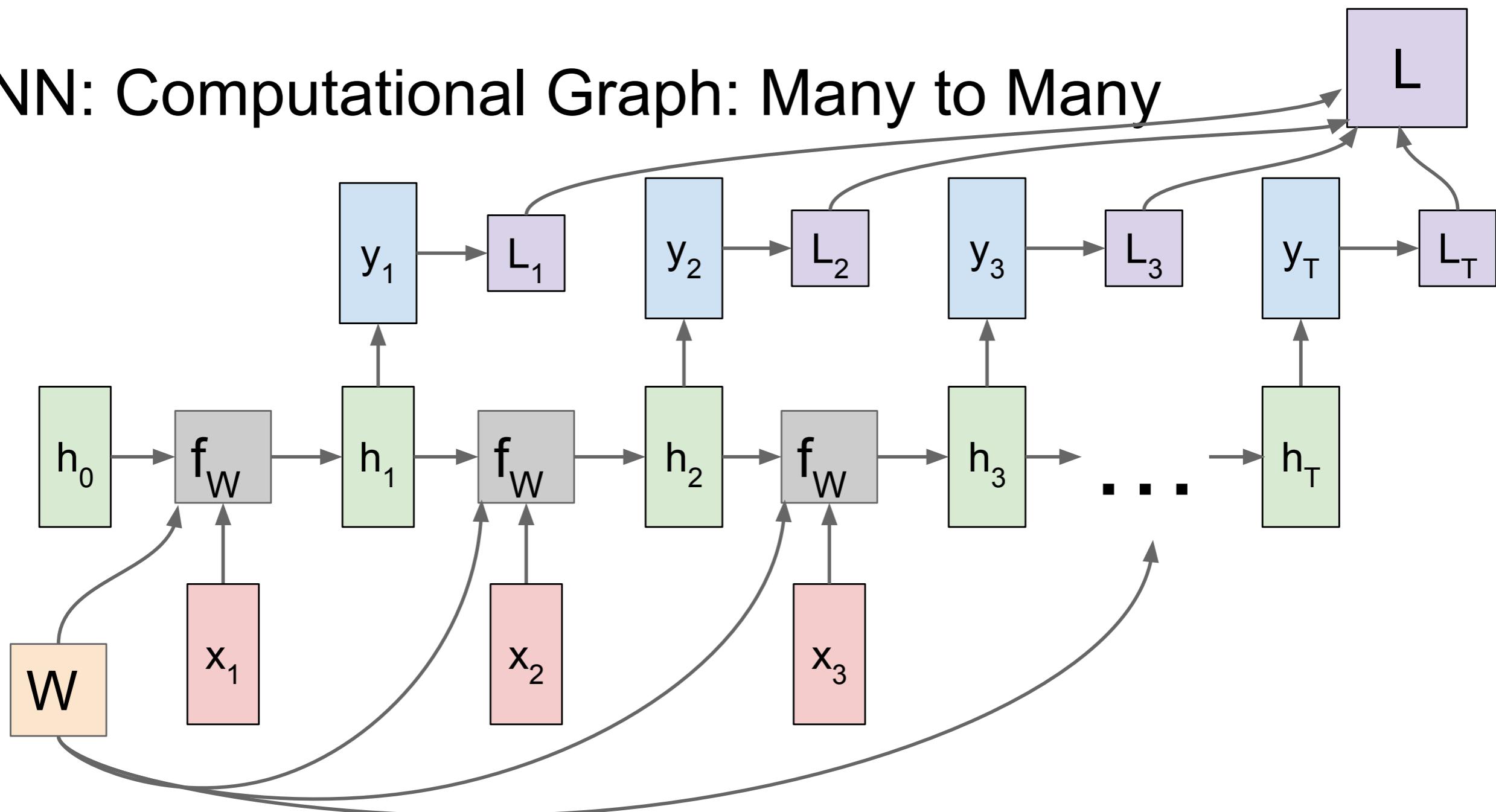
RNN: Computational Graph: Many to Many



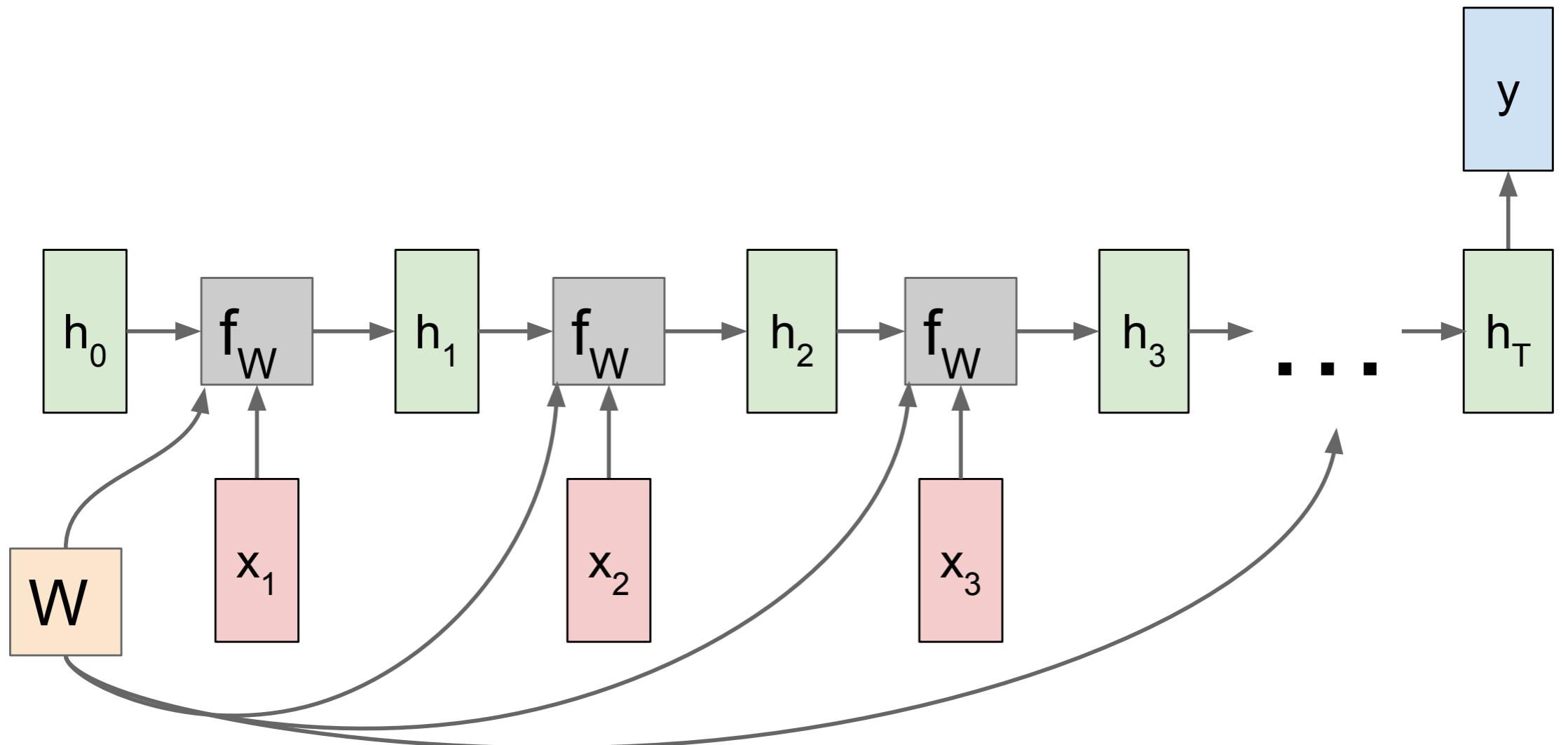
RNN: Computational Graph: Many to Many



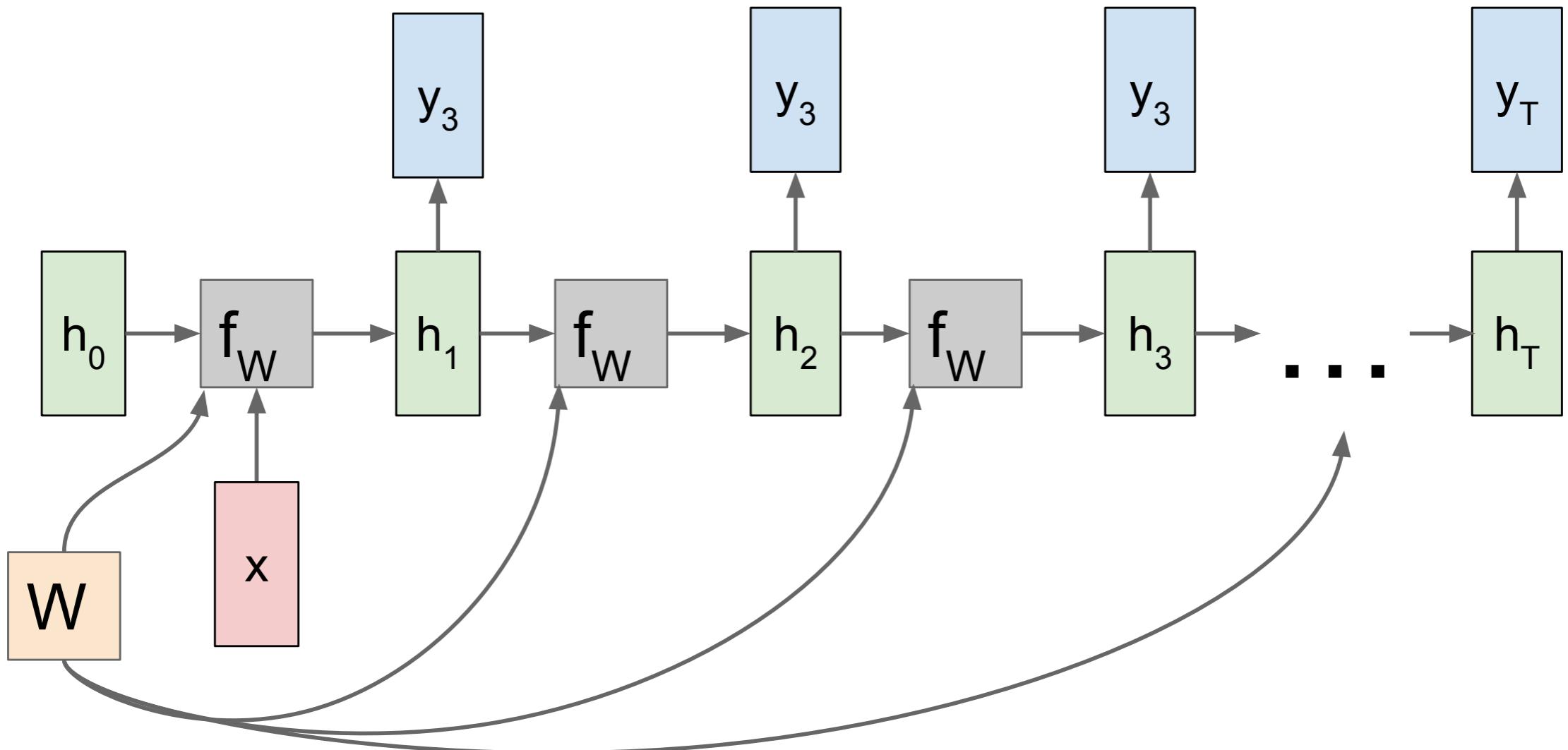
RNN: Computational Graph: Many to Many



RNN: Computational Graph: Many to One

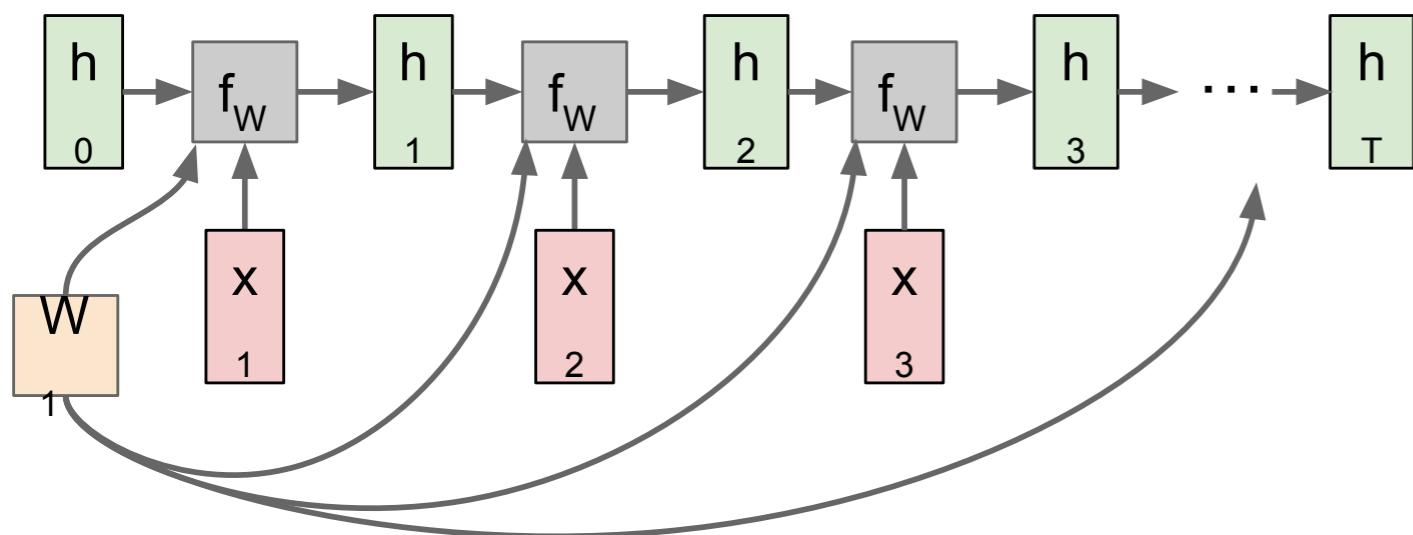


RNN: Computational Graph: One to Many



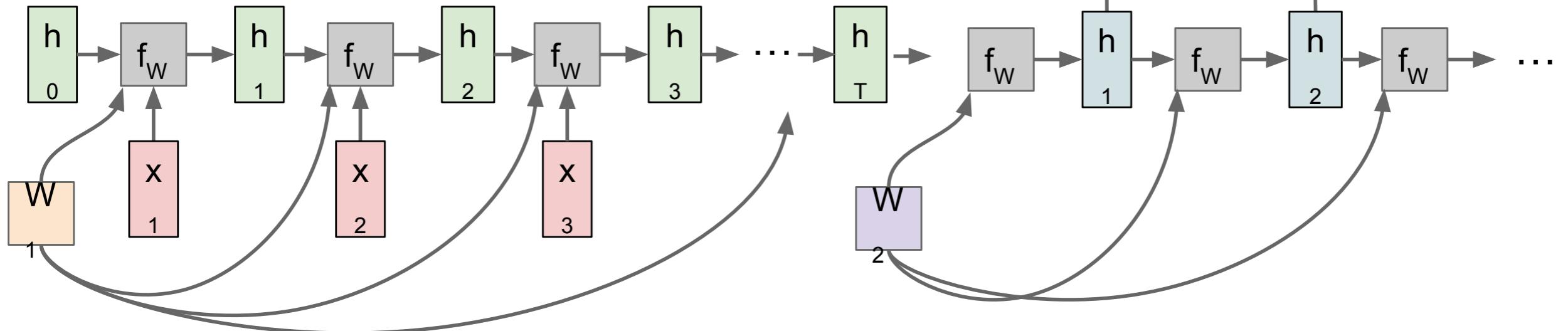
Sequence to Sequence: Many-to-one + one-to-many

Many to one: Encode input sequence in a single vector



Sequence to Sequence: Many-to-one + one-to-many

Many to one: Encode input sequence in a single vector

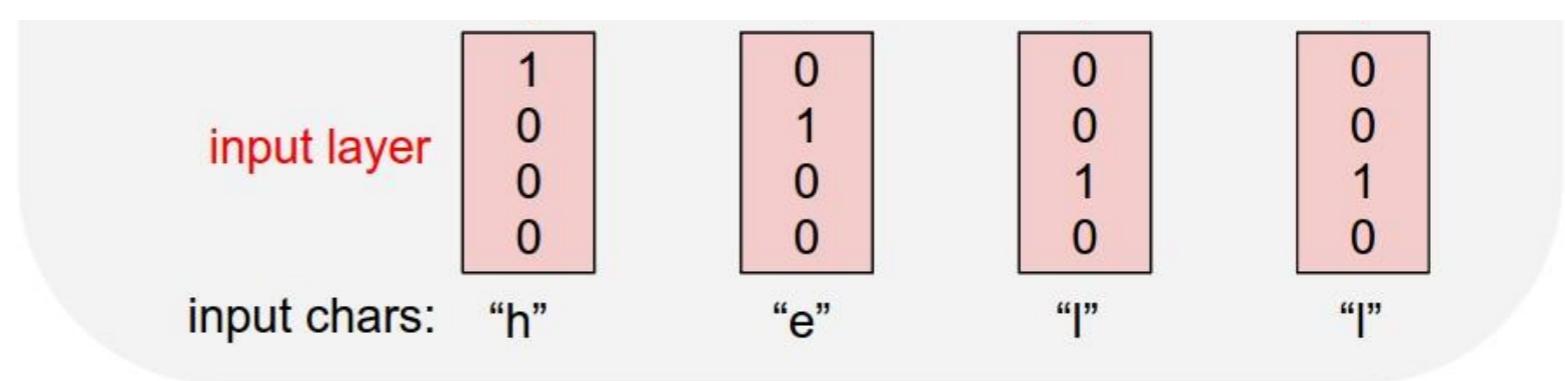


One to many: Produce output sequence from single input vector

Example: Character-level Language Model

Vocabulary:
[h,e,l,o]

Example training
sequence:
“hello”

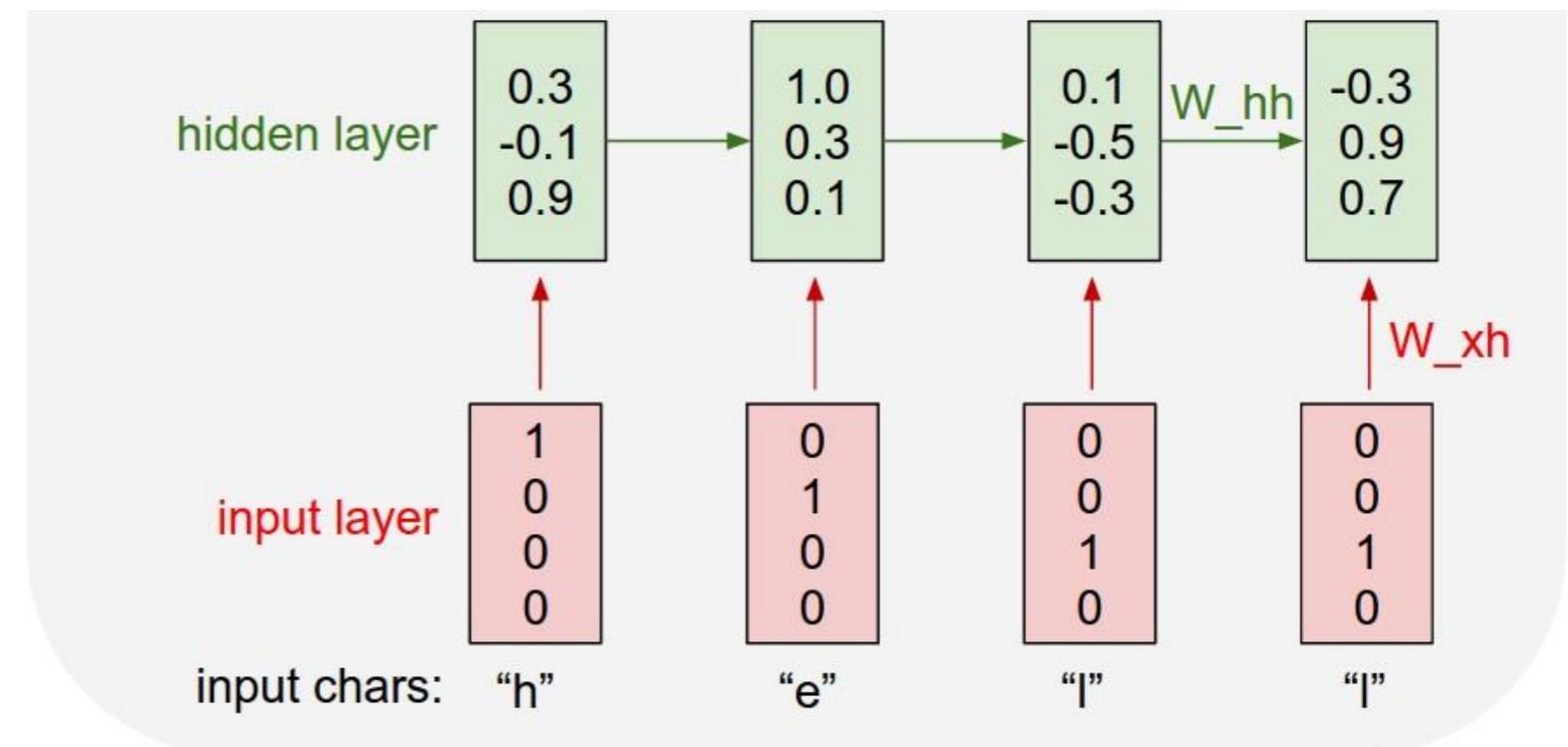


Example: Character-level Language Model

Vocabulary:
[h,e,l,o]

Example training
sequence:
“hello”

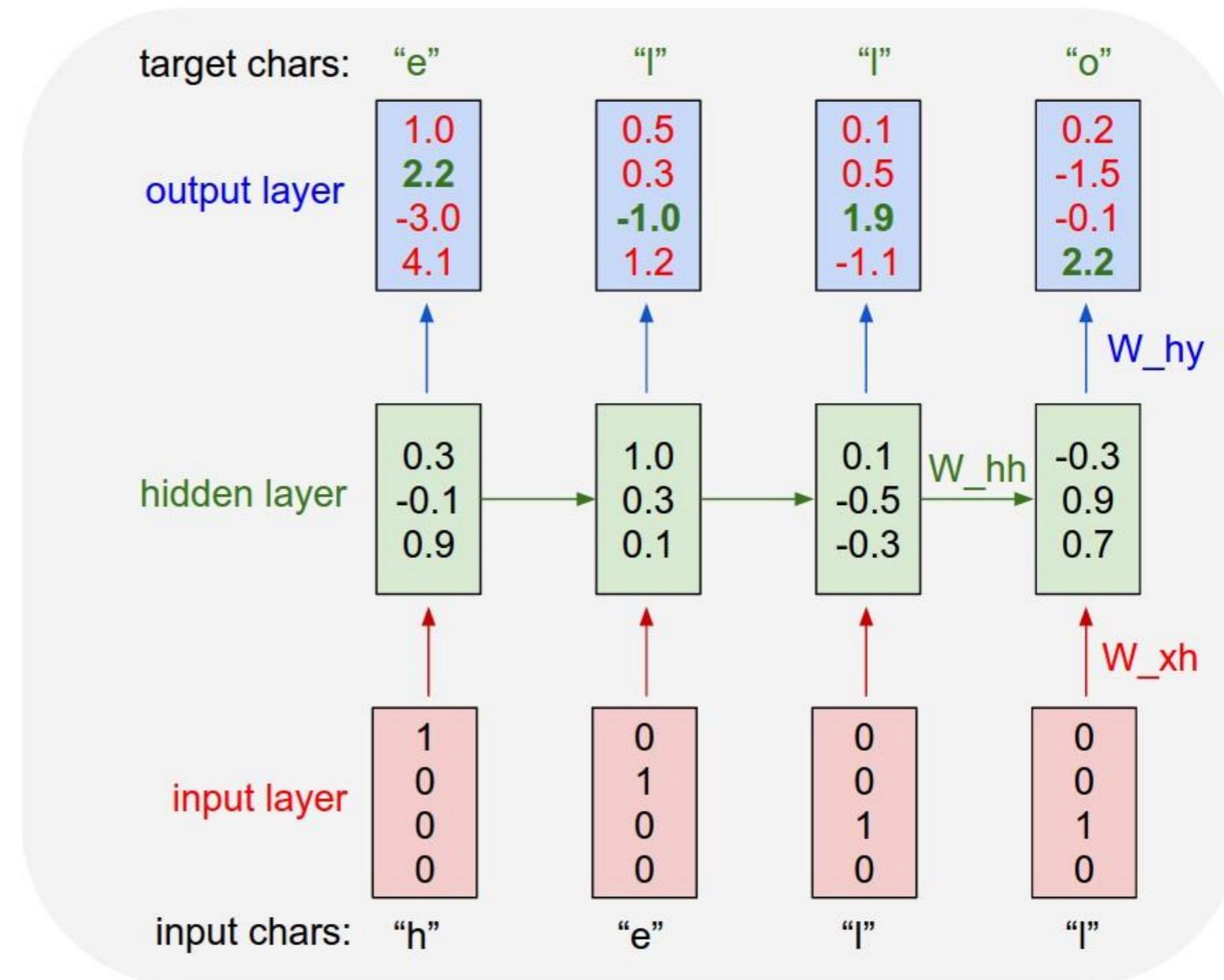
$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t)$$



Example: Character-level Language Model

Vocabulary:
[h,e,l,o]

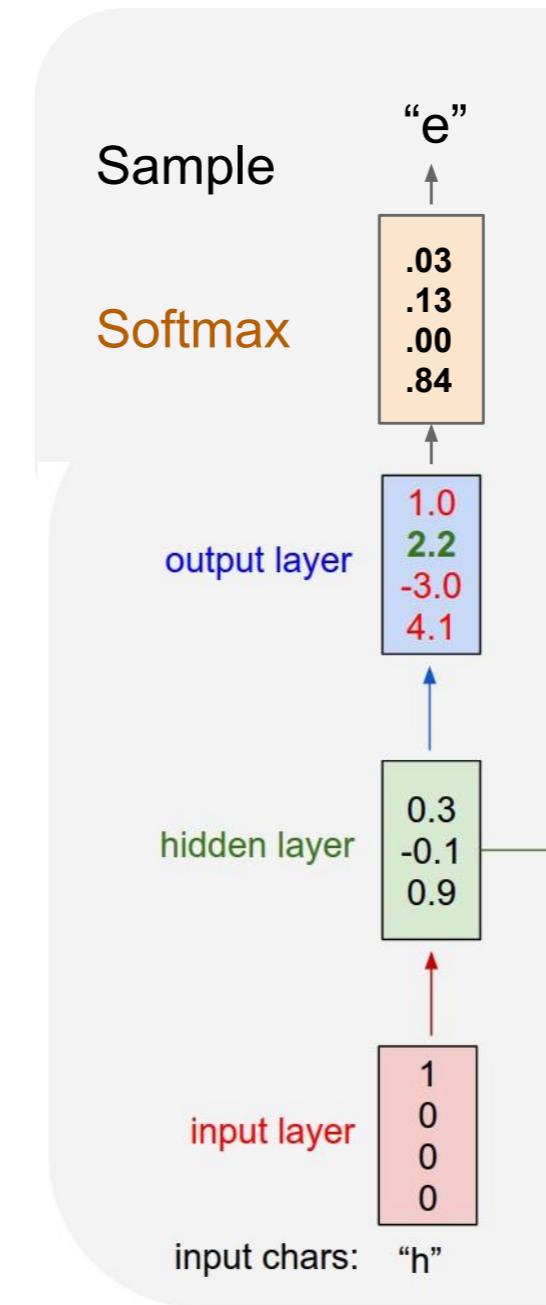
Example training
sequence:
“hello”



Example: Character-level Language Model Sampling

Vocabulary:
[h,e,l,o]

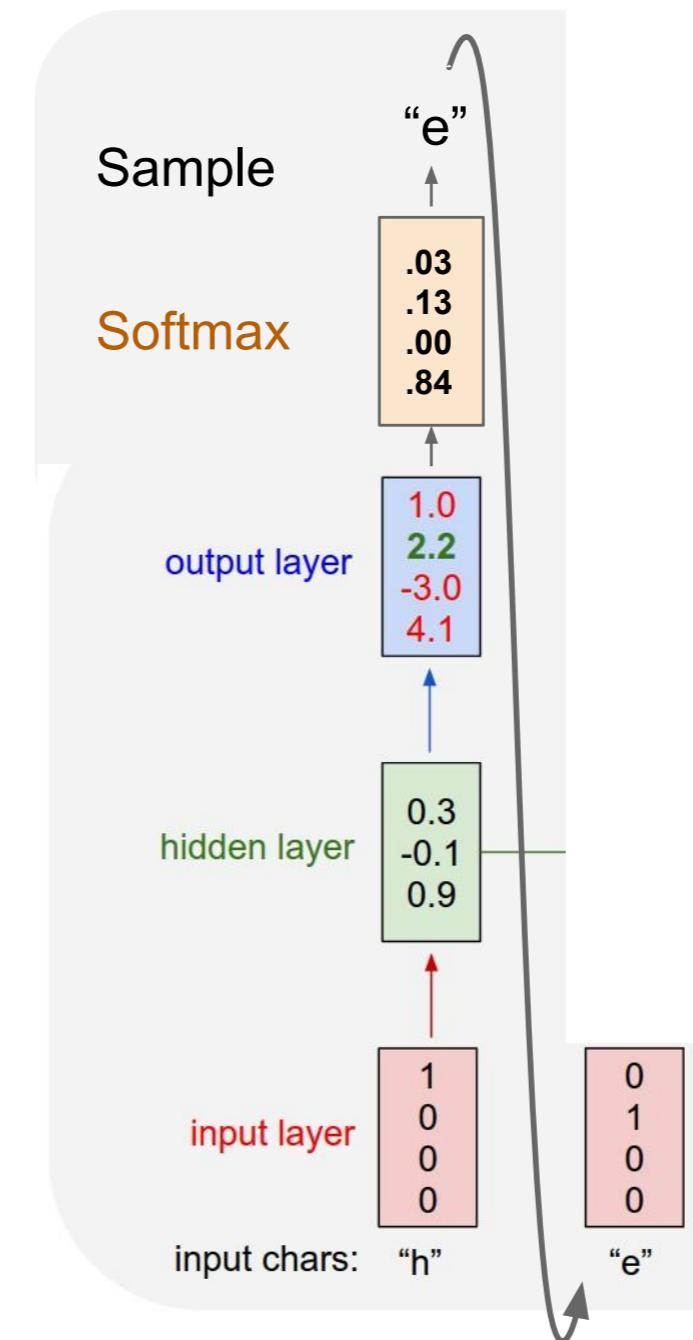
At test-time sample
characters one at a time,
feed back to model



Example: Character-level Language Model Sampling

Vocabulary:
[h,e,l,o]

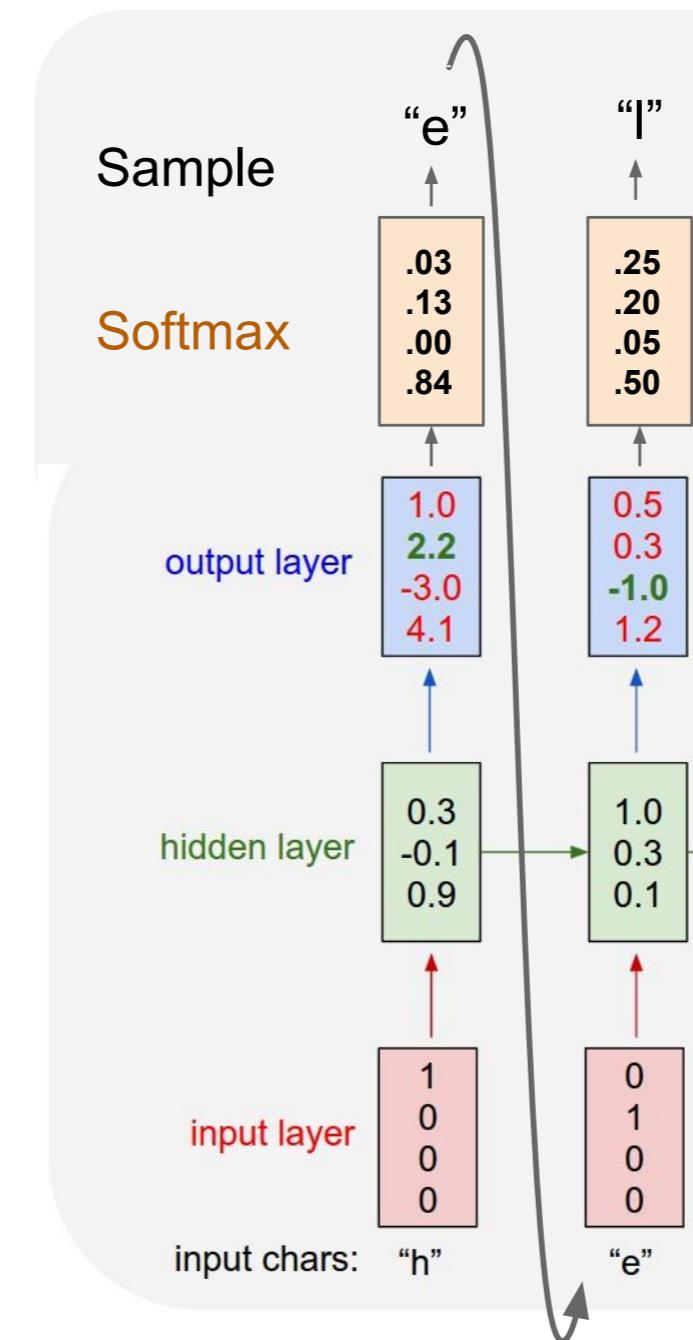
At test-time sample
characters one at a time,
feed back to model



Example: Character-level Language Model Sampling

Vocabulary:
[h,e,l,o]

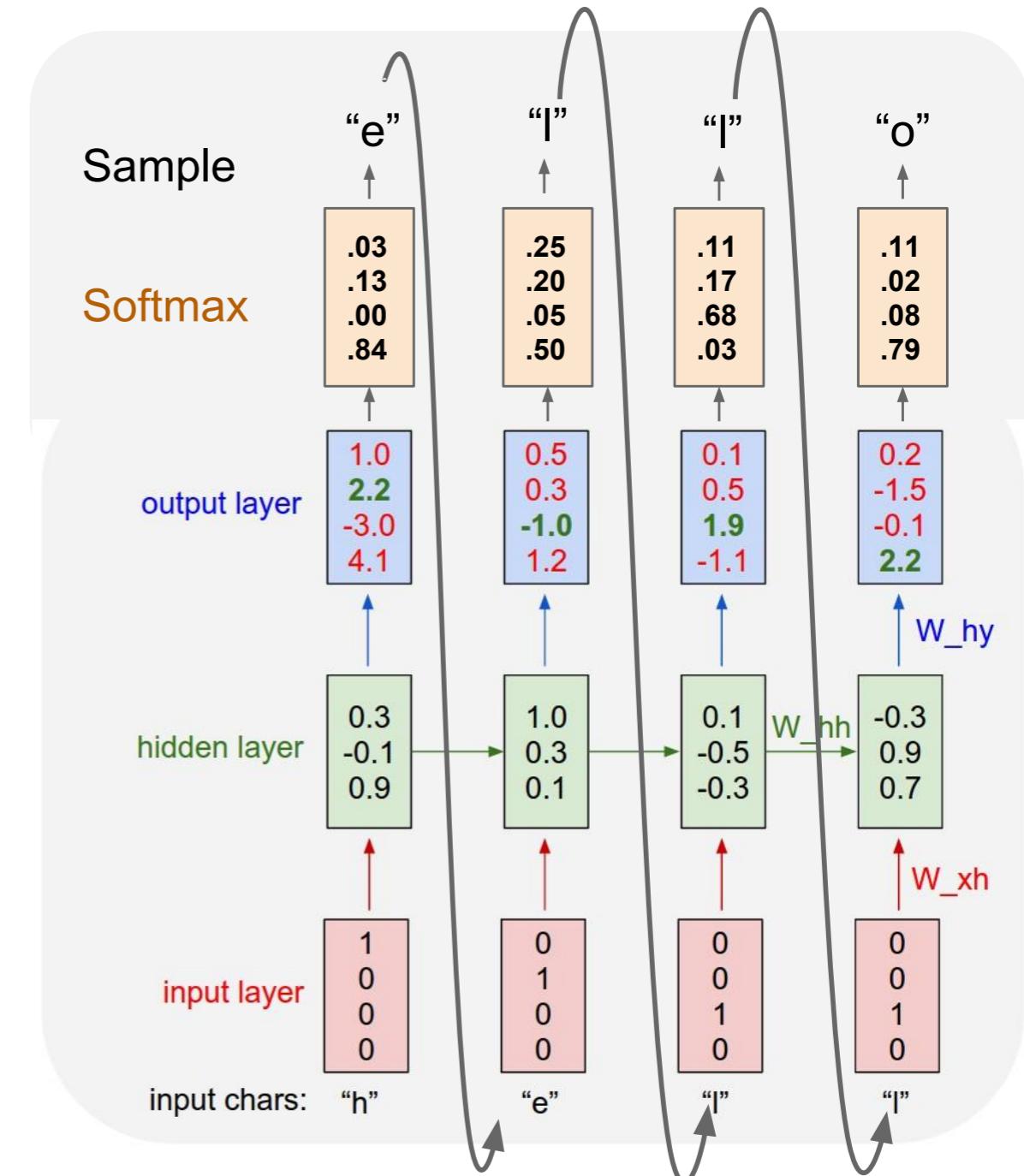
At test-time sample
characters one at a time,
feed back to model



Example: Character-level Language Model Sampling

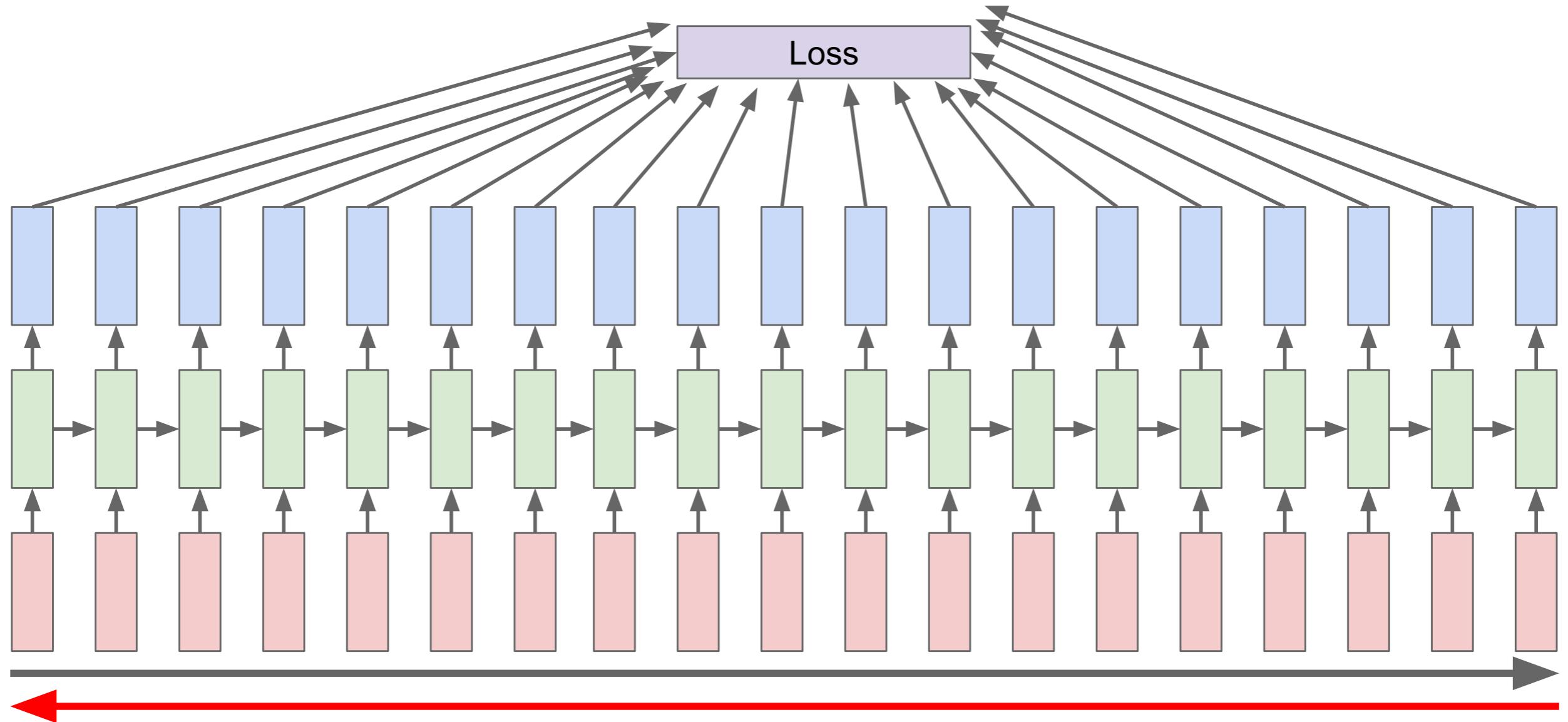
Vocabulary:
[h,e,l,o]

At test-time sample
characters one at a time,
feed back to model

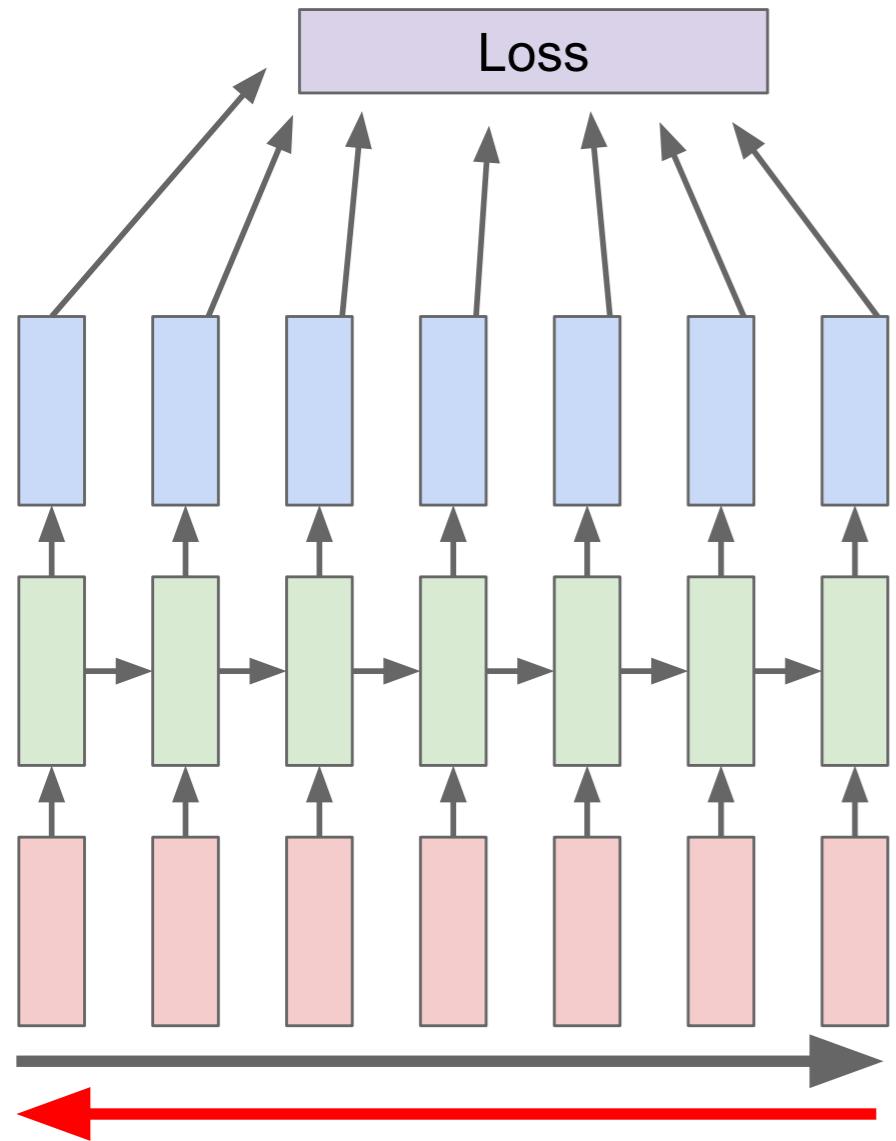


Backpropagation through time

Forward through entire sequence to compute loss, then backward through entire sequence to compute gradient

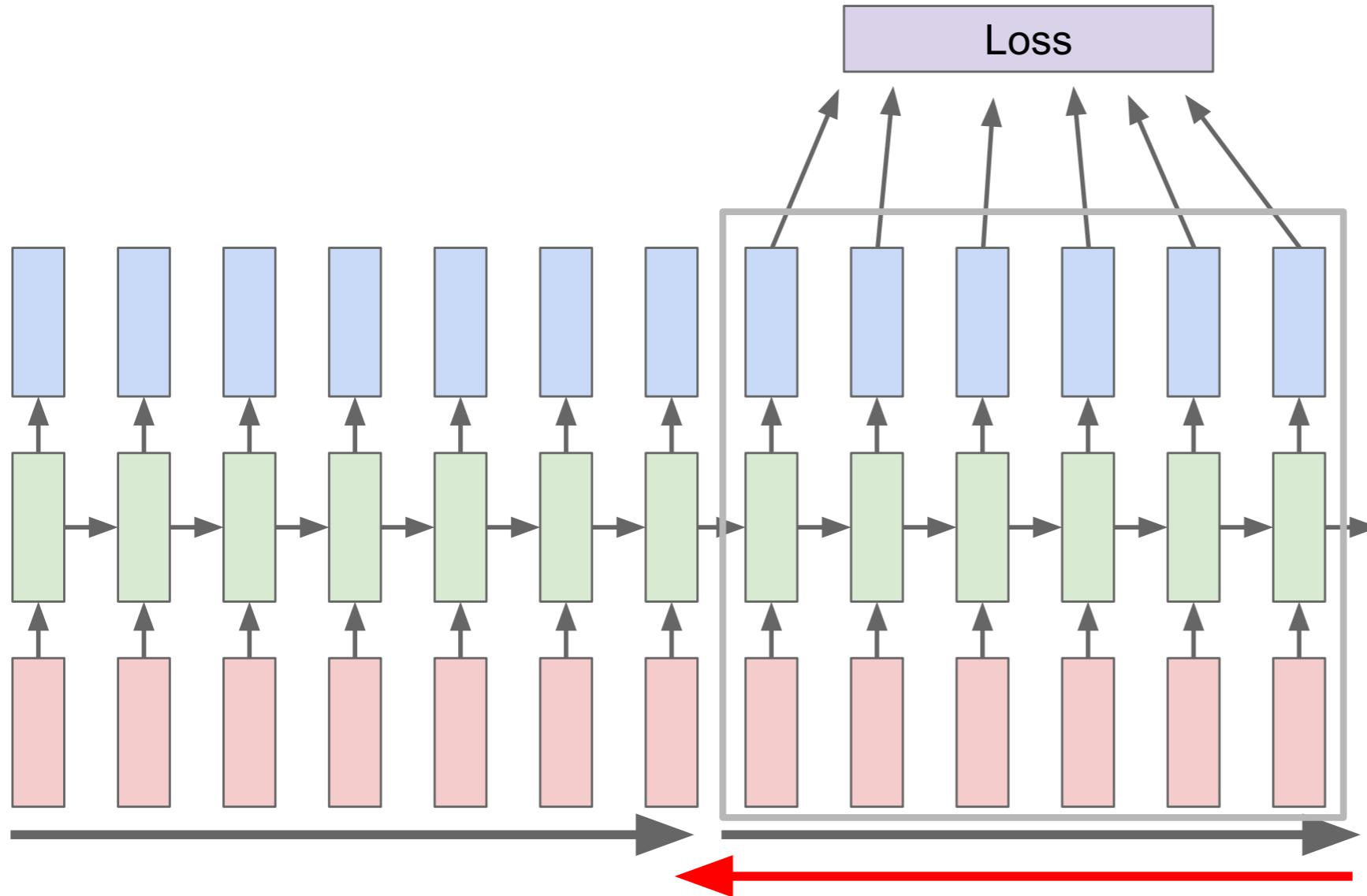


Truncated Backpropagation through time



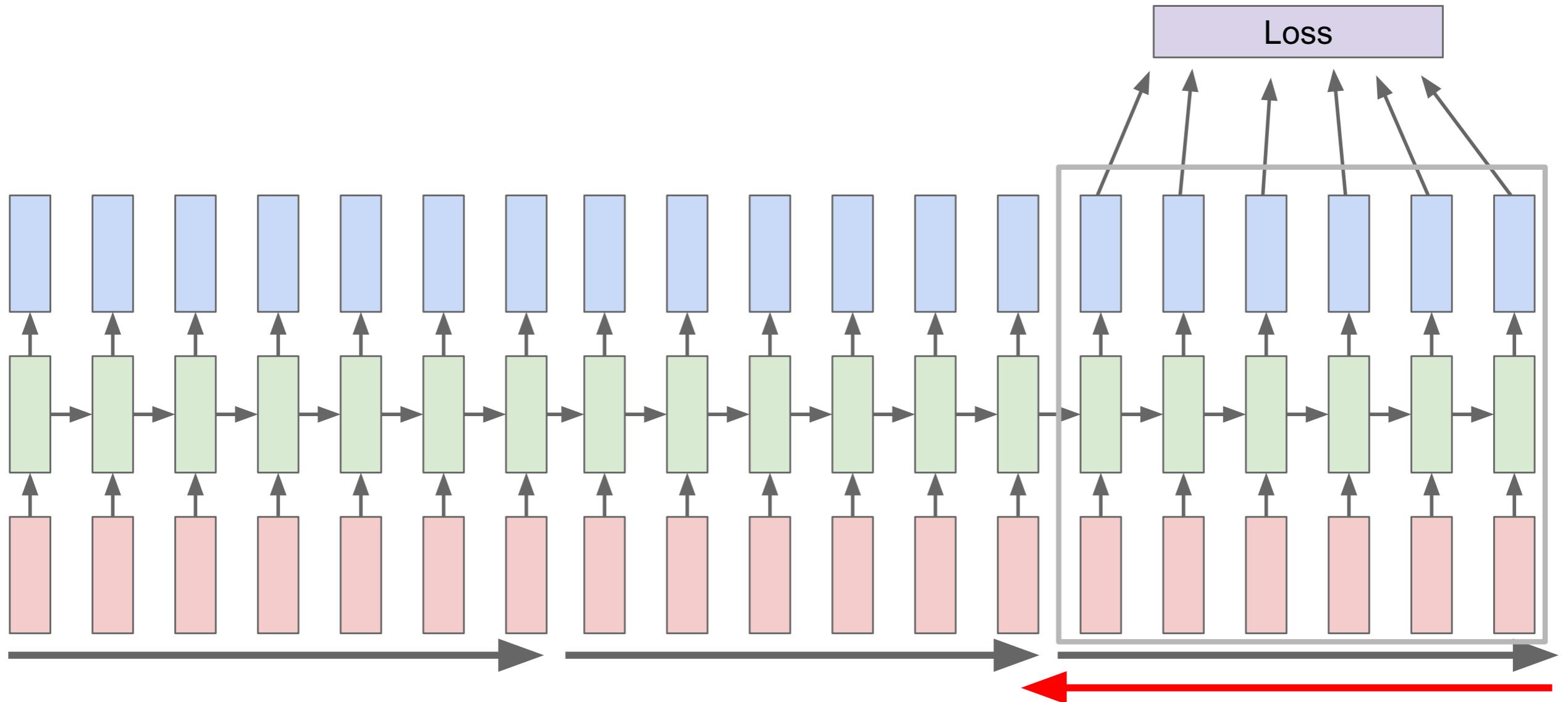
Run forward and backward
through chunks of the
sequence instead of whole
sequence

Truncated Backpropagation through time



Carry hidden states forward in time forever, but only backpropagate for some smaller number of steps

Truncated Backpropagation through time

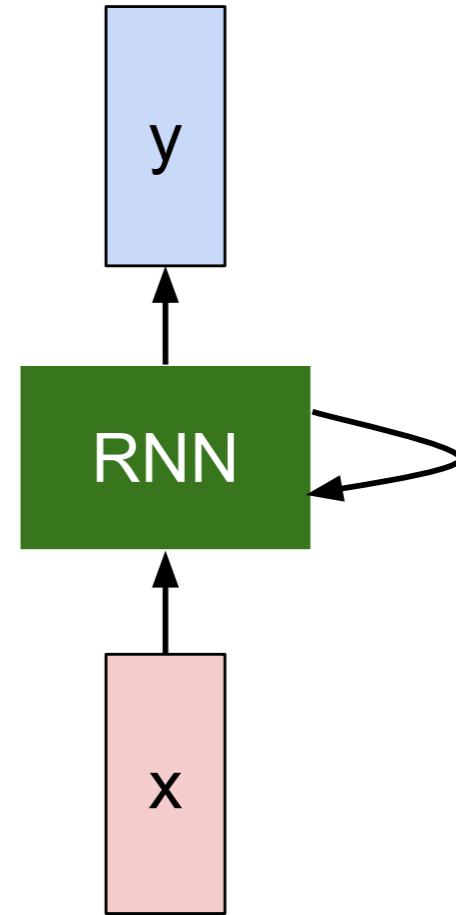


THE SONNETS

by William Shakespeare

From fairest creatures we desire increase,
That thereby beauty's rose might never die,
But as the riper should by time decease,
His tender heir might bear his memory:
But thou, contracted to thine own bright eyes,
Feed'st thy light's flame with self-substantial fuel,
Making a famine where abundance lies,
Thyself thy foe, to thy sweet self too cruel:
Thou that art now the world's fresh ornament,
And only herald to the gaudy spring,
Within thine own bud buriest thy content,
And tender churl mak'st waste in niggarding:
Pity the world, or else this glutton be,
To eat the world's due, by the grave and thee.

When forty winters shall besiege thy brow,
And dig deep trenches in thy beauty's field,
Thy youth's proud livery so gazed on now,
Will be a tatter'd weed of small worth held:
Then being asked, where all thy beauty lies,
Where all the treasure of thy lusty days;
To say, within thine own deep sunken eyes,
Were an all-eating shame, and thriftless praise.
How much more praise desp'rd thy beauty's use,
If thou couldst answer 'This fair child of mine
Shall sum my count, and make my old excuse,'
Proving his beauty by succession thine!
This were to be new made when thou art old,
And see thy blood warm when thou feel'st it cold.



at first:

tyntd-iafhatawiaoahrdemot lytdws e ,tfti, astai f ogoh eoase rrranbyne 'nhthnee e
plia tkldrgd t o idoe ns,smtt h ne etie h,hregtrs nigtike,aoaenns lng

↓ train more

"Tmont thithey" fomesscerliund
Keushey. Thom here
sheulke, anmerenith ol sivh I lalterthend Bleipile shuwy fil on aseterlome
coaniogennc Phe lism thond hon at. MeiDimorotion in ther thize."

↓ train more

Aftair fall unsuch that the hall for Prince Velzonski's that me of
her hearly, and behs to so arwage fiving were to it beloge, pavu say falling misfort
how, and Gogition is so overelical and after.

↓ train more

"Why do what that day," replied Natasha, and wishing to himself the fact the
princess, Princess Mary was easier, fed in had oftened him.
Pierre aking his soul came to the packs and drove up his father-in-law women.

PANDARUS:

Alas, I think he shall be come approached and the day
When little strain would be attain'd into being never fed,
And who is but a chain and subjects of his death,
I should not sleep.

Second Senator:

They are away this miseries, produced upon my soul,
Breaking and strongly should be buried, when I perish
The earth and thoughts of many states.

DUKE VINCENTIO:

Well, your wit is in the care of side and that.

Second Lord:

They would be ruled after this chamber, and
my fair nues begun out of the fact, to be conveyed,
Whose noble souls I'll have the heart of the wars.

Clown:

Come, sir, I will make did behold your worship.

VIOLA:

I'll drink it.

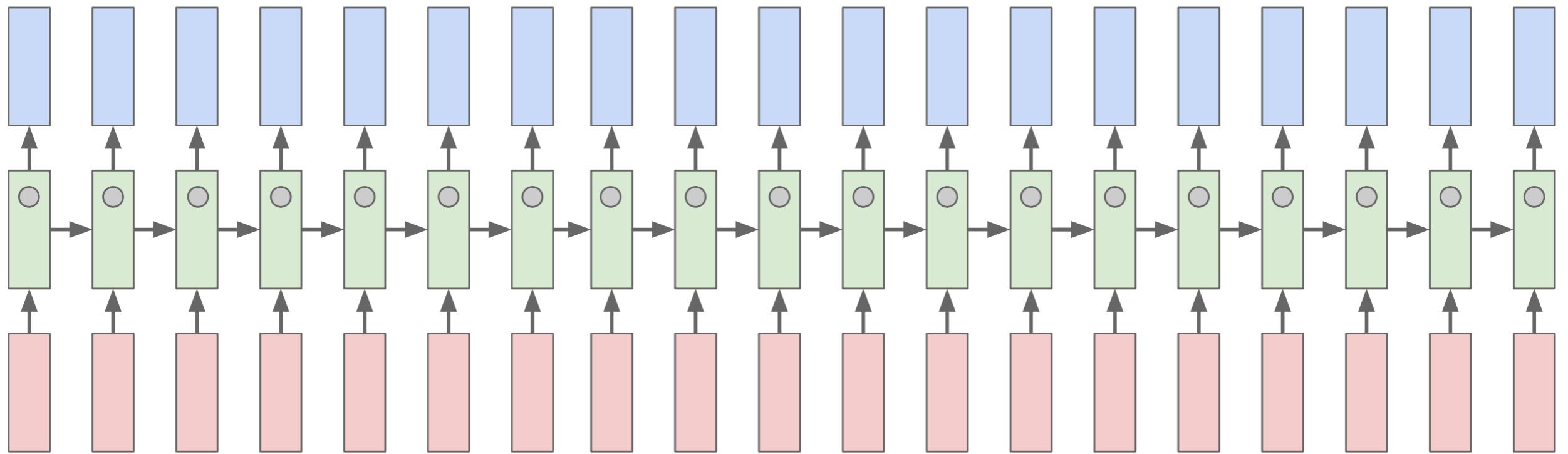
VIOLA:

Why, Salisbury must find his flesh and thought
That which I am not aps, not a man and in fire,
To show the reining of the raven and the wars
To grace my hand reproach within, and not a fair are hand,
That Caesar and my goodly father's world;
When I was heaven of presence and our fleets,
We spare with hours, but cut thy council I am great,
Murdered and by thy master's ready there
My power to give thee but so much as hell:
Some service in the noble bondman here,
Would show him to her wine.

KING LEAR:

O, if you were a feeble sight, the courtesy of your law,
Your sight and several breath, will wear the gods
With his heads, and my hands are wonder'd at the deeds,
So drop upon your lordship's head, and your opinion
Shall be against your honour.

Searching for interpretable cells



Karpathy, Johnson, and Fei-Fei: Visualizing and Understanding Recurrent Networks, ICLR Workshop 2016

Searching for interpretable cells

"You mean to imply that I have nothing to eat out of... On the contrary, I can supply you with everything even if you want to give dinner parties," warmly replied Chichagov, who tried by every word he spoke to prove his own rectitude and therefore imagined Kutuzov to be animated by the same desire.

Kutuzov, shrugging his shoulders, replied with his subtle penetrating smile: "I meant merely to say what I said."

quote detection cell

Karpathy, Johnson, and Fei-Fei: Visualizing and Understanding Recurrent Networks, ICLR Workshop 2016

Figures copyright Karpathy, Johnson, and Fei-Fei, 2015; reproduced with permission

Image Captioning

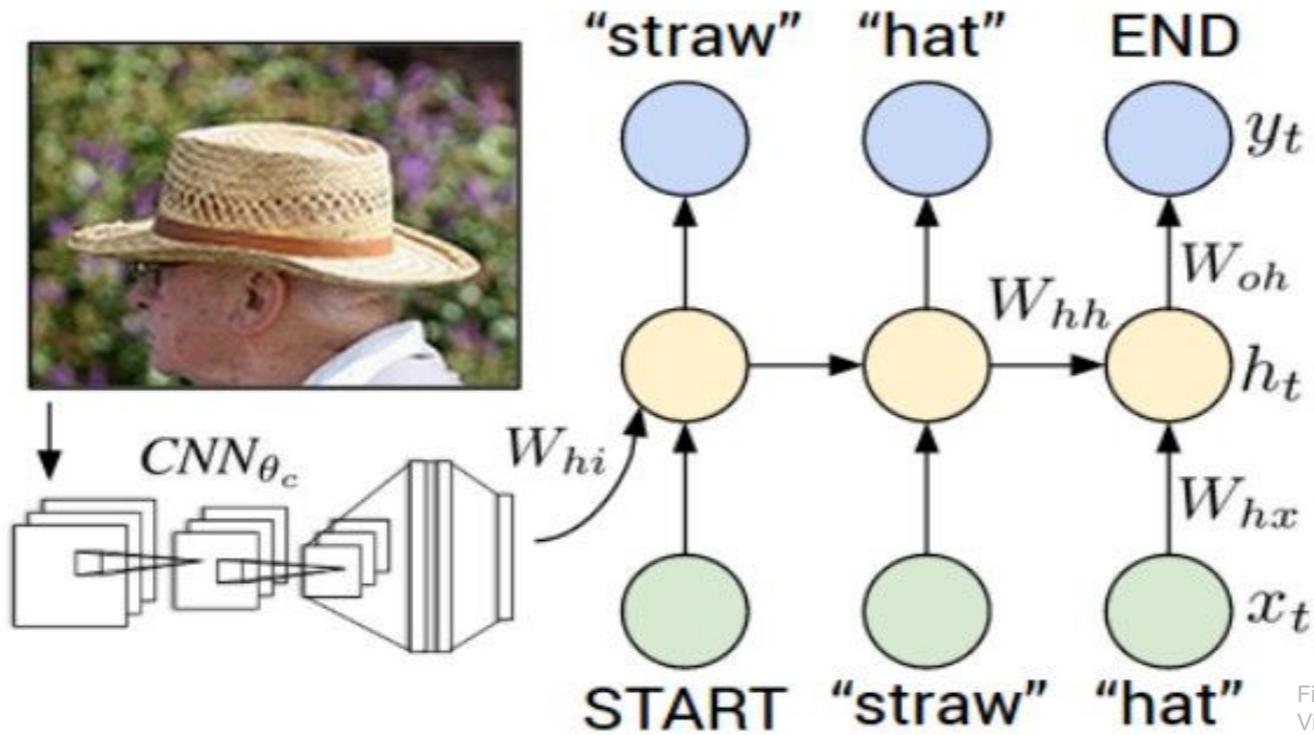


Figure from Karpathy et al, "Deep Visual-Semantic Alignments for Generating Image Descriptions", CVPR 2015; figure copyright IEEE, 2015.
Reproduced for educational purposes.

Explain Images with Multimodal Recurrent Neural Networks, Mao et al.

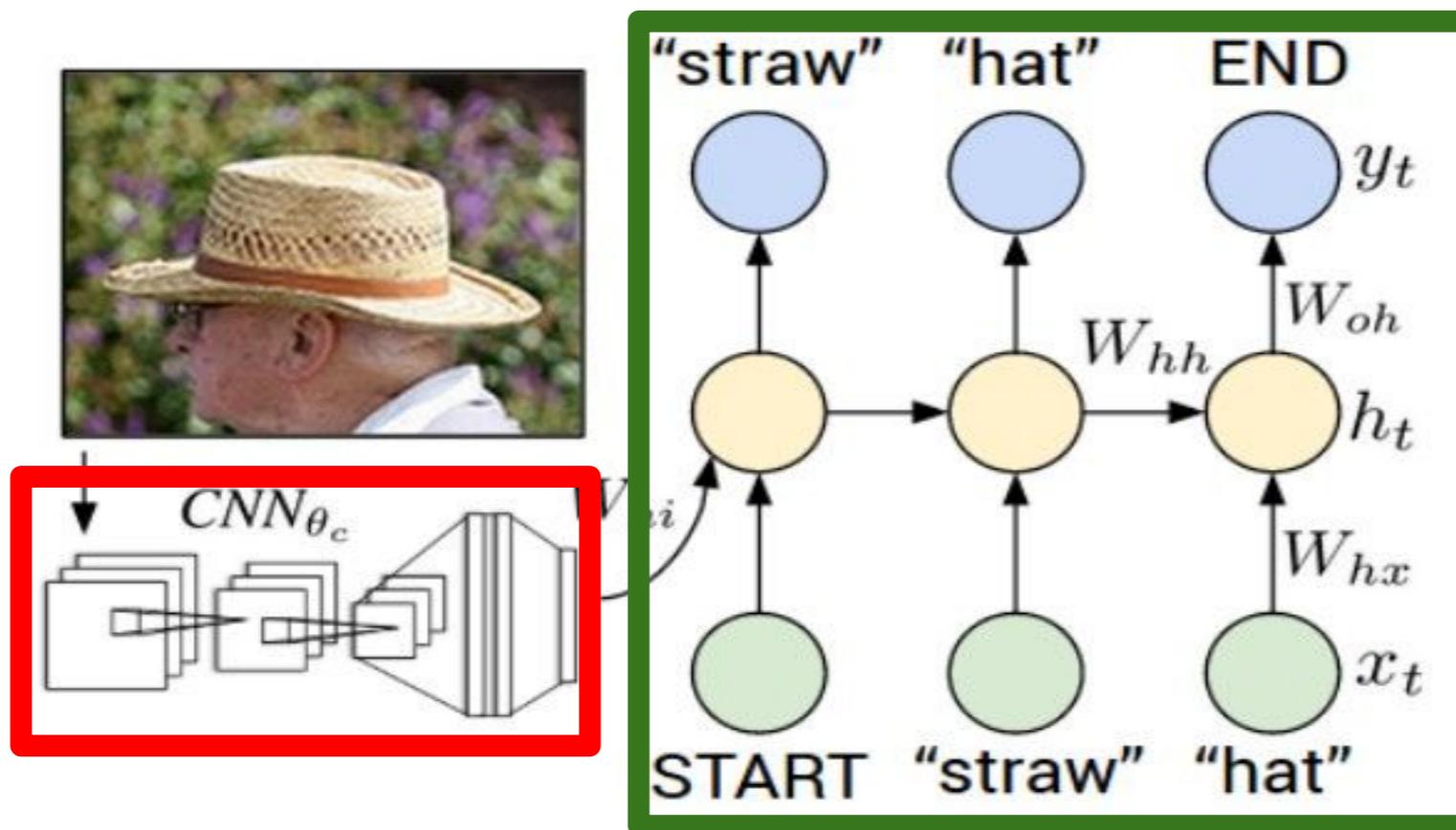
Deep Visual-Semantic Alignments for Generating Image Descriptions, Karpathy and Fei-Fei

Show and Tell: A Neural Image Caption Generator, Vinyals et al.

Long-term Recurrent Convolutional Networks for Visual Recognition and Description, Donahue et al.

Learning a Recurrent Visual Representation for Image Caption Generation, Chen and Zitnick

Recurrent Neural Network



Convolutional Neural Network

test image



[This image is CC0 public domain](#)



test image

image

conv-64

conv-64

maxpool

conv-128

conv-128

maxpool

conv-256

conv-256

maxpool

conv-512

conv-512

maxpool

conv-512

conv-512

maxpool

FC-4096

FC-4096

FC-1000

softmax



test image

image



test image



conv-64

conv-64

maxpool

conv-128

conv-128

maxpool

conv-256

conv-256

maxpool

conv-512

conv-512

maxpool

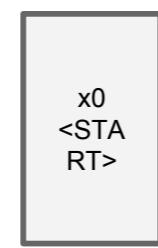
conv-512

conv-512

maxpool

FC-4096

FC-4096



<START>

image



conv-64

conv-64

maxpool

conv-128

conv-128

maxpool

conv-256

conv-256

maxpool

conv-512

conv-512

maxpool

conv-512

conv-512

maxpool

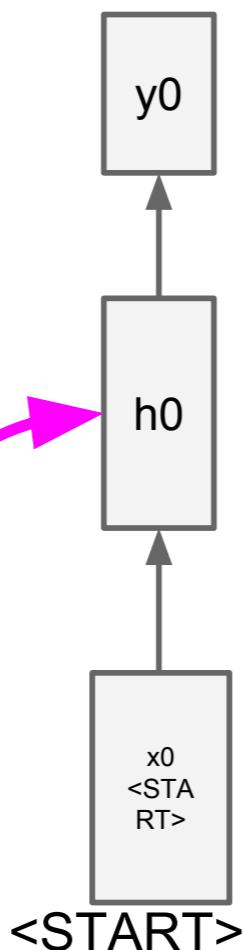
FC-4096

FC-4096

V



test image



before:

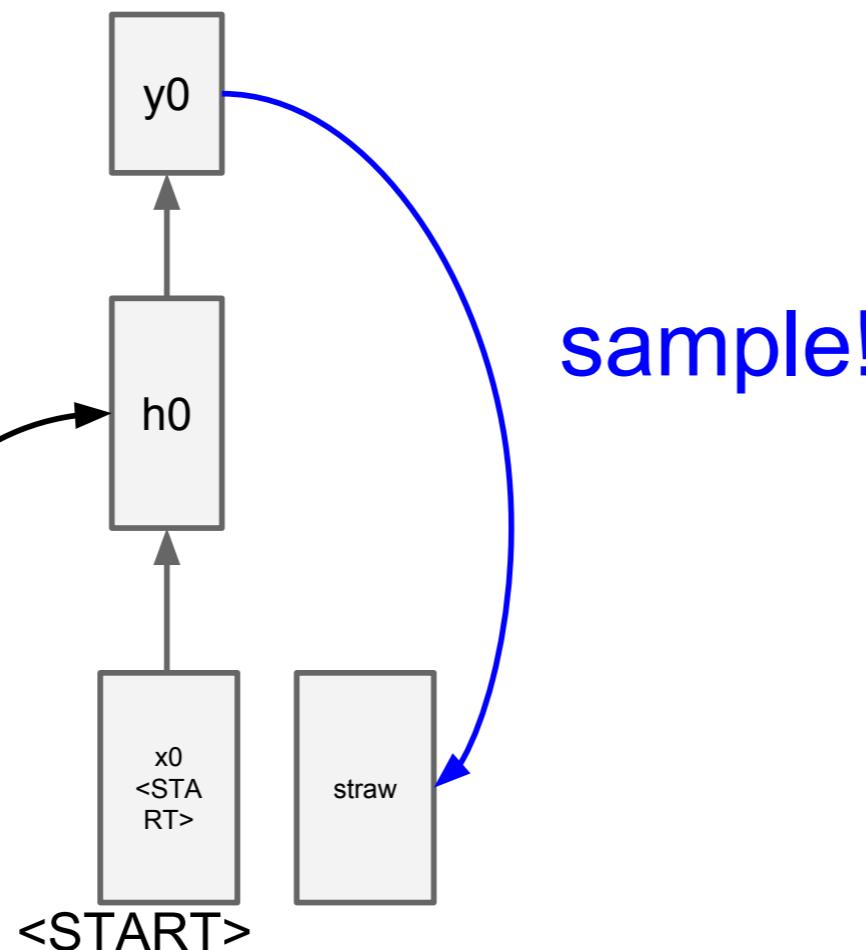
$$h = \tanh(W_{xh} * x + W_{hh} * h)$$

now:

$$h = \tanh(W_{xh} * x + W_{hh} * h + W_{vh} * v)$$



test image



image



conv-64

conv-64

maxpool

conv-128

conv-128

maxpool

conv-256

conv-256

maxpool

conv-512

conv-512

maxpool

conv-512

conv-512

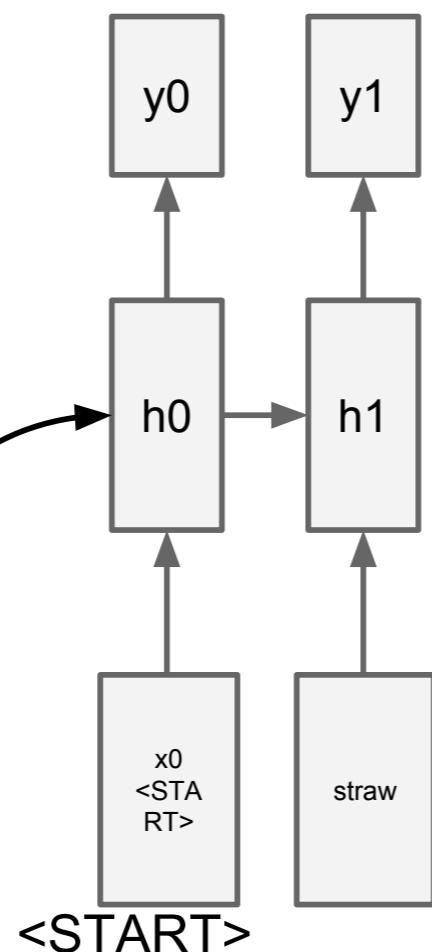
maxpool

FC-4096

FC-4096



test image



image



conv-64

conv-64

maxpool

conv-128

conv-128

maxpool

conv-256

conv-256

maxpool

conv-512

conv-512

maxpool

conv-512

conv-512

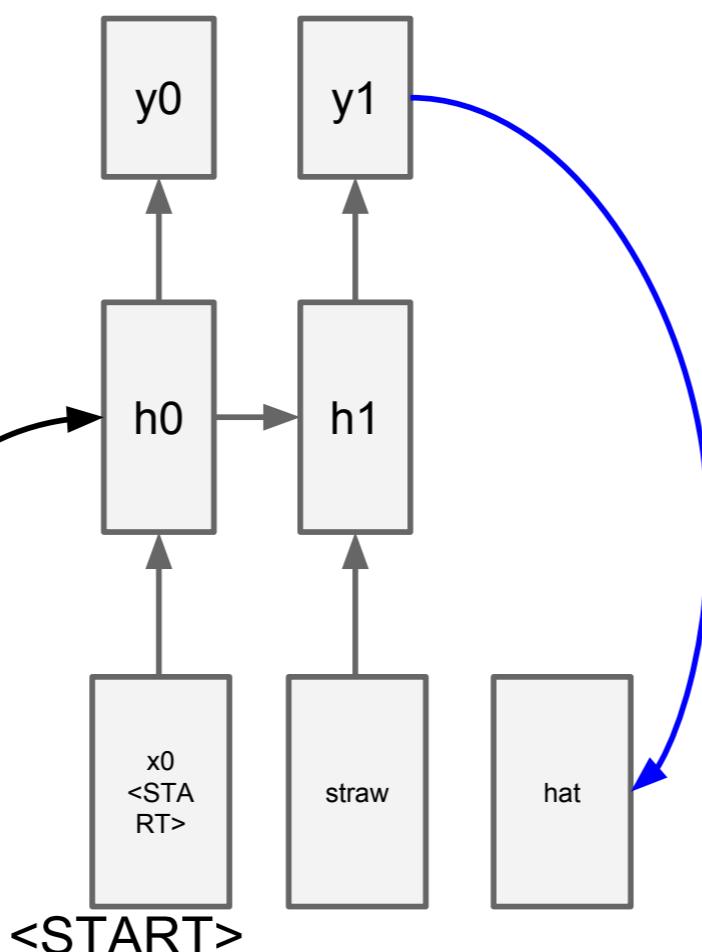
maxpool

FC-4096

FC-4096



test image



image



test image



conv-64

conv-64

maxpool

conv-128

conv-128

maxpool

conv-256

conv-256

maxpool

conv-512

conv-512

maxpool

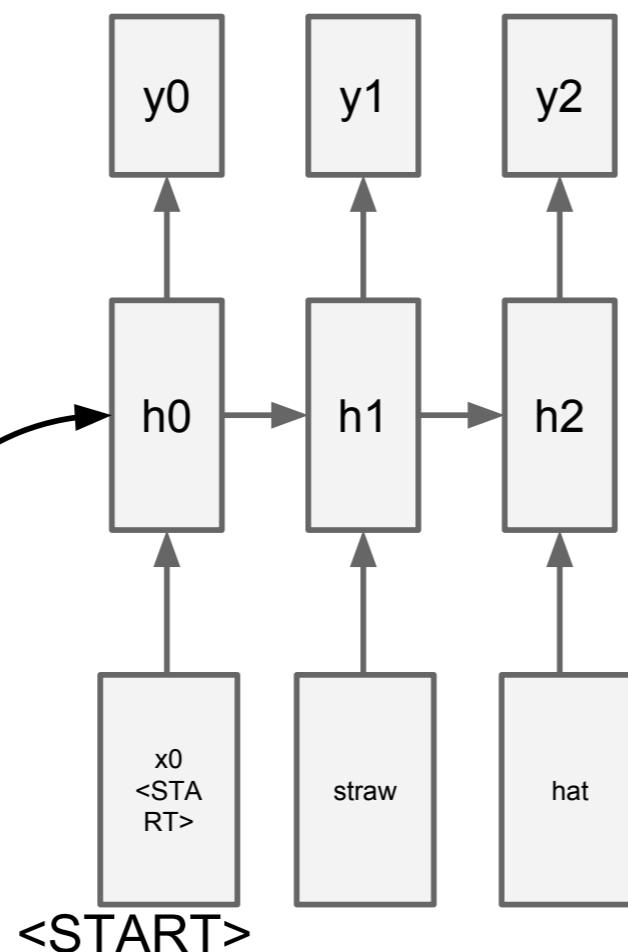
conv-512

conv-512

maxpool

FC-4096

FC-4096



image



conv-64

conv-64

maxpool

conv-128

conv-128

maxpool

conv-256

conv-256

maxpool

conv-512

conv-512

maxpool

conv-512

conv-512

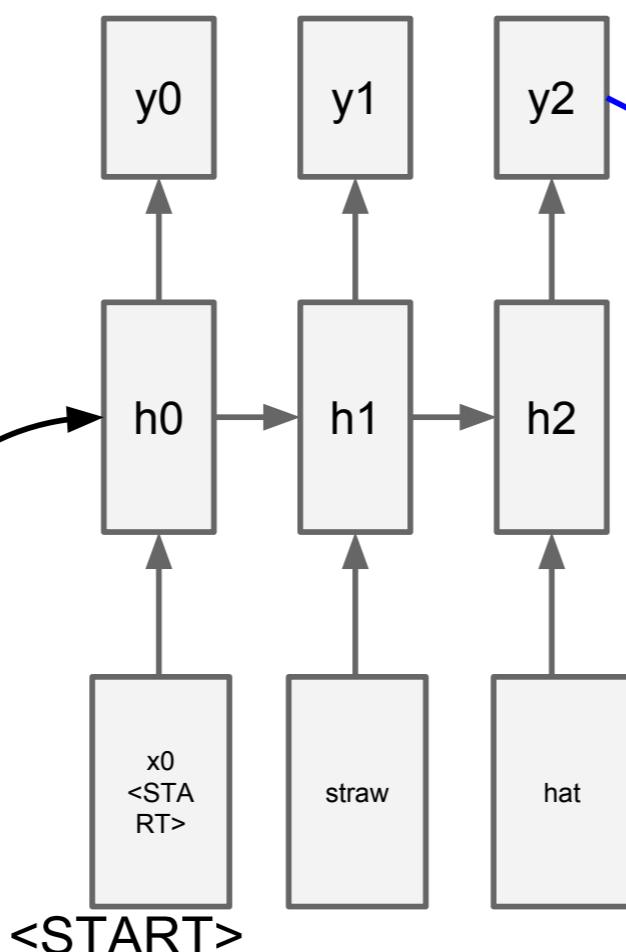
maxpool

FC-4096

FC-4096



test image



sample
<END> token
=> finish.

Image Captioning: Example Results

Captions generated using [neuraltalk2](#)
All images are [CC0 Public domain](#):
[cat suitcase](#), [cat tree](#), [dog](#), [bear](#),
[surfers](#), [tennis](#), [giraffe](#), [motorcycle](#)



A cat sitting on a suitcase on the floor



A cat is sitting on a tree branch



A dog is running in the grass with a frisbee



A white teddy bear sitting in the grass



Two people walking on the beach with surfboards



A tennis player in action on the court



Two giraffes standing in a grassy field

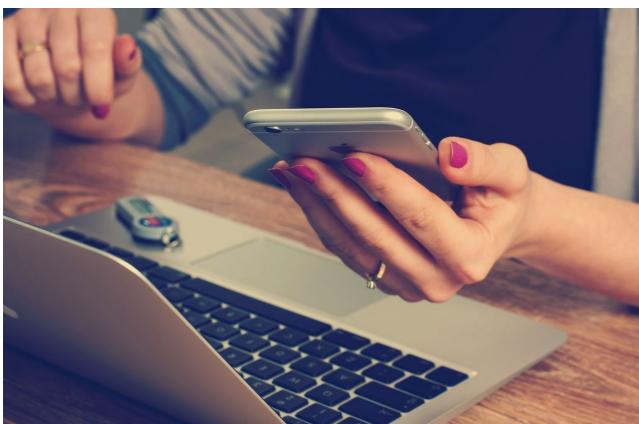


A man riding a dirt bike on a dirt track

Image Captioning: Failure Cases



A woman is holding a cat in her hand



A person holding a computer mouse on a desk



A woman standing on a beach holding a surfboard



A bird is perched on a tree branch



A man in a baseball uniform throwing a ball

Multilayer RNNs

$$h_t^l = \tanh W^l \begin{pmatrix} h_t^{l-1} \\ h_{t-1}^l \end{pmatrix}$$

$h \in \mathbb{R}^n$ $W^l [n \times 2n]$

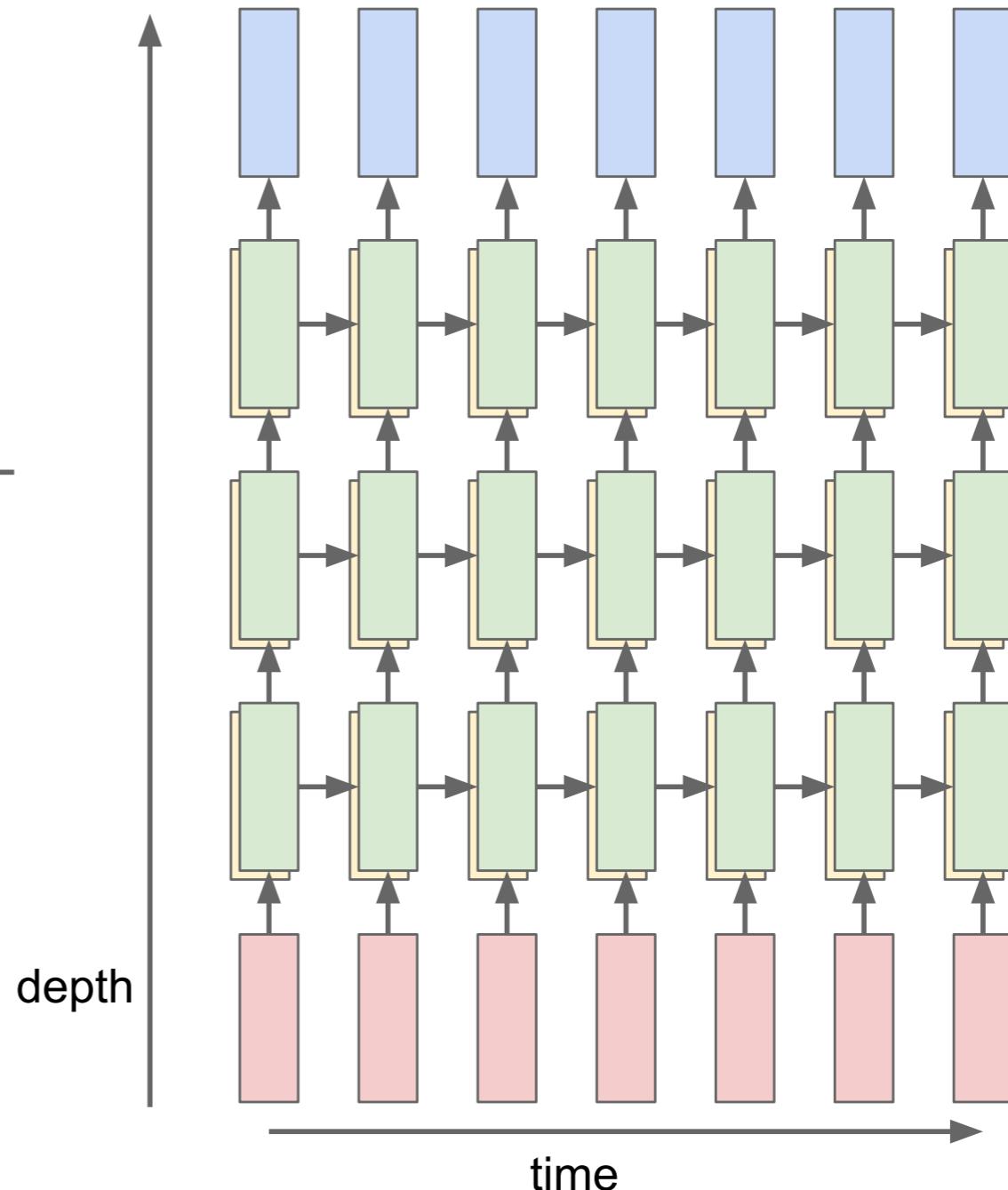
LSTM:

$W^l [4n \times 2n]$

$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \text{sigm} \\ \text{sigm} \\ \text{sigm} \\ \tanh \end{pmatrix} W^l \begin{pmatrix} h_t^{l-1} \\ h_{t-1}^l \end{pmatrix}$$

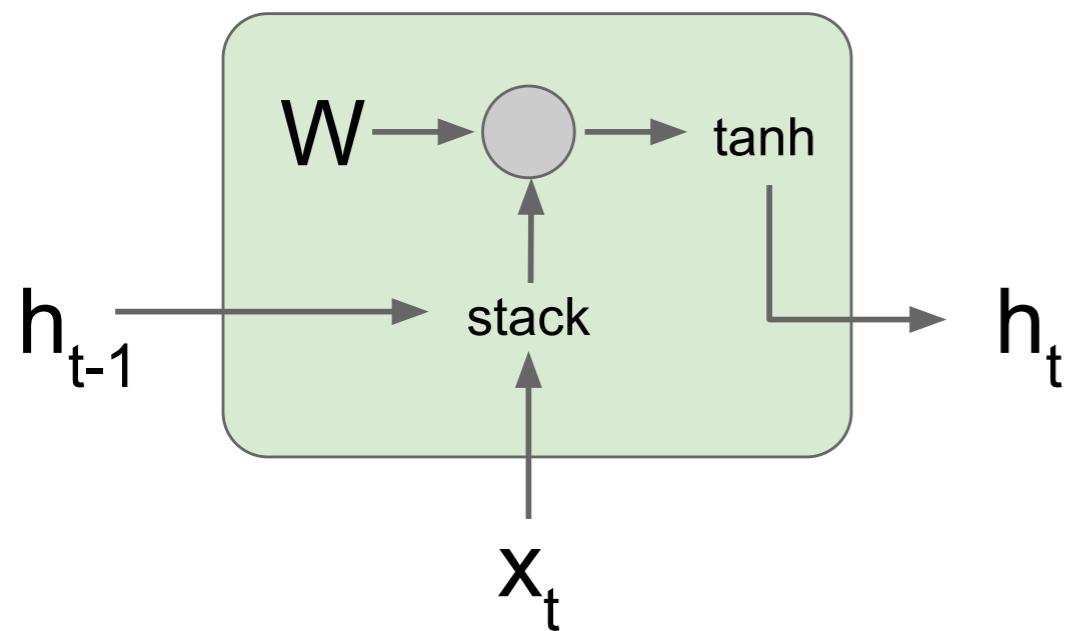
$$c_t^l = f \odot c_{t-1}^l + i \odot g$$

$$h_t^l = o \odot \tanh(c_t^l)$$



Vanilla RNN Gradient Flow

Bengio et al, "Learning long-term dependencies with gradient descent is difficult", IEEE Transactions on Neural Networks, 1994
Pascanu et al, "On the difficulty of training recurrent neural networks", ICML 2013

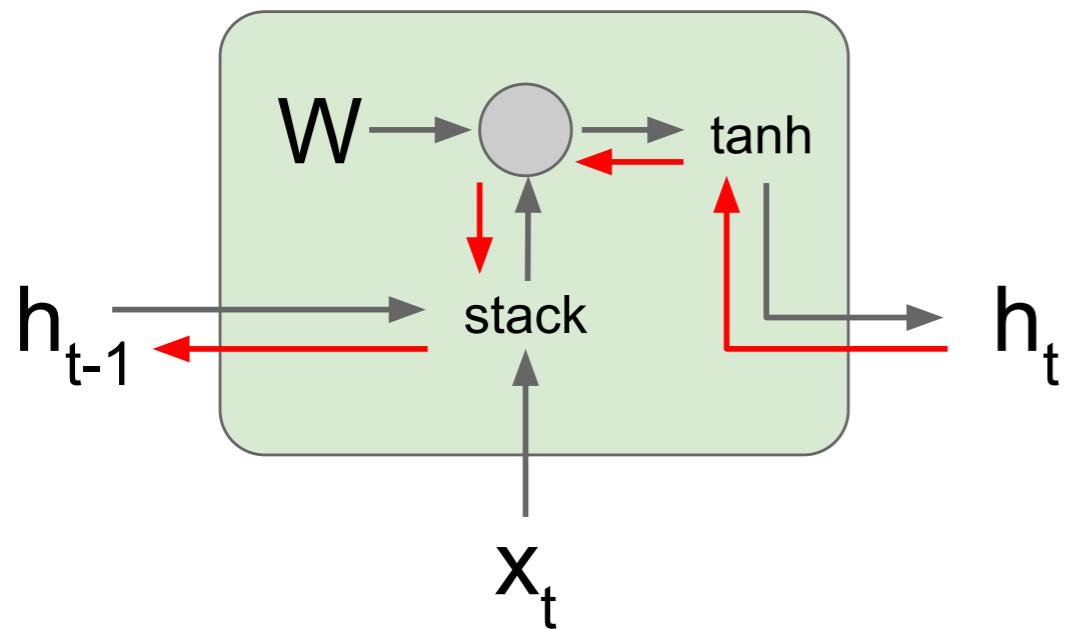


$$\begin{aligned} h_t &= \tanh(W_{hh}h_{t-1} + W_{xh}x_t) \\ &= \tanh \left(\begin{pmatrix} W_{hh} & W_{hx} \end{pmatrix} \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix} \right) \\ &= \tanh \left(W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix} \right) \end{aligned}$$

Vanilla RNN Gradient Flow

Bengio et al, "Learning long-term dependencies with gradient descent is difficult", IEEE Transactions on Neural Networks, 1994
Pascanu et al, "On the difficulty of training recurrent neural networks", ICML 2013

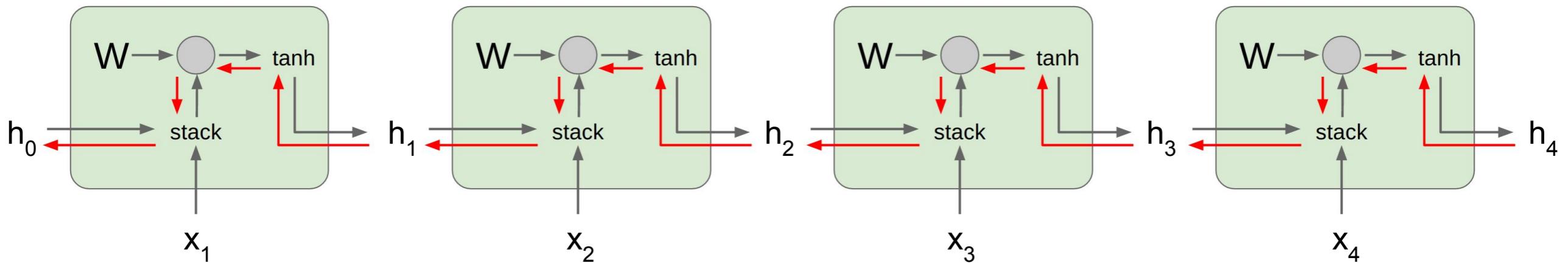
Backpropagation from h_t
to h_{t-1} multiplies by W
(actually W_{hh}^T)



$$\begin{aligned} h_t &= \tanh(W_{hh}h_{t-1} + W_{xh}x_t) \\ &= \tanh \left(\begin{pmatrix} W_{hh} & W_{hx} \end{pmatrix} \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix} \right) \\ &= \tanh \left(W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix} \right) \end{aligned}$$

Vanilla RNN Gradient Flow

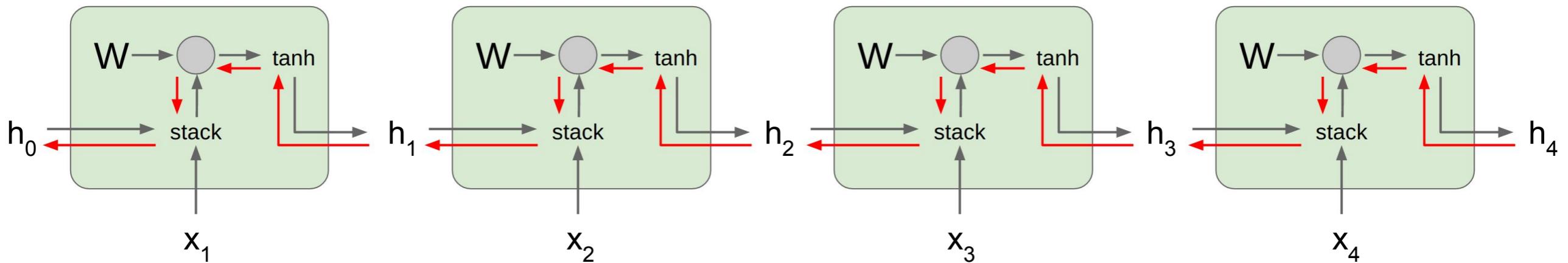
Bengio et al, "Learning long-term dependencies with gradient descent is difficult", IEEE Transactions on Neural Networks, 1994
Pascanu et al, "On the difficulty of training recurrent neural networks", ICML 2013



Computing gradient
of h_0 involves many
factors of W
(and repeated tanh)

Vanilla RNN Gradient Flow

Bengio et al, "Learning long-term dependencies with gradient descent is difficult", IEEE Transactions on Neural Networks, 1994
Pascanu et al, "On the difficulty of training recurrent neural networks", ICML 2013



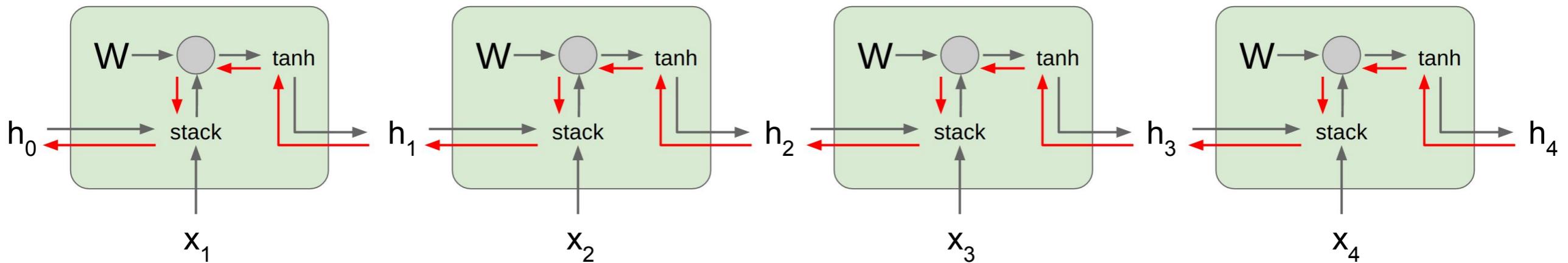
Computing gradient of h_0 involves many factors of W (and repeated tanh)

Largest singular value > 1 :
Exploding gradients

Largest singular value < 1 :
Vanishing gradients

Vanilla RNN Gradient Flow

Bengio et al, "Learning long-term dependencies with gradient descent is difficult", IEEE Transactions on Neural Networks, 1994
Pascanu et al, "On the difficulty of training recurrent neural networks", ICML 2013



Computing gradient of h_0 involves many factors of W (and repeated tanh)

Largest singular value > 1 :
Exploding gradients

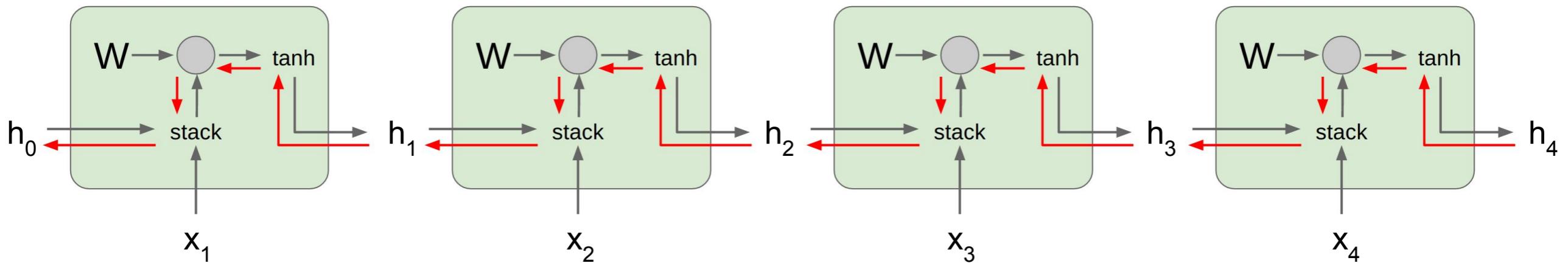
Largest singular value < 1 :
Vanishing gradients

Gradient clipping: Scale gradient if its norm is too big

```
grad_norm = np.sum(grad * grad)
if grad_norm > threshold:
    grad *= (threshold / grad_norm)
```

Vanilla RNN Gradient Flow

Bengio et al, "Learning long-term dependencies with gradient descent is difficult", IEEE Transactions on Neural Networks, 1994
Pascanu et al, "On the difficulty of training recurrent neural networks", ICML 2013



Computing gradient of h_0 involves many factors of W (and repeated tanh)

Largest singular value > 1 :
Exploding gradients

Largest singular value < 1 :
Vanishing gradients

→ Change RNN architecture

Long Short Term Memory (LSTM)

Vanilla RNN

$$h_t = \tanh \left(W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix} \right)$$

LSTM

$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}$$

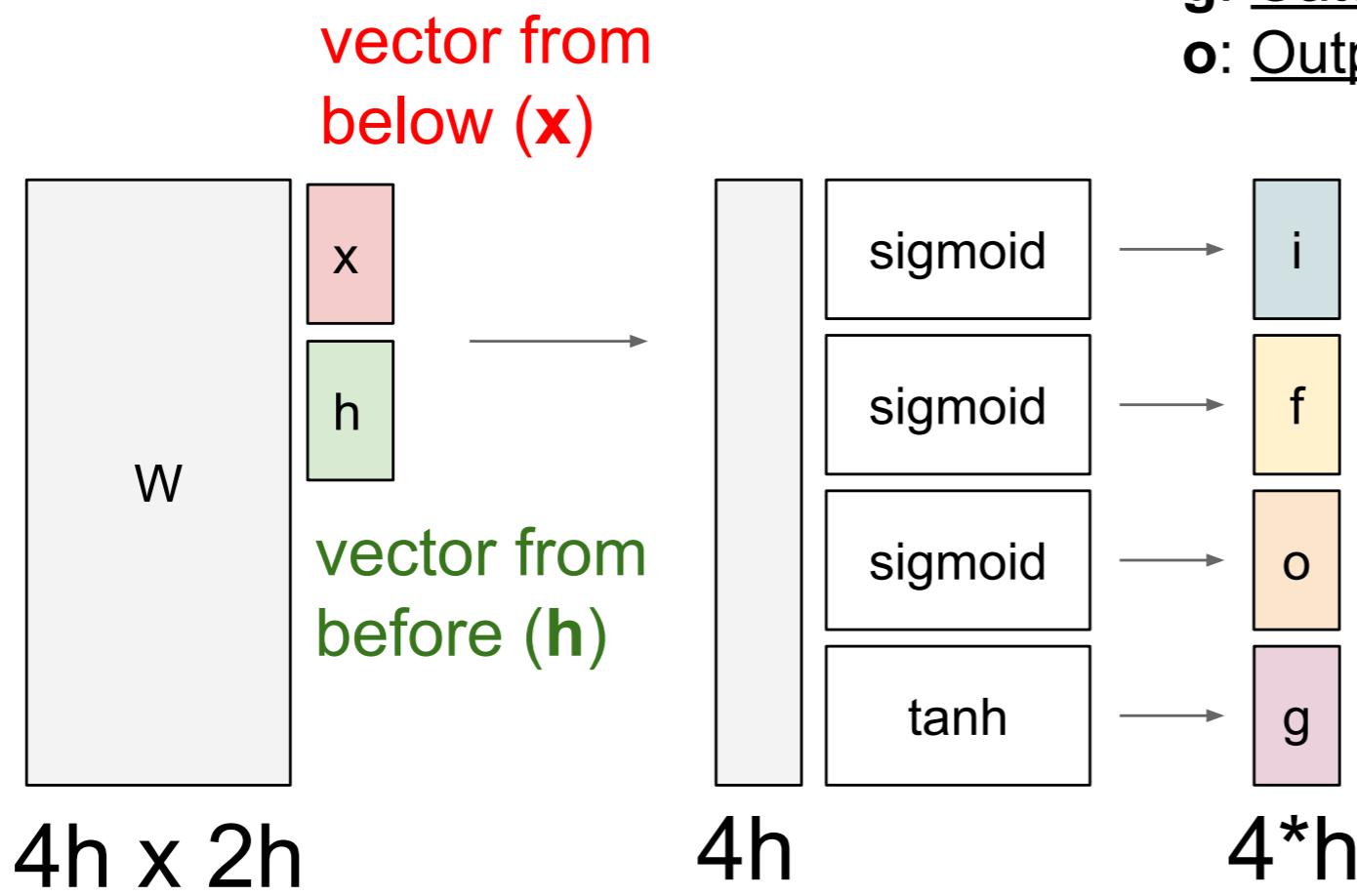
$$c_t = f \odot c_{t-1} + i \odot g$$

$$h_t = o \odot \tanh(c_t)$$

Hochreiter and Schmidhuber, "Long Short Term Memory", Neural Computation
1997

Long Short Term Memory (LSTM)

[Hochreiter et al., 1997]



- f: Forget gate, Whether to erase cell
- i: Input gate, whether to write to cell
- g: Gate gate (?), How much to write to cell
- o: Output gate, How much to reveal cell

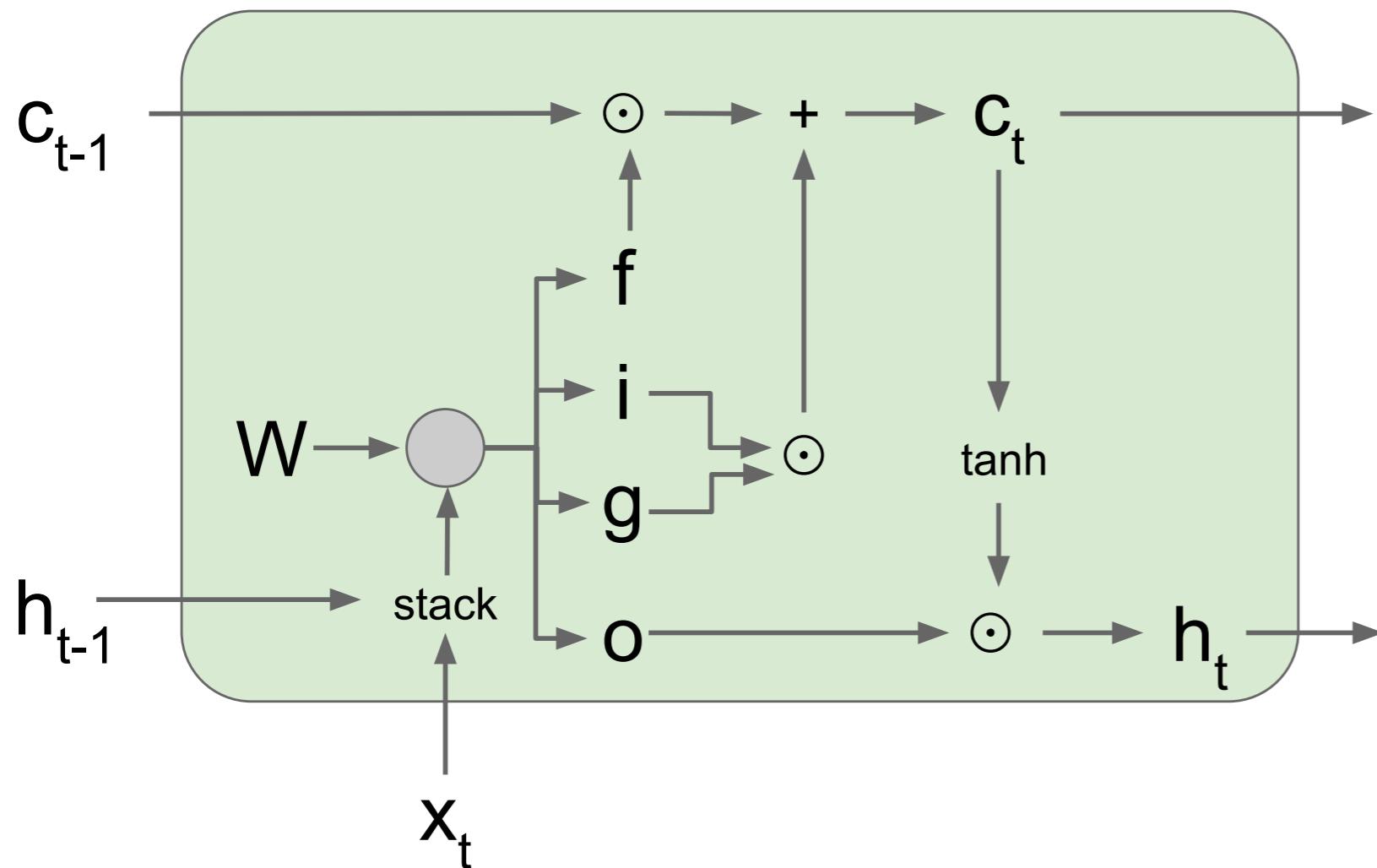
$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}$$

$$c_t = f \odot c_{t-1} + i \odot g$$

$$h_t = o \odot \tanh(c_t)$$

Long Short Term Memory (LSTM)

[Hochreiter et al., 1997]



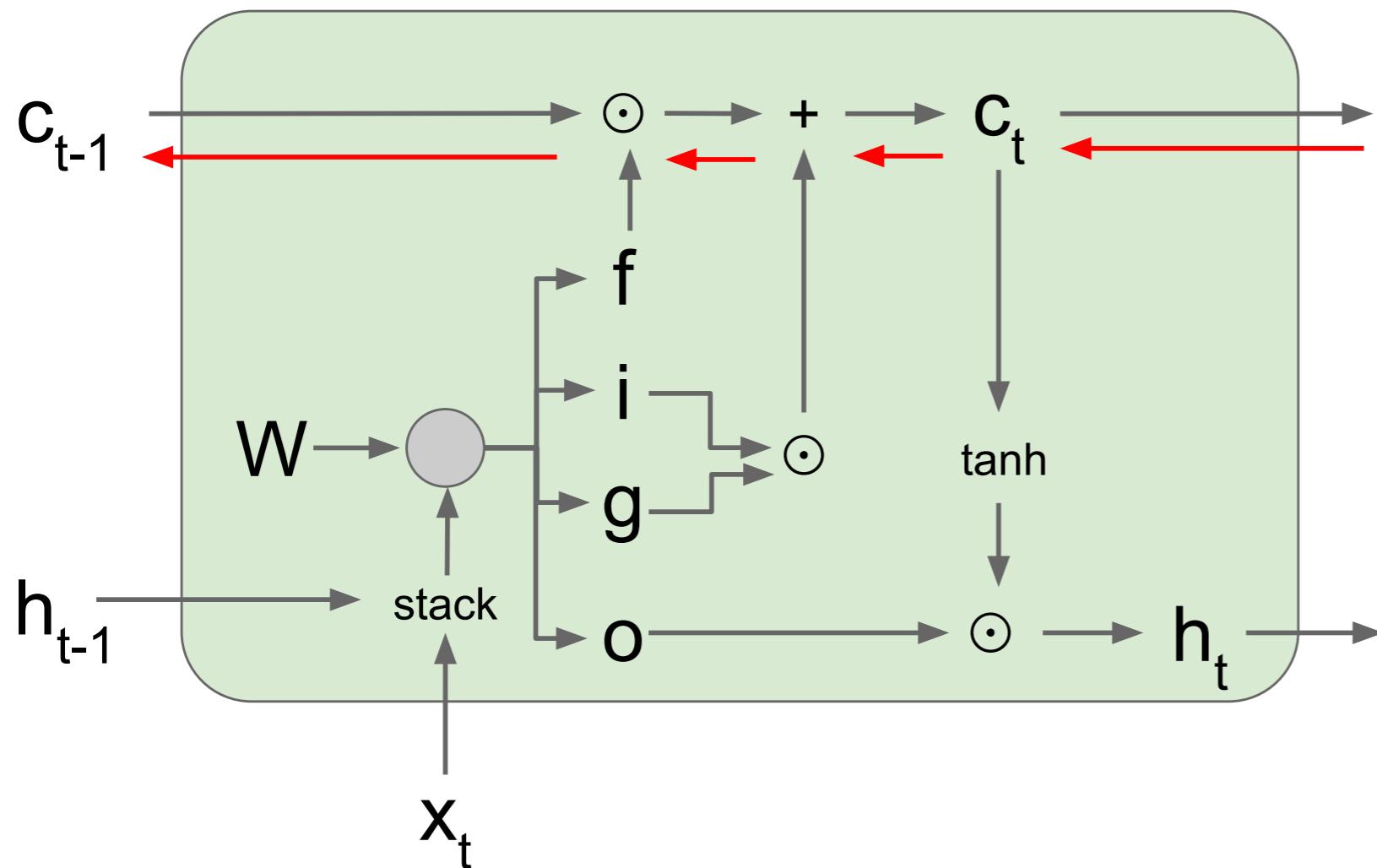
$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}$$

$$c_t = f \odot c_{t-1} + i \odot g$$

$$h_t = o \odot \tanh(c_t)$$

Long Short Term Memory (LSTM): Gradient Flow

[Hochreiter et al., 1997]



Backpropagation from c_t to c_{t-1} only elementwise multiplication by f , no matrix multiply by W

$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}$$

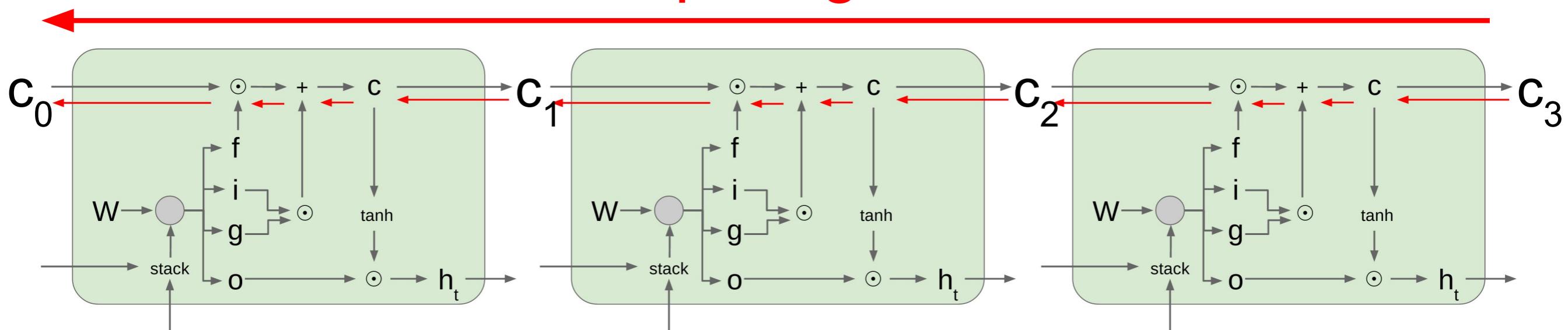
$$c_t = f \odot c_{t-1} + i \odot g$$

$$h_t = o \odot \tanh(c_t)$$

Long Short Term Memory (LSTM): Gradient Flow

[Hochreiter et al., 1997]

Uninterrupted gradient flow!



Summary

- RNNs allow a lot of flexibility in architecture design
- Vanilla RNNs are simple but don't work very well
- Common to use LSTM or GRU: their additive interactions improve gradient flow
- Backward flow of gradients in RNN can explode or vanish. Exploding is controlled with gradient clipping. Vanishing is controlled with additive interactions (LSTM)
- Better/simpler architectures are a hot topic of current research
- Better understanding (both theoretical and empirical) is needed.