



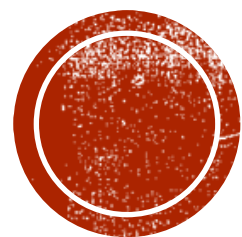
DISCRETE MATHEMATICS IN COMPUTER SCIENCE

**HSIEN-CHIH CHANG
FEBRUARY 18, 2022**

ADMINISTRIVIA

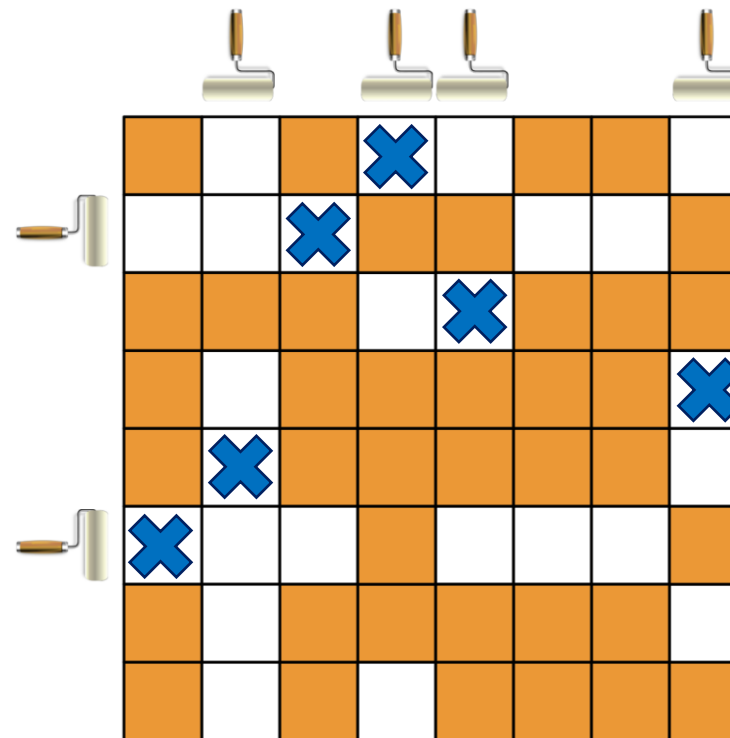
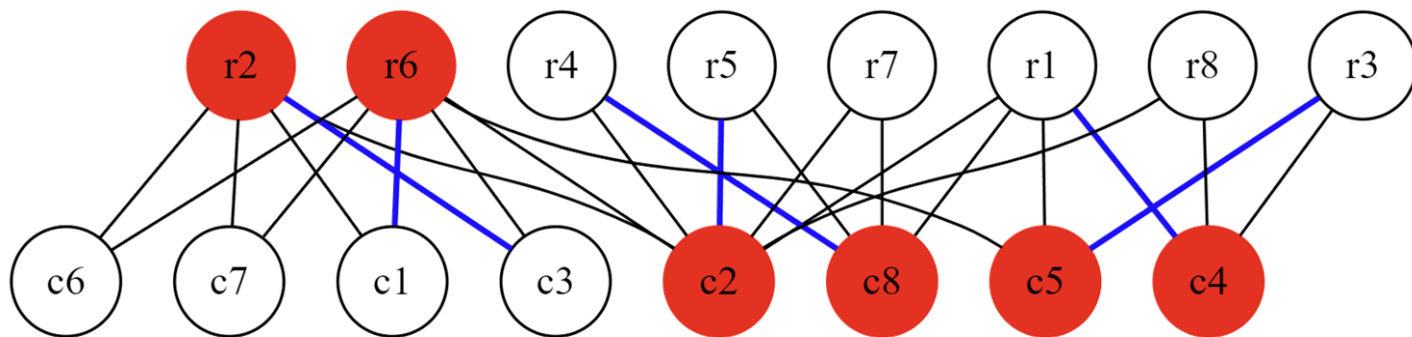
- **Midterm 2**
 - Feb 21 (Mon) 6–9PM
 - Carpenter 013 Herb West Lecture Hall
- **Conflict Midterm 2**
 - Feb 22 (Tue) 6–9PM
- **SAS/Conflict Conflict/COVID**
 - Come talk to me
- **Closed-book written exam**
- **Scope: Module G on graphs**
- **One-page two-sided cheatsheet**
 - Must be hand-written





MANY MODELS FOR COUNTING

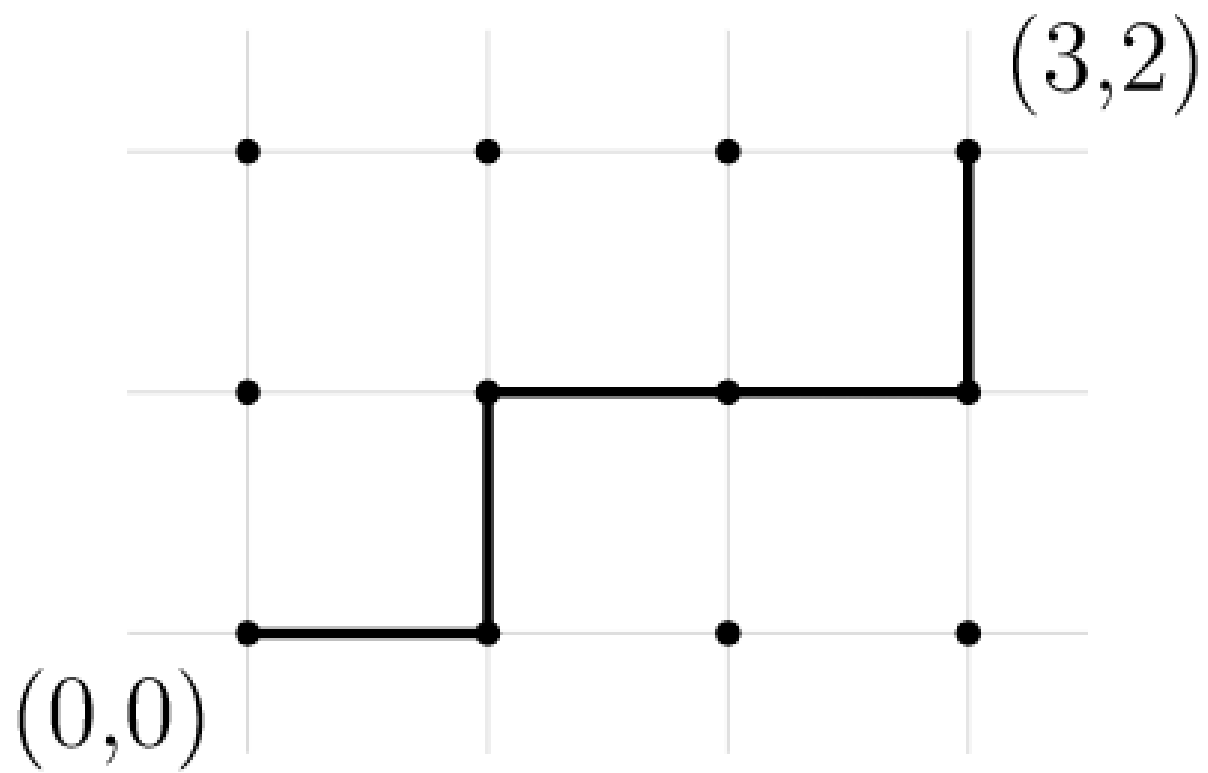




COMBINATORIAL EQUIVALENCE

IF THERE IS A BIJECTION
BETWEEN SET A AND B, THEN
 $|A| = |B|$





LATTICE PATHS

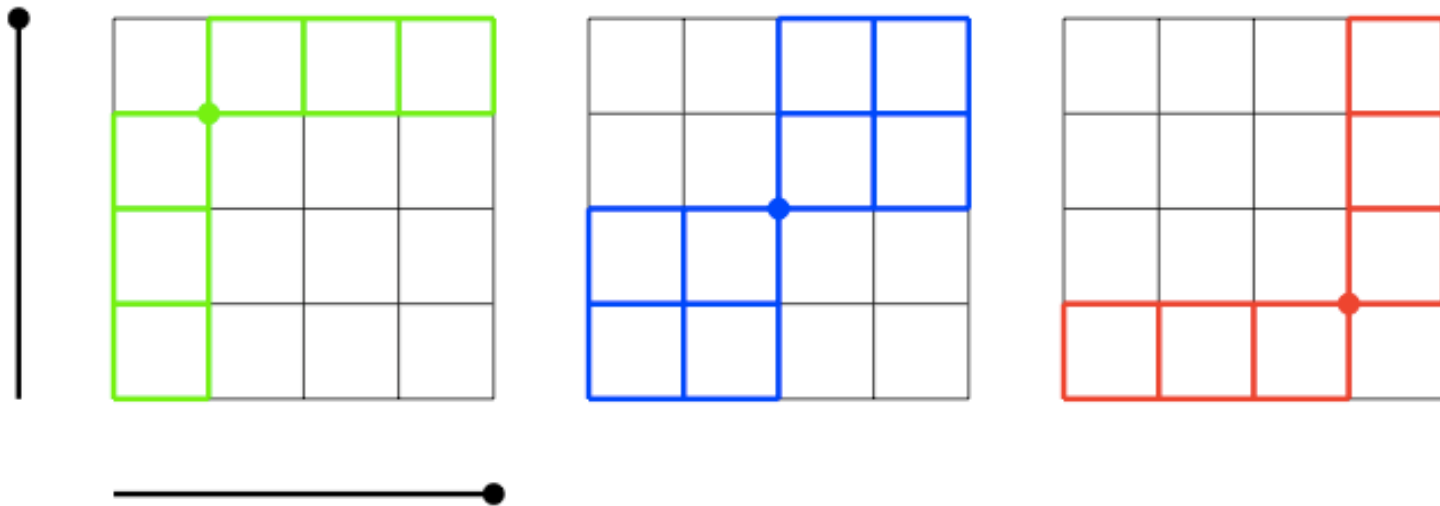


$$\sum_{k=0}^n \binom{n}{k}^2 = \binom{2n}{n}$$

LATTICE PATHS

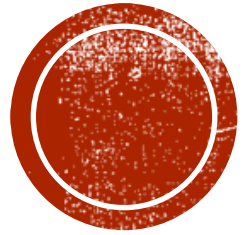


$$\sum_{k=0}^n \binom{n}{k}^2 = \binom{2n}{n}$$



LATTICE PATHS



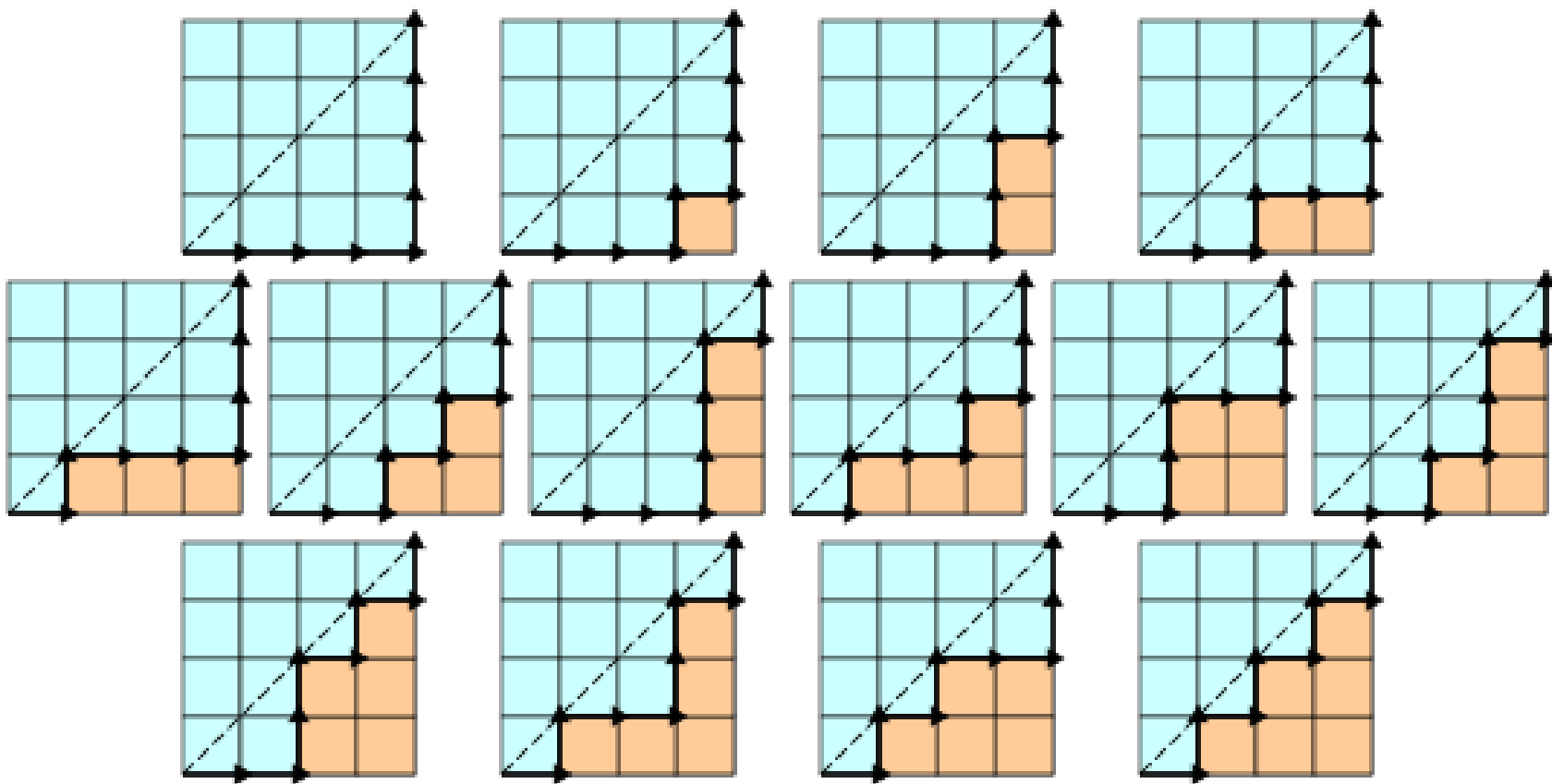


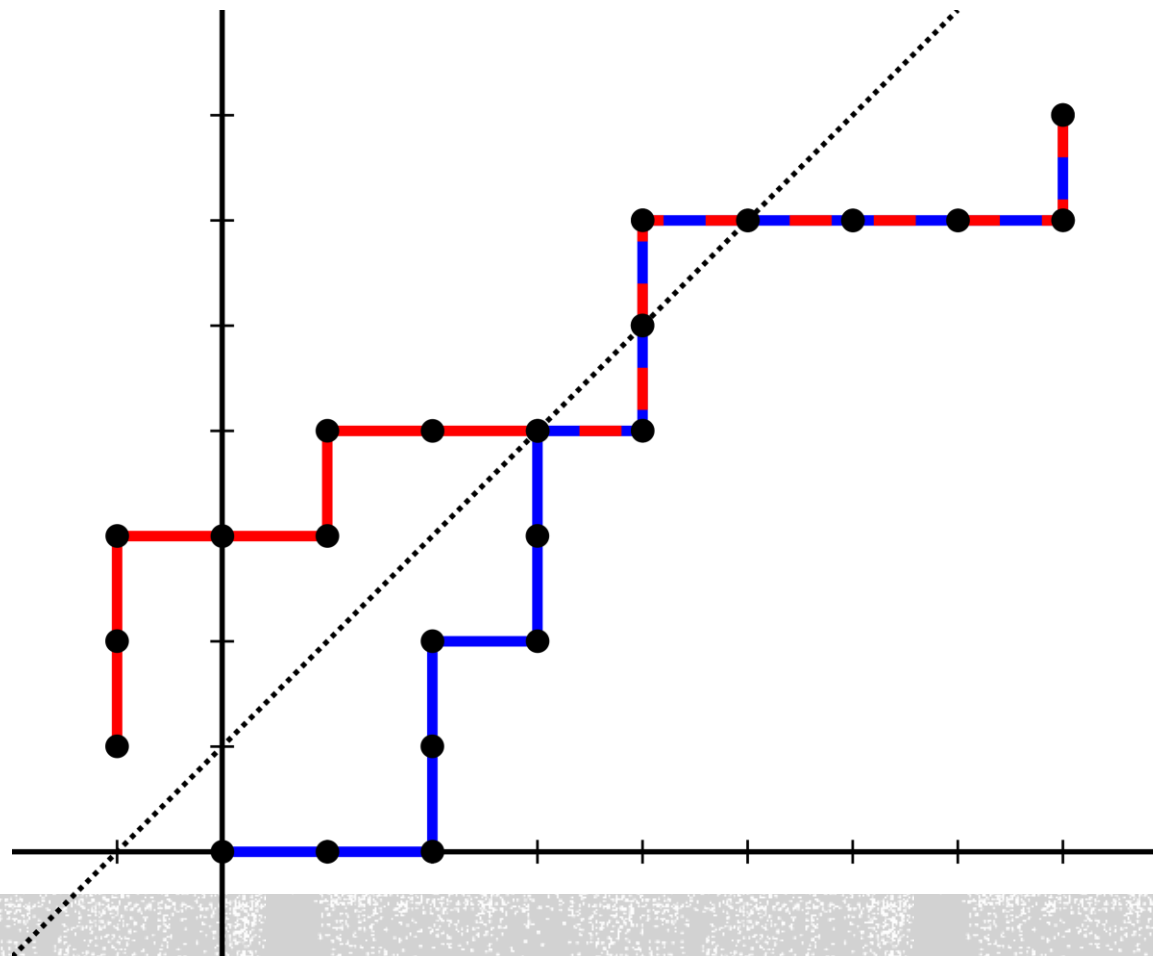
CATALAN NUMBERS



HOW MANY WAYS TO ORDER x 0s AND y 1s SUCH THAT
EVERY PREFIX HAS AT LEAST AS MANY 1s AS 0s?

BERTRAND'S
BALLOT

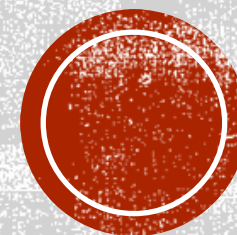




BERTRAND'S BALLOT

[Bertrand 1887]

$\binom{x+y}{x} - \binom{x+y}{x+1} = \# \text{ways to order } x \text{ 0's and } y \text{ 1's such that}$
any prefix has at least as many 1s as 0s

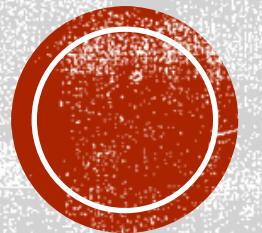


$$C_n = \frac{1}{n+1} \binom{2n}{n} = \frac{(2n)!}{(n+1)! n!} = \prod_{k=2}^n \frac{n+k}{k}$$

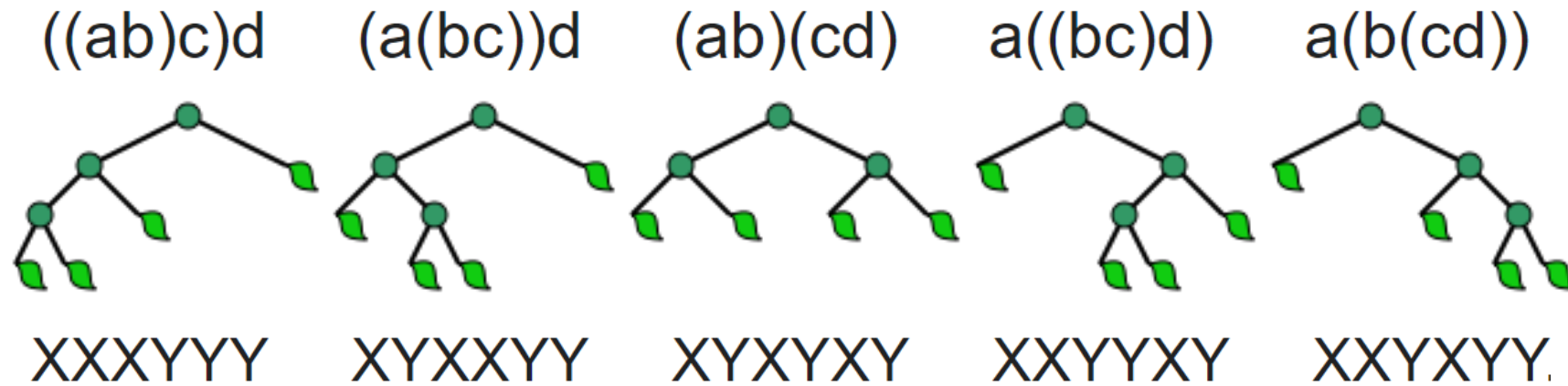
1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, ...

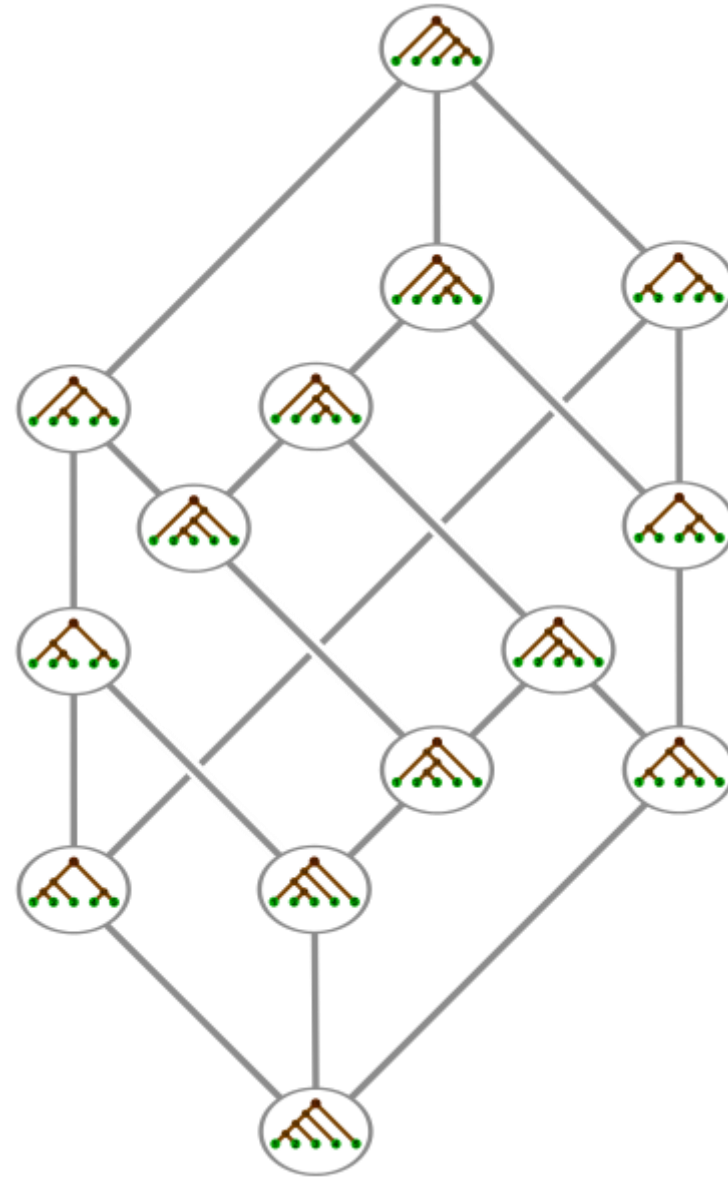
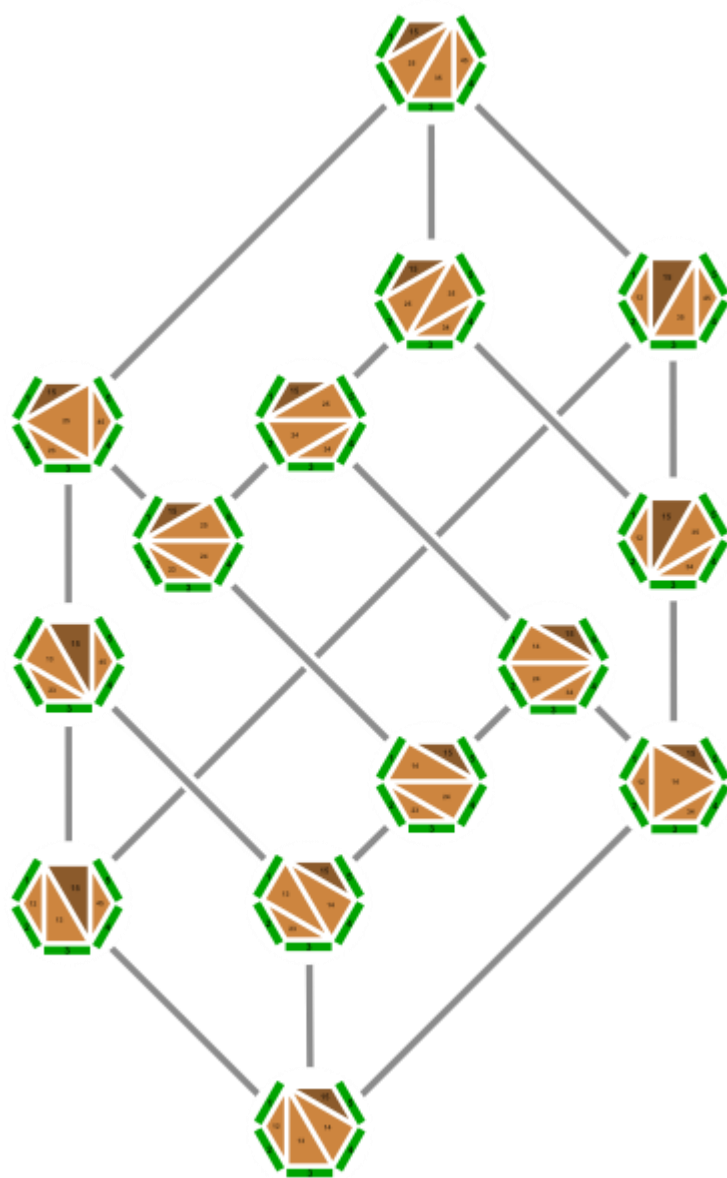
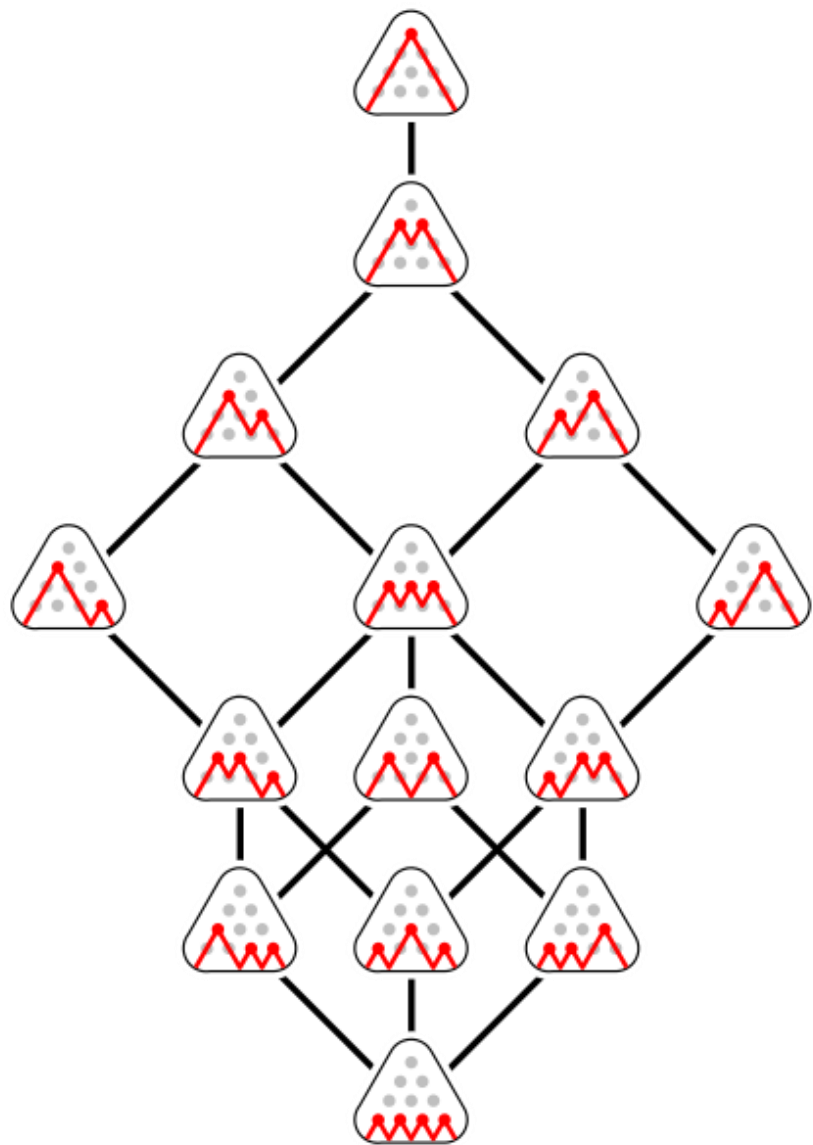
CATALAN NUMBERS [Netto 1901, Catalan 1838, Segner 1759]

#ways to order n 0's and n 1's such that
any prefix has at least as many 1s as 0s



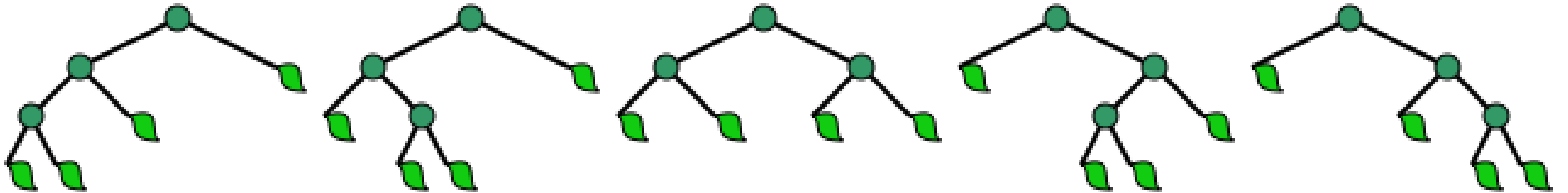
MODELS FOR CATALAN NUMBER





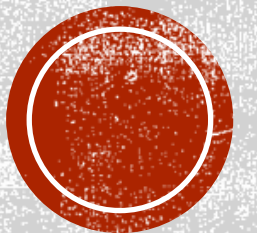
MODELS FOR CATALAN NUMBER



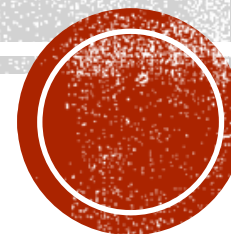


CATALAN RECURRENCE

$$C_n = \sum_k C_{k-1} \cdot C_{n-k}, \quad C_0 = 1$$



How much is $\frac{1}{n+1} \binom{2n}{n}$?



GENERATINGFUNCTIONALOGY.

NEXT TIME.
SUM AND ESTIMATIONS

