# Discrete Mathematics

## in Computer Science

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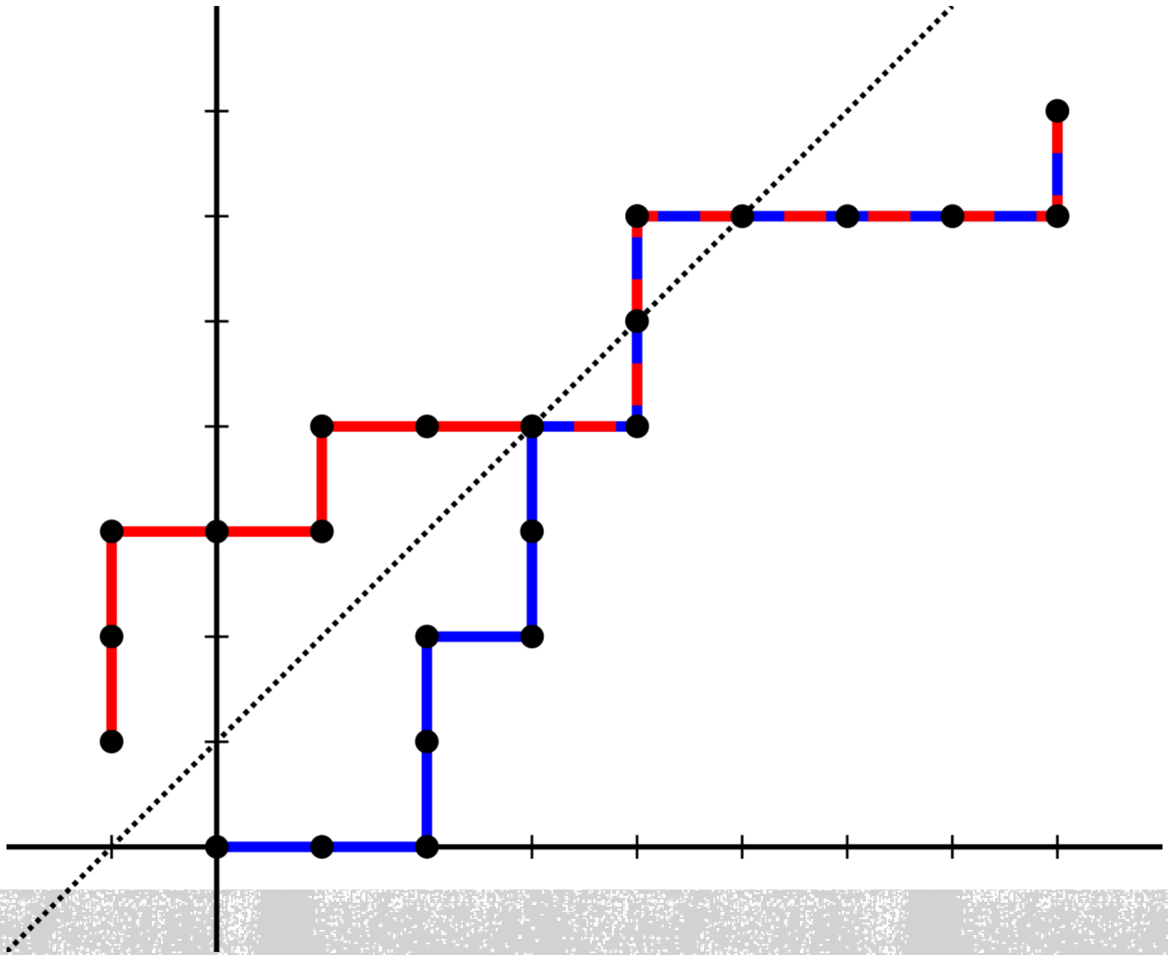
# Administrivia

- Midterm 2 grading in progress
- Homework 7 due Feb 27 (Sun)

- 5 lectures left!  Time flies.



# Bertrand's Ballot [Bertrand 1887]

$$\binom{x+y}{x} - \binom{x+y}{x+1} = \text{\#ways to order x 0's and y 1's such that any prefix has at least as many 1s as 0s}$$

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \frac{(2n)!}{(n+1)!\,n!} = \prod_{k=2}^{n} \frac{n+k}{k}$$

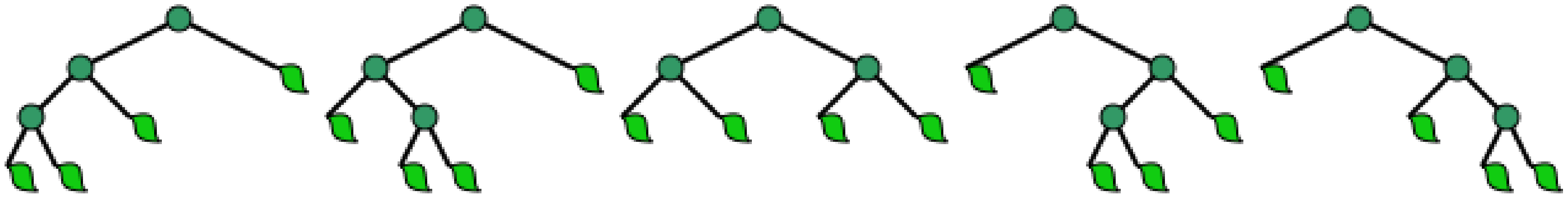
1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, ...

# Catalan Numbers [Netto 1901, Catalan 1838, Segner 1759]

#ways to order n 0's and n 1's such that any prefix has at least as many 1s as 0s

# Models for Catalan number

((ab)c)d (a(bc))d (ab)(cd) a((bc)d) a(b(cd))



XXXYYY XYXXYY XYXYXY XXYYXY XXYXYY

# How much is $\frac{1}{n+1}\binom{2n}{n}$?

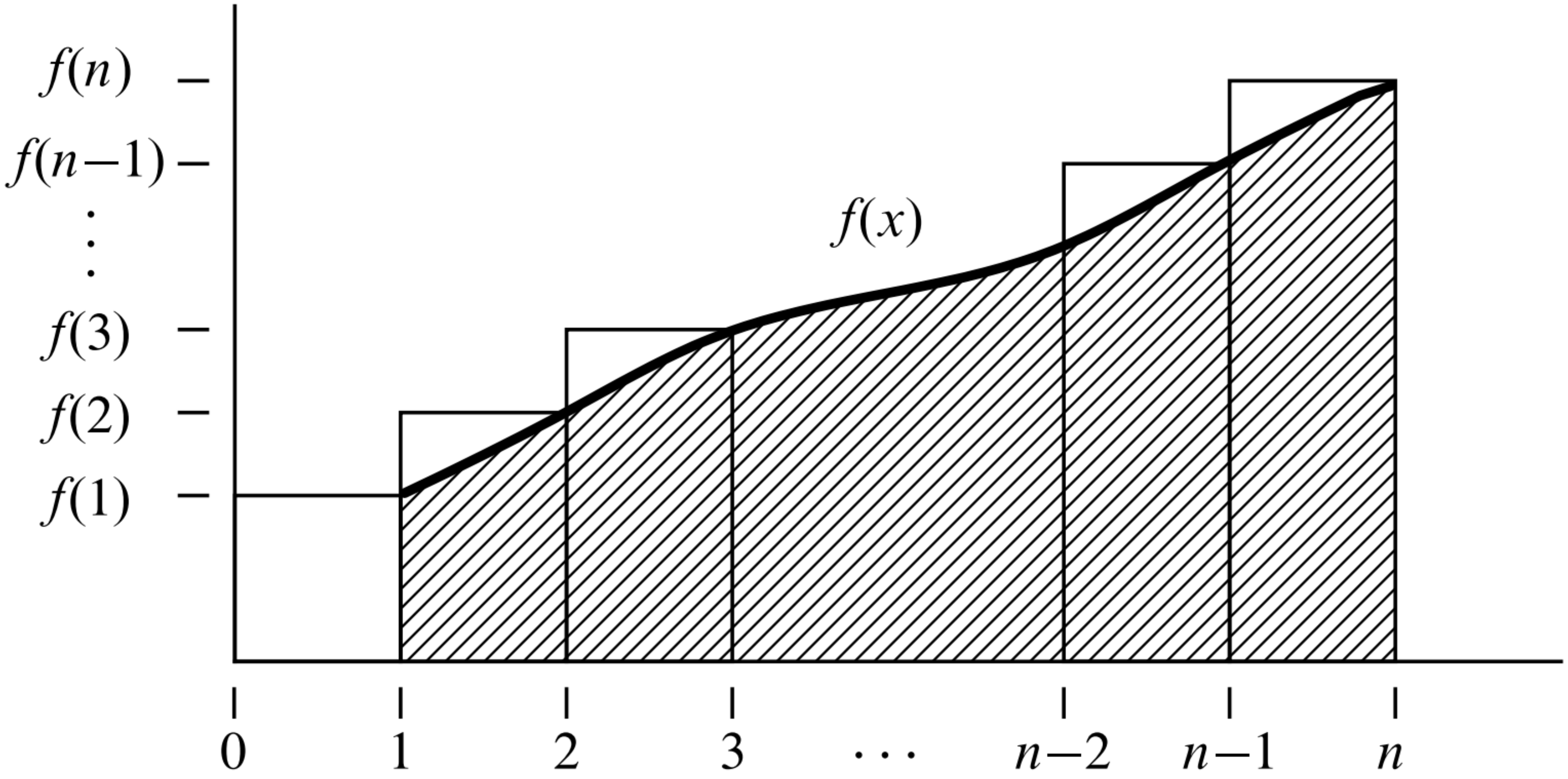
# Sums and Estimates

$$\sum_{n=a}^{b} f(n) \sim \int_a^b f(x)\,dx + \frac{f(b)+f(a)}{2}$$

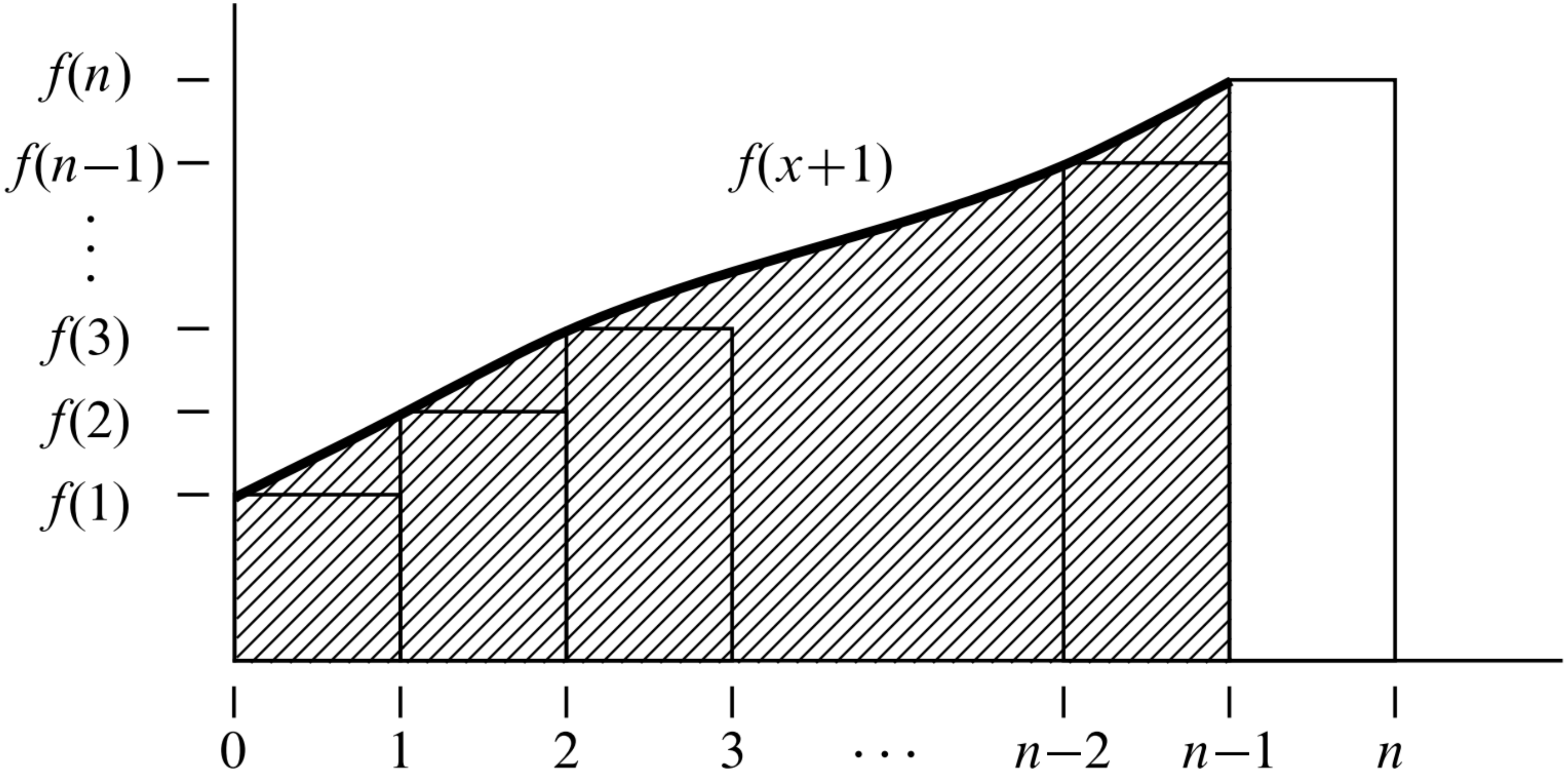
$$+ \sum_{k=1}^{\infty} \frac{B_{2k}}{(2k)!}\left(f^{(2k-1)}(b) - f^{(2k-1)}(a)\right)$$

# Euler-Maclaurin Formula [Euler 1735, Maclaurin 1736]

Approximating sum by integral




# Riemann Sum




# Riemann Sum

# Example

$$\sum_{i=1}^{n} i$$

$$\sum_{i=1}^{n} i\,x^{i}$$

# Example

$$\sum_{i=1}^{n} \frac{1}{i}$$

# Example

# Example

$$\prod_{i=1}^{n} i$$

$$\binom{n}{k}$$

# Example

$$\frac{1}{n+1}\binom{2n}{n}$$

# Example

# It's all lazy Calculus.

## Next time.

Asymptotics, solving recurrence