Let G be an (either undirected or directed) graph. A ***Hamiltonian path*** in G is a path that visits every single vertex in G exactly once. A ***Hamiltonian cycle*** in G is a closed Hamiltonian path that starts and ends at the same vertex. Consider the following problems:

HAMILTONIANPATH
- ***Input:*** *A directed graph G*
- ***Output:*** *Is there a Hamiltonian path in G?*

UNDTHAMILTONIANPATH
- ***Input:*** *An undirected graph G*
- ***Output:*** *Is there a Hamiltonian path in G?*

HAMILTONIANCYCLE
- ***Input:*** *A directed graph G*
- ***Output:*** *Is there a Hamiltonian cycle in G?*

1. Describe a polynomial-time reduction from UNDTHAMILTONIANPATH to HAMILTONIANPATH.
2. Describe a polynomial-time reduction from HAMILTONIANPATH to UNDTHAMILTONIANPATH.
3. Describe a polynomial-time reduction from HAMILTONIANPATH to HAMILTONIANCYCLE.
4. Describe a polynomial-time reduction from HAMILTONIANCYCLE to HAMILTONIANPATH. *[Hint: Might be easier to describe an oracle reduction.]*

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5. ***After class:*** Read the proof from Erickson's note that HAMILTONIANPATH is NP-complete.