Text, table

Description automatically generatedA.

1 - 0.9285x + 0.6985x3 for [0,0.25)

0.7788 – 0.7975(x-0.25) + 0.5239(x-0.25)2 - 0.3610(x-0.25)3 for [0.25,0.50)

0.6065 – 0.6032(x-0.5) + 0.2531(x-0.5)2+ 0.0530(x-0.5)3 for [0.50,0.75)

0.4724 – 0.4668(x-0.75) + 0.2929(x-0.75)2 – 0.3905(x-0.75)3 for [0.25,0.50)

S =

S’(0.5) = -0.6032 f’(0.5) = -0.6065; |S’(0.5) – f’(0.5)| = 0.0033

S’’(0.5) = 0.5062 f’’(0.5) = 0.6065; |S’’(0.5) – f’’(0.5)| = 0.1003

B.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| h | S’(0.5) | Absolute error | S’’(0.5) | Absolute error |
| 2-3 = 0.125 | -0.6064124097 | 1.182500×10-4 | 0.5986995694 | 0.0078310903 |
| 2-4 = 0.0625 | -0.606530608 | 5.171263×10-8 | 0.606333247 | 1.974127×10-4 |
| 2-5 = 0.03125 | -0.606530656 | 3.712633×10-9 | 0.606481302 | 4.93577×10-5 |
| 2-6 = 0.015625 | -0.60653066 | 2.873666×10-10 | 0.60651832 | 1.233971×10-5 |
| 2-7 = 0.0078125 | -0.60653066 | 2.873666×10-10 | 0.606527575 | 3.084713×10-6 |

I could calculate p using log(absolute error)/log(h). Therefore, I could get p as described in the table and I can observe that p is converging to the 4 and 2 for S’(0.5) and S’’(0.5) respectively.

The errors are O(h4) and O(h2) for approximations to f’(0.5) and f’’(0.5) respectively.

Table

Description automatically generatedC.

1 – 1.0000x +0.4805x2 – 0.0788x3 for [0,0.25)

0.7788 – 0.7745(x-0.25) + 0.4214(x-0.25)2 - 0.3187(x-0.25)3 for [0.25,0.50)

0.6065 – 0.6236(x-0.5) + 0.1824(x-0.5)2+ 0.6611(x-0.5)3 for [0.50,0.75)

0.4724 – 0.4084(x-0.75) + 0.6782(x-0.75)2 – 2.8650 (x-0.75)3 for [0.25,0.50)

S =

S’(0.5) = - 0.6236 f’(0.5) = -0.6065; |S’(0.5) – f’(0.5)| = 0.0171

S’’(0.5) = 0.3648 f’’(0.5) = 0.6065; |S’’(0.5) – f’’(0.5)| = 0.2417

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| h | S’(0.5) | Absolute error | p | S’’(0.5) | Absolute error | p |
| 2-3 = 0.125 | -0.6064124097 | 0.0141027 | 7.598 | 0.5986995694 | 0.181395 | 2.33 |
| 2-4 = 0.0625 | -0.606530608 | 7.27452×10-5 | 6.05 | 0.606333247 | 0.00205933 | 3.08 |
| 2-5 = 0.03125 | -0.606530656 | 5.14474×10-9 | 5.60 | 0.606481302 | 4.94569×10-5 | 2.86 |
| 2-6 = 0.015625 | -0.60653066 | 2.00836×10-10 | 5.28 | 0.60651832 | 1.23398×10-5 | 2.72 |
| 2-7 = 0.0078125 | -0.60653066 | 1.25542×10-11 | 4.53 | 0.606527575 | 3.08497×10-6 | 2.62 |

The errors are O(h3) and O(h) for approximations to f’(0.5) and f’’(0.5) respectively.

natural cubic spline

h Abs Err of f‘(0.5) Abs Err of f‘’(0.5)

2^(-3) 0.00011825 0.00783109

2^(-4) 3.5434e-07 0.000233748

2^(-5) 3.21717e-09 4.9359e-05

2^(-6) 2.00836e-10 1.23398e-05

2^(-7) 1.25542e-11 3.08497e-06

p for first derivative: 8.38, 6.78, 4 and 3.999

p for second derivative: 5.06, 2.24, 1.99 and 1.99 again

clamped cubic spline

h Abs Err of f‘(0.5) Abs Err of f‘’(0.5)

2^(-3) 0.0141027 0.181395

2^(-4) 7.27452e-05 0.00205933

2^(-5) 5.14474e-09 4.94569e-05

2^(-6) 2.00836e-10 1.23398e-05

2^(-7) 1.25542e-11 3.08497e-06

p for first derivative: 7.598, 13.787, 4.679, and 3.999

p for second derivative: 6.46, 5.38, 2.003 and 1.99

D. The natural cubic spline is better to approximate f(x) = e-x because it has higher order of big-oh.