A List of Topics for the First Midterm

Here's a list of things you should be comfortable doing for the exam.

1. **Three-Dimensional Coordinate Systems (Chapter 12.1)**
   (a) Plot points in three dimensions.
   (b) Compute the distance between two points in $\mathbf{R}^3$.
   (c) Recognize equations for cylinders and spheres.
2. **Vectors (Chapter 12.2)**
   (a) Recognize vectors written in a variety of forms.
   (b) Find a vector from one point to another.
   (c) Add, subtract, and scale vectors, either geometrically or algebraically.
   (d) Compute the length of a vector.
3. **The Dot Product (Chapter 12.3)**
   (a) Compute the dot product between two vectors.
   (b) Determine when two vectors are parallel or perpendicular.
   (c) Find the angle between two vectors.
   (d) Compute $\text{proj}_{\mathbf{a}}(\mathbf{b})$ and $\text{comp}_{\mathbf{a}}(\mathbf{b})$.
4. **The Cross Product (Chapter 12.4)**
   (a) Compute the cross product of two vectors in $\mathbf{R}^3$.
   (b) Understand the connection between the directions of $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{a} \times \mathbf{b}$.
   (c) Find the area of a triangle or parallelogram using the cross product.
5. **Lines & Planes (Chapter 12.5)**
   (a) Find the equation for a line given a point and a direction vector.
   (b) Find the equation for a plane given a point and a normal vector.
   (c) Solve all sorts of problems involving lines & planes, including but not limited to:
      - Check whether two lines are parallel, intersecting, or skew.
      - Find the intersection of two planes.
      - Find the intersection of a line and a plane.
      - Find a plane through three points.
      - Find a plane through a point and a line.
      - Find the distance from a point to a plane.
      - Find the angle between two planes.

6. **Quadric Surfaces (Chapter 12.6)**
   (a) Complete the square to write the equation for a quadric surface in standard form.
   (b) Recognize various quadric surfaces from their equations.
   (c) Draw the traces of a quadric surface.
   (d) Find the intersection(s) of a line with a quadric surface.
   (e) Find the equation of a quadric surface given information about that surface.
7. **Vector Functions and Space Curves (Chapter 13.1)**
   (a) Compute limits of vector functions.
   (b) Check whether the space curves of two vector functions intersect, and if so where.
   (c) Locate the intersection of a space curve and a quadric surface.
   (d) Find a vector function to represent the intersection of two surfaces.
8. **Derivatives and Integrals of Vector Function (Chapter 13.2)**
   (a) Take the derivative of a vector function.
   (b) Find the tangent vector to a space curve at a given point.
   (c) Compute antiderivatives of vector functions.
9. **Arc Length and Curvature (Chapter 13.3)**
   (a) Compute arc length for vector functions in three or more dimensions.
   (b) Find κ, $\mathbf{T}$, and $\mathbf{N}$ for a given vector function.
10. **Velocity & Acceleration (Chapter 13.4)**
   (a) Compute velocity and acceleration vectors for an object using its position vector.
   (b) Integrate to find the position vector using the acceleration vector.
   (c) Apply the equation $\mathbf{F} = m\mathbf{a}$.
   (d) Decompose an acceleration vector into its normal and tangential components.

Some Useful Equations

- Distance from (x_1, y_1, z_1) to (x_2, y_2, z_2): $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$
- Sphere in $\mathbb{R}^3$ with center (x_0, y_0, z_0) and radius r: $(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$
- Magnitude of a vector: $|\langle a_1, a_2, \ldots, a_n \rangle| = \sqrt{a_1^2 + a_2^2 + \cdots + a_n^2}$
- The dot product: $\langle a_1, a_2, \ldots, a_n \rangle \cdot \langle b_1, b_2, \ldots, b_n \rangle = a_1 b_1 + a_2 b_2 + \cdots + a_n b_n$
- If the angle between $\mathbf{a}$ and $\mathbf{b}$ is θ, then the dot product satisfies $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos(\theta)$.
- Component of $\mathbf{b}$ in the $\mathbf{a}$ direction: $\text{comp}_{\mathbf{a}}\mathbf{b} = \dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|}$
- Projection of $\mathbf{b}$ onto $\mathbf{a}$: $\text{proj}_{\mathbf{a}}\mathbf{b} = \left(\dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|^2} \right) \mathbf{a}$

- The cross product: $\langle a_1, a_2, a_3\rangle \times \langle b_1, b_2, b_3\rangle = \langle a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1\rangle$
- If the angle between $\mathbf{a}$ and $\mathbf{b}$ is θ, then the cross product satisfies $|\mathbf{a}\times\mathbf{b}| = |\mathbf{a}||\mathbf{b}|\sin(\theta)$.
- Parametric equations for a line through (x_0, y_0, z_0) with direction $\langle a, b, c\rangle$:

$$\begin{aligned} x &= x_0 + at \\ y &= y_0 + bt \\ z &= z_0 + ct \end{aligned}$$

- Symmetric form for a line through (x_0, y_0, z_0) with direction $\langle a, b, c\rangle$:

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

- Plane through (x_0, y_0, z_0) with normal vector $\langle a, b, c\rangle$: $a(x-x_0)+b(y-y_0)+c(z-z_0) = 0$
- Standard forms for quadric surfaces (up to rotation):

$$\begin{aligned} \text{Ellipsoid:} \quad & \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \\ \text{Elliptical Paraboloid:} \quad & \frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c} \\ \text{Elliptical cone:} \quad & \frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2} \\ \text{Hyperboloid of one sheet:} \quad & \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1 \\ \text{Hyperboloid of two sheets:} \quad & \frac{z^2}{c^2} - \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \\ \text{Hyperbolic Paraboloid:} \quad & \frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{z}{c} \end{aligned}$$

- Arc length of the space curve $\mathbf{r}(t)$ from $t = a$ to $t = b$: $\displaystyle\int_a^b |\mathbf{r}'(t)|\, dt$.
- Curvature of a curve $\mathbf{r}(t)$ in $\mathbb{R}^3$: $\kappa = \dfrac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$.
- Unit tangent vector for $\mathbf{r}(t)$: $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}$.
- Unit normal vector for $\mathbf{r}(t)$: $\mathbf{N}(t) = \dfrac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|}$.
- Relation between force $\mathbf{F}$, mass m, and acceleration $\mathbf{a}$: $\mathbf{F} = m\mathbf{a}$.
- Tangential component of acceleration for $\mathbf{r}(t)$: $a_T = \dfrac{\mathbf{r}'(t) \cdot \mathbf{r}''(t)}{|\mathbf{r}'(t)|}$.
- Normal component of acceleration for $\mathbf{r}(t)$: $a_N = \dfrac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|}$.