- The homework is due on May 23, 23:59pm. Please submit your solutions to Gradescope.
- Starting from Homework 1, all homework sets allow *group submissions* up to 2 people. Please write down the names of the members *very clearly* on the first page of your solutions.
- Answer the questions in a way that is clear, correct, convincing, and concise. The level of details to aim for is that your peers in this class should be convinced by your solutions.
- You can use any statements proved during the working sessions/lectures without proofs in your solutions.
- You might notice the difficulty of the homework problems are much higher than the worksheets. *This is by design.* These problems are meant to stretch your ability and solidify your understanding of the core concepts.
- You are expected to spend a reasonable amount of time (measured in hours) working on these problems. Remember you are allowed to utilize any resources. Make sure to cite all the people/webpages/source of infomation that helped.
- Some problems are marked with a *star*; these are more challenging (and fun) extra credit problems. They are optional and do not count toward raw grades.

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1. ***No fast automata checker.*** Consider the following problem about NFAs.

   NFA-REJECT
   - ***Input:*** *An n-state NFA N.*
   - ***Output:*** *Does NFA N reject at least one string? In other words, is $L(N) \neq \Sigma^*$?*

   Prove that the NFA-REJECT problem is NP-hard.

   [We have demonstrated in class that analyzing the behavior of a program in generally undecidable; here we see that even when the program has no access to memory we cannot expect to have efficient algorithms!]

2. ***Unfinished coursework.*** A ***dominating set*** in graph G is set of vertices S so that every vertex is in S or a neighbor of S.

   DOMSET
   - ***Input:*** *An undirected graph G, a parameter q*
   - ***Output:*** *Does G have a dominating set of size at most q?*

   We can also separate the parameter q out from the definition of DOMSET:

   q-DOMSET
   - ***Input:*** *An undirected graph G*
   - ***Output:*** *Does G have a dominating set of size at most q?*

   Obviously for each *fixed* q, q-DOMSET can be solved in $O(n^{q+1})$ time. (Why?)

(a) Prove that DOMSET does not have sub-exponential time algorithm under ETH.

(b) Prove that q-DOMSET cannot be solved in $O(n^{q-\delta})$ time for any $\delta > 0$ under SETH, by reducing an arbitrary n-variable sparse k-SAT instance (where number of clauses is $O(n)$) to a q-DOMSET instance with $O(q2^{n/q} + m)$ nodes.

★3. Many puzzles, board games, and video games are NP-hard; the list are so long that I cannot list it here fully. (See this course webpage of Erik Demaine.)

Prove one of your favorite games is NP-hard. (To get full credit, you need to choose a game that has not been proven to the NP-hard before.)