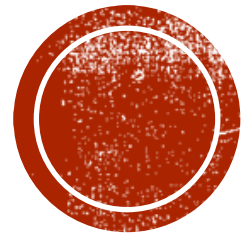




# **DISCRETE MATHEMATICS IN COMPUTER SCIENCE**

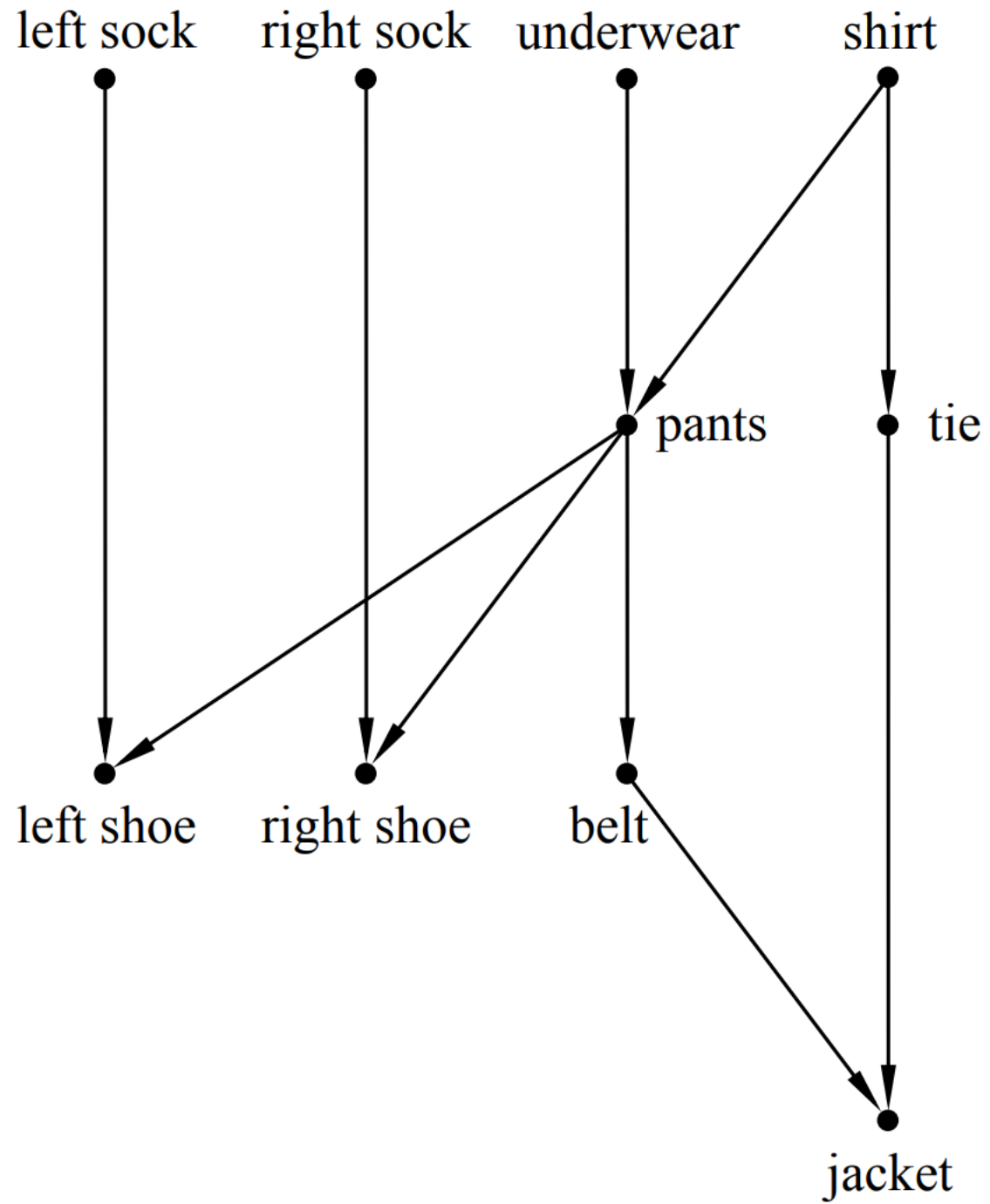
**HSIEN-CHIH CHANG  
FEBRUARY 9, 2022**



# **DIRECTED ACYCLIC GRAPHS**

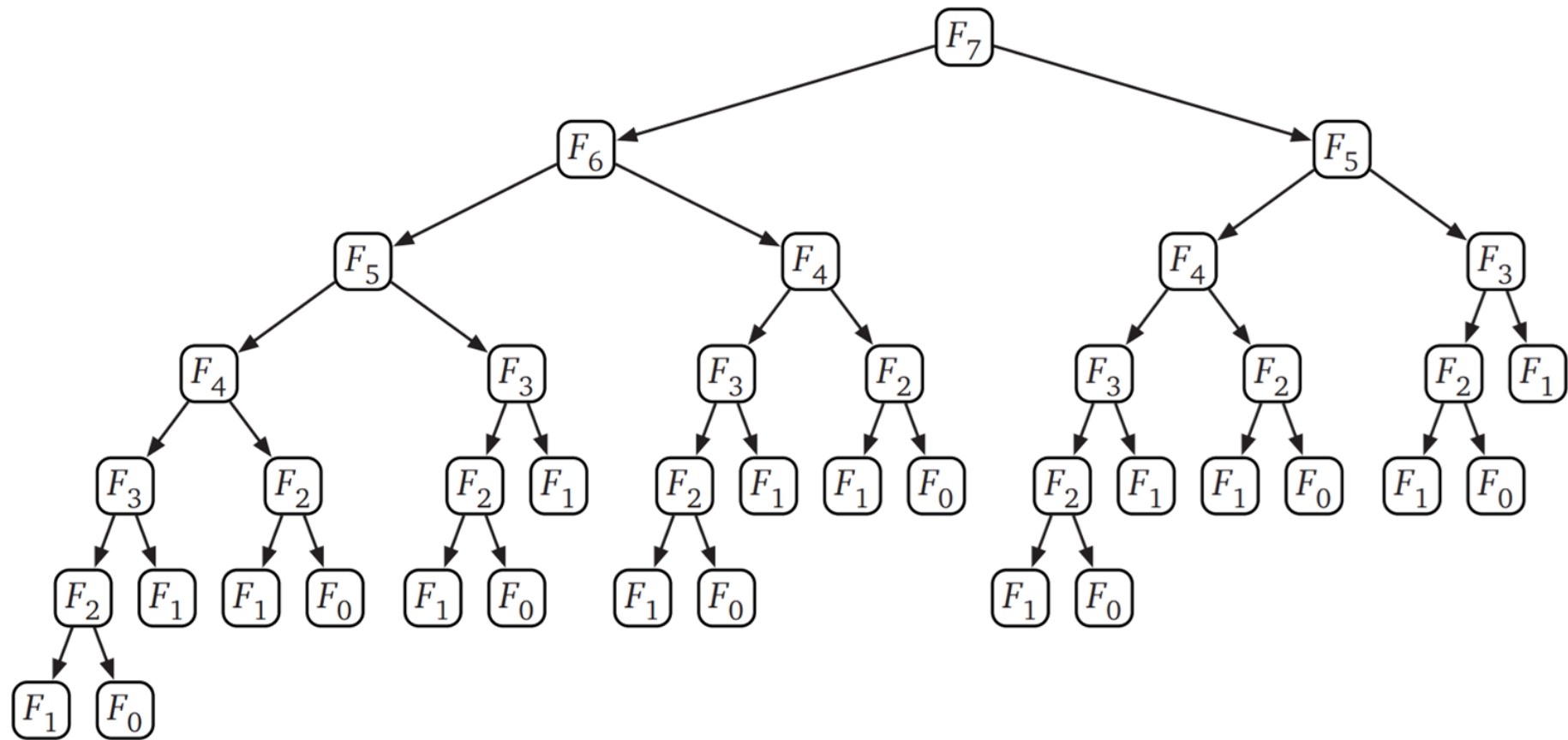




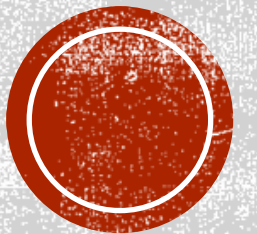


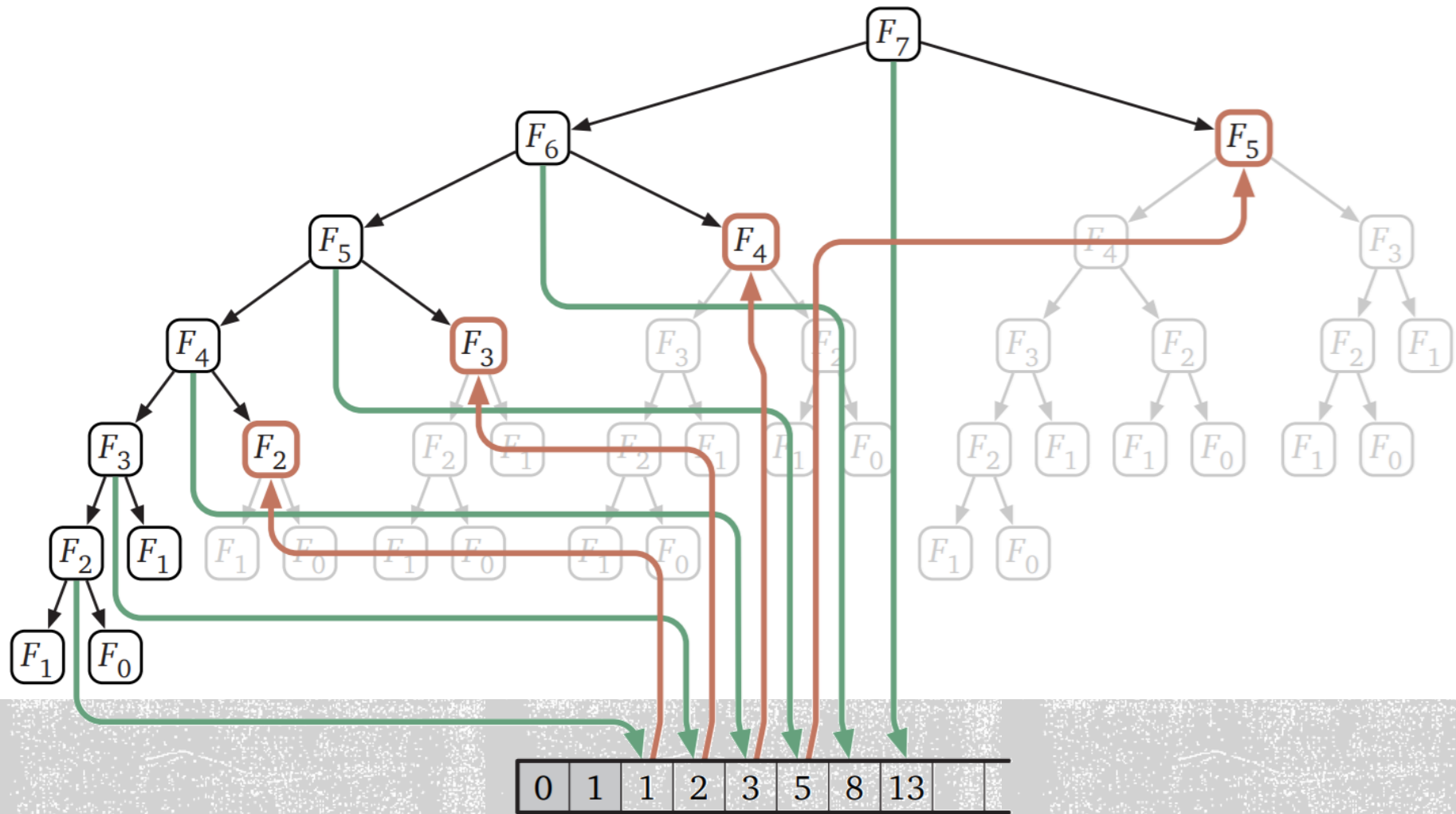
# DEPENDENCY GRAPH





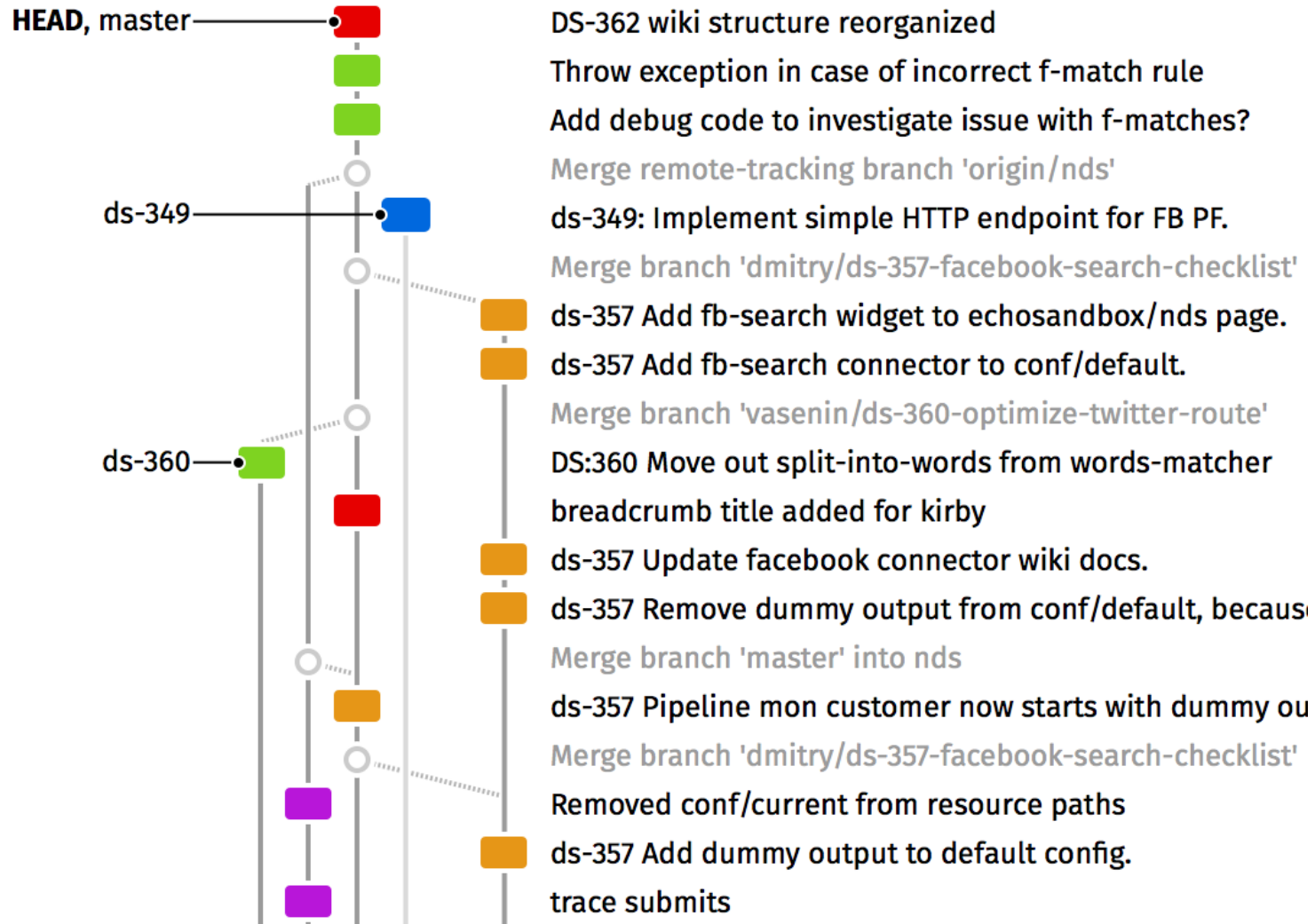
# RECURSION TREE





# RECURSION DAG



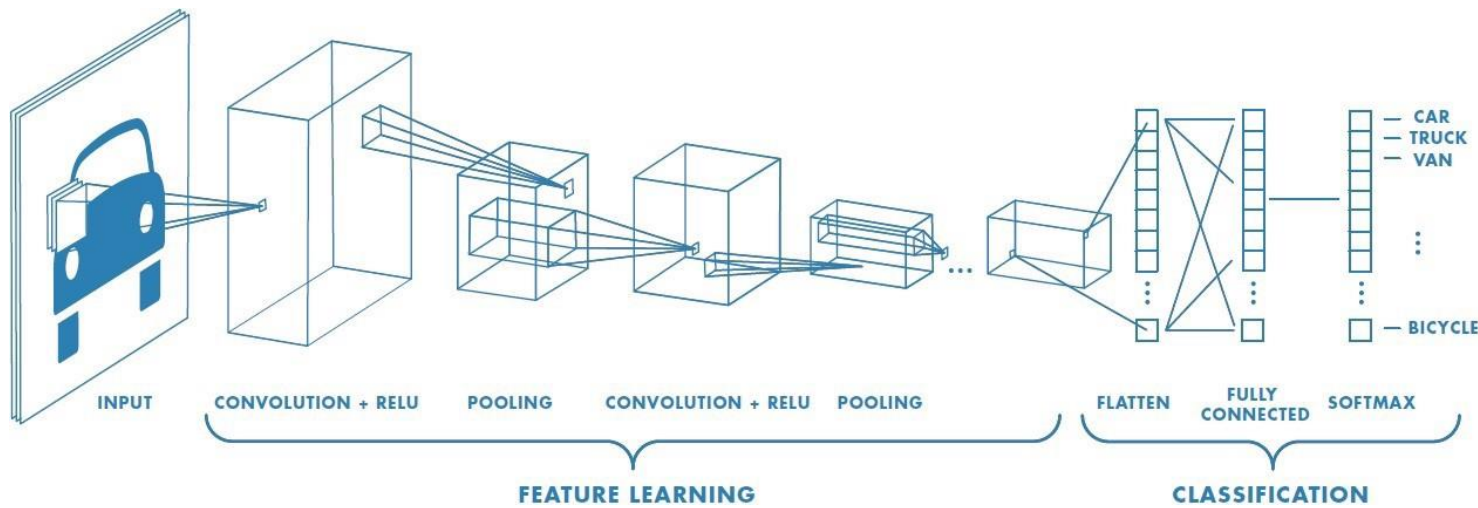


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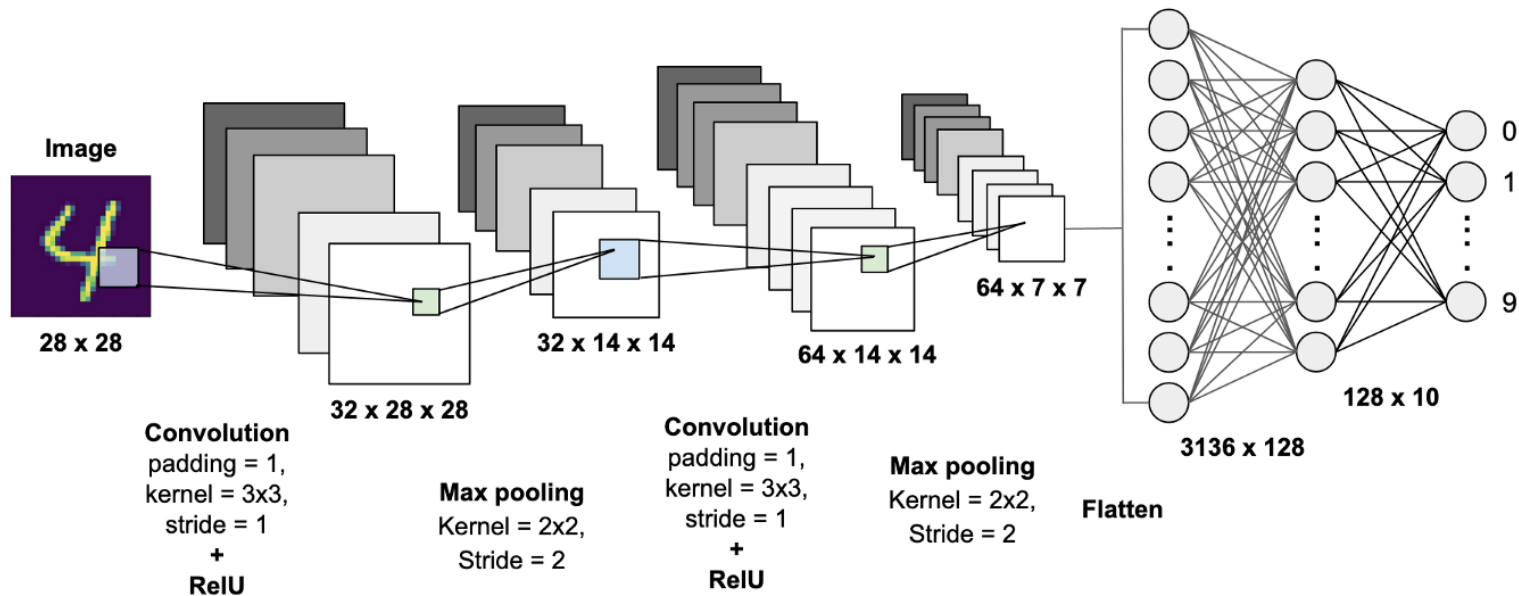
Git commits







Sumit Saha, "A Comprehensive Guide to Convolutional Neural Networks — the ELI5 way"

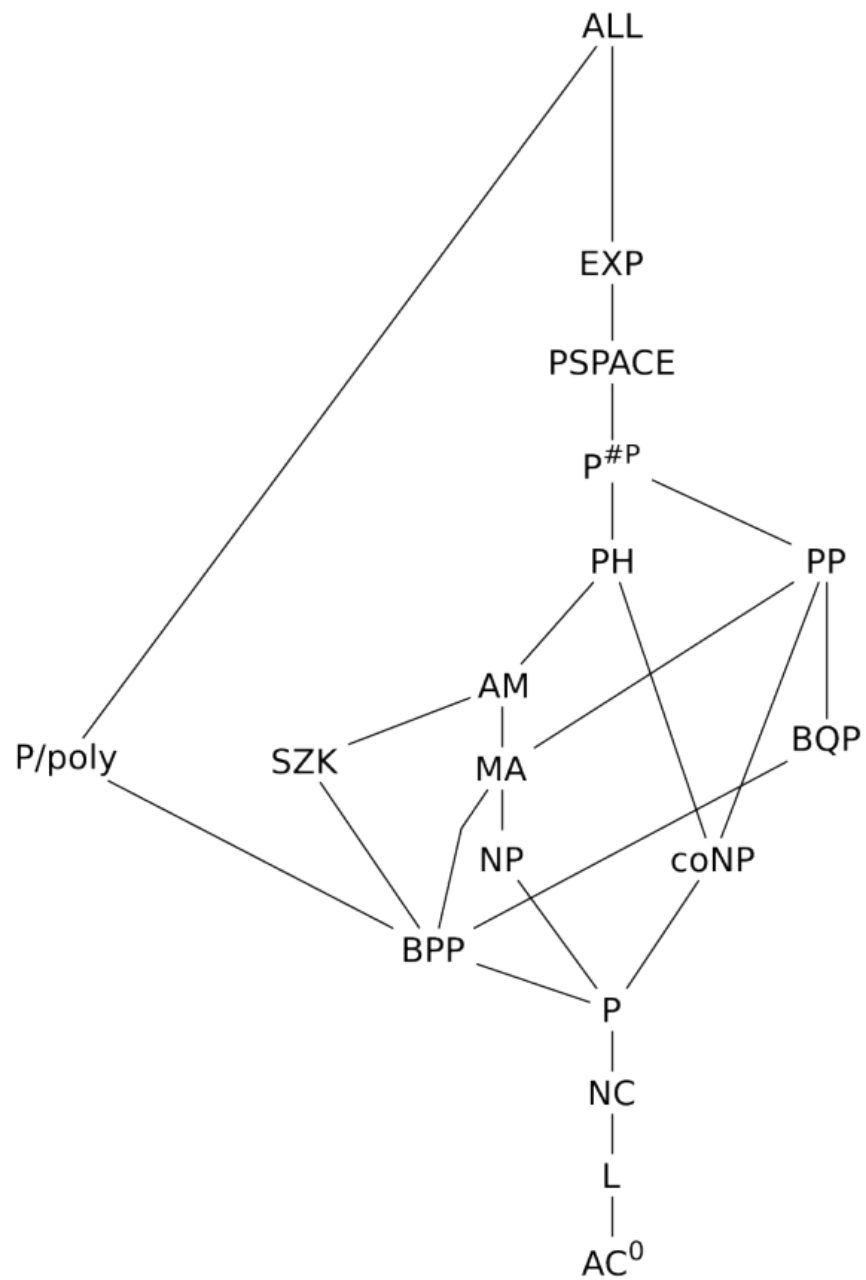


Krut Patel, "MNIST Handwritten Digits Classification using a Convolutional Neural Network"

# DEPENDENCY GRAPH

Neural networks



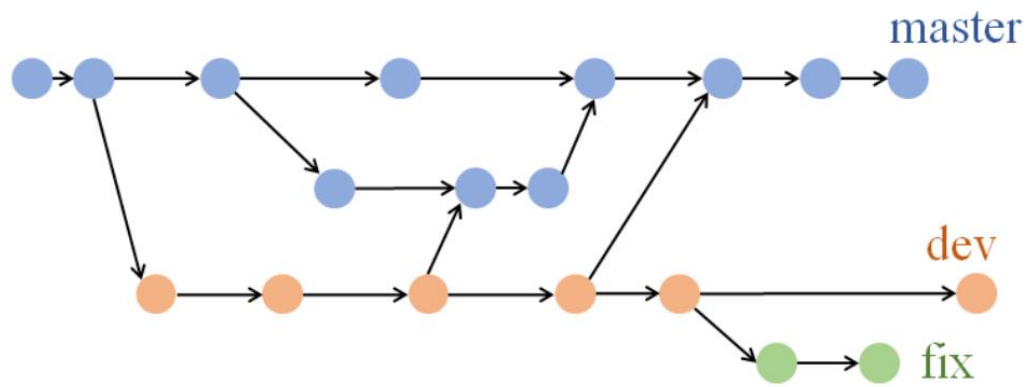


# INCLUSION GRAPH

Complexity classes







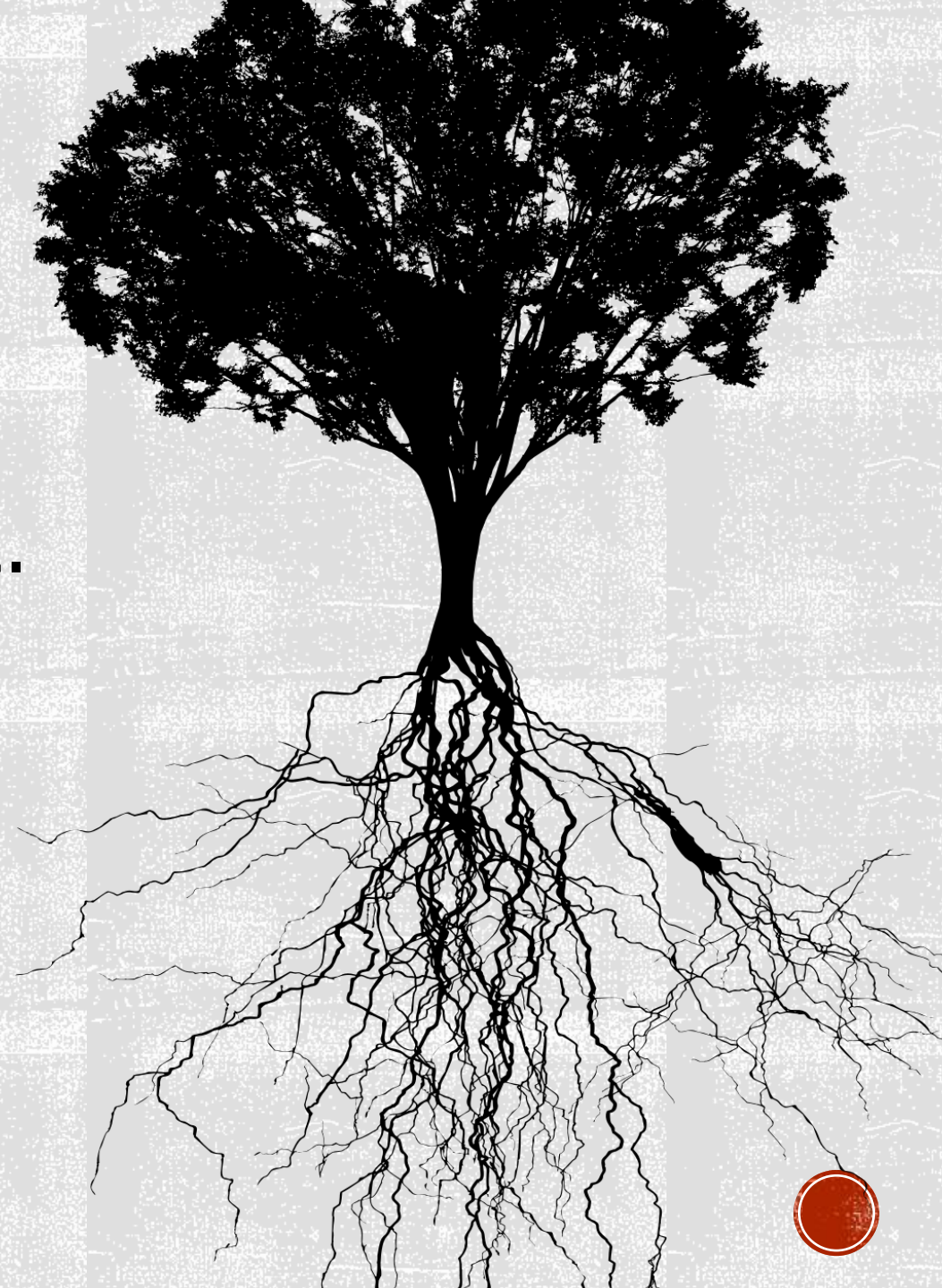
# NO PRE-REQ THEOREM

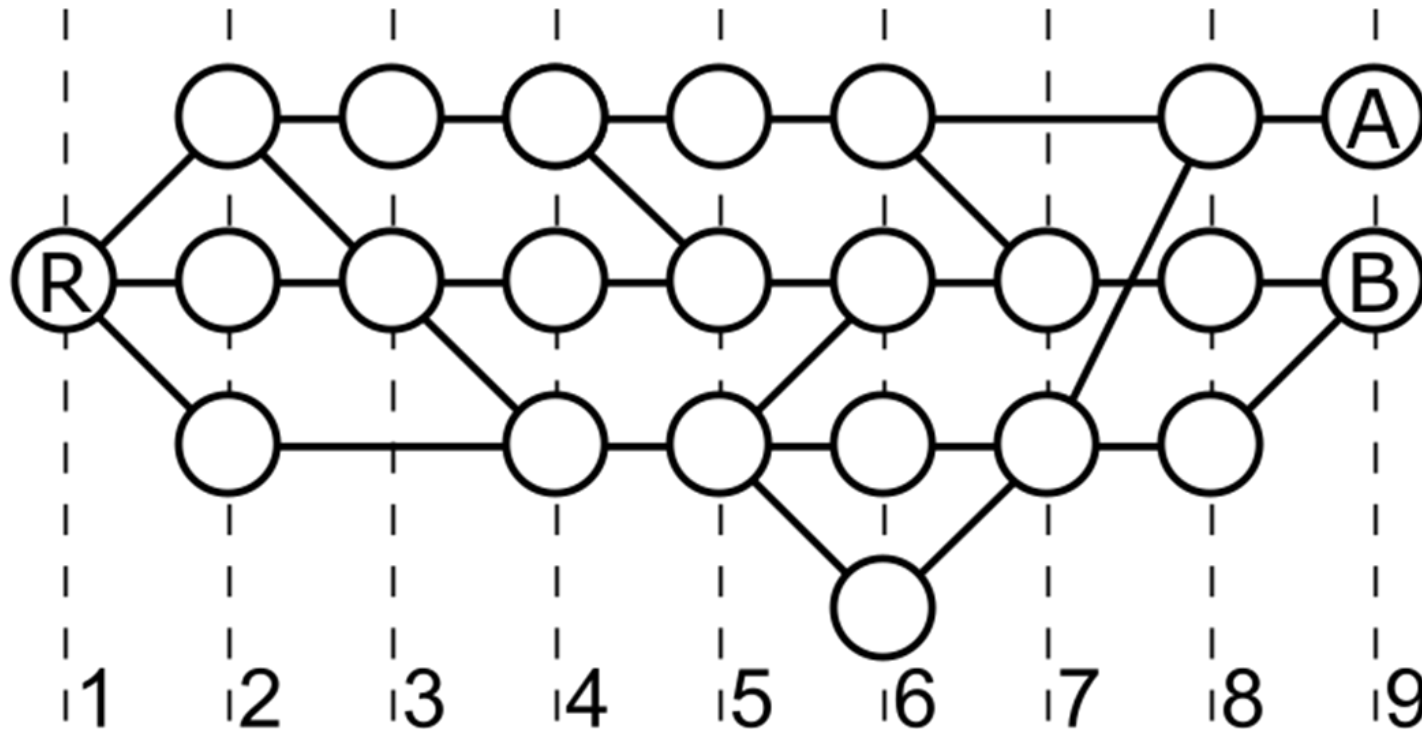
Every directed acyclic graph must have a source.



# ELIMINATION ORDER

- There is a way to **direct** the edges of  $T$  so that every non-root vertex has **out-degree** 1.
- Every tree can be **rooted**.





# Jargon

chain

anti-chain

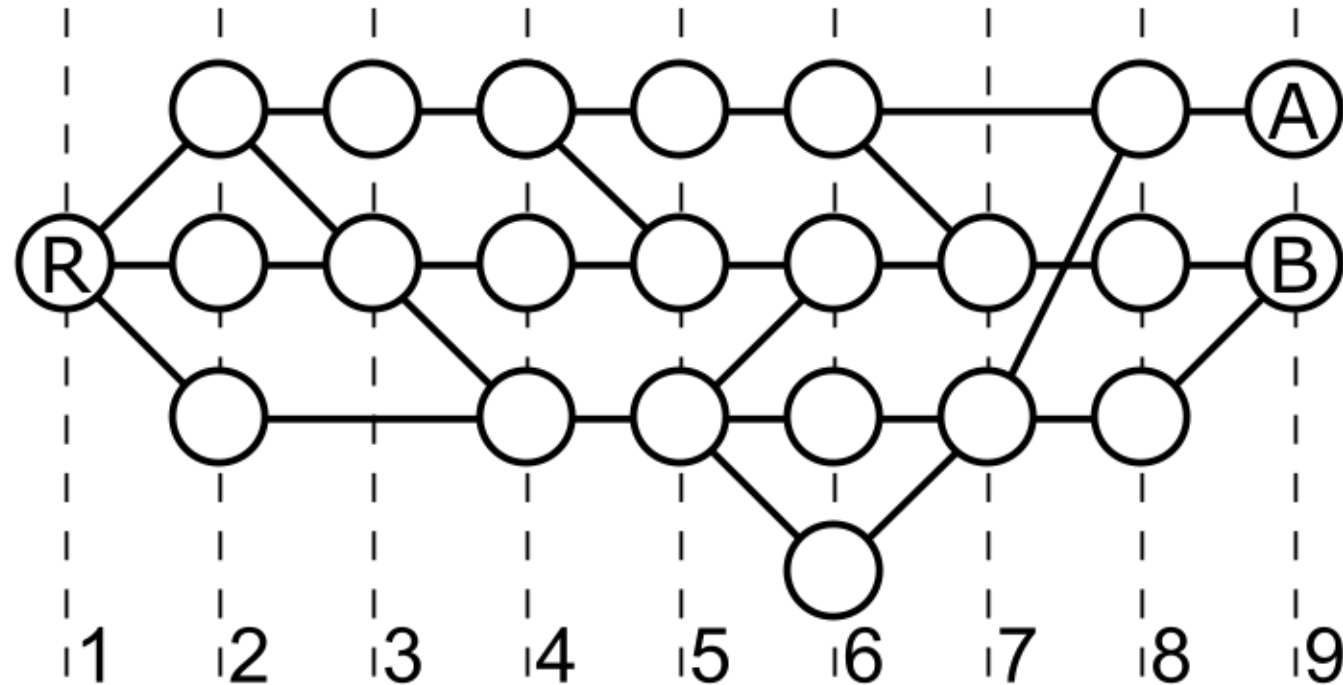
height

width





IF DAG  $D$  HAS HEIGHT  $t$ ,  
THEN  $D$  DECOMPOSES INTO  $t$  ANTICHAINS.



## MIRSKY'S THEOREM



**FOR ANY DAG  $D$  AND ANY POSITIVE INTEGER  $T$ ,  
EITHER  $\text{HEIGHT}(D) > T$ , OR  $\text{WIDTH}(D) \geq N/T$**

## **COROLLARY**

**EVERY DAG  $D$  HAS EITHER A CHAIN OF SIZE  $> \sqrt{N}$ ,  
OR AN ANTICHAIN OF SIZE  $\geq \sqrt{N}$ .**



# COMPARISON

## ■ Mirsky's Theorem

If dag  $D$  has **height**  $t$ , then  
 $D$  decomposes into  $t$  **antichains**.

## ■ Dilworth's Theorem

If dag  $D$  has **width**  $t$ , then  
 $D$  decomposes into  $t$  **chains**.





**DAG IS WHERE THE INDUCTION FAIRY IS.**

**NEXT TIME.**  
**RELATION, PARTIAL ORDER, EQUIVALENCE**

