- You know the drill now: Find students around you to form a ***small group***; use ***all resources*** to help to solve the problems; ***discuss*** your idea with other group member and ***write down*** your own solutions; raise your hand and pull the ***course staffs*** to help; ***submit*** your writeup through Gradescope in *24 hours*.

Our topic for this working session is on *NP-hardness reductions via gadget constructions*.

Key takeaway when reducing from 3SAT to your favorite problem R:

- Look for structure in instance of R that give you a ***choice*** for assignments; use them to build *variable gadget*.
- Look for structure in instance of R that give you a ***interaction with multiple objects***; use them to build *clause gadget*.
- Look for ***constraints*** in the definition of R; use them to model variable assignments *(not) satisfying* clauses.

An ***induced cycle*** in an undirected graph G a subgraph $G[S]$ induced on the vertices in S, that happens to be a cycle. (Intuitively, an induced cycle is a cycle in G without *chords*.)

INDUCEDCYCLE
- ***Input:*** *An undirected graph G, two vertices u and v*
- ***Output:*** *Is there an induced cycle in G passing through u and v?*

1. Why doesn't the following construction work?

Given 3CNF formula ϕ, for each variable create a gadget:

For each clause create a gadget:

Add an edge between node f_i and node $c_{j,\ell}$ if x_i is the ℓ-th literal of clause C_j; add an edge between node t_i and node $c_{j,\ell}$ if $\overline{x_i}$ is the ℓ-th literal of clause C_j. Link the gadgets in the following way:

Now there is an induced cycle passing through u and v if and only if there are variable assignments for $\langle x_i \rangle$ that satisfy all clauses.

2. Prove that INDUCEDCYCLE is NP-hard.

To think about later: (No submissions needed)

3. Prove that the following problem is NP-hard:

INDUCEDCYCLE
- ***Input:*** *An undirected graph G, one vertex u*
- ***Output:*** *Is there an induced cycle in G of even length passing through u?*