1. ***Introduce yourself.*** Why do you take the course? What is your personal goal and expectations? What parts of topology might be potentially useful to you? More broadly, introduce yourself a bit and how much time and energy do you plan to invest this term.

2. ***Torus games.*** The *Torus Games*, developed by Jeff Weeks, is a collection of common board games (including Tic-tac-toe and Gomoku), played on a toroidal or Klein-bottle board.

   Play some Torus Games. Tell me about your findings! *Bonus question*: Find and play some other topology games and post it on the discussion forum. Yes, Pac-Man and Asteroids count, but ideally something lesser-known.

3. ***Fun with groups.*** A *group* $(\Sigma, \bullet)$ is a set Σ equipped with a binary relation $\bullet : \Sigma \times \Sigma \to \Sigma$, satisfying the following properties:

   - *Associativity*: for any three elements x, y, and z in Σ, one has $(x \bullet y) \bullet z = x \bullet (y \bullet z)$;
   - *Identity*: there is an element e in Σ such that $e \bullet x = x \bullet e = x$ for any x in Σ;
   - *Inverses*: for any x in Σ, there is an element x^{-1} in Σ such that $x^{-1} \bullet x = x \bullet x^{-1} = e$.

   A *group homomorphism* ϕ from group $(\Sigma, \bullet)$ to group $(\Pi, \circ)$ is a map $\phi : \Sigma \to \Pi$ respecting the group operations; that is, for every pair of elements x and y in Σ,

   $$\phi(x \bullet y) = \phi(x) \circ \phi(y).$$

   The *kernel* of a group homomorphism, denoted as $\ker \phi$, is the subset of elements in Σ that maps by ϕ to the identity e_Π of Π; in notation,

   $$\ker \phi := \{x \in \Sigma \colon \phi(x) = e_\Pi\}.$$

   Let G be arbitrary graph. There are two natural objects associated with G:

   - The *vertex space* Σ_V, containing all possible subsets of vertices of G;
   - The *edge space* Σ_E, containing all subgraphs of G.

   Both the vertex space and the edge space can be made into a group under element-wise symmetric difference operation (verify this yourself!).

   (a) Prove that the map from Σ_E to Σ_V, taking a subgraph H of G and returns the subset of *odd-degree* vertices in H (that is, vertices that incident to an odd number of edges), is a group homomorphism. What is the kernel of this map?

   *(b) As a vector space over the two-element field $\mathbb{Z}_2$, what is the dimension of $\ker \phi$?

   (Groups will appear several times in this class, so it would be helpful to equip yourself with basic knowledge about groups. There is no need to go through the whole text on groups in an abstract algebra book; sufficient knowledge on group homomorphisms and quotient groups will be enough. Remember, we only need to pick up *just enough* tools to build our projects.)

4. ***Coin game.*** Let G be a connected undirected graph. Suppose we start with three coins on three arbitrarily chosen vertices of G, and we want to move the coins so that they lie on the same vertex using as few moves as possible. At every step, each coin *must* move to an adjacent vertex.

   Describe and analyze an algorithm to compute the minimum number of steps to reach a configuration where all three coins are on the same vertex, or to report correctly that no such configuration is reachable. The input to your algorithm consists of a graph G and three vertices u_1, u_2, u_3 in G (which may or may not be distinct).

*5. ***Spaghetti search.*** Consider the following search algorithm SPAGHETTI. Given a connected undirected graph G as input, we separate each edge vw of G into two arcs $v{\to}w$ and $w{\to}v$, pointing in opposite directions. We also assume that all vertices are unmarked at first, and all arcs of G has the color white.

   Prove that SPAGHETTI(G, s), performed on an arbitrary vertex s in G, terminates and visits all the vertices in G.

```
SPAGHETTI(G, v):
  mark v
  if there is a white arc v→w
      if w is unmarked
          color w→v green
      color v→w red
      SPAGHETTI(G, w)
  else if there is a green arc v→w
      color v→w red
      SPAGHETTI(G, w)
```

Figure 1. Pseudocode for Spaghetti search.