DISCRETE MATHEMATICS

IN COMPUTER SCIENCE

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Administrivia

- Homework 2 due Friday
- Confusion in terminology: hyperplanes, winning strategy

Induction

Every integer n>1 has a prime divisor.

Proof by Contradiction?

Every integer n>1 has a prime divisor.

Proof by Contradiction?

Smallest Example:

Jump to the smallest n that has no prime divisor.

Taking the Contrapositive

$\exists n : P(n)$ is FALSE

implies

$\exists n : (P(1)$ and … and $P(n-1)$ and $\neg P(n))$ is TRUE

$\forall n : (P(1)$ and … and $P(n-1)$ and $\neg P(n))$ is FALSE

implies

$\forall n : P(n)$ is TRUE

Axiom of Induction

(P(1) and … and P(n-1) implies P(n)) is TRUE for all n

implies

P(n) is TRUE for all n

Meet the Recursion Fairy

Boilerplate for Induction

Theorem. P(x) holds for every object x.

Let x be an arbitrary object.
Assume P(y) is true for every smaller $y < x$.
[Assume recursion fairy is with us.]

- If x is … [base case]
- If x is … [inductive case]
  The induction hypothesis implies …
  [Recursion fairy says …]

Thus P(x) is true.

Every integer n>1 has a prime divisor.

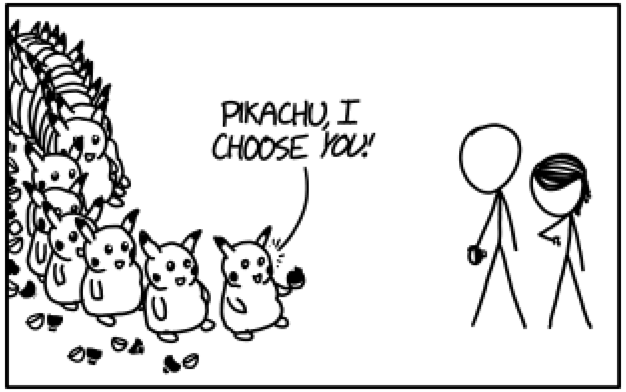
Example

$$\sum_{0 \le i \le n} i = n(n+1)/2$$

Example

Never, NEVER do induction with the $P(n) \Rightarrow P(n+1)$ template

Sudden-Death



Inductive Definition

Definition of an object using a smaller instance of itself.

Example: Fibonacci Number

Fibonacci number F_n:

- $F_n = F_{n-1} + F_{n-2}$ if $n>1$
- $F_0 = 0$
- $F_1 = 1$

F_n is even if and only if n is divisible by 3.

Example: Fibonacci Number

Fibonacci number F_n:

$F_n = F_{n-1} + F_{n-2}$ if $n>1$

$F_0 = 0$

$F_1 = 1$

Theorem(?)
All cows have the same color

Next time.

Induction IS Recursion IS Induction IS Recursion IS Inductio

Example: Strings

String w over the alphabet set Σ:

- empty string ε, or
- concatenation a · x between symbol a in Σ and another string x over Σ.

For every strings w and z,
$|w \cdot z| = |w| + |z|$.

Induction using Inductive Defn.

String w over Σ:

- empty string ε, or
- concatenation $a \cdot x$ for some string x and some symbol a