

# DISCRETE MATHEMATICS IN COMPUTER SCIENCE

HSIEN-CHIH CHANG MARCH 4, 2022

#### ADMINISTRIVIA

- Final exam
  - Mar 13 (Sun) 8—11AM
  - LSC 100 (this room)
- SAS/Conflict/COVID
  - Come talk to me

- Closed-book written exam
- Scope: Module C on counting
- One-page two-sided cheatsheet
  - Must be hand-written



## CONDITIONAL PROBABILITY

#### Jargon

conditioning independence





## TWO CHILDREN PROBLEM





## RANDOM VARIABLES AND EXPECTATION

#### Jargon

random variable indicator variable expectation





GAMBLING GAMES (NOT A GOOD IDEA)



#### EXPECTED NUMBER OF TOSSES?

```
VonNeumannCoin():
  x \leftarrow \text{BiasedCoin}()
  y \leftarrow \text{BiasedCoin}()
  if x \neq y
        return x
  else
        return VonNeumannCoin()
```

#### EXAMPLE

Removing coin bias



## HOW MANY TOSSES ON AVERAGE BEFORE GETTING THE FIRST HEAD?

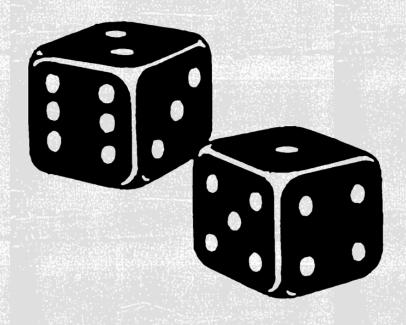
#### EXAMPLE



#### LINEARITY OF EXPECTATION

 $-\mathbf{E}[\mathbf{\Sigma}_{\mathbf{k}}\mathbf{X}_{\mathbf{k}}] = \mathbf{\Sigma}_{\mathbf{k}}\mathbf{E}[\mathbf{X}_{\mathbf{k}}]$ 

-Equality holds even when the variables are dependent to each other!







HOW MANY DIFFERENT POKÉMON DO WE HAVE AFTER BUYING N CARDS, GIVEN THERE ARE N POKÉMON IN TOTAL?

## GOTTA CATCH 'EM ALL



## HOW CARDS WE EXPECTED TO BUY TO COLLECT ALL THE POKÉMON?

## GOTTA CATCH 'EM ALL



## HOW CARDS WE EXPECTED TO BUY TO COLLECT ALL THE DIFFERENT POKÉMON?

## GOTTA CATCH 'EM ALL



### GAMBLER'S RUIN.

NEXT TIME.
LAST LECTURE: PROBABILISTIC METHOD

