- The homework is due on May 23, 23:59pm. Please submit your solutions to Gradescope.
- Starting from Homework 1, all homework sets allow *group submissions* up to 2 people. Please write down the names of the members *very clearly* on the first page of your solutions.
- Answer the questions in a way that is clear, correct, convincing, and concise. The level of details to aim for is that your peers in this class should be convinced by your solutions.
- You can use any statements proved during the working sessions/lectures without proofs in your solutions.
- You might notice the difficulty of the homework problems are much higher than the worksheets. *This is by design.* These problems are meant to stretch your ability and solidify your understanding of the core concepts.
- You are expected to spend a reasonable amount of time (measured in hours) working on these problems. Remember you are allowed to utilize any resources. Make sure to cite all the people/webpages/source of infomation that helped.
- Some problems are marked with a *star*; these are more challenging (and fun) extra credit problems. They are optional and do not count toward raw grades.

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1. ***No fast automata checker.*** Consider the following problem about NFAs.

   NFA-REJECT
   - ***Input:*** *An n-state NFA N.*
   - ***Output:*** *Does NFA N reject at least one string? In other words, is $L(N) \neq \Sigma^*$?*

   Prove that the NFA-REJECT problem is NP-hard.

   [We have demonstrated in class that analyzing the behavior of a program in generally undecidable; here we see that even when the program has no access to memory we cannot expect to have efficient algorithms!]

2. ***Unfinished coursework.*** A ***dominating set*** in graph G is set of vertices S so that every vertex is in S or a neighbor of S.

   DOMSET
   - ***Input:*** *An undirected graph G, a parameter q*
   - ***Output:*** *Does G have a dominating set of size at most q?*

   We can also separate the parameter q out from the definition of DOMSET:

   q-DOMSET
   - ***Input:*** *An undirected graph G*
   - ***Output:*** *Does G have a dominating set of size at most q?*

   Obviously for each *fixed* q, q-DOMSET can be solved in $O(n^{q+1})$ time. (Why?)

(a) Prove that DOMSET does not have sub-exponential time algorithm under ETH.

(b) Prove that q-DOMSET cannot be solved in $O(n^{q-\delta})$ time for any $\delta > 0$ under SETH, by reducing an arbitrary n-variable sparse k-SAT instance (where number of clauses is $O(n)$) to a q-DOMSET instance with $O(q2^{n/q} + m)$ nodes.

★3. Many puzzles, board games, and video games are NP-hard; the list are so long that I cannot even. (See this course webpage of Erik Demaine.) Most signature examples includes Baba is You (actually undecidable), Candy Crush Saga, Chess, Cookie Clicker, Doom, Go, Grand Theft Auto, Legend of Zelda, Magic the Gathering (actually undecidable), Minesweeper, Pokémon, Portal, Resident Evil, Rush Hour, Sudoku, Super Mario Bros., Tetris, and more.

Prove one of your favorite games is NP-hard. (To get full credit, you need to choose a game that has not been proven to be NP-hard before.)