- Find students around you to form a ***small group***.
- Work on the exercise problems in sequence. You are allowed to use ***all resources*** (including textbook and search engines) to help you solve the problems.
- After solving each problem, ***discuss*** your solution with other group members and exchange your ideas. You should try to poke holes in each other's argument and defend your own; this is a good way to learn about how to communicate.
- In daily life we are used to *not argue* with people and overlook the gaps in the statements. Well, not for this course. Being critical and patient when listening to others, ask questions to see if you understand them correctly, and provide well-thoughtout counter-arguments when you disagree. Be civilized and argue about ideas but don't make it personal.
- As you are trying to solve the problems you might have questions. Ask the questions within the group and see if others know the answer. If no ones knows, or the whole group is stuck and don't know how to proceed, Raise your hand and pull one of the ***course staff*** to help.
- After reaching concensus, ***write down*** the solutions *in your own words*. You are not allowed to copy sentences from others or TAs; everything you deliver must be from *you*. Use the writing tips provided on Canvas to help with your presentation.
- ***Submit*** your writeup with the proofread signature through Gradescope. You have *24 hours* to do so. The TA will grade them and provide feedback.

Our topic for this working session is the *NFA construction*.

NFAs allow *ε transitions* (multi-touch) and *multiple starting and accepting states*. In the end, as long as one finger is at an accepting accept, the NFA accepts the input string.

The best way to interpret nondeterminism (aka multiple fingers) is that the automata now has the ability to *guess*. In other words, given a positive input string, only one guess (or more) has to be correct; given a negative input string, no guesses should be correct. We are going to learn about how to design NFAs with the new gained power of guessing.

Construct nondeterministic finite automata (NFAs) for each of the following languages. No formal proofs required; explain your construction in English.

1. Given an arbitrary NFA M, design another NFA $\hat{M}$ that recognizes the same language, but with a single accepting state.

2. Construct an NFA to recognize the language generated by the expression $\texttt{1}^*(\texttt{00} + \texttt{011}^*)^*$.

3. For a string $w \in \{\texttt{0}, \texttt{1}\}^*$, $\text{rev}(w)$ is w written backwards (or ε if $w = \varepsilon$). For a language L recognized by some DFA D, define $\text{rev}(L) := \{\text{rev}(x) : x \in L\}$. Design an NFA to recognize $\text{rev}(L)$.

4. In many programming languages, there is a notion of a "comment". Given a finite alphabet Σ, we can extend it by adding the symbols `/` and `*`. A comment is then a string that starts with the characters `/*` and ends with the first appearance `*/` *after* the initial `/*`. Anything can be in between *except* the ending sequence `*/` itself. For example, `/*//*/` is a valid comment, while `/*/` is **not** a valid comment.

   Design an NFA to accept all valid comments.

To think about later: (No submissions needed)

5. Construct an NFA to recognize the language

$$L_5 := \left\{ w \in \{\texttt{0}, \texttt{1}\}^* : w \text{ contain } \texttt{1111000110} \text{ as a substring} \right\}.$$

   [Hint: Don't use KMP pattern matching! There is a simpler way with NFAs.]

6. Construct an NFA to recognize the language

$$L_6 := \left\{ w \in \{\texttt{0}, \texttt{1}\}^* : \begin{array}{l} w \text{ has length at least 3, does not contain } \texttt{110} \text{ as substring,} \\ \text{the third symbol being } \texttt{0}, \text{ and the number of } \texttt{1} \text{ modulo 5 is 0} \end{array} \right\}.$$

   [Hint: Sure you can do product constructions repeatedly. But can you make a smaller NFA?]

Conceptual question: Recall the complement of an automatic language still automatic. Can you repeat this argument for languages recognized by NFAs? Are lanuages recognized by NFAs closed under complement?