1. ***Gauss code.*** A ***Gauss code*** is a cyclic string of $2n$ symbols where each symbol occurs exactly two times; it is ***signed*** if in addition each symbol `x` is attached with a plus/minus sign `+`/`-`, one for each occurrence of `x`. A Gauss code is ***planar*** if it encodes the sequence of crossings we see as we traverse an n-vertex planar curve γ; the signing of the Gauss code correspond to the Gauss signs of the crossings of γ.

   Describe and analyze an algorithm whether a given signed Gauss code is planar.

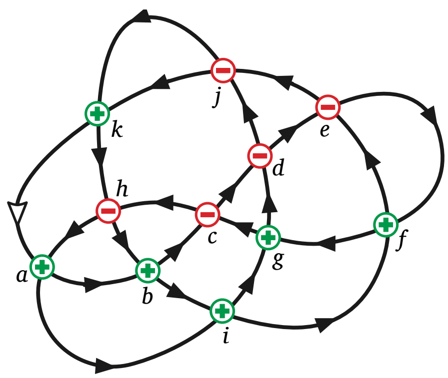

Figure 1. A planar curve with Gauss code `[abcdefgchaigdjkhbifejk]` and signing `[++---+++--+-+-++---++-]`.

2. ***Counting saddles.*** A ***terrain*** is a plane graph G together with a function $h : V(G) \to \mathbb{R}$, mapping each vertex v to a real number $h(v)$, called the ***height*** of v. Without loss of generality let's assume all vertices have different heights. An edge uv incident to v is

   - ***upward*** if $h(u) > h(v)$, and
   - ***downward*** if $h(u) < h(v)$.

   (Notice that an edge uv is upward for v if and only if it is downward for u.)

   We say a vertex is a ***source*** if all the incident edges are downward, and a ***sink*** if all the incident edges are upward. A vertex is a ***saddle*** if among all its incident edges, four of them are alternating between being upward and downward; in other words, there are 4 neighbors $u_1, \ldots, u_4$ around v in cyclic order, where u_1v, u_3v are upward edges and u_2v, u_4v are downward edges.

   Prove that the number of saddles in a terrain is at most $s + t - 2$, where s is the number of sources and t is the number of sinks. *[Hint: Try to case when the terrain has a unique source and a unique sink first.]*