



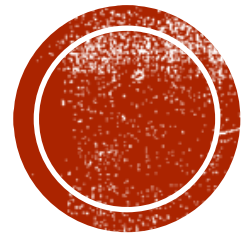
DISCRETE MATHEMATICS IN COMPUTER SCIENCE

**HSIEN-CHIH CHANG
FEBRUARY 11, 2022**

ADMINISTRIVIA

- **Midterm 2**
 - Feb 21 (Mon) 6–9PM
 - Carpenter 013 Herb West Lecture Hall
- **Conflict Midterm 2**
 - To be updated
- **SAS/Conflict Conflict/COVID**
 - Come talk to me
- **Closed-book written exam**
- **Scope: Module G on graphs**
- **One-page two-sided cheatsheet**
 - Must be hand-written





RELATION, PARTIAL ORDER, AND EQUIVALENCE



Jargon

relation

transitive

reflexive

symmetric

antisymmetric

relation graph

bipartite graph

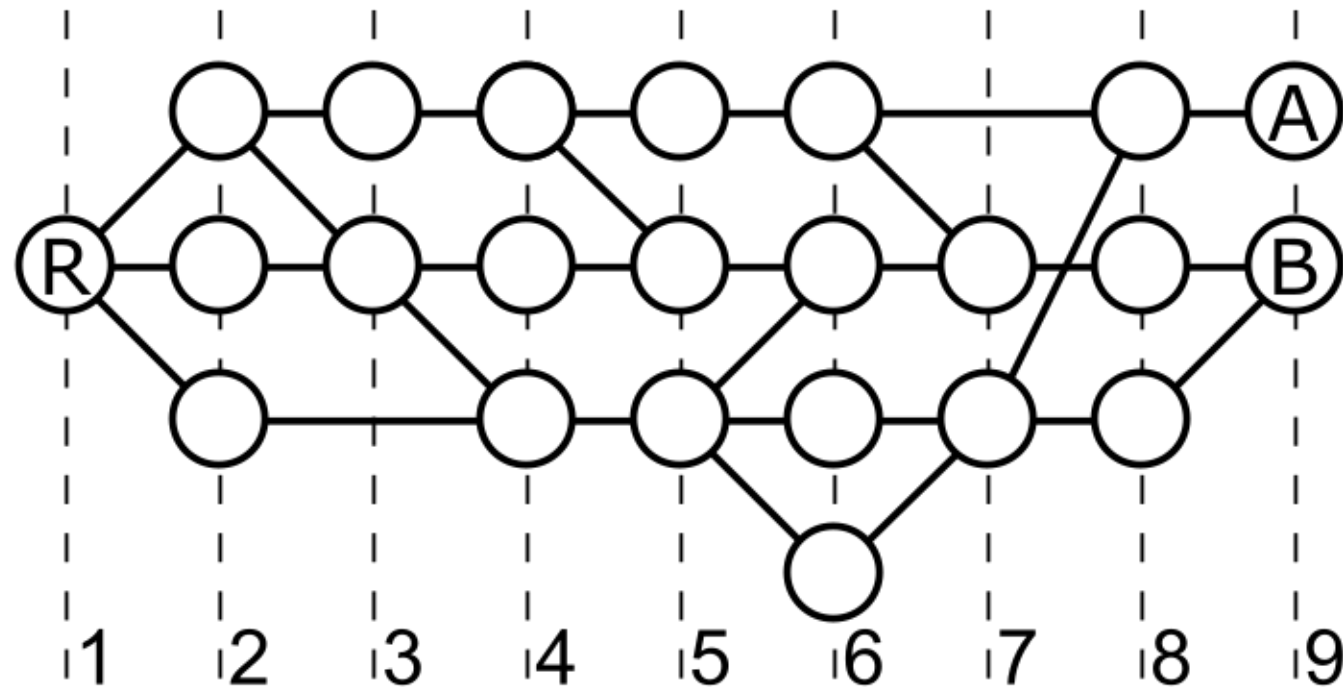


PARTIAL ORDER

- A **partial order** on set S is a relation from S to S that is
 - reflexive
 - transitive
 - antisymmetric



“WALK” RELATION IN A DAG IS A PARTIAL ORDER.



EXAMPLE

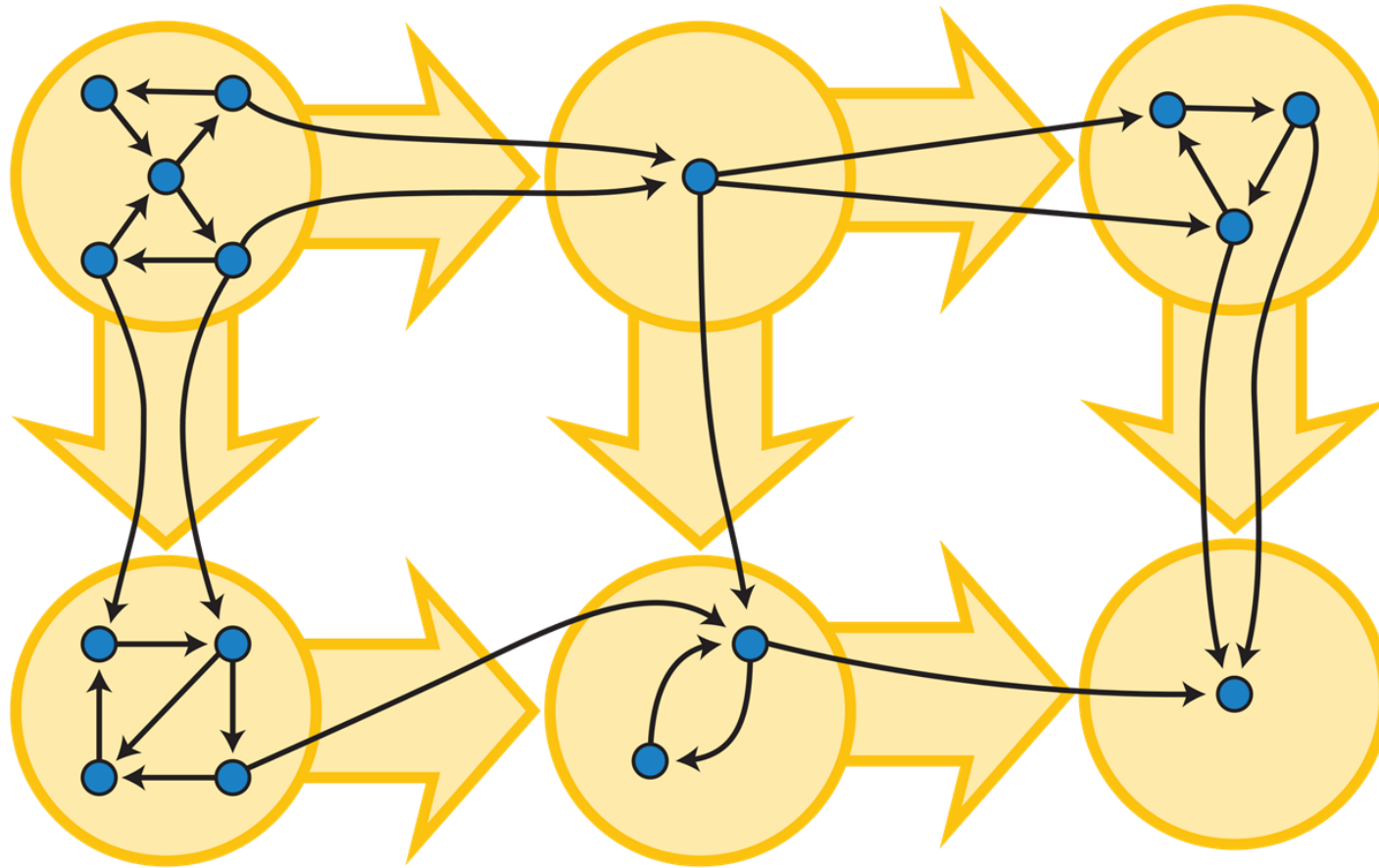


EQUIVALENCE

- An **equivalence** on set S is a relation from S to S that is
 - reflexive
 - transitive
 - symmetric



“STRONGLY-CONNECTED” RELATION IN A DIRECTED GRAPH IS AN EQUIVALENCE.



EXAMPLE



COMPARISON

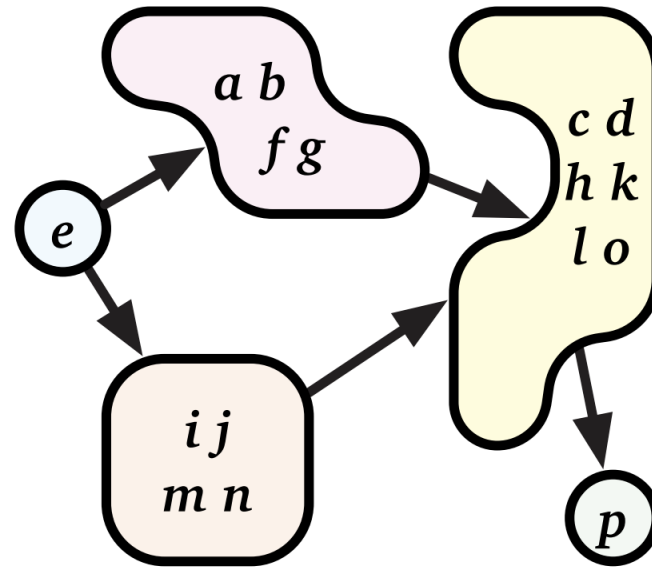
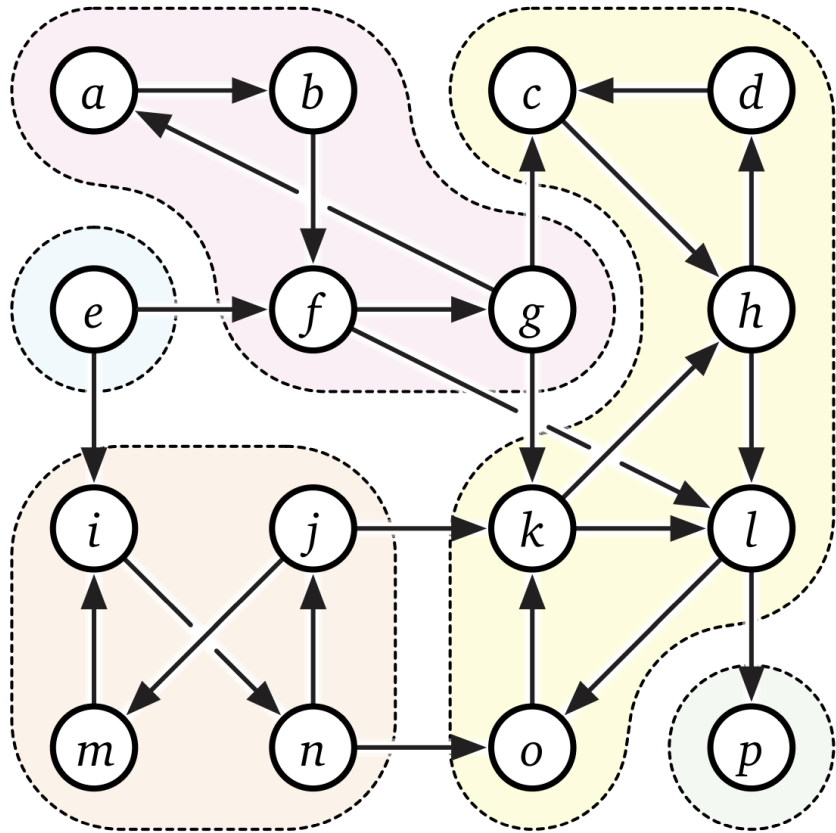
■ Partial order

- reflexive
- antisymmetric
- transitive

■ Equivalence

- reflexive
- symmetric
- transitive

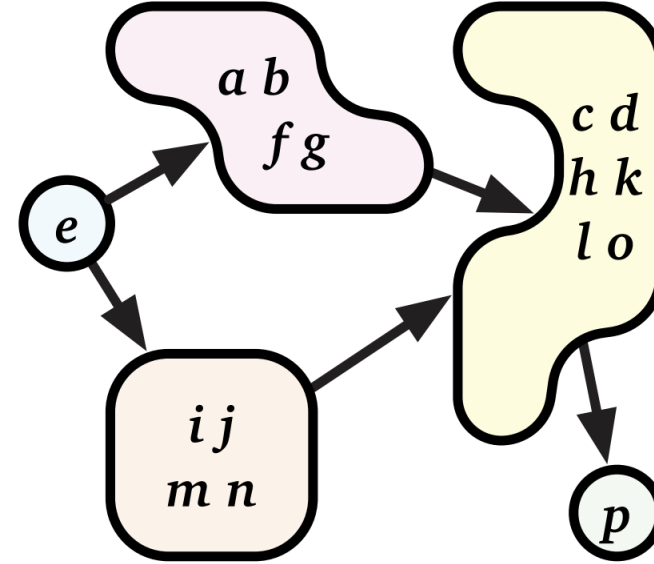
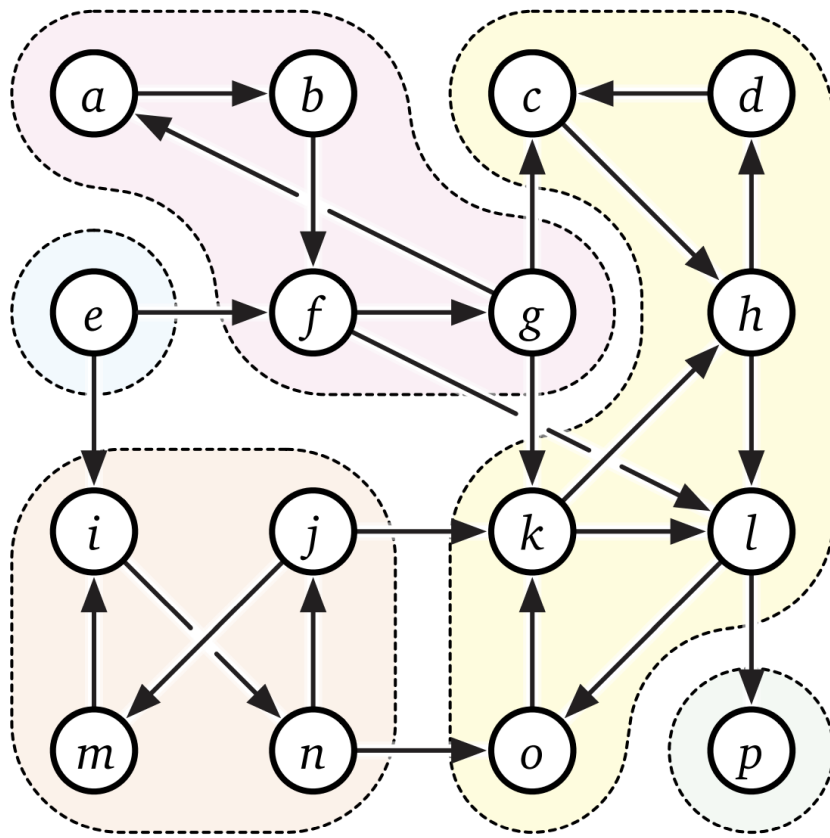




STRONG COMPONENT GRAPH

meta-graph
condensation

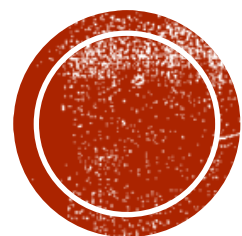




STRONG COMPONENT DECOMPOSITION.

The strong component graph of any directed graph must be a dag.

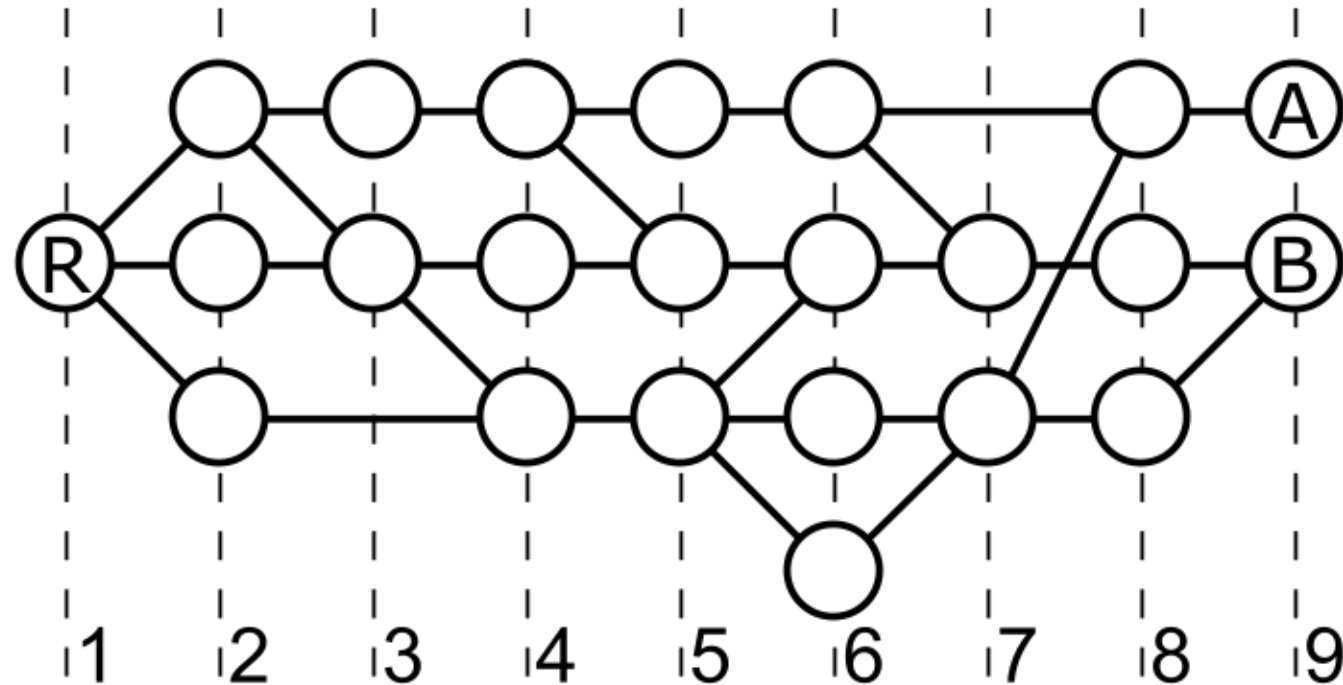




DILWORTH'S THEOREM



IF DAG D HAS WIDTH t ,
THEN D CAN BE COVERED BY t CHAINS.



DILWORTH'S THEOREM



COMPARISON

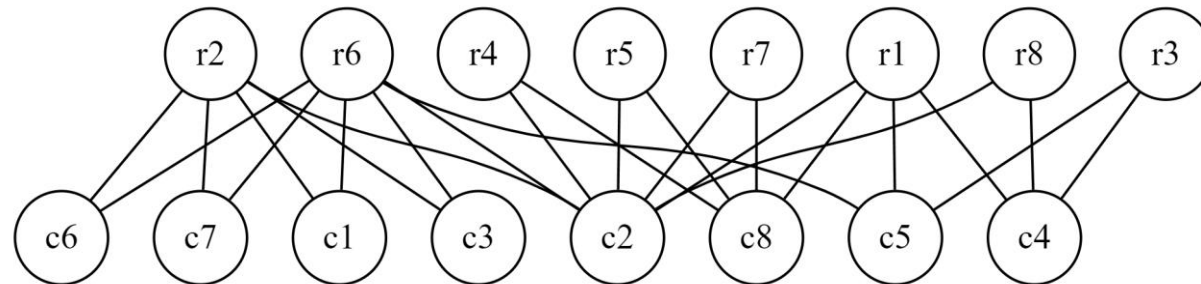
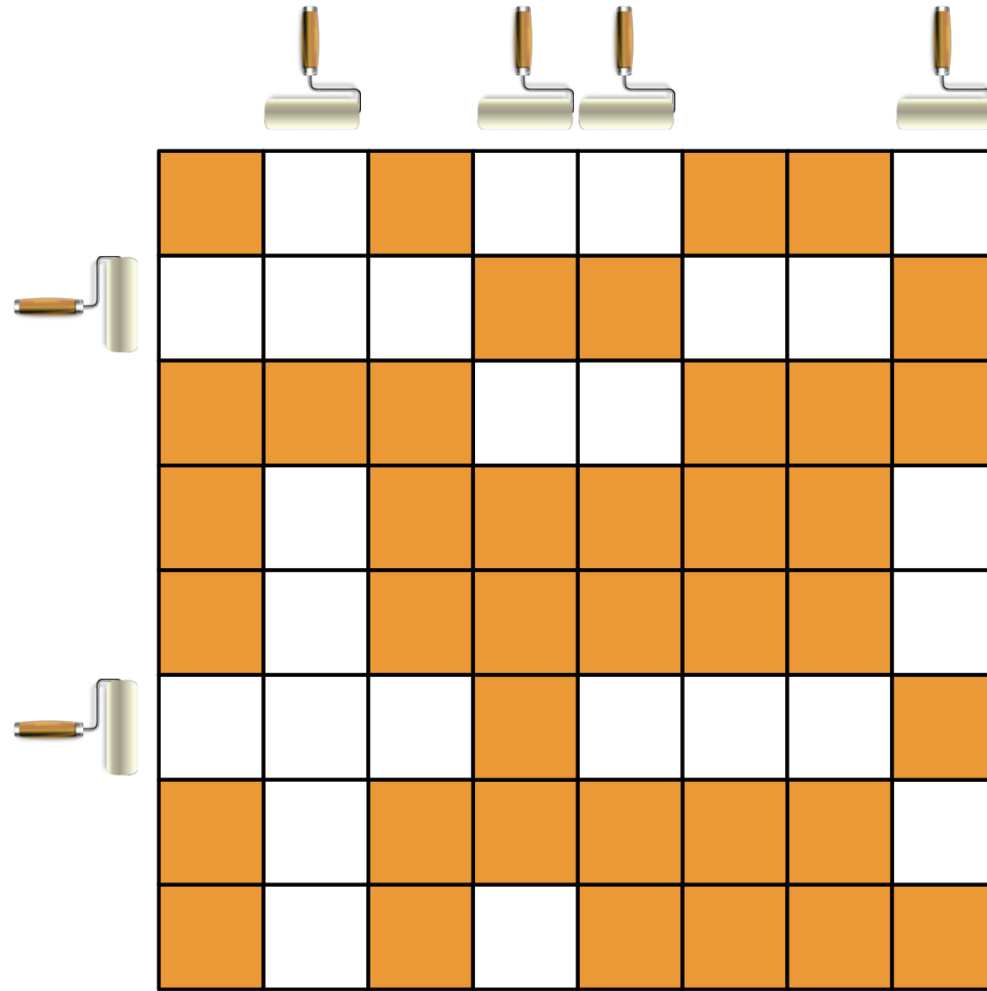
■ Mirsky's Theorem

If dag D has **height** t , then
 D decomposes into t **antichains**.

■ Dilworth's Theorem

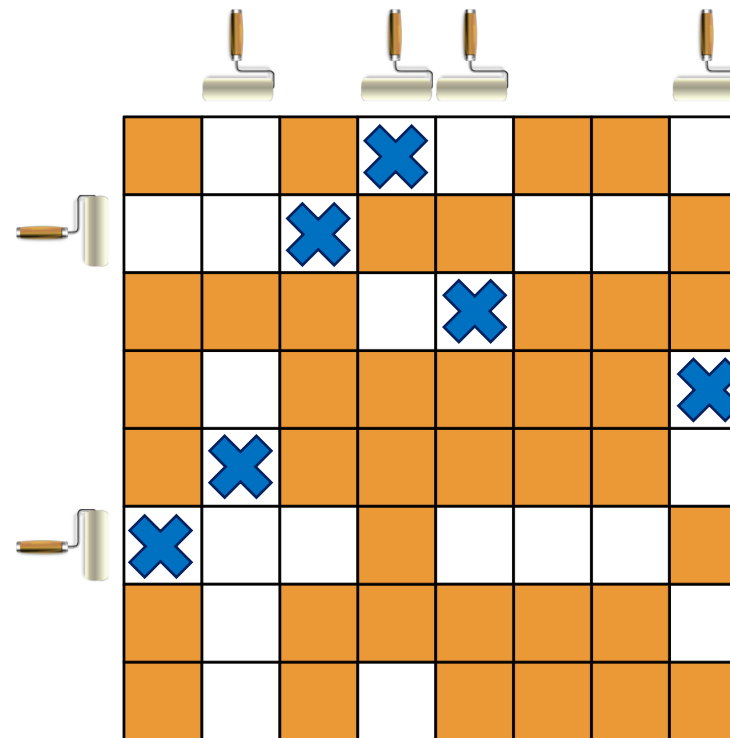
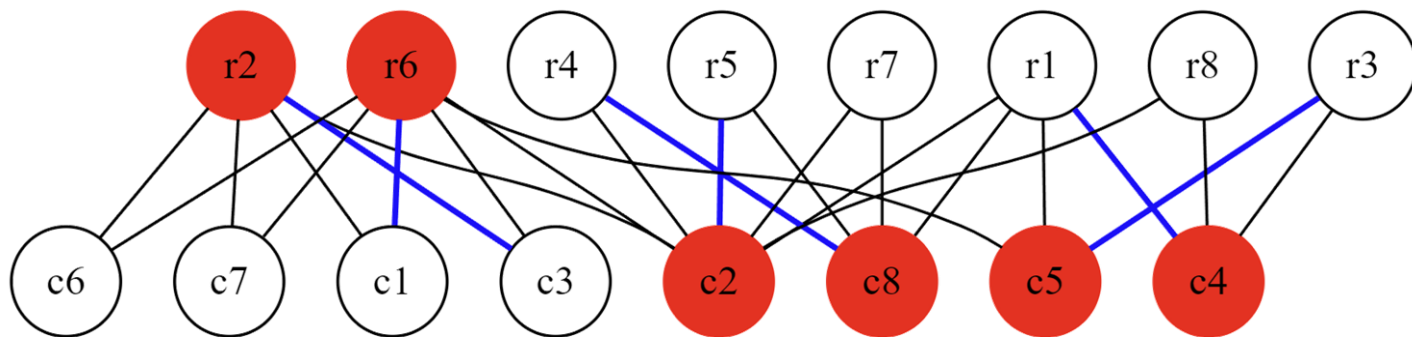
If dag D has **width** t , then
 D can be covered by t **chains**.





VERTEX COVER PROBLEM





KÖNIG-EGERVÁRY THEOREM

IN ANY BIPARTITE GRAPH,
MAXIMUM SIZE OF A MATCHING =
MINIMUM SIZE OF A VERTEX COVER.



ALWAYS LOOK FOR THE UNDERLYING STRUCTURES

**NEXT TIME.
NEW MODULE: COUNTING.**

