DISCRETE MATHEMATICS

IN COMPUTER SCIENCE

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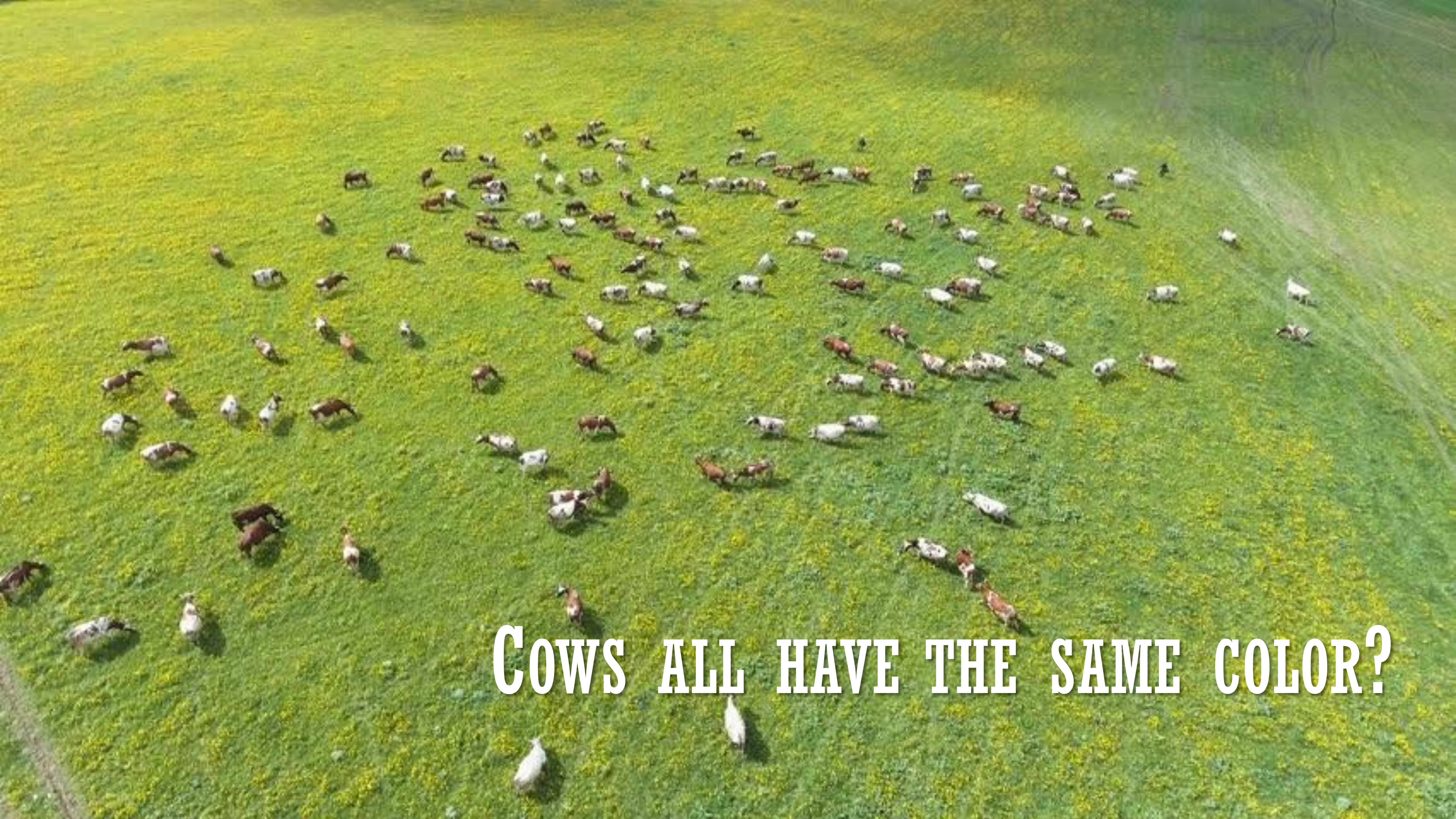
Administrivia

- Homework 2 due today
- Homework 3 will be out
- Midterm 1 to be announced

- Confusion in terminology: hyperplanes


Cows all have the same color?

More Induction

Boilerplate for Induction

Theorem. P(x) holds for every object x.

Let x be an arbitrary object.
Assume P(y) is true for every smaller $y < x$.
[Assume recursion fairy is with us.]

- If x is … [base case]
- If x is … [inductive case]
  The induction hypothesis implies …
  [Recursion fairy says …]

Thus P(x) is true.

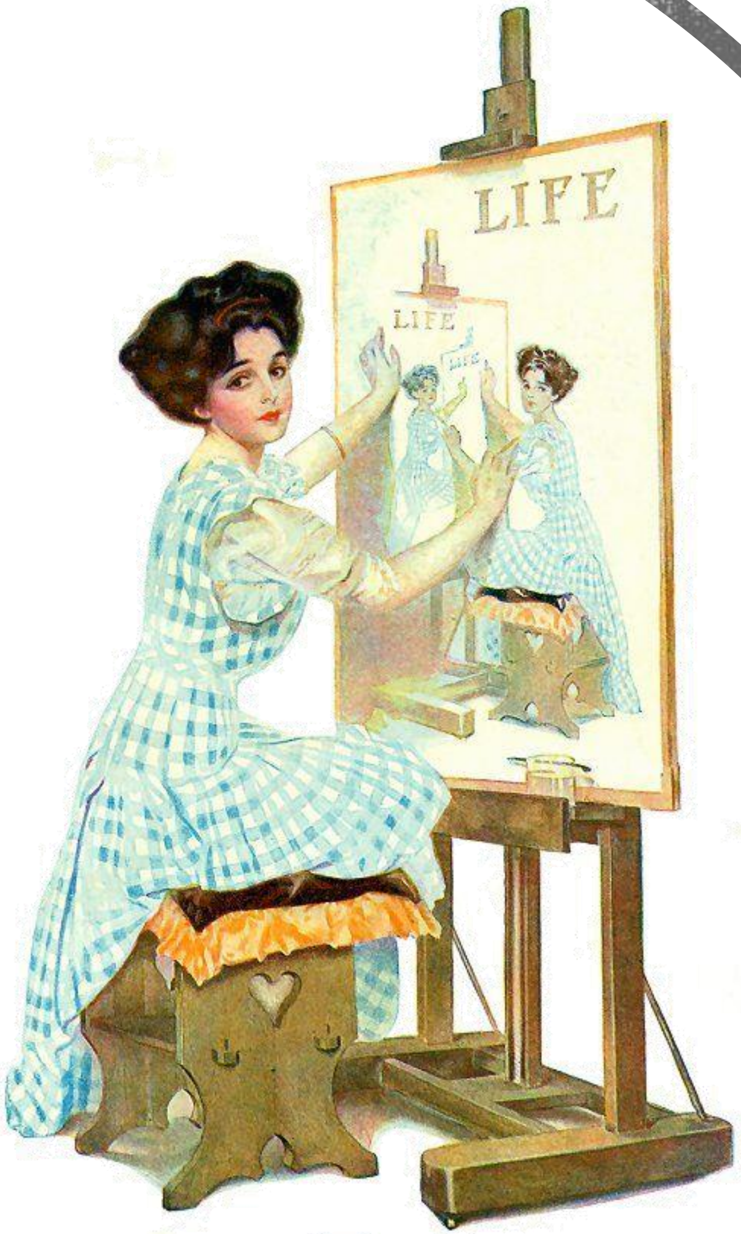
Every non-negative integer can be written as the sum of distinct powers of 2.

Proof 1

Every non-negative integer can be written as the sum of distinct powers of 2.

```
Binary(n):
    if n = 0:
        return ε
    else (if n > 0):
        2^k ← largest power of 2s at most n
        return  2^k + Binary(n-2^k)
```

Proof 2



FROM THE MIRROR.

Induction

IS

Recursion

Every non-negative integer can be written as the sum of distinct powers of 2.

Let S be a bag containing n copies of 2^0s.

```
Binary(S):
    if S has ≥ 2 copies of any 2^i:
        remove 2 copies of 2^i from S
        insert 1 copy of 2^(i+1) into S
        return Binary(S)
    else: return S
```

Proof 3

Every non-negative integer can be written as the sum of distinct powers of 2.

```
BINARY(n):
   w = ε
   while n > 0:
      if n is even:
         n ← n/2,  w ← 0·w
      else (if n is odd):
         n ← (n-1)/2,  w ← 1·w
   return w
```

Proof 4

May the Recursion Fairy be with you.

Next time.

New Module: Graphs!