- You know the drill now: Find students around you to form a ***small group***; use ***all resources*** to help to solve the problems; ***discuss*** your idea with other group member and ***write down*** your own solutions; raise your hand and pull the ***course staffs*** to help; ***submit*** your writeup through Gradescope in *24 hours*.

Our topic for this working session is *fooling set construction*.

Example. Find a fooling set of infinite size for the following language L, thus showing that L is *non-regular*:

- $L := \{\mathtt{0}^n\mathtt{1}^n : n \in \mathbb{N}\}$

Solution: Let $F = \{\mathtt{0}^n : n \in N\}$; notice that F has infinite size. For two distinct prefixes $x = \mathtt{0}^i$ and $x' = \mathtt{0}^j$ in F (where $i \neq j$), define the distinguishing suffix y be $\mathtt{1}^i$.

- $xy = \mathtt{0}^i\mathtt{1}^i$, thus we have xy in F.
- $x'y = \mathtt{0}^j\mathtt{1}^i$; because $i \neq j$, we have $x'y$ not in F.

This implies that F is a fooling set of infinite size, and thus L is not regular.

Find a fooling set of infinite size for each of the following languages, thus showing that all of them are *non-regular*:

1. $L_1 := \{ww : w \in \{0,1\}^*\}$
2. $L_2 := \{0^m 1^n : n \text{ divides } m \text{ and } m \geq 0,\, n > 0\}$
3. $L_3 := \{ww^R : w \in \Sigma^*\}$. (These are even-length parlinedromes. A ***parlindrome*** is a string that reads the same forward and backward.)

To think about later: (No submissions needed)

1. $L_4 := \{w \in \{(,)\}^* : w \text{ is a balanced parenthesis}\}$
2. L_5 consists of all strings in which substrings 00 and 11 appear the same number of times.
3. $L_6 := \{\#0^n 1^n : n \in \mathbb{N}\} \cup \{\#^k w : k \in \mathbb{N}, k \geq 2, w \in \{0,1\}^*\}$. (This language is non-regular, but cannot be certified by pumping lemma; thus the reason why we choose to work with fooling sets.)

Conceptual question: Wait. For Q3 why can't we set another pointer to the end of the string, and verify that the input reads the same forward and backward by checking the symbols one by one? Doesn't this contradict HW2 Q1(a)?