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From human to humanoid locomotion—an inverse optimal control approach

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Abstract The purpose of this paper is to present inverse optimal control as a promising approach to transfer biological motions to robots. Inverse optimal control helps (a) to understand and identify the underlying optimality criteria of biological motions based on measurements, and (b) to establish optimal control models that can be used to control robot motion. The aim of inverse optimal control problems is to determine—for a given dynamic process and an observed solution—the optimization criterion that has produced the solution. Inverse optimal control problems are difficult from a mathematical point of view, since they require to solve a parameter identification problem inside an optimal control problem. We propose a pragmatic new bilevel approach to solve inverse optimal control problems which rests on two pillars: an efficient direct multiple shooting technique to handle optimal control problems, and a state-of-the art derivative free trust region optimization technique to guarantee a match between optimal control problem solution and measurements. In this paper, we apply inverse optimal control to establish a model of human overall locomotion path generation to given target positions and orientations, based on newly collected motion capture data. It is shown how the optimal control model can be implemented on the humanoid robot HRP-2 and thus enable it to autonomously generate natural locomotion paths.

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1 Introduction

1.1 Identifying the optimality of human locomotion

The general hypothesis that natural structures and processes are optimal is one of the fundamentals of bionics. It is also a common assumption that locomotion of animals and humans is optimal (Alexander 1984, 1996). However, the specific optimization criterion for locomotion in different situations is generally unknown, and it can be expected that in most cases a combination of multiple criteria is used. The optimal behavior during a particular locomotion task can be observed and measured by motion capture, EMG measurements etc.

From a mathematical perspective, locomotion of animals and humans can be formulated as optimal control problem, i.e. an optimization problem which has to satisfy constraints in the form of differential equations and where the unknowns are input ("control") and state functions of the locomotor system. As stated above, however, we do not know the exact formulation of this optimal control problem—in particular we do not know the objective function—but we know the solution to this problem from measurements (or at least the observable part of the solution). This setting—a (partly) known optimal solution, but an unknown optimization criterion that produced it—leads to problems of inverse optimization or, in the case of dynamic processes, to inverse optimal control problems. It has to be clearly distinguished from the classical forward optimization or optimal control problem where the problem formulation is fully known and

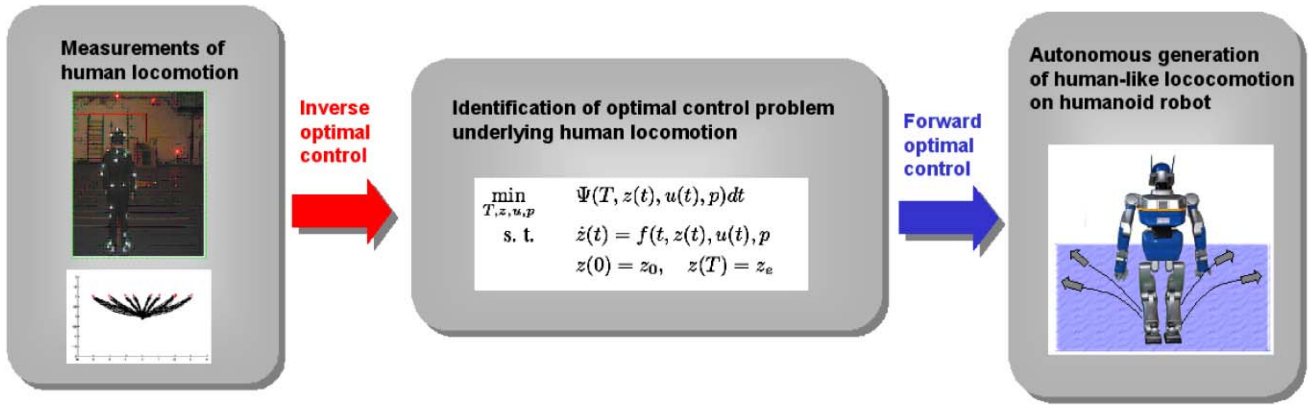


Fig. 1 The inverse optimal control approach helps to (a) understand optimality of human locomotion and (b) to generate natural humanoid locomotion

the solution has to be determined. Inverse optimal control problems are much harder than standard identification problems since optimization and data fitting have to be handled simultaneously.

We see the understanding of the optimality principles of human locomotion as one of the keys to generate biologically inspired locomotion on autonomous robots. If the human optimization criterion of locomotion can correctly be formulated in mathematical terms, it is straightforward to mimic this optimization approach on a humanoid robot. Figure 1 gives an overview of the inverse optimal control approach that we present in this paper. In a first step, inverse optimal control is used to identify human optimality criteria for locomotion from motion capture measurements. Based on this knowledge, the optimal control model can be formulated. In the next step, this optimal control model can then be implemented on the humanoid robot to enable it to autonomously generate locomotion trajectories in a human-like manner, each time solving an optimal control problem for the individual task to be performed.

There are different perspectives from which human and humanoid locomotion can be investigated—the biomechanics or the neuroscience point of view. Most researchers in biomechanics study locomotion on joint level along a given straight or bent overall path on the floor to be followed. The study of the selection and optimal generation of this overall path has however been widely neglected in humanoid robotics and also in biomechanics so far. If humans are asked to walk towards a given end position and orientation in an empty space with no obstacles, they will select a very specific path, out of an infinity number of possibilities. This choice is not so much influenced by biomechanical properties but rather by neuroscience aspects. In the attempt to control humanoids in a biologically inspired manner, it would be desirable to understand and imitate that behavior of humans.

In this paper, we show how the inverse optimal control approach is used to generate natural overall locomotion trajectories from an initial rest position an orientation to a given target rest position and orientation. For this purpose, we are not interested in studying the individual trajectories of all joints. Instead, the locomotor system can be described by its overall position and orientation in the plane. However, inverse optimal control problems are prevalent and can basically be found everywhere in natural sciences, and the proposed approach is very general. Consequently, it can also be used to analyze motions on joint level.

1.2 State of the art

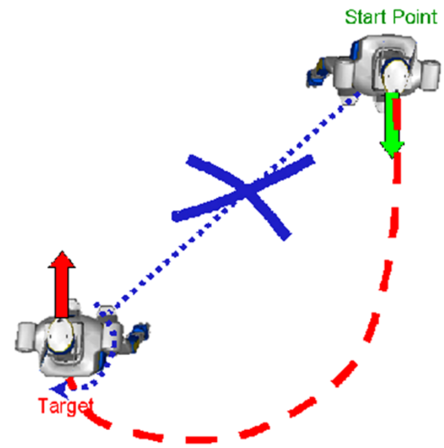
1.2.1 *Previous work on inverse optimization*

Several authors have studied imitation problems for humanoid robots, i.e. the task of reproducing a human movement within the kinematic and dynamic ranges of the particular humanoid, and obtained impressive results (e.g. Nakazawa et al. 2002; Ikeuchi 2009; Suleiman et al. 2008; Billard and Mataric 2001). They use different approaches such as learning techniques or optimization methods to identify unknown parameters in the model. But they all have in common that they only focus on imitating a particular observed motion and do not ask the question for the underlying optimality principles. The problem becomes much harder when imitation and the search for optimality criteria is combined for a dynamic system. Then a parameter identification problem has to be handled simultaneously with an optimal control problem—a class of problems that have been named "inverse optimal control problems" and which have not yet extensively been investigated.

Liu et al. (2005) study realistic generation of character motion by physics-based models. They assume a given objective function of minimizing joint torques squared and identify unknown model parameters from measurement sequences by a nonlinear inverse optimization technique. Heuberger provides a detailed overview of inverse optimization in combinatorial problems (Heuberger 2004) describing solutions only for linear and network problems. Inverse optimal control problems formulated as bilevel problems can also be treated as MPEC (Mathematical programs with equilibrium constraints). Here the optimal control problem is replaced by the corresponding first order optimality conditions which become constraints of the parameter estimation problem (see the book Luo et al. 1996). Several very theoretical papers have been published on MPECs and corresponding optimality conditions and constraints qualifications, such as Ye (2005). The approach is however very hard to implement in practice. In a recent thesis, it has been applied to very simple dynamical models (Hatz 2008).

1.2.2 Generation of human-like locomotion paths

In robotics, the problem of generating the overall path (i.e. the trace on the floor) has been extensively studied for mobile wheeled robots (see e.g. Latombe 1991; Laumond 1998; LaValle 2006). In this case the focus is on finding a feasible path, in particular in the presence of obstacles, and the path is generally nonholonomic, i.e. the orientation of the robot and the direction of motion are directly coupled. No biological inspiration is required for motion planning of wheeled robots. In humanoid robot research, several authors have studied real time path planning and adaption based on sensor information, looking at the same time at the shape of the path and an appropriate choice of footholds (e.g. Stasse et al. 2006; Chestnutt et al. 2005; Gutmann et al. 2005; Yoshida et al. 2008). The problem of natural off-line locomotion path planning has not received much attention yet. In Mombaur et al. (2008), we have proposed a heuristic optimal control model to generate naturally shaped locomotion paths for humanoid robots. In computer graphics, offline planning for biped locomotion has been studied (Choi et al. 2003; Pettré et al. 2003; Brogan and Johnson 2003). Several authors have investigated the shape of human locomotion paths, e.g. Hicheur et al. (2007). It has recently been shown (Arechavaleta et al. 2008b, 2008a; Laumond et al. 2007) that human locomotion in many cases is nonholonomic as wheeled motion, i.e. that people tend to move in forward direction. This general preference may easily be understood from the anatomy of the feet and legs. But in certain situations it also seems very natural to abandon this behavior and to include sideward or oblique steps in the locomotion, i.e. move in a holonomic way. This not only occurs when obstacles must be avoided but also in the case of very close goals (compare Fig. 2). In Mombaur et al. (2008), we have made a first attempt to establish a model that continuously selects between holonomic and nonholonomic locomotion in a realistic way.



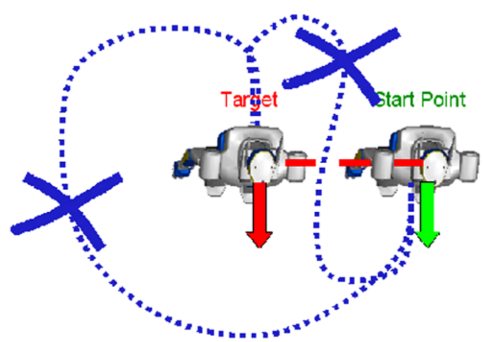


Fig. 2 Natural locomotion paths for humanoid robots: Examples of realistic and unrealistic paths (*dashed lines* vs. *dotted lines*) for two different targets

1.3 Contribution of this article

The contribution of this article is twofold. First, we present inverse optimal control as a general approach to transfer biological motions to robots. Inverse optimal control not only helps to understand the underlying optimization objectives of recorded biological motion. It also leads to the generation of mathematical forward optimal control models that can be applied to control humanoid robot motions in a natural way. In this paper, we describe the general form of inverse optimal control problems as well as a very flexible numerical technique for their solution. The second contribution of this article is to present a dynamic model and a unique objective function of the overall locomotion path generation to close targets (defined by their position and orientation) if the system starts and ends at rest. For this, we started from our previously mentioned research Mombaur et al. (2008) where a qualitative model was created and parameters were selected by manual tuning. In contrast to this, the goal in the present paper is to truly identify the weights of the proposed optimal control model from human motion capture data. In addition, the inverse optimal control approach allows us now to simplify the formulation by establishing a unique model with constant weight factors that is valid for a whole domain of targets.

The remainder of the paper is split into three sections. In Sect. 2, the general inverse optimal control approach is discussed by presenting problem statement and numerical solution techniques. The third section deals with the application of inverse optimal control to the problem of generating natural human-like locomotion paths. We describe motion capture experiments, basic model formulation, computational results, and the implementation of the optimal control model on the humanoid robot HRP-2. In the final section, we summarize results and discuss future work.

2 Inverse optimal control: a general approach to understand natural processes

The goal of inverse optimal control problems is to determine the formulation of an optimal control problem—and in particular its cost function—that is able to best reproduce the available experimental data. Inverse optimal control problem arise in many areas of natural sciences. Human locomotion is a very important example. We present the general statement of inverse optimal control problems in Sect. 2.1. A numerical technique for the solution of inverse optimal control problems that we have recently developed is presented in Sect. 2.2.

2.1 Problem statement

We consider optimal control problems of the form

$$\min_{z(\cdot),u(\cdot),T} \int_0^T \Phi(z(t),u(t))dt \tag{1}$$

$$\text{s.t.} \quad \dot{z} = f(t,z(t),u(t)) \tag{2}$$

$$z(0) = z_0 \tag{3}$$

$$z(T) = z_e \tag{4}$$

where $z(t)$ are the state variables and $u(t)$ the control variables. It is assumed that $\Phi(z(t),u(t))$ in the objective function formulation (1) is a priori unknown and remains to be identified by inverse optimal control. The dynamic model (2), and initial and final conditions (3), (4) are given. We are interested in two different cases:

- the optimal solution $z^*(t) \in \mathbb{R}^{n_z}$, $u^*(t) \in \mathbb{R}^{n_u}$ is known at m evenly spaced discrete points t_i, and the optimal duration T^* is known,

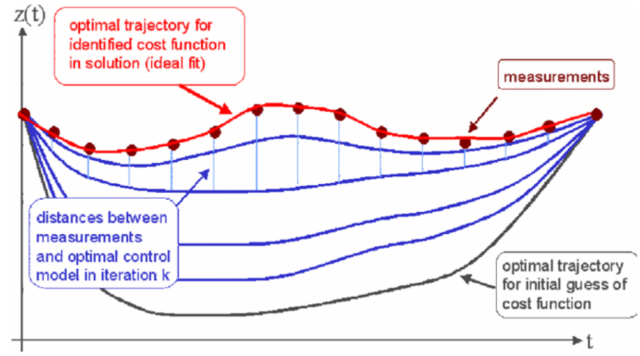


Fig. 3 Goal of inverse optimal control: identify cost function that best approximates measured data

- only some components of the optimal states and controls $z^*_{red}(t) \in \mathbb{R}^{n_{zr}}$ and $u^*_{red} \in \mathbb{R}^{n_{ur}}$ (where $0 < n_{zr} < n_z$ and $0 < n_{ur} < n_u$) are known at m discrete points, as well as T^*.

From a practical point of view, the second case is more relevant, since typically not the full solution is observable.

As shown in Fig. 3, the inverse optimal control problem now consists in determining the exact objective function $\Phi(\cdot)$ that produces the best fit to the measurements in the least squares sense. For the objective function we make the assumption that it can be expressed as a weighted sum of a series of n base functions $\phi_i(t)$ with corresponding weight parameters α_i:

$$\Phi(z(t),u(t),\alpha) = \int_0^T \left[\sum_{i=0}^{n-1} \alpha_i \phi_i(z(t),u(t))\right] dt \tag{5}$$

The problem of determining the “best” objective function $\Phi(\cdot)$ thus is transformed into the problem of determining the best weight factors α_i. In a combined objective function the relative size of the weight factors is crucial since they determine the influence of the respective term on the overall sum: the larger the weight, the more the corresponding term is punished and therefore is likely to be reduced in the overall context. In the other extreme case, individual α_i can be set to zero.

Note that in this objective function formulation we use, for simplicity of presentation, only integral objective functions (called of Lagrange type), but this also includes Mayer type functions depending on end values since the two forms are theoretically equivalent and can be transformed into each other. The base functions $\phi_i(x(t),u(t))$ describe reasonable potential components of the objective function, It is important to choose a non-redundant set of objective functions since different base function leading to exactly the same behavior would be impossible to identify.

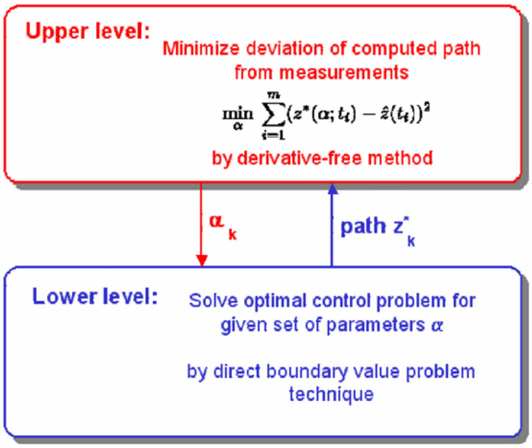


Fig. 4 Solution of inverse optimal control problem as bilevel optimization problem

Using (5), we can formulate the inverse optimal control problem as bilevel problem:

$$\min_{\alpha} \sum_{j=1}^{m} \|v^*(t_j;\alpha) - v_M(t_j)\|^2 \tag{6}$$

where $v^*(t;\alpha)$ is the solution of

$$\min_{z,u,T} \int_0^T \left[\sum_{i=1}^{n} \alpha_i \phi_i(z(t),u(t))\right] dt \tag{7}$$

$$\text{s.t.} \quad \dot{z} = f(t,z(t),u(t)) \tag{8}$$

$$z(0) = z_0 \tag{9}$$

$$z(T) = z_e \tag{10}$$

Here z summarizes the full or reduced vector of states and controls, $v(t)^T = (z(t)^T, u(t)^T)$ or $v(t)^T = (z_{red}(t)^T, u_{red}(t)^T)$, depending on the case treated, i.e. the available measurements. v_M denotes the measured values.

2.2 Numerical solution of inverse optimal control problems

In this section we present the numerical solution of inverse optimal control problems treated as bilevel problem (compare Fig. 4). The upper level handles the iteration over α such that the fit between measurements and optimal control problem solution is improved. Each upper level iteration includes one call to the lower level where a forward optimal control problem is solved for the current set of α_i. The optimal solution of this problem is then communicated back to the upper level such that the least squares fit between measurements and computations can be evaluated for this iteration.

As described in Mombaur (2009), we have implemented and tested a method to solve inverse optimal control problems on the basis of two powerful numerical techniques. We propose a combination of efficient direct techniques for the solution of the lower level optimal control problems, and of an efficient derivative-free method or the solution of the upper-level least-squares problem. Both techniques that we have combined in our modular software environment will be briefly described in this section.

For the solution of the lower level optimal control problem we have applied the direct boundary value problem approach using multiple shooting developed by Bock and coworkers (MUSCOD Bock and Plitt 1984; Leineweber et al. 2003a, 2003b). Optimal control problems in the form given above are infinite-dimensional (since $z(t)$ and $u(t)$ are variables in function space), but can be transformed into finite dimensional problems by means of discretization. The MUSCOD method uses a direct approach (also called a first-discretize-then-optimize approach), i.e. instead of allowing arbitrary control functions, they are discretized using base functions with local support, such as piecewise constant, linear or cubic functions. State parameterization is performed by the multiple shooting technique which transforms the original boundary value problem into a set of initial value problems with corresponding continuity and boundary conditions. The same grid is used for control discretization and multiple shooting state discretization. The resulting structured nonlinear programming problem (NLP) is solved by an efficient tailored sequential quadratic programming (SQP) algorithm. It is important to note that this approach still includes a simulation of the full problem dynamics on each of the multiple shooting intervals. This is performed simultaneously to the NLP solution using fast and reliable integrators also capable of an efficient and accurate computation of trajectory sensitivity information (Bock 1987).

For the solution of the upper-level least squares problem, we apply a derivative-free optimization technique, i.e. it only requires function evaluations and does not need gradient information. Derivative-free optimization is always favorable if function evaluations are expensive and noisy and derivative information can therefore not be generated in a reliable manner. In the case of our bilevel problem, each function evaluation of the upper-level problem corresponds to a solution of the lower-level optimal control problem, so it would definitely be difficult to generate numerical derivatives of this function. We only have to handle simple box constraints on the weight parameters in the upper level, all other constraints are handled by the optimal control code in the lower level. We use the newly released derivative-free optimization code BOBYQA by Powell (2008, 2009). BOBYQA stands for Bound Optimization BY Quadratic Approximation. It is an extension of Powell's well known code NEWUOA, and can additionally handle simple bounds on the variables. Many derivative-free methods are direct search methods, either using search directions, e.g. along coordinate axis, or using a search simplex or polytope, such as the well known Nelder-Mead method (Nelder and Mead 1965). In contrast to this, BOBYQA and NEWUOA are interpolation-based trust region techniques of derivative-free optimization which establish a quadratic polynomial model of the objective function, based on function evaluations only. In each iteration only one point of the polynomial is replaced to form a new approximation. In our bilevel environment, BOBYQA has replaced our own implementation of a variant of the Nelder-Mead direct search algorithm that also could handle box constraints and that we have originally used for this purpose. The use of BOBYQA has lead to a significant increase of computational efficiency (factor 5–10).

It should be noted that the determination of weight parameters is only possible up to a common factor, since only the relative size of parameters is important in a multi-criteria objective function. Our practical approach to this issue is to fix one of the parameters a priori to 1.0 and to determine the remaining parameters. If ever this choice turns out to be bad (i.e. the chosen parameter in fact should be zero in the solution), this becomes very obvious from the behavior of the numerical iterations, and the choice then should be revised.

3 Study of human locomotion by inverse optimal control

In this section, we show how the inverse optimal control approach has been applied to identify a suitable optimal control problem formulation to describe human behavior and to implement this optimal control model on the robot to generate natural motions. We describe human locomotion experiments in Sect. 3.1 and the basic locomotion model and objective base functions in Sect. 3.2. Numerical results will be presented in Sect. 3.3, and the generation of actual robot motions will briefly be shown in Sect. 3.4.

3.1 Experiments: human locomotion trajectories

The purpose of the experiments was to capture representative overall locomotion trajectories for given start and end positions and orientations. In particular, close-by targets in a radius of ~3.5 m have been investigated. Initial and final total velocity is enforced to be zero.

The experiments have been performed by ten healthy male subjects with height between 1.71 m and 1.83 m (average 1.77 m), aged between 23 and 31 years (average 27). All subjects gave their informed consent to perform the experiments.

The subjects were asked to perform 204 motions to close-by targets (102 different target scenarios, i.e. defined position and orientation targets, with two repetitions were randomly ordered and prescribed). An arrow on the floor indicated target position and orientation of each trial. For the experimental setup, and an overview of all target positions and

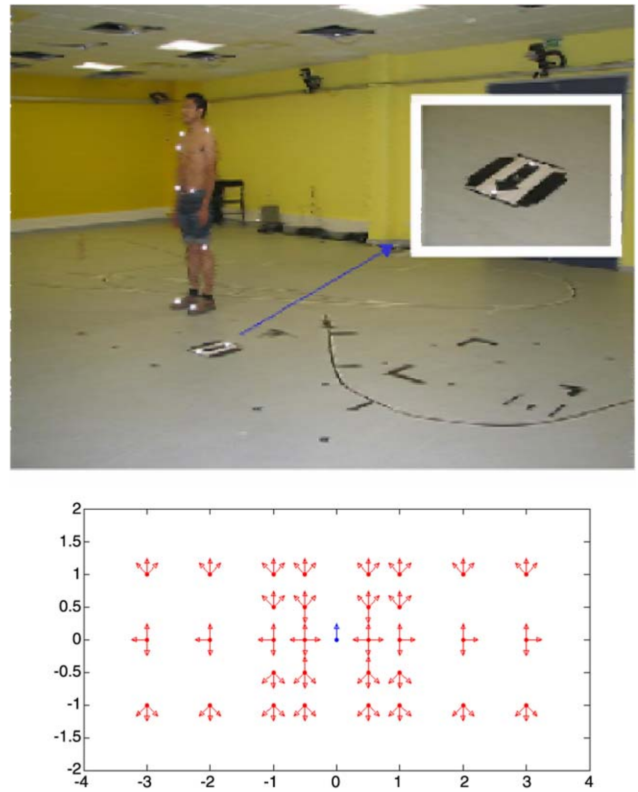


Fig. 5 Experimental setup to capture human locomotion trajectories (*top*) and overview of the 102 different target scenarios

orientations, see Fig. 5. The subjects received the instruction to go to the respective target of the trial, at their own timing and freely selecting their trajectory on the floor. They had no prior training to perform the experiments. The arrow was manually moved to the new position between runs, which gave the subject enough time to be informed about the location of the new target, even if it was located in their back. We recall that the purpose of these experiments was to study objectives of planned human locomotion, not the reaction to surprise situations.

We have used a Motion Analysis motion capture system with 10 cameras, all with a sampling frequency of 100 Hz.

We were interested in recording the time histories of the overall positions $x(t)$ and $y(t)$ and orientations $\theta(t)$ of the subjects. This could be achieved using two markers on the subjects shoulders, as shown in Fig. 6. Our experience has shown that shoulder orientation is the best simple approximation to the "overall" orientation of the subject.

There were six additional markers on the subjects—two on the pelvis (on the forward iliac crest), two on the feet, and additionally two asymmetric markers, one on the right half of the thorax and one on the left knee to help identifying the order of the other markers and the forward orientation of the subject.

The recorded time histories of the two shoulder markers were filtered to eliminate oscillations at step frequency. It

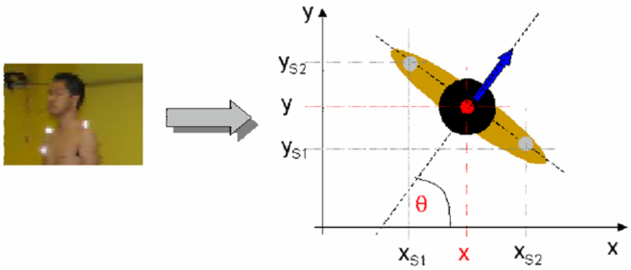


Fig. 6 Global position and orientation histories of the subjects are determined using markers on the shoulders

is a natural phenomenon that even while walking forward at constant speed humans perform oscillations at step frequency: the actual forward velocity of the trunk oscillates about the average forward velocity, the trunks also oscillates sidewards about a middle position while taking left and right steps, and shoulders and arm perform rotational motions to compensate the rotations of pelvis ad legs. These oscillations were eliminated in order to produce good measurements of the average development of position and orientation of the central body. The filtering was performed using Matlab 7.4.0. We designed first a fourth order low pass digital Butterworth filter with normalized cutoff frequency 0.01. Then a two-way Butterworth filter was used with a filter order that is double the order of the above Butterworth filter. In addition to the forward-reverse filtering, it attempts to minimize start-up and ending transients by adjusting initial conditions. The same filtering process was applied to all experimental data.

In these experiments we could observe stereotypical behavior of the ten subjects for most of the recorded trajectories. The absolute deviations were roughly the same in all cases, leading to larger relative deviations in the case of very short trajectories. Another publication describing the experiments in more detail and providing a detailed statistical analysis of the collected data is currently in preparation.

3.2 A general optimal control model of the human locomotion path

The purpose of this section is to present an optimal control model that gives a realistic description of overall locomotion trajectories for rest-to-rest motions in the near range. It consists of a set of differential equations as well as the objective function resulting from the combination of different reasonable base functions. The aim of this optimal control model is not to describe locomotion on the detailed level of joint trajectories, but to provide a good description of the essential locomotion objectives.

Variables x, y and θ describe position and orientation of the locomotor system in the global reference frame. But since humans generally do not perceive and plan their movement in a general fixed coordinate system but rather in a local coordinate system attached to their body, we use velocities and accelerations expressed in the body system to describe the motion. In the system attached to the body, we can distinguish translational velocities in forward and sideward—called orthogonal—direction, v_{forw} and v_{orth}, as well as rotational velocity ω. It is much more reasonable to formulate a model based on velocities v_{forw} and v_{orth} instead of $\dot{x}$ and $\dot{y}$ which have no unique meaning with respect to the body.

Accelerations are split into the corresponding three directions, and are used as input variables of the locomotion model $u = (u_1, u_2, u_3)^T = (a_{forw}, a_{rot}, a_{orth})^T$. The locomotion model becomes:

$$\begin{aligned}
\dot{x} &= \cos\theta\, v_{forw} - \sin\theta\, v_{orth} \\
\dot{y} &= \sin\theta\, v_{forw} + \cos\theta\, v_{orth} \\
\dot{\theta} &= \omega \\
\dot{v}_{forw} &= u_1 \\
\dot{\omega} &= u_2 \\
\dot{v}_{orth} &= u_3
\end{aligned} \tag{11}$$

This full holonomic locomotion model still includes nonholonomic motions as a special case for $v_{orth} \equiv 0$, i.e. $u_3 \equiv 0$ and $v_{orth}(0) = 0$.

The choice of base functions for the objective function was guided by some intuitive ideas:

- the total time of the path is not defined in advance ($\rightarrow$ end time T is a free variable);
- humans will generally prefer faster over slower paths ($\rightarrow$ end time should be minimized);
- smooth paths are desired, and large variations of velocities should be avoided, i.e. accelerations should be minimized (or rather squares of accelerations to equally treat accelerations and decelerations, and to punish in particular large values);
- the cost of forward and orthogonal components of motions must be computed differently, i.e. the associated weights should be independent;
- there seems to be strong tendency to adjust the orientation of the body towards the goal which starts right at the beginning of the motion (minimization of an angle dependent term).

These ideas result in the following basic formulation of the objective function as a combined weighted minimization of total time, the integrated squares of the three acceleration components, and the integrated squared difference of body orientation angle and direction towards the goal $\Psi(t)$

$$\begin{aligned}
&\Phi(T, z(t), u(t)) \\
&\quad = \int_0^T \left[\sum_0^4 \alpha_i \phi_i(z(t), u(t)) \right] dt
\end{aligned}$$

$$= \int_0^T [\alpha_0 + \alpha_1 u_1(t)^2 + \alpha_2 u_2(t)^2 + \alpha_3 u_3(t)^2 + \alpha_4 \Psi(z(t), z_e)^2] dt$$
$$= \alpha_0 \cdot T + \alpha_1 \int_0^T u_1(t)^2 dt + \alpha_2 \int_0^T u_2(t)^2 dt + \alpha_3 \int_0^T u_3(t)^2 dt + \alpha_4 \int_0^T \Psi(z(t), z_e)^2 dt \qquad (12)$$

with

$$\Psi(z(t), z_e) = \arctan\left(\frac{y_e - y(t)}{x_e - x(t)}\right) - \theta(t)$$
$$-\pi \le \Psi(z(t), z_e) \le \pi \qquad (13)$$

It should be noted that the first four terms alone, which we have already used in the preliminary version of this paper (Mombaur et al. 2009), would produce trajectories that exhibit the property of origin-target symmetry in time (see the definition in Appendix). We have however realized based on experimental observations that the rest-to-rest locomotion trajectories studied here exhibit a clear asymmetry in that sense, in particular since the initial re-orientation towards the goal is much stronger than the final adjustment of the angle, due to the final angle anticipation when reaching the target. This was the reason to introduce the last term, minimizing the integral of Ψ^2, the difference between body orientation and angular difference to target. The basic idea of this is related to the funneling controller in Boulic (2008), the formulation is however completely different, in our case with a combination of constraints guaranteeing correct final position and orientation and an optimization criterion steering the behavior towards the target.

The five weight parameters corresponding to the five base functions of the objective function are a priori unknown. Other than in Mombaur et al. (2008), where the parameters were determined by manual tuning for a humanoid robot model, we will here use inverse optimal control to properly identify the size of the parameters from human locomotion data.

In contrast to the qualitative robot model proposed in Mombaur et al. (2008), we will show in this paper, that it is not necessary to each time adjust the parameter α_3 of the orthogonal acceleration component, according to the distance and orientation change of the target. We will show that is possible to approximate the human behavior in the whole area of close-by targets investigated in the experiments by a unique set of $\alpha_0 - \alpha_4$. The model weights will change for far away targets and long motion segments without any rest position where the motion can be expected to be in general nonholonomic. So instead of the continuous model proposed in Mombaur et al. (2008) we identify here a model which only requires a split into few domains.

3.2.1 Discussion of other potential terms of the objective function

The objective function with the five base function terms formulated above does not contain all the possibilities that we investigated, but only the ones on which we finally determined to be relevant criteria. In addition to these five terms, we have studied the influence of some objectives frequently discussed in the biomechanical literature, but then discarded them again based on inverse optimal control results. We will briefly summarize our findings here:

– Velocity dependent terms: adding terms that are linear or quadratic in the velocities v_{forw}, v_{orth} and ω to objective function (15) does not contribute to a better match between experiments and optimal control model, but makes it worse. Inverse optimal control determines the corresponding weight factors to be zero. We therefore conclude that the objective function of overall human locomotion trajectories does not depend on these velocity terms.
– Jerk dependent terms: The same observation has been made for the jerk. An inverse optimal control with quadratic terms in the three jerk components j_{forw}, j_{orth} and j_{rot} in addition to (15) also reduces their weights to zero. It should be noted that for the study of these jerk dependent terms, we had to work with an augmented model with nine equations

$$\begin{aligned} \dot{x} &= \cos\theta\, v_{forw} - \sin\theta\, v_{orth} \\ \dot{y} &= \sin\theta\, v_{forw} + \cos\theta\, v_{orth} \\ \dot{\theta} &= \omega \\ \dot{v}_{forw} &= a_{forw} \\ \dot{\omega} &= a_{rot} \\ \dot{v}_{orth} &= a_{orth} \\ \dot{a}_{forw} &= u_1 \\ \dot{a}_{rot} &= u_2 \\ \dot{a}_{orth} &= u_3 \end{aligned} \qquad (14)$$

using state variables $z^T = (x, y, \theta, v_{forw}, \omega, v_{orth}, a_{forw}, a_{rot}, a_{orth})$, and the jerks as controls variables $u^T = (u_1, u_2, u_3) = (j_{forw}, j_{orth}, j_{rot})$. We conclude from this, that minimization of jerks does not appear to be a contributing factor in the human selection of locomotion paths (even though it may be a good model to explain multi-joint movements of the arm, as proposed by Flash and Hogan 1984).
– Energy, work or efficiency related terms: Since the kinetic (translational or rotational) energy of the whole system (computed based on its total mass or an approximate total moment of inertia and the motion of the central body) is

proportional to the corresponding squared velocity terms and since these are no contributing factors to the objective function, kinetic energy in that sense can't be either. We recall that with our model we can not (and do not want to) study the articulated motion of the human body as multi-body system and the energy input or work performed in all its joints or the sum of the kinetic energy of the motion of all segments. While it is generally agreed upon that energy or efficiency related issues play an important role for motions on the joint level (compare e.g. Alexander 1984; Collins et al. 2005), we conclude that energy issues do not seem to play a dominant role in the objective function of the overall locomotion trajectory.

3.3 Computational results: identification of the objectives of human locomotion

In this section we present computational results of applying inverse optimal control to identify the objective function of problem (5) to match the human locomotion trajectories described in Sect. 3.1.

We present numerical evidence to support the hypothesis that locomotion objectives can be approximated by a simple unique model in all of the domain we investigated experimentally.

Concerning the choice of trajectories or trajectory combinations there is of course a wide range of possibilities, due to the large amount of data collected. We have made our selections along the following reasoning:

- Since our goal was to establish a model of the average stereotypical behavior of human locomotion, it did not seem reasonable to investigate individual subject's trajectories. For each investigated locomotion scenario (a "scenario" is characterized by a target position and orientation), we decided to arbitrarily select trajectories of five different subjects to be approximated simultaneously.
- In a first set of computations, we performed inverse optimal control runs per scenario (each time taking trajectories of five subjects as described above). We did this for a number of different scenarios, and each time identified the set of parameters that best approximated the respective measurements. The resulting parameters in all cases were very similar—a first hint to support our hypothesis that the optimization criterion actually is the same in all cases. We do however not present these results here due to reasons of space, but move to the next set of computations the results of which are even stronger.
- We then performed inverse optimal control computations to approximate several scenarios simultaneously. We arbitrarily selected five different scenarios (and again the trajectories of five subjects for each of these scenarios, i.e. a total of 25 trajectories) and determined the set of objective functions parameters α_i that produced the best overall fit between optimization model and measurements for all five cases. The results of these computations are presented in Sect. 3.3.1.
- In order to validate the model that was established above by means of five different scenarios, we have arbitrarily chosen several other test scenarios and evaluated the performance of the model in comparison with the measurements of these scenarios. The results of these tests are shown in Sect. 3.3.2.

3.3.1 *Identification of objective function by simultaneous inverse optimal control of multiple scenarios*

Here we describe the results of inverse optimal control which was simultaneously applied to five different locomotion scenarios.

As stated in the experimental section, we measure position variables x and y and orientation θ of the system, i.e. three of the six state variables. Neither velocities (the three remaining state variables) nor accelerations (the three control variables) are directly measured. The variable vector z in the bilevel inverse optimal control problem formulation therefore has dimension three: time histories of x, y and θ are approximated at the same time.

The set of objective function parameters identified by inverse optimal control are

$$\alpha_0 = 1$$
$$\alpha_1 = 1.2$$
$$\alpha_2 = 1.7$$
$$\alpha_3 = 0.7$$
$$\alpha_4 = 5.2$$

where α_0 was the parameter fixed a priori. The objective function of type (5) that best reproduces measurements therefore becomes

$$\begin{aligned}\Phi(T, x(t), u(t), p) &= T + 1.2\int_0^T u_1^2 dt + 1.7\int_0^T u_2^2 dt + 0.7\int_0^T u_3^2 dt \\ &\quad + 5.2\int_0^T \Psi(z(t), z_e)^2 dt \qquad (15)\end{aligned}$$

The weight factor corresponding to the orthogonal direction is about 1.5 the weight factor of the forward direction which, in combination with the orientation towards the goal induced by the last term, leads to a clear preference of forward walking, but leaves the possibility for orthogonal motions whenever they are more efficient in this measure. The weight factor of the rotational term is smaller, i.e. large accelerations in rotational direction are less punished, but it is present to

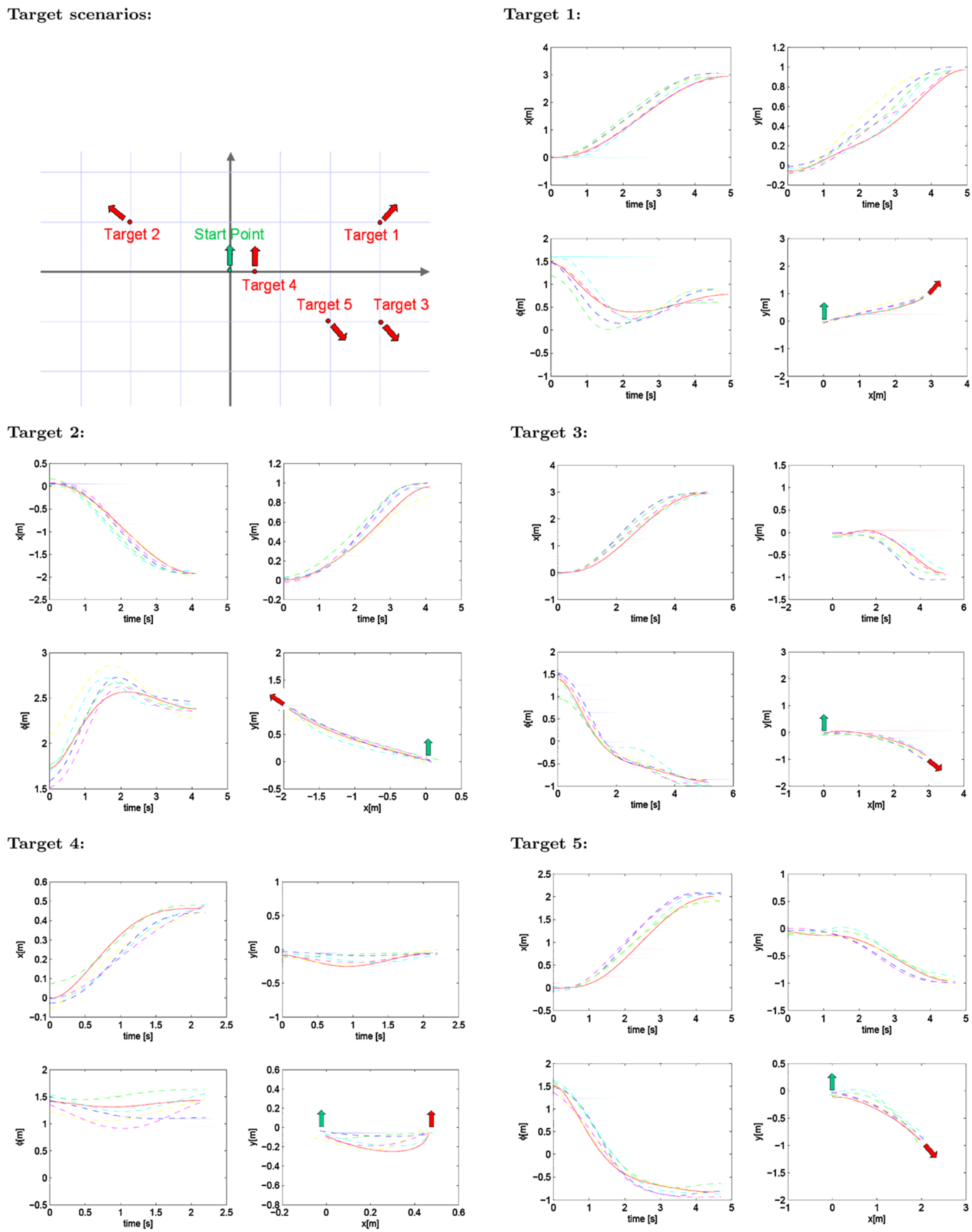


Fig. 7 Results of inverse optimal control performed simultaneously for five scenarios. *The top left sub-figure* presents the five arbitrarily selected scenarios. The other sub-figures show the fit between the measurements (5 *dashed lines* in each case, representing 5 different subjects) with the respective optimal trajectory (*solid line*) produced by the objective function identified by inverse optimal control

balance too large accelerations produced by the last term. Compared with previous results without α_4 (see Mombaur et al. 2009), the parameters have changed, since this term has a strong, but different influence on the three acceleration components.

The top left part of Fig. 7 shows the five arbitrarily chosen scenarios. The remainder of the figure shows the results of inverse optimal control for all five cases. The solid line in all sub-figures represents the respective computed optimal trajectory for the identified set of objective function parameters. The five dashed lines denote the measured trajectories of the five subjects used as bases for the computation. The fit in all cases is very good, taking into account that the model equations and optimization functions are always a simplification, and that no perfect fit can be achieved. In comparison with earlier results without the last term (Mombaur et al. 2009), it can be seen that this objective function component minimizing the angle difference to the target and thus introducing the discussed asymmetry in the objective function, leads to a considerable improvement of the fit. Figure 8 shows the corresponding profiles of forward, orthogonal and rotational velocity in all five cases. The figure also shows the time histories of the angles between body orientation and direction of motion which can also be computed via the relative size of forward and orthogonal velocity

$$\xi = \begin{cases} \arctan(\frac{v_{orth}}{v_{forw}}) & \text{if } v_{forw} \neq 0 \\ 0.5\Pi & \text{if } v_{forw} = 0 \text{ and } v_{orth} > 0 \\ -0.5\Pi & \text{if } v_{forw} = 0 \text{ and } v_{orth} < 0 \\ := 0 & \text{if } v_{forw} = v_{orth} = 0 \end{cases} \tag{16}$$

If this angle is zero, the motion is perfectly nonholonomic. Angles below a magnitude of 15° can be considered nonholonomic from a practical point of view (compare the bounds in the angle plots in Fig. 8). It can clearly be seen that in most cases, the motions start with a holonomic behavior, due to the re-orientation movement, which usually is a combination of a rotation and a sidestep. It then becomes quickly non-holonomic in some cases, and remains longer in the holonomic region for others.

3.3.2 *Validation of identified cost function on additional test scenarios*

Figure 9 shows the fit of the optimal control model established above, applied to two arbitrarily chosen test cases, and the respective measurement data. The targets chosen are $(x_e, y_e, \theta_e) = (1, 0, 0)$ and $(x_e, y_e, \theta_e) = (-1, 1, \Pi/4)$. The model still represents a good approximation of the data.

3.4 Using inverse optimal control results to control humanoid robots

In the previous section, we have used inverse optimal control to identify the optimal control model describing collected

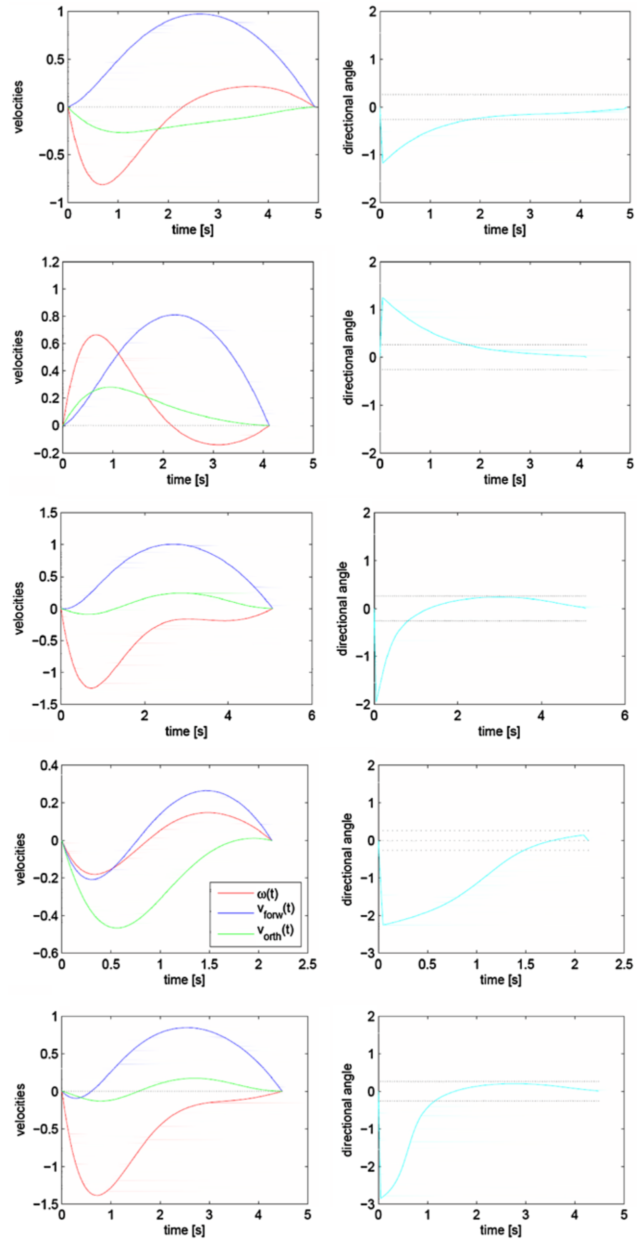


Fig. 8 Forward, orthogonal and rotational velocities (*left plots*) and resulting angle between direction of motion and forward orientation of the body (*right plots*) for all five targets presented in Fig. 3. *Dashed lines* in right plots indicate quasi-nonholonomic angle ranges

human locomotion data. In this section we briefly present how this optimal control model can be used to enable the humanoid robot HRP-2 (Kaneko et al. 2004) at LAAS to autonomously generate locomotion trajectories.

As described extensively in the previous sections, the focus of this paper is on the generation of bio-inspired overall locomotion trajectories, i.e. the appropriate choice of the trace of the robot on the floor. Our interest here is neither the selection of foot patterns about the path nor the generation

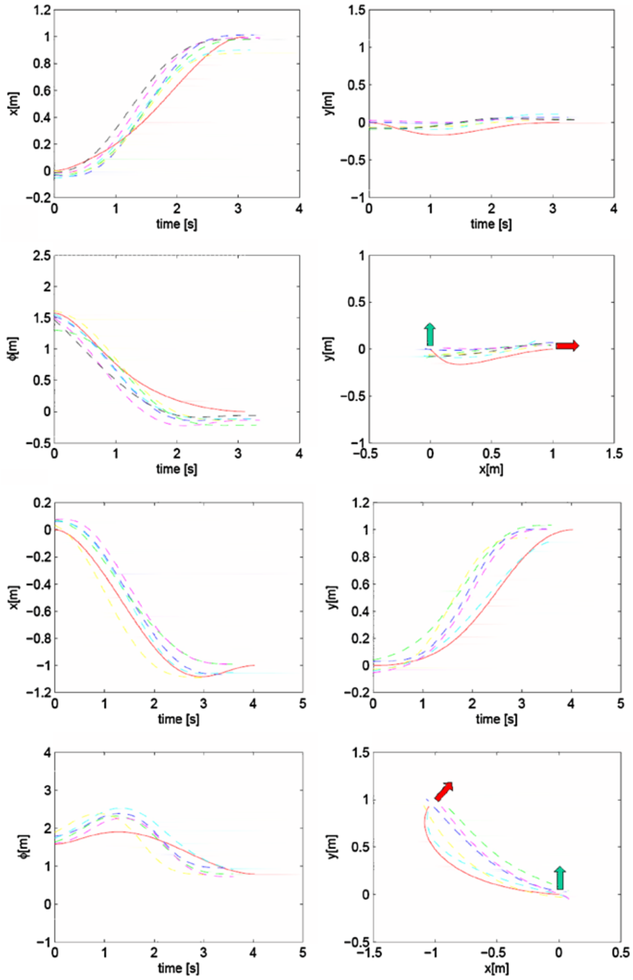


Fig. 9 Validation of optimal control model on two test scenarios

of trajectories of all internal joints. For this purpose we rely on existing approaches for the robot HRP-2.

For any given locomotion target to be reached by the humanoid robot, the following steps are performed:

1. An optimal control model (1)–(4) with modified parameters α_i (with respect to the human model (15)) must be formulated to take into account that size and velocity characteristics of the robot differ from those of a human. While solutions generated with the original problem pose no burden to humans and do not take them to their physical limits, they usually exceed the speed limits of the humanoid robot HRP-2. Taking into account those constraints would completely modify the nature of the solutions from an unconstrained minimum of (15) to a severely constrained optimum. The objective function parameters therefore have to be modified such that the resulting solution automatically stays inside the velocity bounds without reaching them. The same balance between time minimization and the fast angle change term on one side and the terms minimizing acceleration of the other side must again be restored. For a simple scaling rule, we approximate that humans are about 4 times as fast as the HRP-2 robot, and therefore scale the weight parameters $\alpha_1 - \alpha_3$ for the robot by this factor

$$\tilde{\alpha}_i = 4 \cdot \alpha_i \quad i = 1, \ldots, 3$$

2. The optimal control problem (1)–(4) with the modified parameters $\tilde{\alpha}_i$ then is solved. The solution of this optimal control problem describes the overall path to be followed.
3. Linear and angular velocities of the path computed in 1 are passed to the pattern generator. We use the walking pattern generator by Kajita et al. (2003), which is based on preview control of zero moment point (ZMP) using the table-cart inverted pendulum model, and which produces appropriate footprints and generates a desired ZMP trajectory. Leg joint angles are computed by inverse kinematics from the CoM trajectory and the footprints. The resulting biped walking motion is dynamically stable in the ZMP sense.

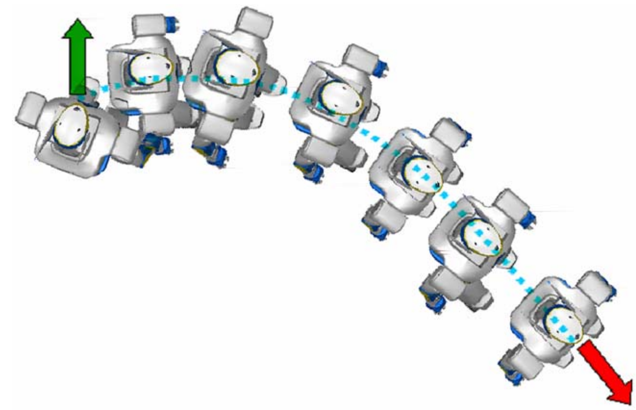


Fig. 10 Implementation of natural locomotion trajectory for target 5 on the humanoid robot HRP-2

Figure 10 shows a visualization of the resulting robot motion for one example, using the humanoid simulator and controller software OpenHRP (Kanehiro et al. 2004) for the humanoid robot HRP-2. Due to identical interfaces of OpenHRP towards simulation and the real robot, the same motions can easily be transferred to the robot.

So far, the computations described above have been performed offline. But since they are very fast—compared to the standard start delays of a humanoid before it actually starts to walk—these routines could easily be implemented on the robot and could be called each time the robot has to autonomously decide about a locomotion trajectory.

4 Conclusion and outlook

In this paper, we have presented an inverse optimal control approach allowing to identify underlying optimization objectives of processes such as biological motions from measured data. A flexible numerical technique has been described which allows the solution of a large class of problems.

In addition, we have proposed a model to describe human locomotion in rest-to rest motions to close targets. This model does not aim at giving a detailed account of all biomechanical motions on joint level, but rather studies locomotion from a neuroscience perspective, based on the idea of a central steering element. It describes the motion of a subject as a single body with position and orientation in the plane. Using inverse optimal control, it was possible to establish a simple and unique optimal control model that seems to represent a good approximation of the collected data.

Obviously, a very crucial element in the formulation of inverse optimal control problems is the selection of appropriate base functions for the objective function. Any solution of the inverse optimal control problem can only get as good as its base functions permit, and with a bad selection of base functions, the fit would still be unsatisfactory after applying inverse optimal control. In our research, we have investigated a series of possible objective functions, and found that only some of them seem to be adequate to reproduce the measured data.

We present a simple objective function that has only five components—total time, as well as square terms of the accelerations in forward, orthogonal and rotational direction, and a last term minimizing the squared difference between body orientation and direction to the target. It is generally agreed upon that acceleration is an important quantity for human motion if studied from a global perspective, and that sensing of acceleration is with the vestibular organ is crucial for all human movement.

Our research also has shown that many obvious ideas for objective functions—widely studied functions in the biomechanical community—do not work to explain overall locomotion trajectories in this case. As discussed in Sect. 3.2.1, neither jerk nor velocity nor kinetic energy related terms seem to be plausible objective functions for this problem.

Besides explaining human motion, this optimal control model is also very useful in bio-inspired robotics. More classical approaches in bio-inspired robotics, based on passive-dynamic walking (e.g. Collins et al. 2005) or on central pattern generators (e.g. Ijspeert 2001) study natural motions on joint level. However, the overall locomotion trajectory along which the mechanism moves is very often a straight line, or at least assumed to be predefined. The approach discussed in this paper could therefore be ideally combined with other bio-inspired techniques, by providing natural locomotion trajectories generated by optimization along which energy optimal or efficient joint motions are generated.

Our current work aims at extending the experimental basis of human locomotion as well as the optimal control models to targets that are further away from the origin. We are currently also extending our computational and experimental research on the shape of natural locomotion paths towards the inclusion of obstacles—fixed obstacles as well as moving obstacles, e.g. other persons. A hypothesis to be verified by our current research is that the locomotion trajectories in the presence of obstacles are optimized using the same optimization criterion as in free space, but additionally satisfy obstacle avoidance constraints.

As mentioned previously, the generality of the presented inverse optimal control approach allows its application to a variety of other problems, such as the identification of optimization criteria of locomotion on joint level. Based on our previous work on (forward) optimal control of human-like running motions (Schultz and Mombaur 2009; Mombaur 2008) and the multi-body system models developed in this research, we are currently applying inverse optimal control to identify objective functions of different human running motions based on motion capture data.

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A short preliminary version of this article has been presented at the International Symposium of Robotics Research 2009 in Lucerne (Mombaur et al. 2009).

Appendix: Symmetry properties of locomotion trajectories

Definition 1 (Origin-target symmetry of two trajectories) We define two corresponding trajectories $q(t)$ and $\bar{q}$ which result from a permutation of origins and targets

$q(t) \in [0, T]$

$$\text{with } q(0)^T = (x_1, y_1, \theta_1),\ q(T)^T = (x_2, y_2, \theta_2) \quad \text{and}$$

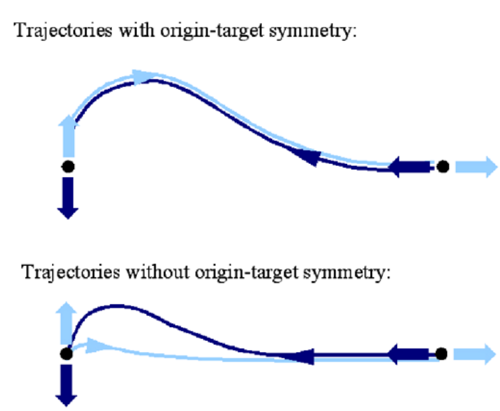

Fig. 11 Trajectories with and without origin-target symmetry

$$\bar{q}(t) \in [0, \bar{T}]$$
$$\text{with } \bar{q}(0)^T = (x_2, y_2, \theta_2 + \pi),\ \bar{q}(\bar{T})^T = (x_1, y_1, \theta_1 + \pi)$$

to be *origin-target symmetric in time* if they have identical durations $T = \bar{T}$ and if

$$\bar{q}(t) = q(T - t) + (0, 0, \pi)^T$$

Otherwise the two trajectories are *origin-target asymmetric in time* (compare the illustrations in Fig. 11).

While such a symmetry may be present in certain locomotion scenarios, it becomes very clear from experimental observation that the rest-to-rest locomotion trajectories studied in this paper are asymmetric in that sense. This must be taken care of for the choice of the objective function base functions, as explained in Sect. 3.2.

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