|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| n | Equation A | Equation B | Equation C | Equation D |
| 0 | 1 | 1 | 1 | 1 |
| 1 | 343 | 7 | 2.2 | 1.5 |
| 2 | -2.25393e25 | -335.857 | 1.819763677 | 1.450520833 |
| 3 | -3.38385e253 | 3.78844e7 | 1.583474830 | 1.498749661 |
| 4 | -Inf | -5.43726e22 | 1.489460974 | 1.451903535 |
| 5 | NaN | 1.60746e68 | 1.476022436 | 1.497577067 |
| 6 | NaN | -Inf | 1.475773246 | 1.453192290 |
| 7 | NaN | NaN | 1.475773162 | 1.496475364 |
| 8 | NaN | NaN | 1.475773162 | 1.454396119 |
| 9 | NaN | NaN | 1.475773162 | 1.495438587 |
| 10 | NaN | NaN | 1.475773162 | 1.455522810 |

A)

The table represents the values of approximation at each of 10 iterations; Equation C could get to the close enough approximation only after 6 iterations and Equation D is also getting close to the actual value, but it requires 163 iterations to have error within 1e-3. In other words, the convergence of Equation C is faster than the convergence of Equation D. Furthermore, Equation A and B do not converge as you can observe in the table.

**Speed of convergence**

(C), (D)

B)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| n | Equation A | Equation B | Equation C | Equation D |
| 0 | 0.475773162 | 0.475773162 | 0.475773162 | 0.475773162 |
| 1 | 341.5242268 | 5.524226838 | 0.724226838 | 0.024226838 |
| 2 | -2.25393×1025 | 337.3327732 | 0.343990515 | 0.025252329 |
| 3 | 3.38385×10253 | 37884398.52 | 0.107701668 | 0.022976499 |
| 4 | Inf | 5.43726×1022 | 0.013687812 | 0.023869627 |
| 5 | NaN | 1.60746×1068 | 2.4927x10-4 | 0.021803905 |
| 6 | NaN | Inf | 0 | 0.022580872 |
| 7 | NaN | NaN | 0 | 0.020702202 |
| 8 | NaN | NaN | 0 | 0.021377043 |
| 9 | NaN | NaN | 0 | 0.019665425 |
| 10 | NaN | NaN | 0 | 0.020250352 |

p = 7(1/5) = 1.475773162

This table shows the absolute error |pn – p| against n.

The convergence can be determined for the 4 different equations by looking at the table.

Equation A; **diverges;** the abs.error gets larger as n increases.

Equation B; **diverges**; the abs.error gets larger as n increases.

Equation C; **converges**;the abs.error gets smaller towards 0 as n increases.

Equation D; **converges**; the abs.error gets smaller towards 0 as n increases.

Text, letter

Description automatically generated  
C) As Equation\_A and Equation\_B diverge, it is not possible for them to estimate the order of convergence α and the asymptotic error constant λ because the definition 2.7 has an assumption saying that if wanting to have positive constants λ and α exist, then the sequence must converge to p. Therefore, α and λ are going to be estimated only for Equation\_C and Equation\_D by the definition 2.7 in the textbook, = λ. This definition can be manipulated to solve for α and λ.

|  |  |  |  |
| --- | --- | --- | --- |
| n | Pn | α | λ |
| 1 | 2.2 | -1.7719 | 0.1942 |
| 2 | 1.819763677 | 1.5598 | 0.5690 |
| 4 | 1.489460974 | 1.9418 | 1.0365 |
| 5 | 1.476022436 | 1.9955 | 1.3050 |

Text, letter

Description automatically generatedTheorem 2.9 says that if g’(p) =0 and g’’(p) is then the sequence is quadratically convergent with asymptotic error constant of . It is shown that first derivative of Equation C is 0 at p and the second derivative is continuous having non-zero at p; therefore, the sequence is quadratically convergent having α =2 and error constant of 1.355. This have been proven by using the manipulated formula to compute α and λ at each iteration. (I omit some n)

Table

Description automatically generated with medium confidence

|  |  |  |  |
| --- | --- | --- | --- |
| n | Pn | α | λ |
| 1 | 1.5 | -0.0139 | 0.0240 |
| 163 | 1.4752763892 | 1.00000 | 0.9764 |

Theorem 2.8 says that if g’(p)0, the fixed-point iteration exhibits linear convergence with asymptotic error constant |g’(p)|. It is shown that the first derivative of Equation D is continuous having non-zero at p; therefore, the sequence is linearly convergent having α =1 and error constant of 0.9764. This also have been proven by using the manipulated formula to compute α and λ at each iteration. (I omit some n)

|  |  |  |
| --- | --- | --- |
|  | α | λ |
| Equation A | N/A | N/A |
| Equation B | N/A | N/A |
| Equation C | 2 | 1.355 |
| Equation D | 1 | 0.9764 |