



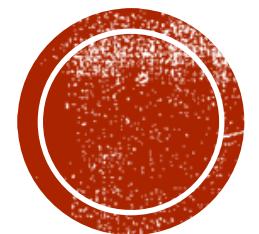
INTRODUCTION TO COMPUTATIONAL TOPOLOGY

HSIEN-CHIH CHANG
LECTURE 14, OCTOBER 28, 2021

ADMINISTRIVIA

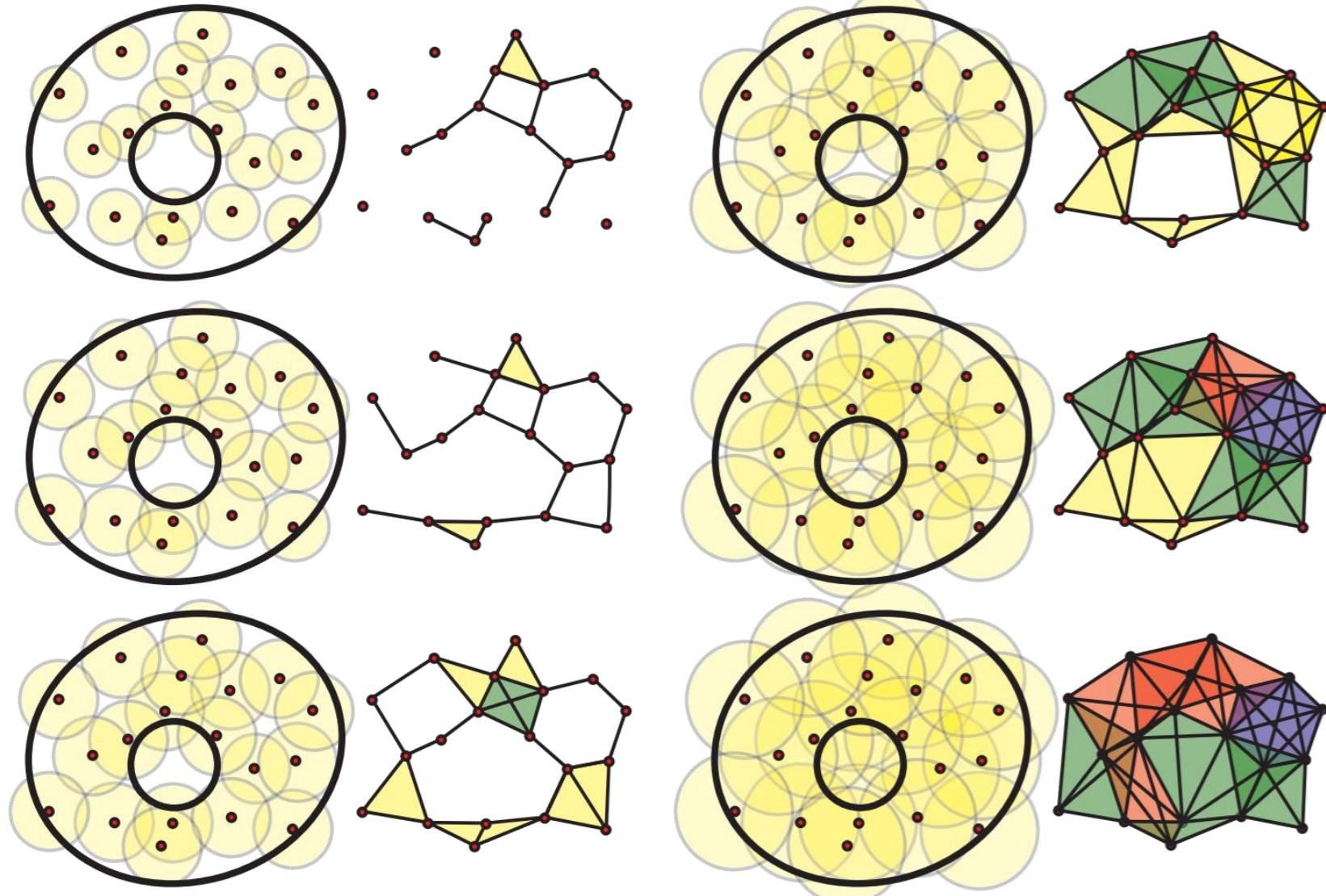
- Homework 4 is out, due 11/8 (Mon)
- Come talk to me about the project (as a group)





PERSISTENCE DIAGRAM

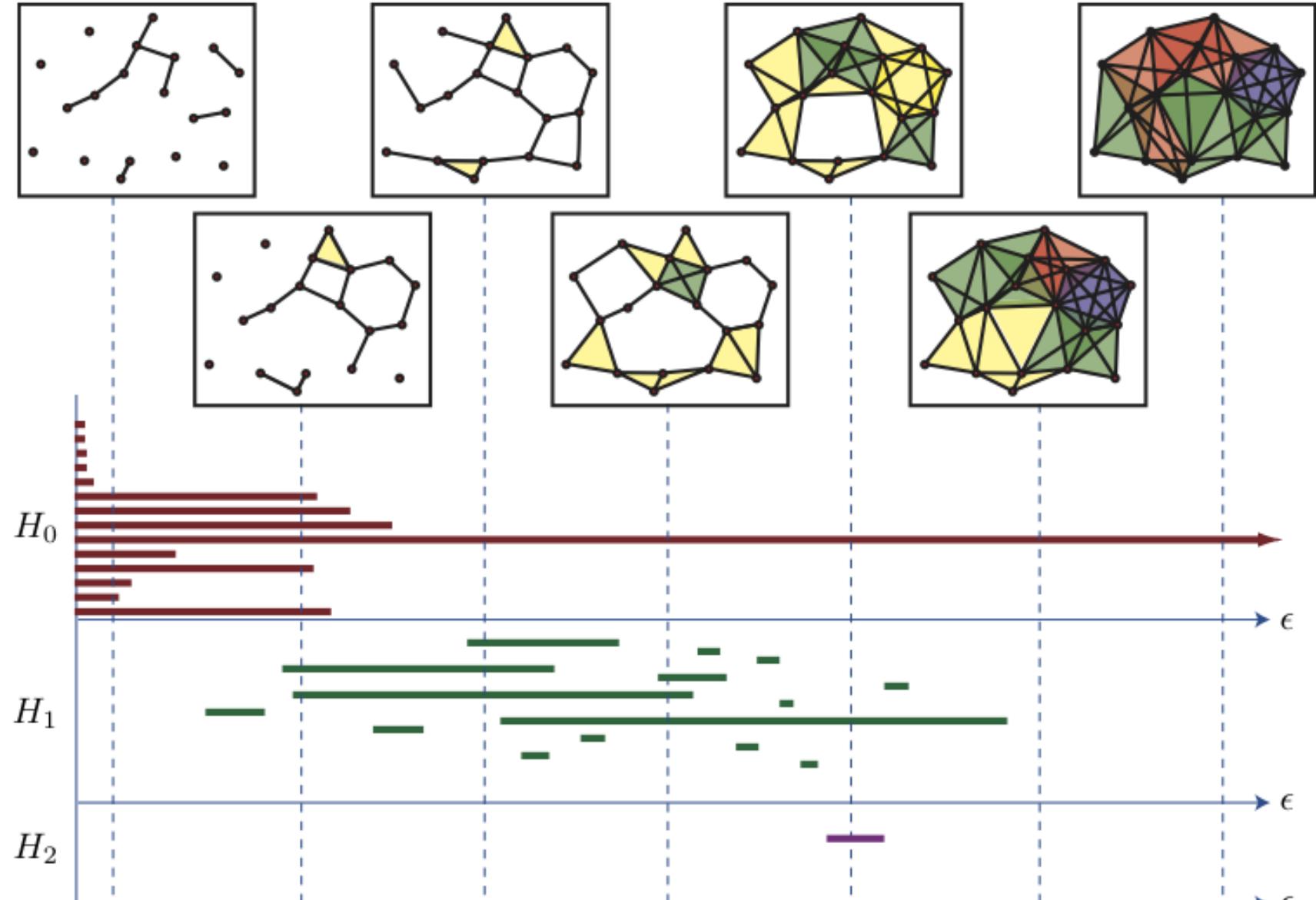




[Ghrist 2008]

COOL IDEA

- Don't guess ε ; take all!



[Ghrist 2008]

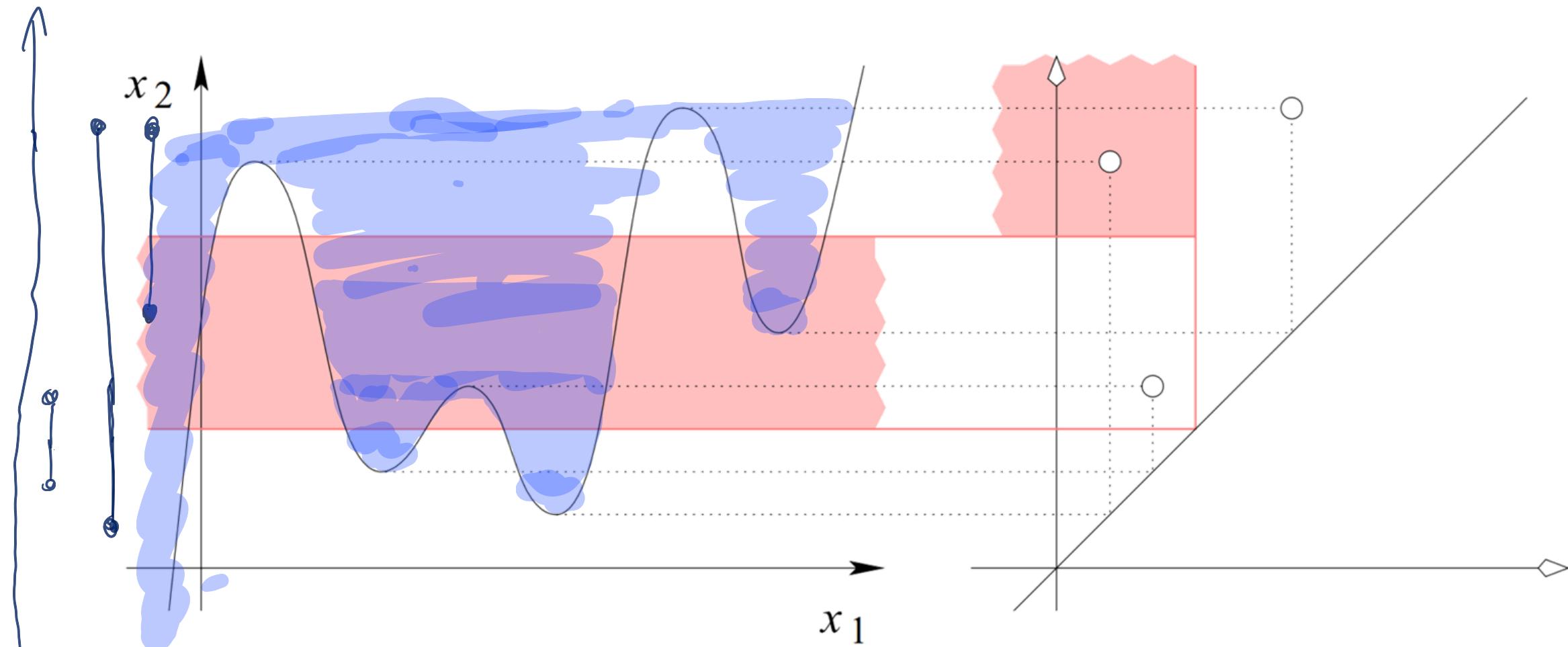
PERSISTENCE

- $\text{Pers}[\gamma] = \varepsilon_j - \varepsilon_i$



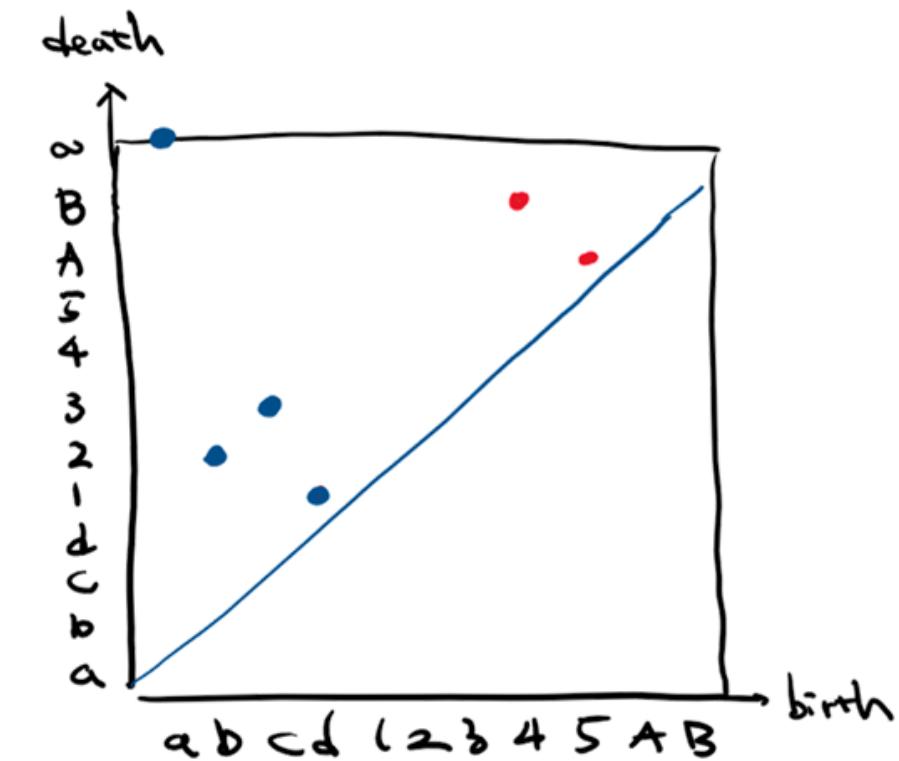
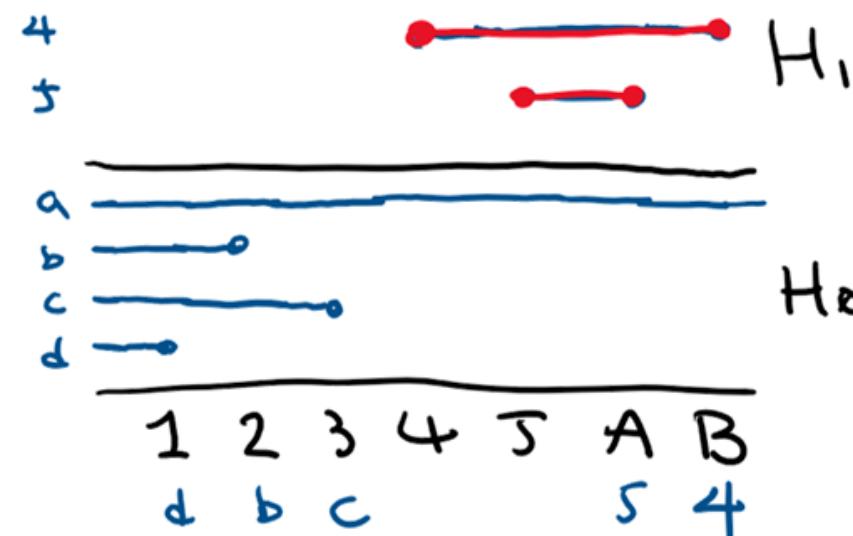
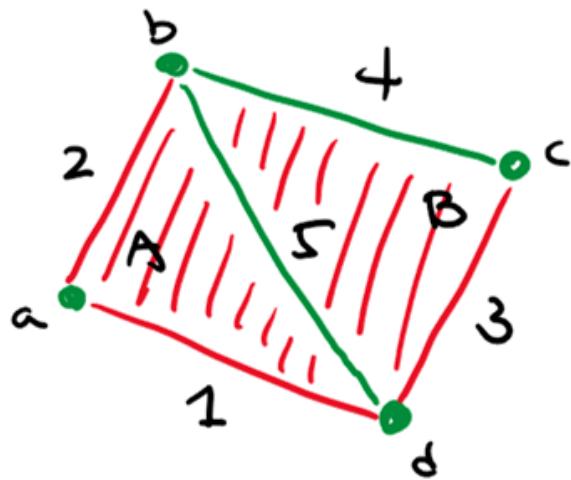
PERSISTENCE DIAGRAM

[CohenSteiner-Edelsbrunner-Harer 2005]

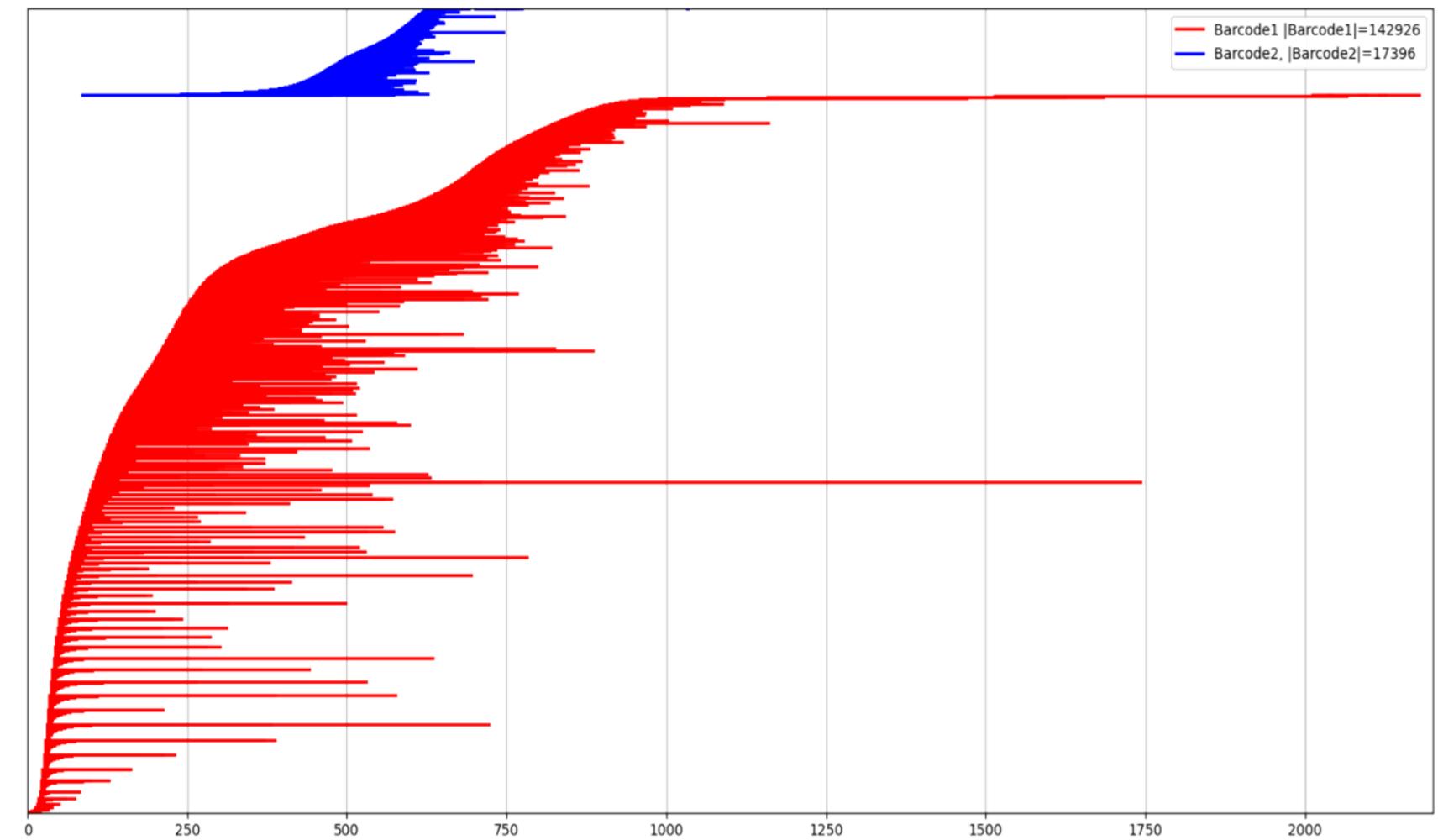


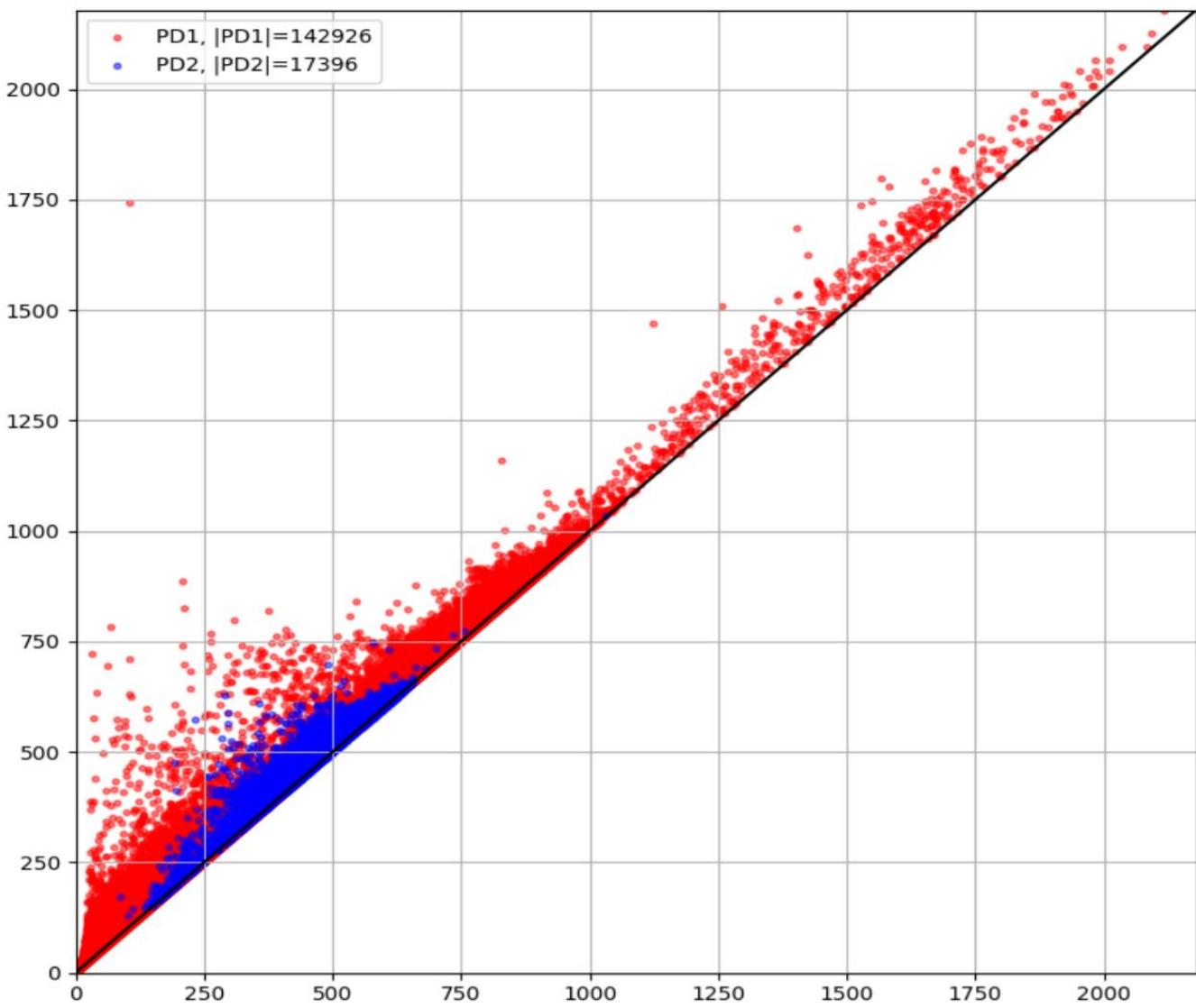
PERSISTENCE DIAGRAM

[CohenSteiner-Edelsbrunner-Harer 2005]



PERSISTENCE DIAGRAM





PERSISTENCE DIAGRAM

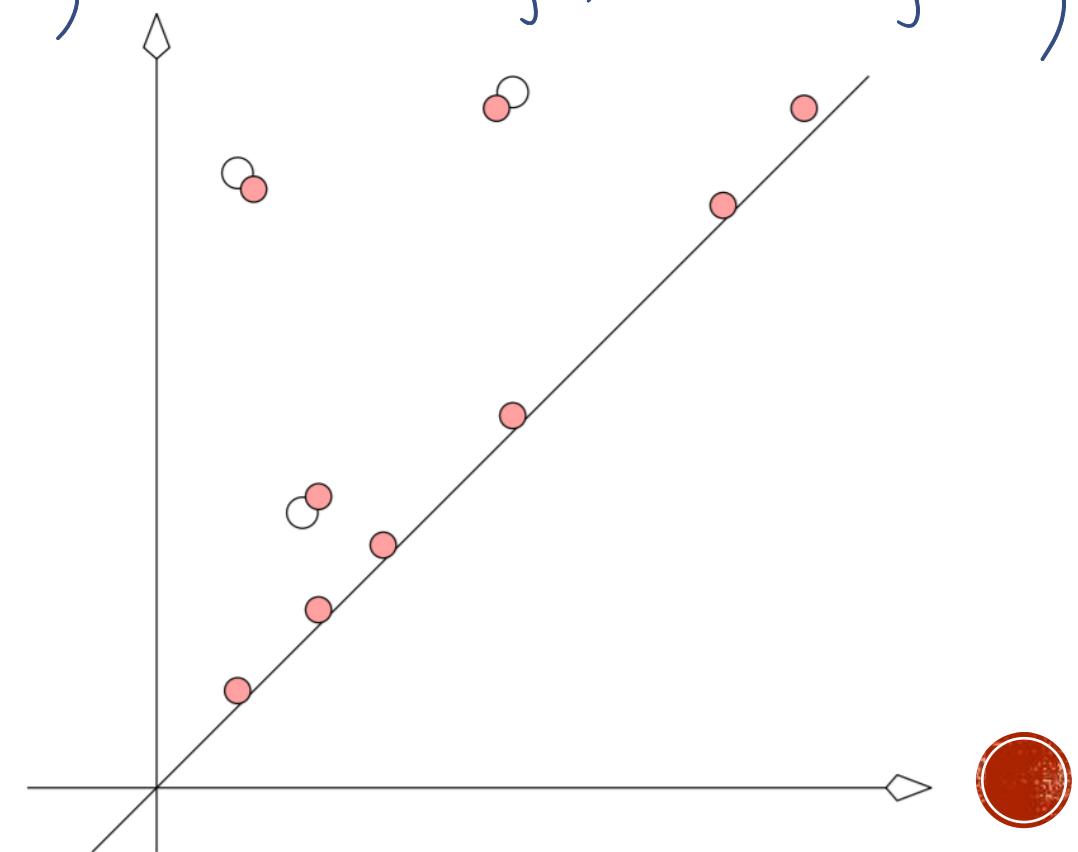
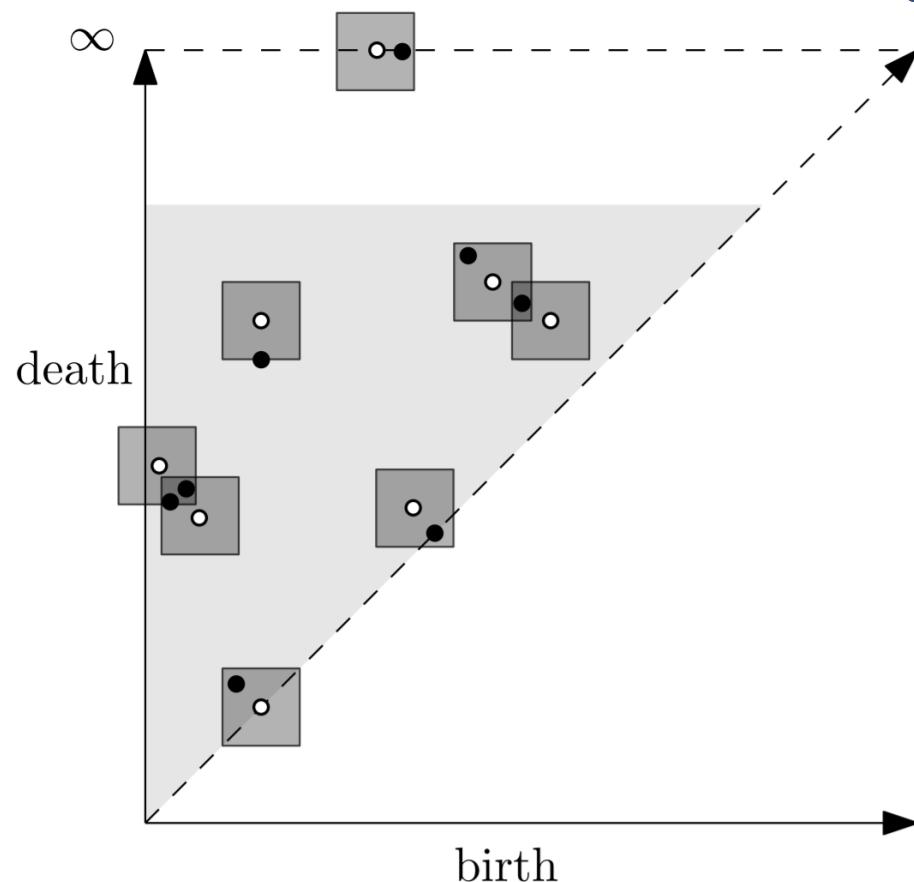


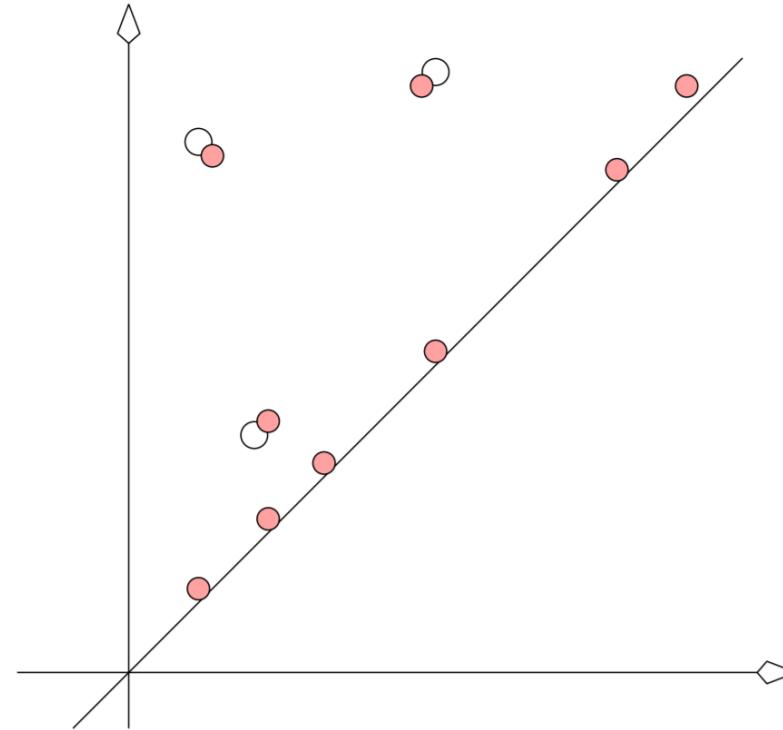
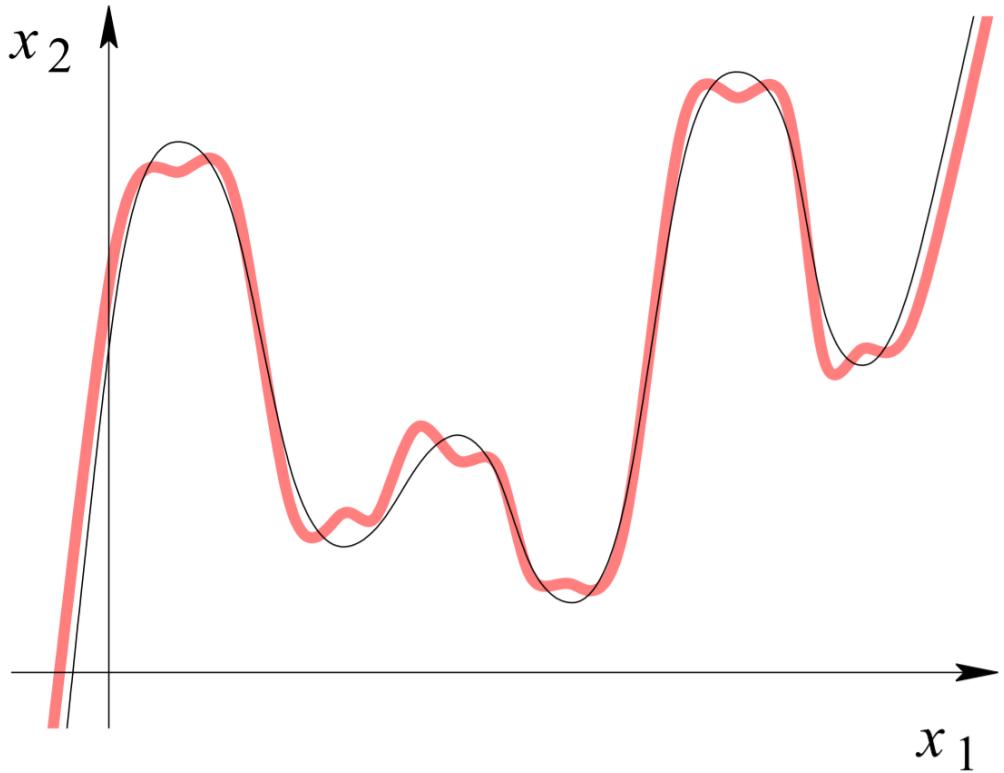
BOTTLENECK DISTANCE

[CohenSteiner-Edelsbrunner-Harer 2005]

$$d_B(P_g(f), P_g(g)) := \min_{\pi: \text{perfect matching}} \max \left(b_{\text{reh}}(f_i) - b_{\text{reh}}(\pi(f_i)), \right.$$

$$\left. d_{\text{reh}}(f_i) - d_{\text{reh}}(\pi(f_i)) \right)$$

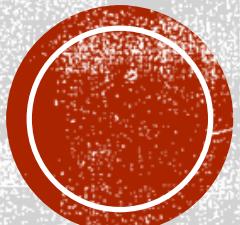




STABILITY THEOREM

[CohenSteiner-Edelsbrunner-Harer 2005] [CS-E-Morozov 2006]

$$d_B(D(f), D(g)) \leq \| f - g \|_{\infty}$$

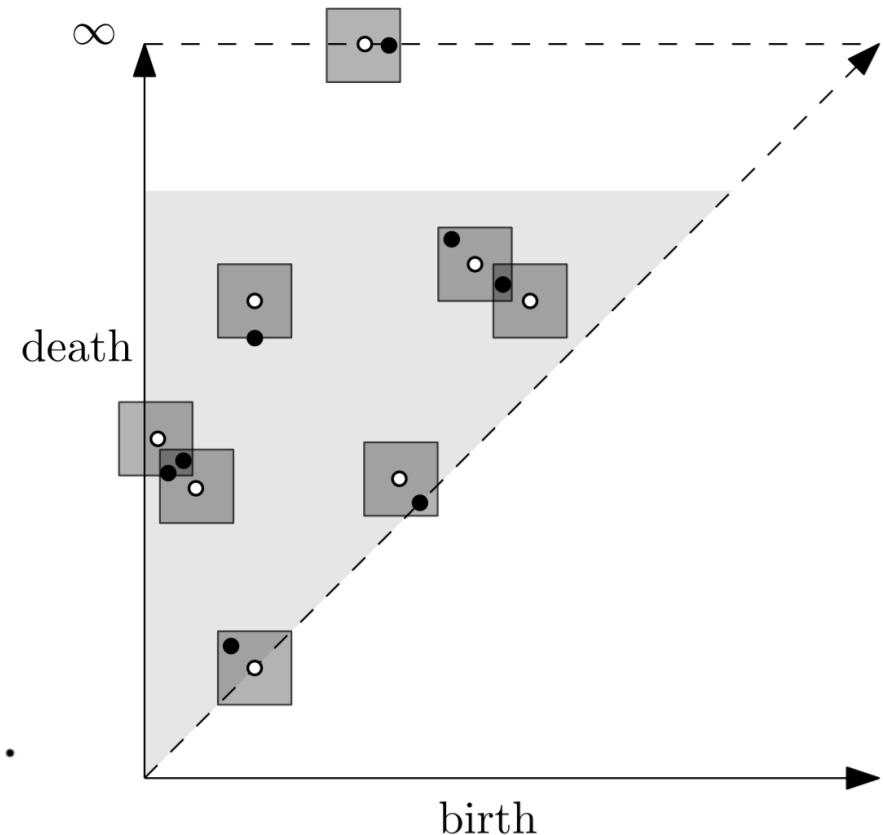


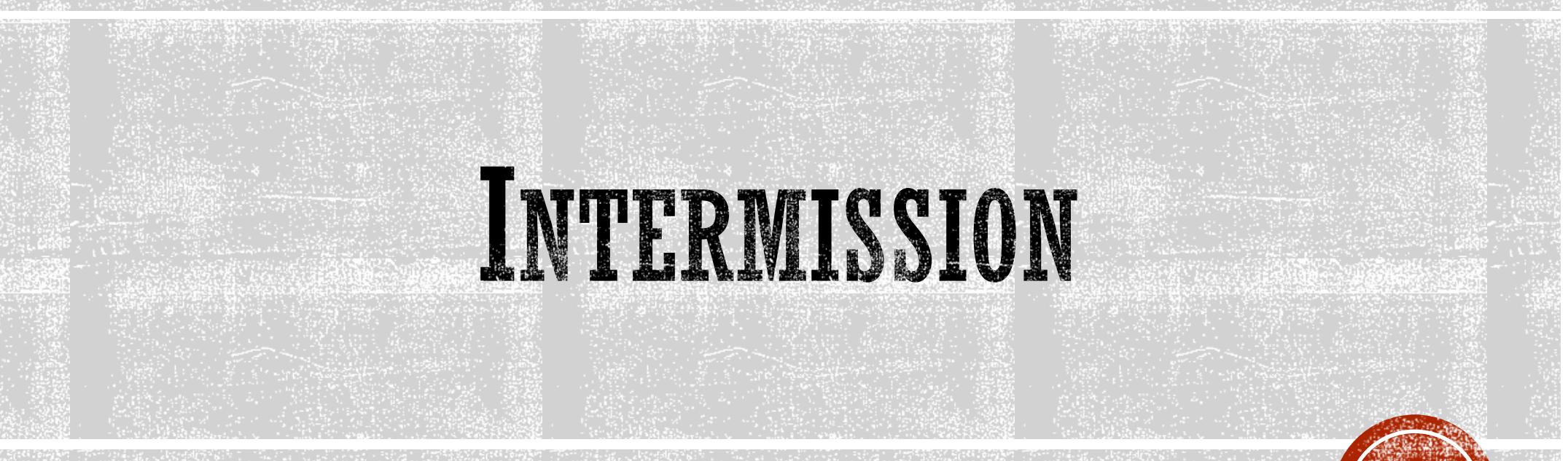
PROOF OF STABILITY THEOREM.

$$\begin{aligned} d_B(\text{D}_p(\bar{f}_r), \text{D}_p(\bar{f}_s)) &\leq \|\bar{f}_r - \bar{f}_s\|_\infty \\ &= (s - r) \cdot \|\bar{f} - \bar{g}\|_\infty. \end{aligned}$$

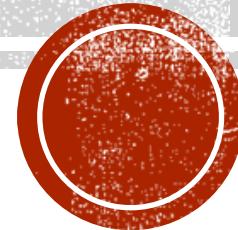
$$d_B(\text{D}_p(\bar{f}), \text{D}_p(\bar{g})) \leq \sum_{i=0}^k d_B(\text{D}_p(\bar{f}_{t_i}), \text{D}_p(\bar{f}_{t_{i+1}}))$$

$$\leq \|\bar{f} - \bar{g}\|_\infty \sum_{i=0}^k (t_{i+1} - t_i).$$

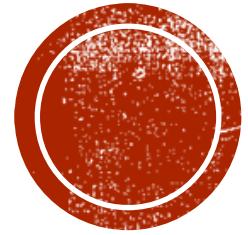




INTERMISSION



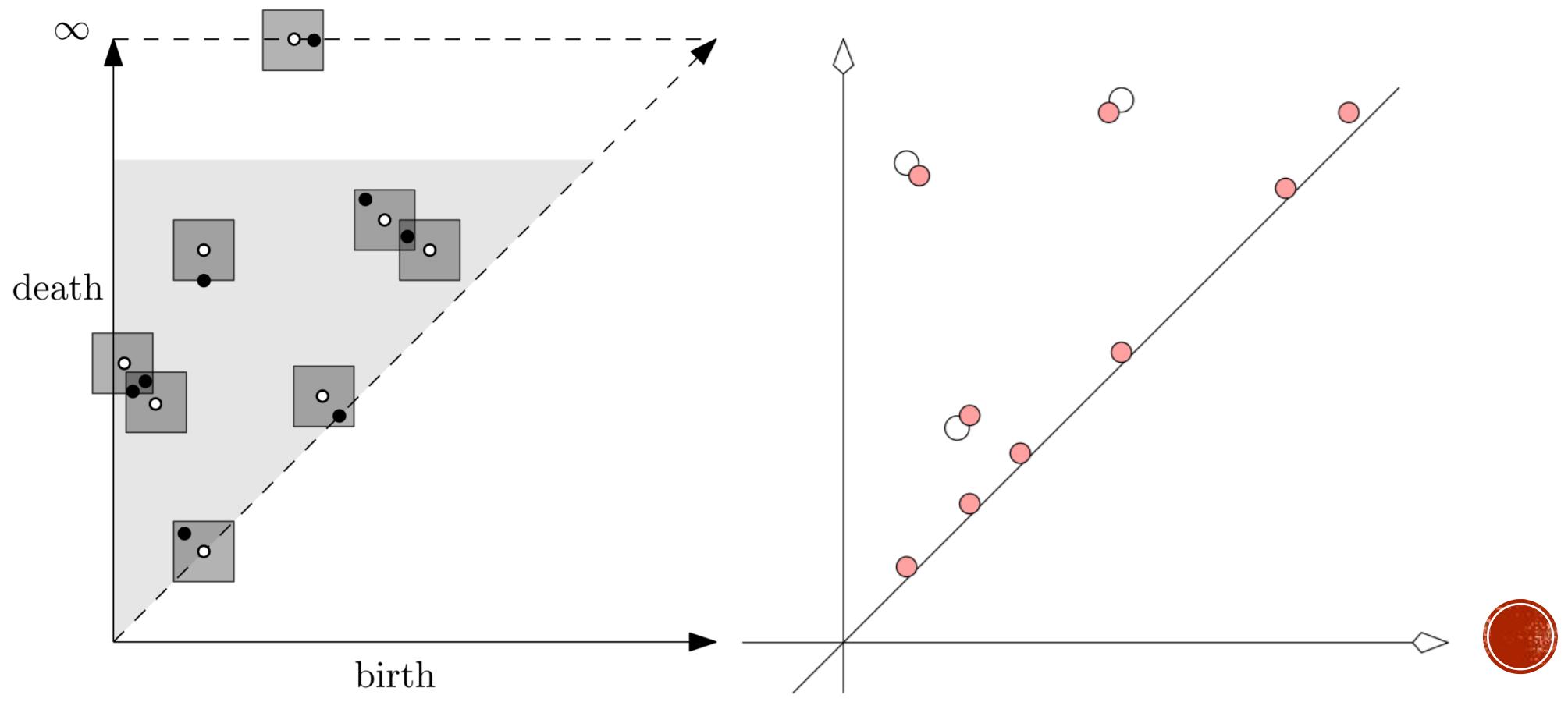
FOOD FOR THOUGHT.
Pros and cons of using persistence diagrams?

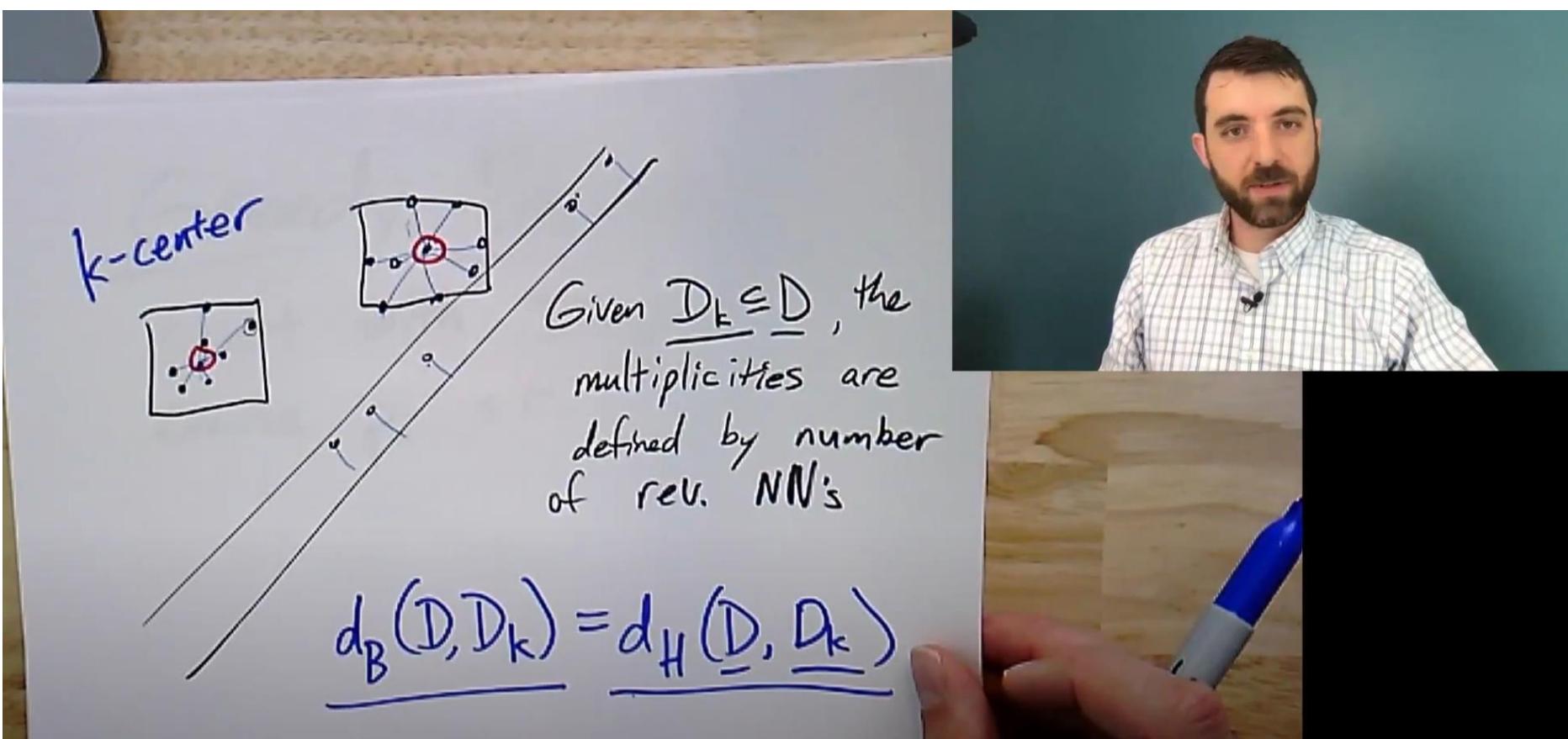


COMPUTATION AND GENERALIZATION



COMPUTING PERFECT MATCHING

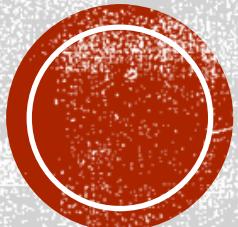




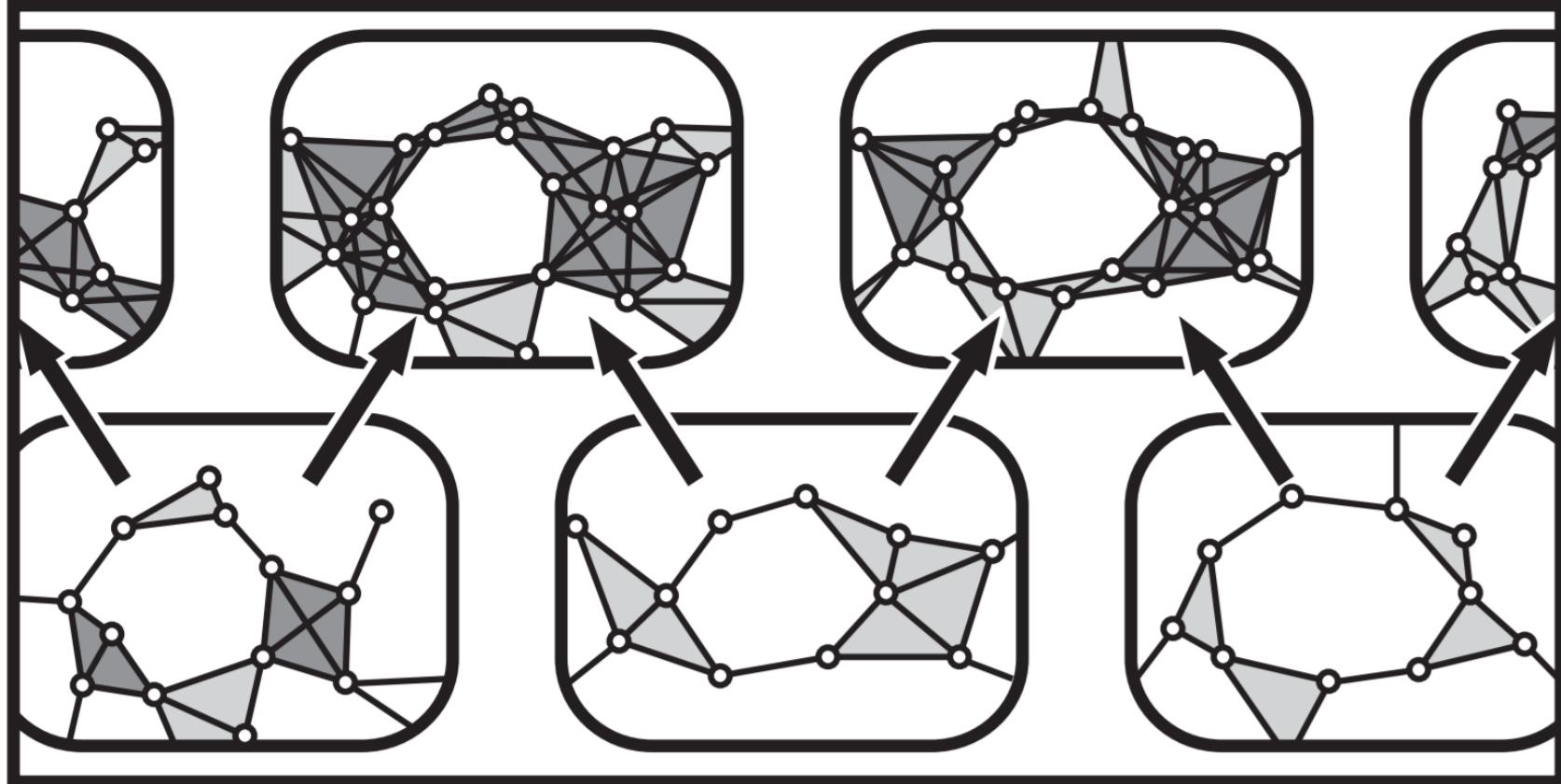
SKETCHING PERSISTENCE DIAGRAMS

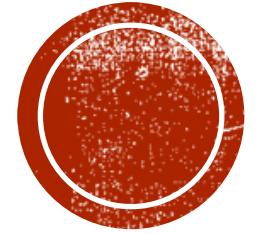
Computing sequence of diagrams of k points converging to optimal, each a 2-approximation to OPT_k

[Sheehy-Sheth 2020]



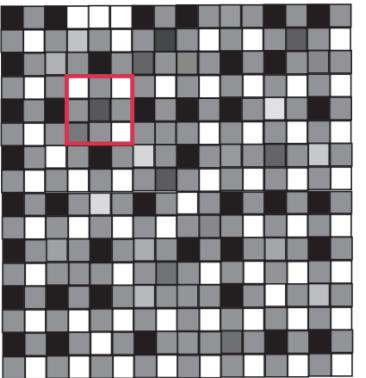
ZIGZAG PERSISTENCE





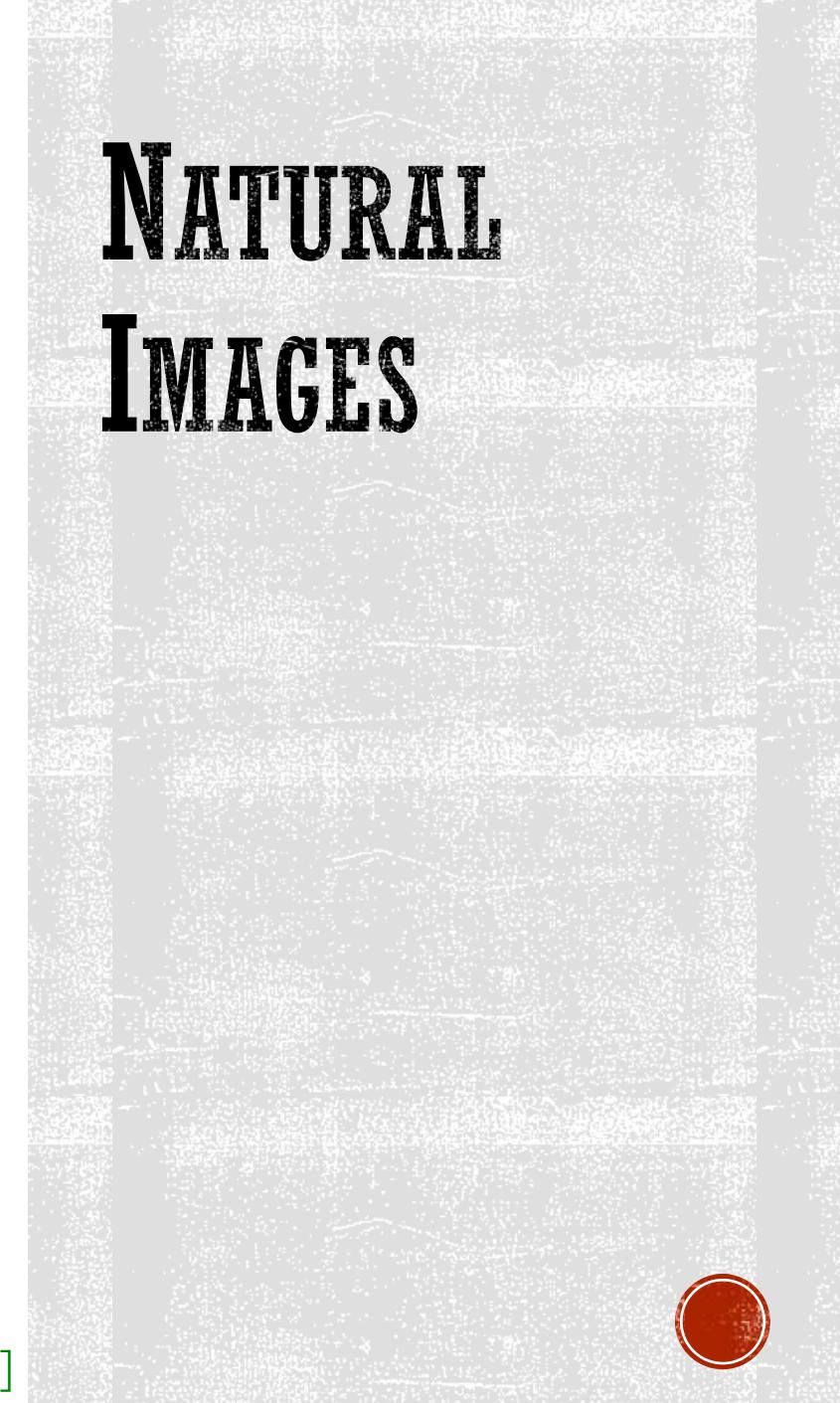
APPLICATIONS

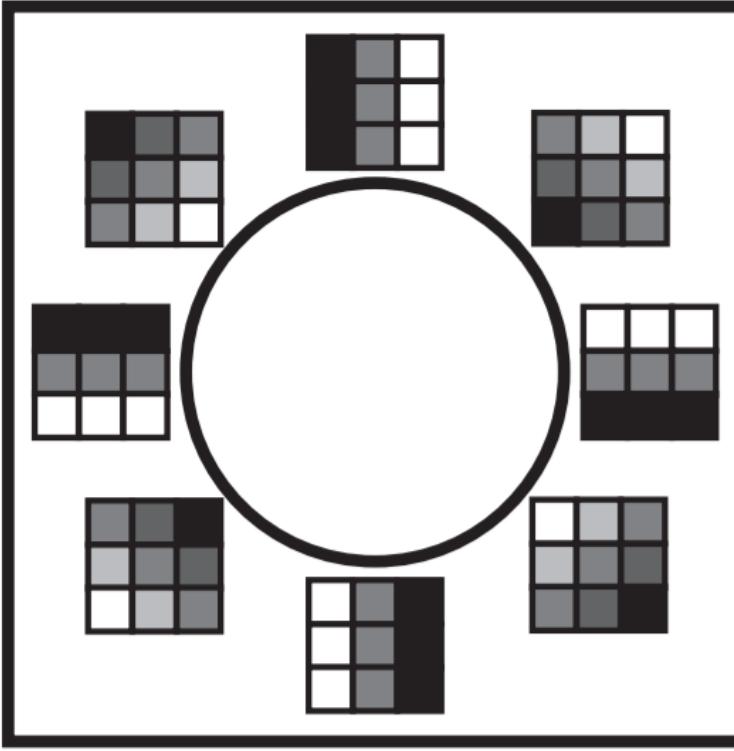
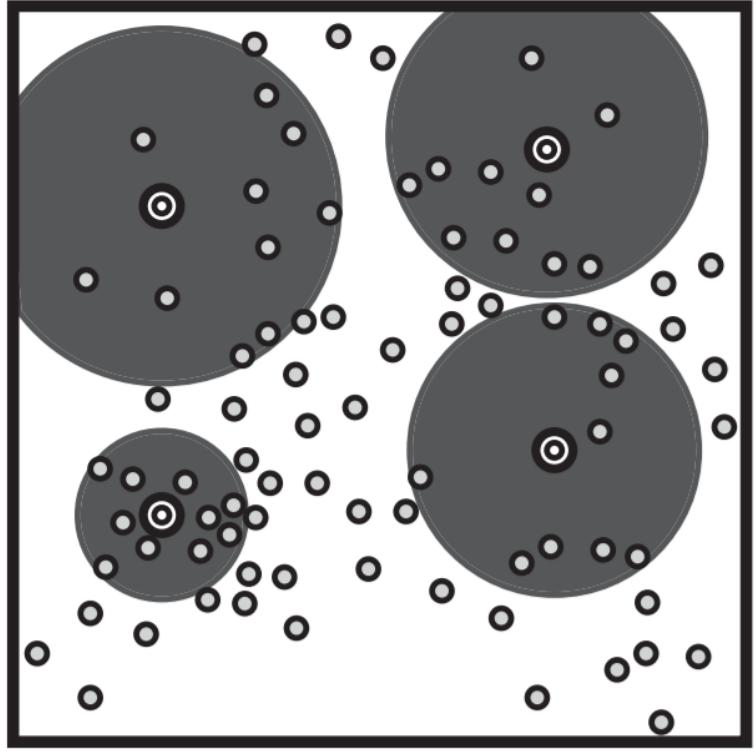




[Lee-Mumford-Pedersen 2003]
[Carlsson-Ishkhanov-DeSilva-Zomorodian 2008]

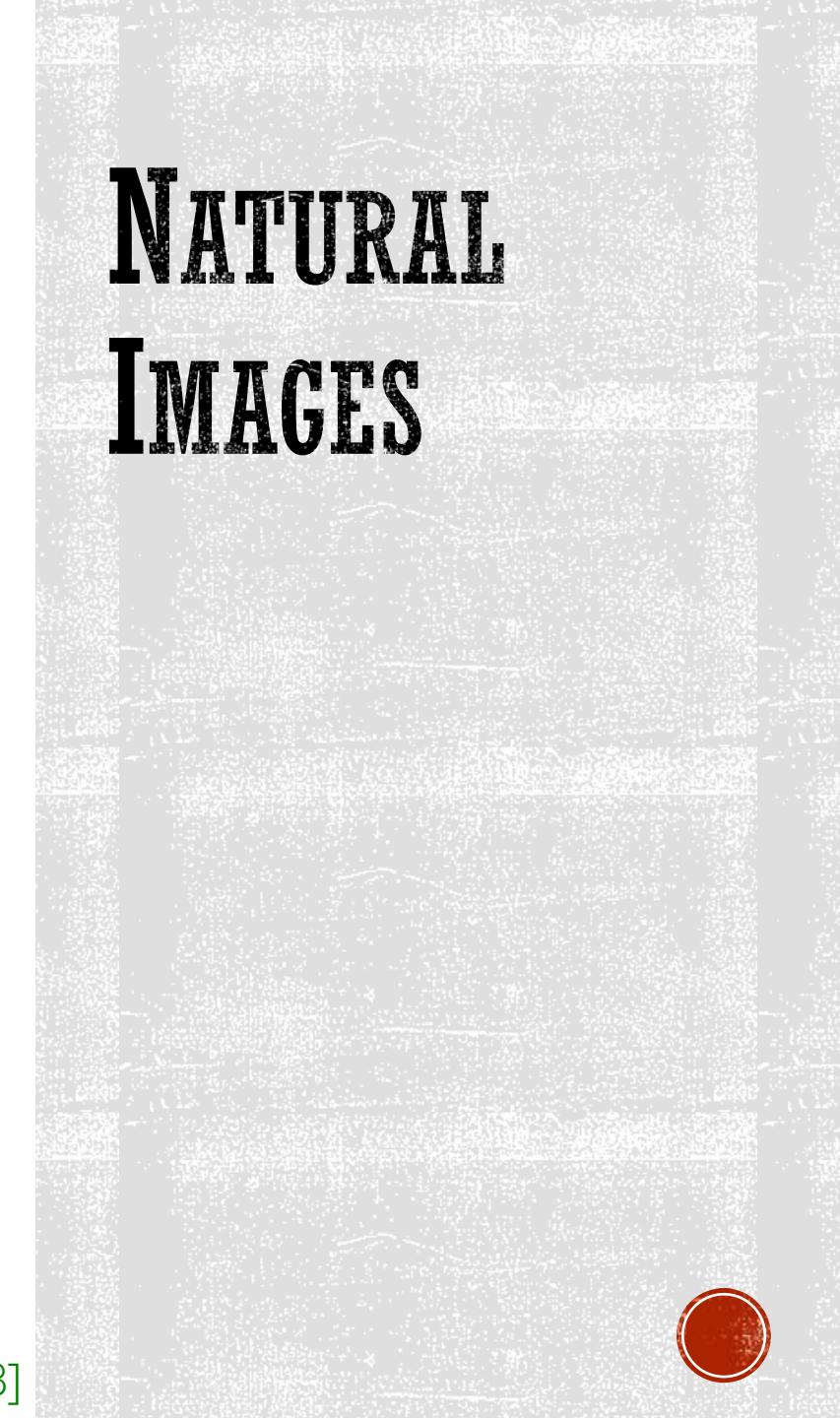
NATURAL IMAGES

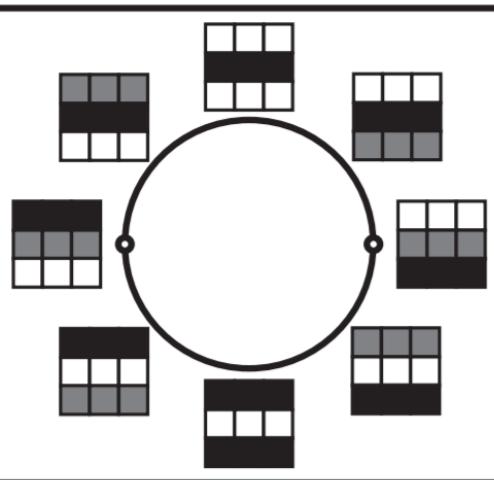
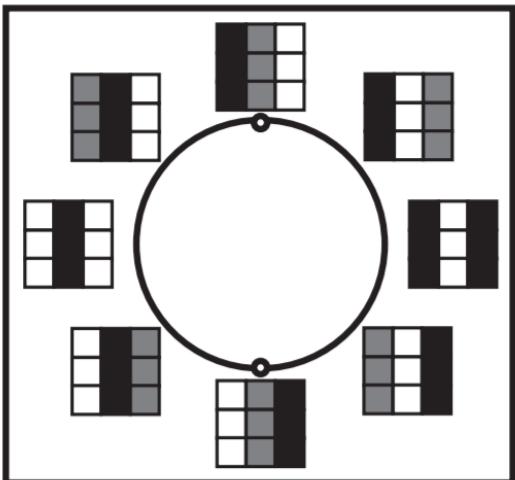
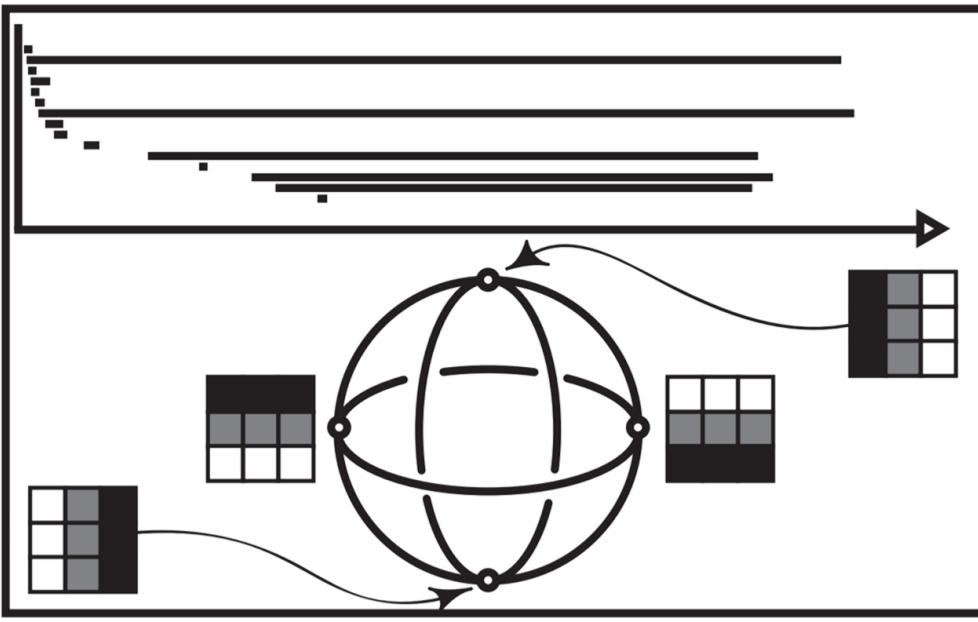
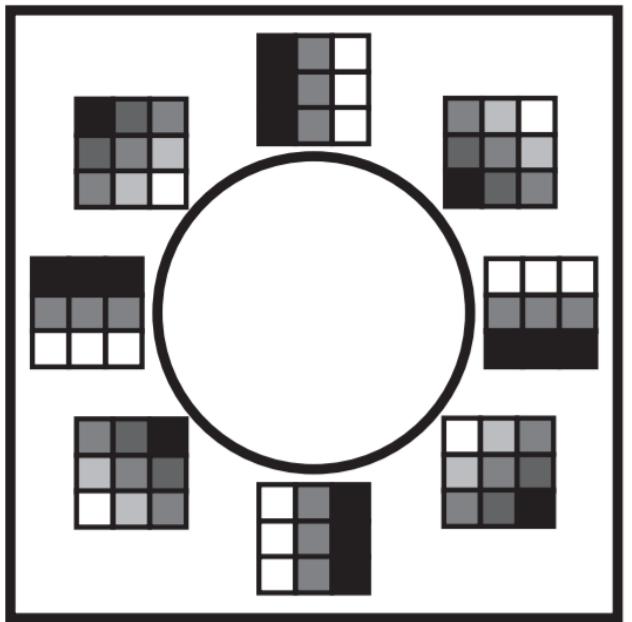




[Lee-Mumford-Pedersen 2003]
[Carlsson-Ishkhanov-DeSilva-Zomorodian 2008]

NATURAL IMAGES



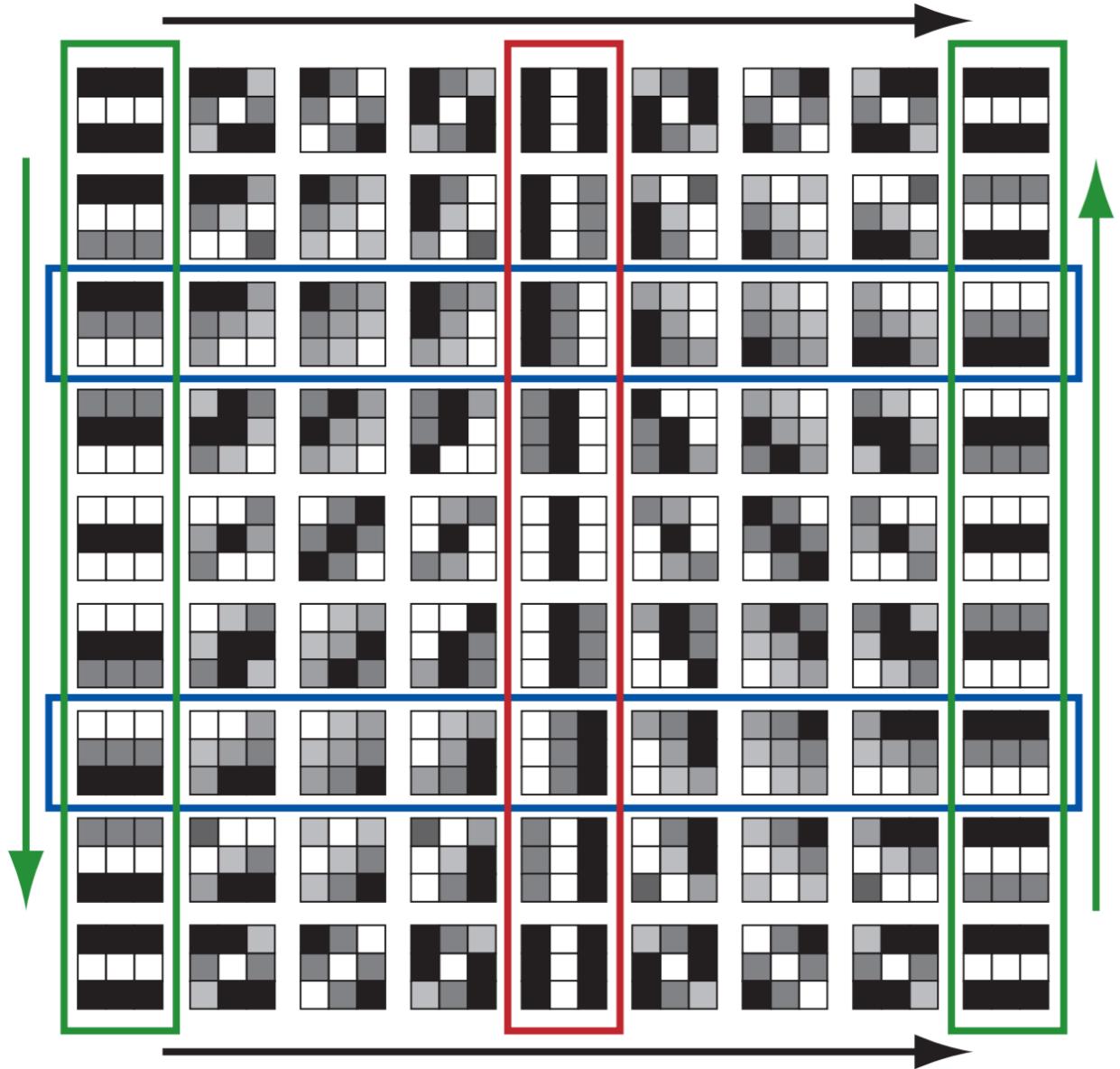


[Lee-Mumford-Pedersen 2003]

[Carlsson-Ishkhanov-DeSilva-Zomorodian 2008]

NATURAL IMAGES

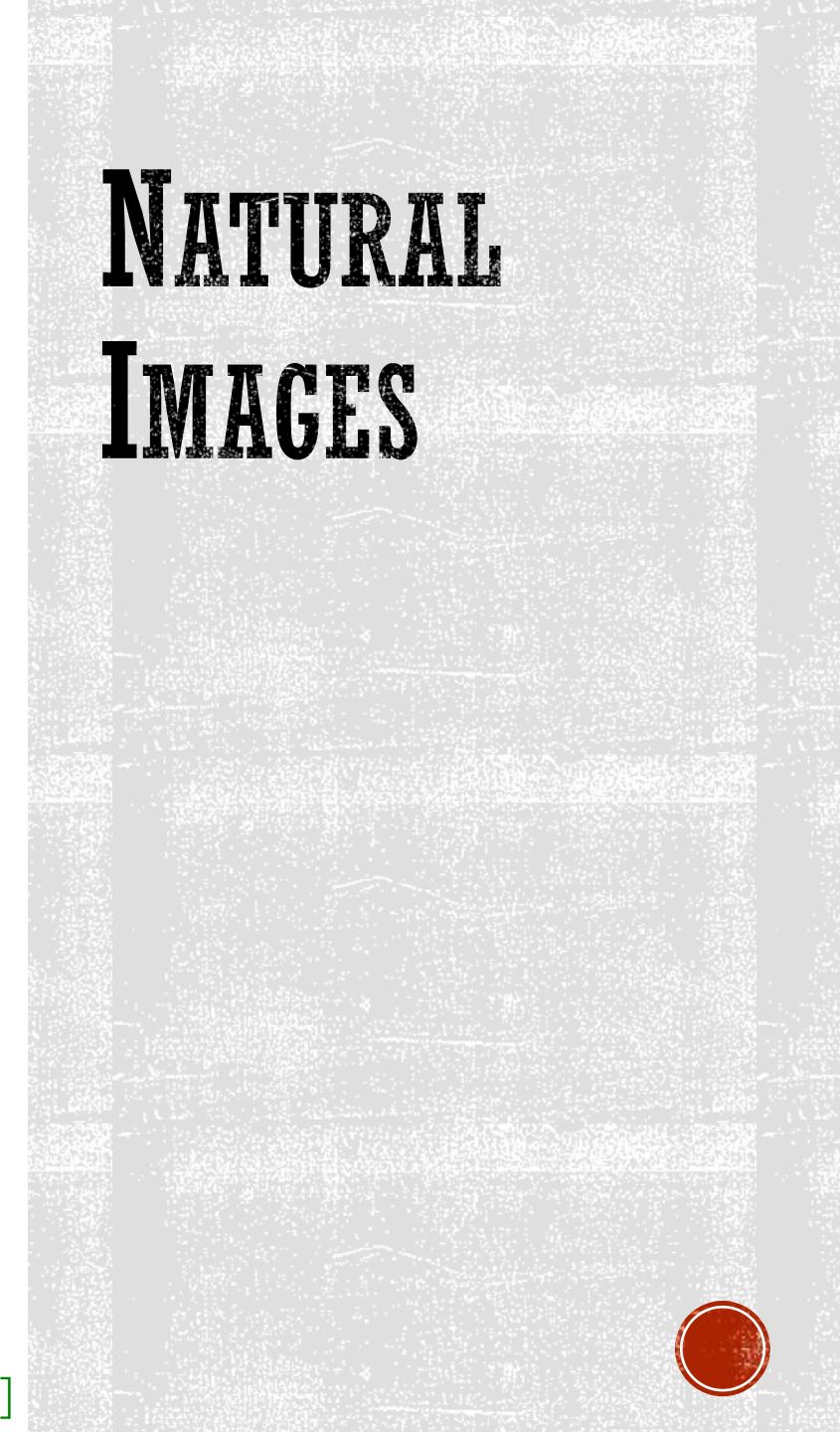




[Lee-Mumford-Pedersen 2003]

[Carlsson-Ishkhanov-DeSilva-Zomorodian 2008]

NATURAL IMAGES



(d)

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7	6	5	5	5	4	3	2	1	-1	-2	-1	1
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-2

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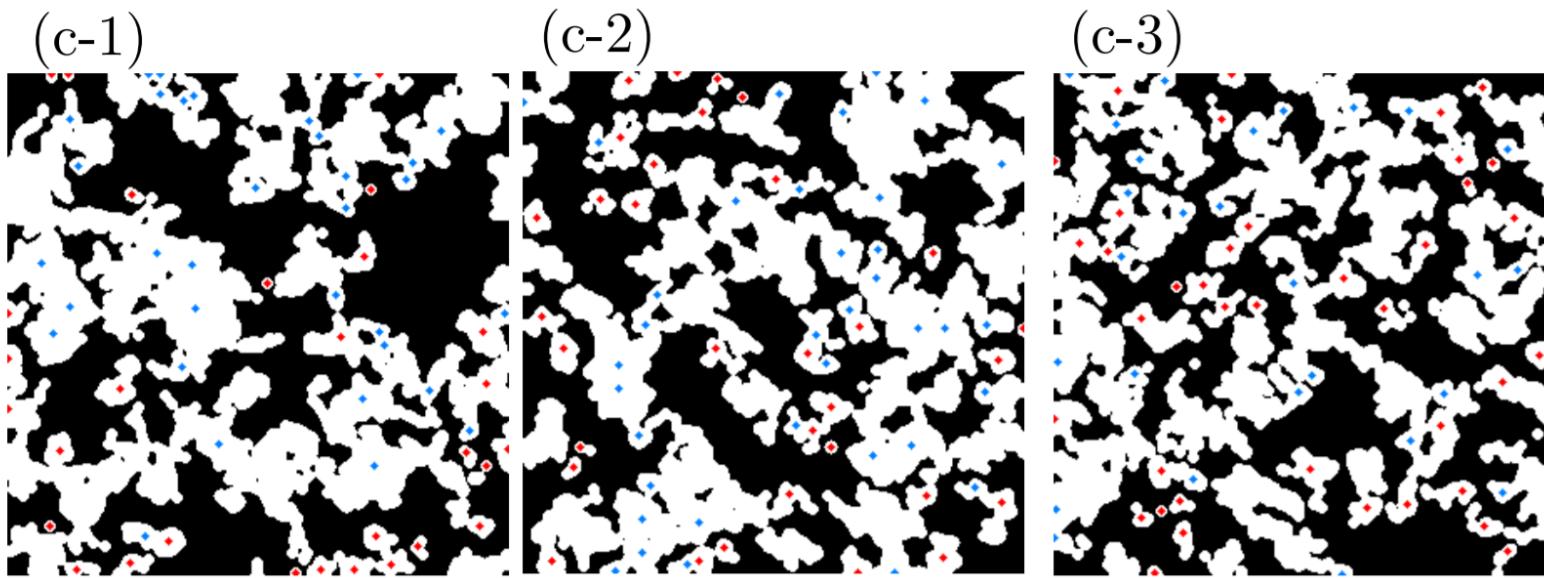
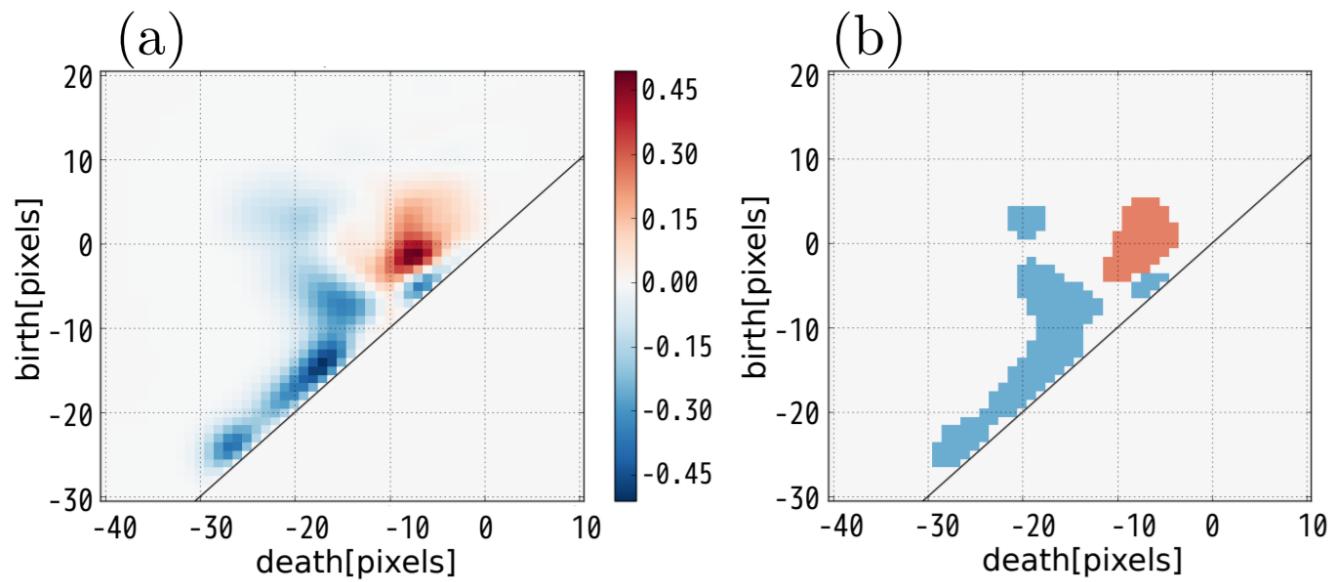
3

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8	7	6	6	6	5	4	3	2	1	-1	1	2

[Obayashi-Hiraoka 2017]

PERSISTENCE IMAGES

PERSISTENCE IMAGES



[Obayashi-Hiraoka 2017]



data

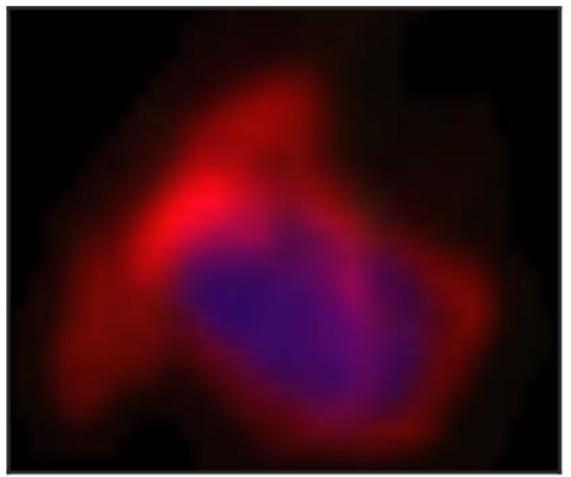


diagram B

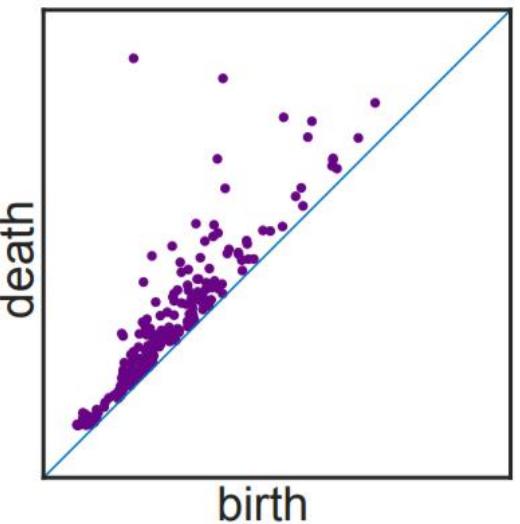
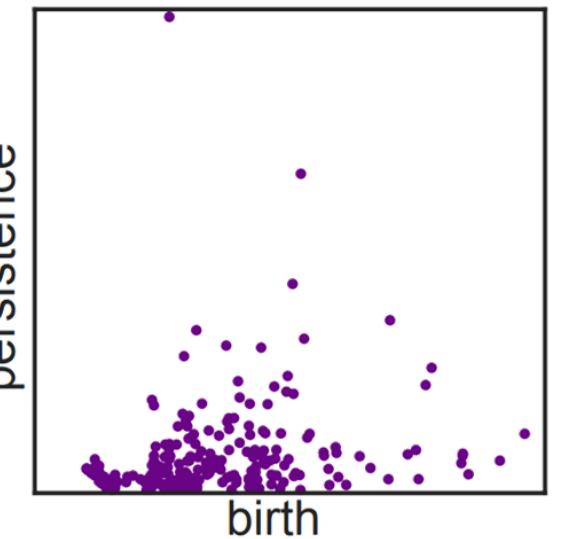
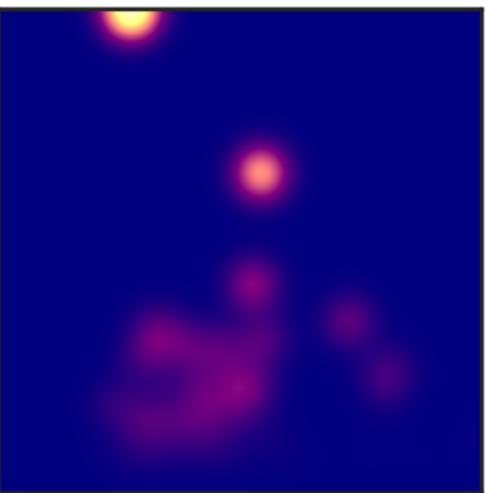


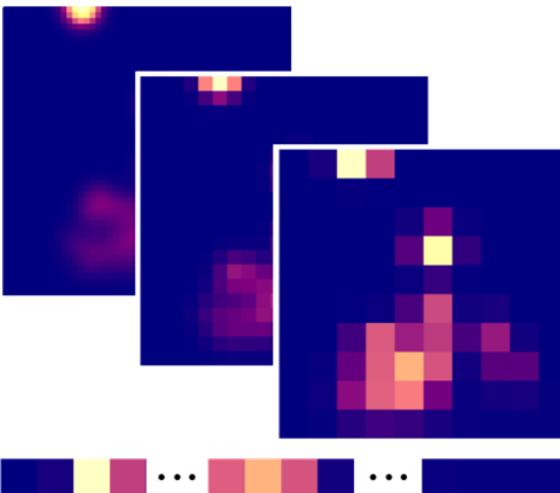
diagram $T(B)$



surface



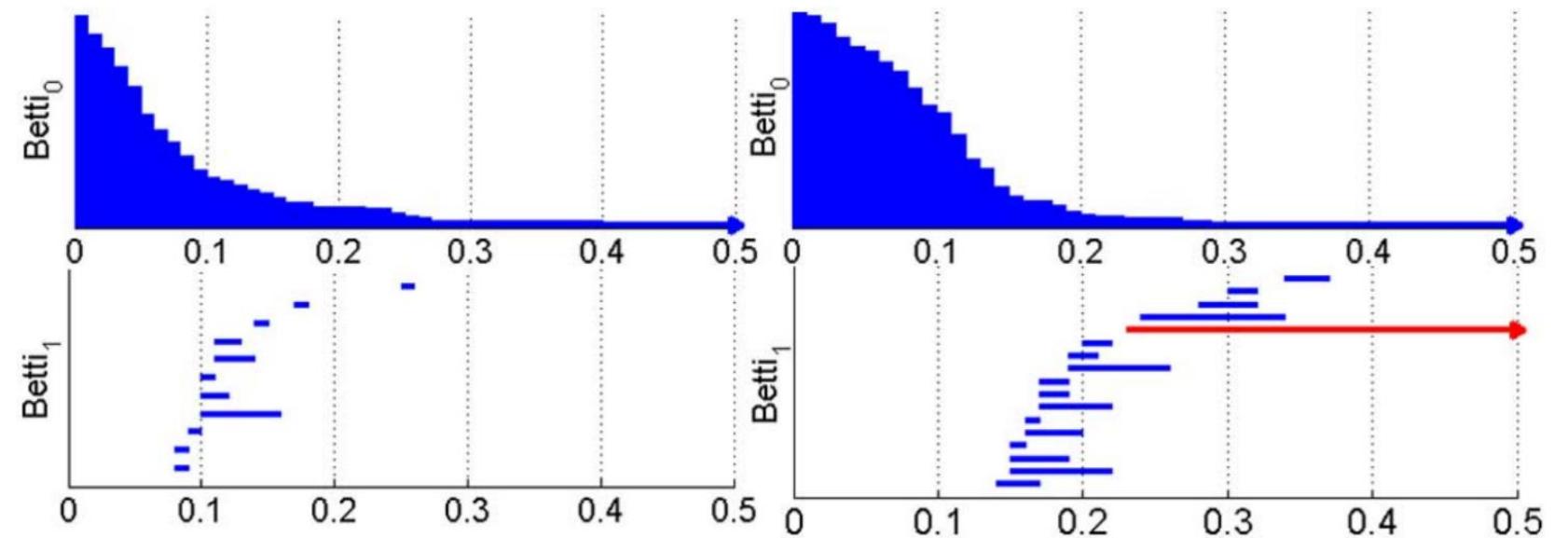
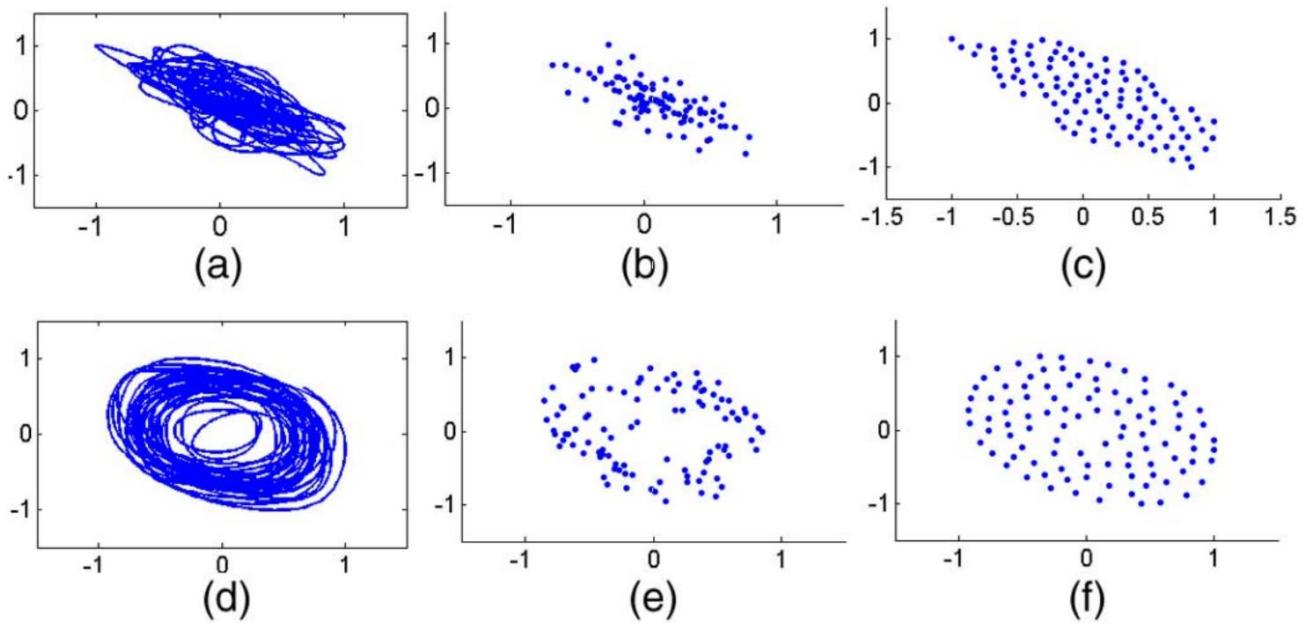
image



[Adams et al. 2017]

PERSISTENCE IMAGES

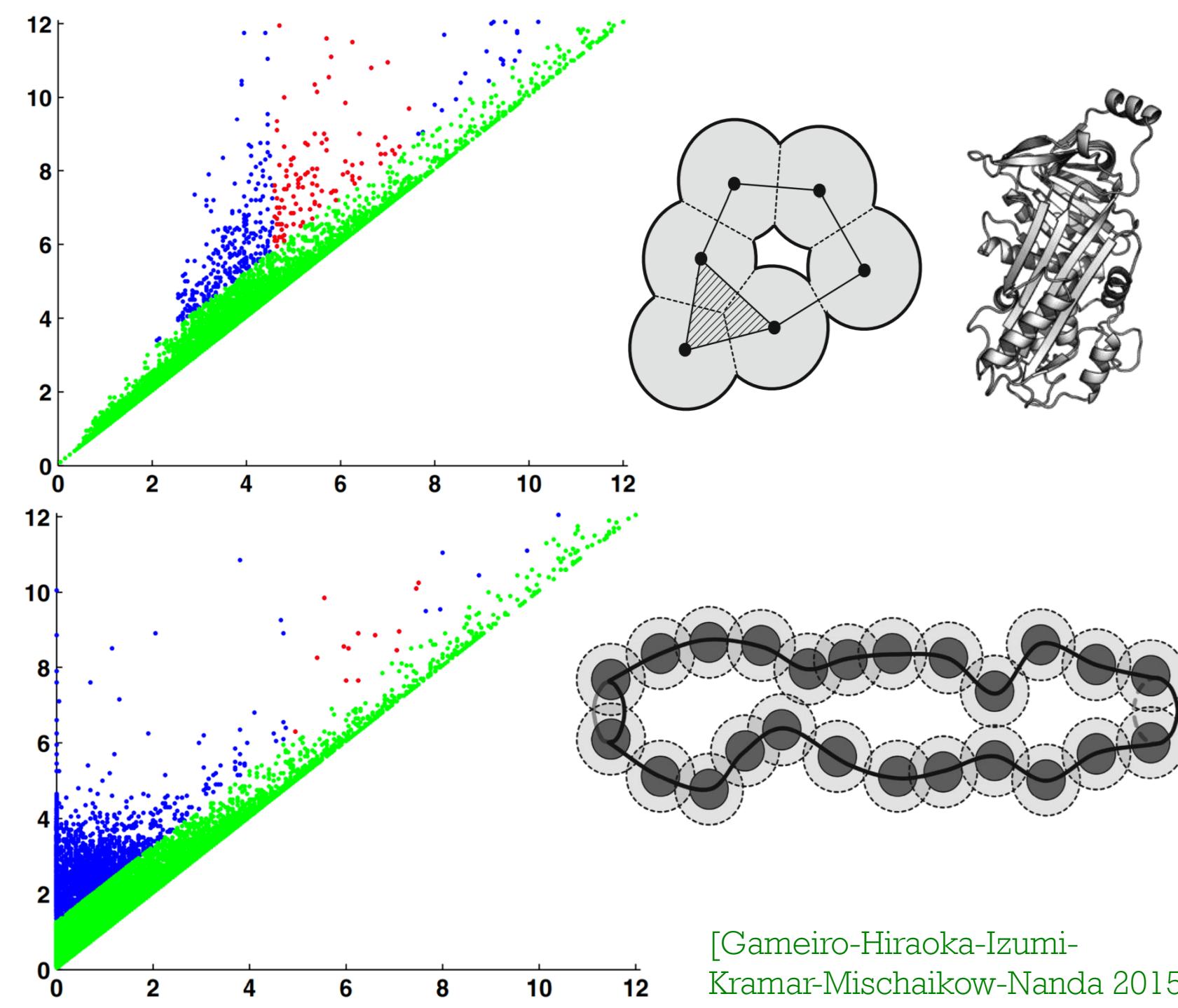




[Emrani-Gentimis-Krim 2014]

WHEEZE DETECTION





PROTEIN COMPRESSABILITY



SUMMARIZING DATA WITH A SINGLE IMAGE

NEXT TIME.
Water-rising is a cool idea.

