



DISCRETE MATHEMATICS IN COMPUTER SCIENCE

**HSIEN-CHIH CHANG
JANUARY 26, 2022**

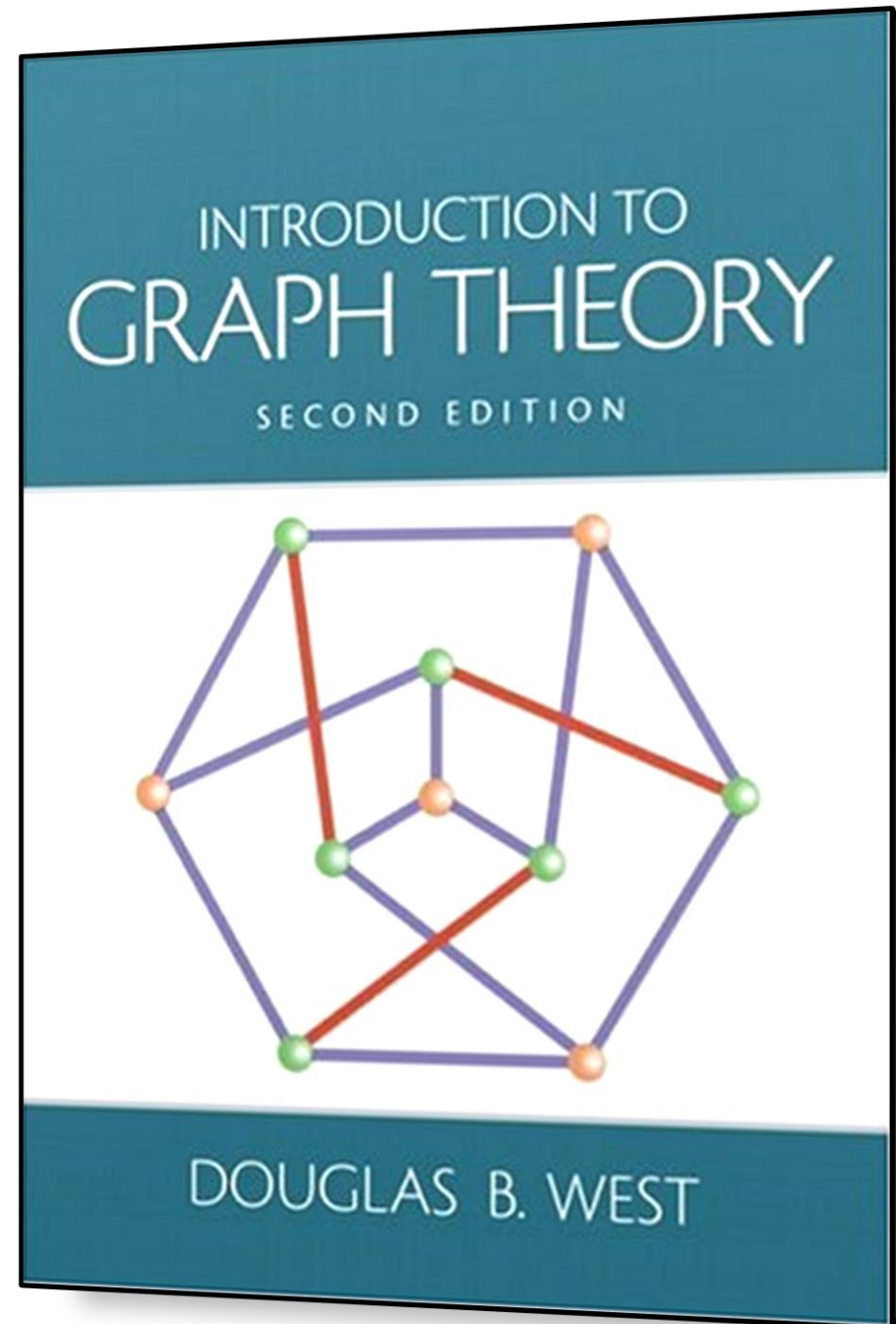
ADMINISTRIVIA

- **Midterm 1**
 - Jan 31 (Mon) 6–9PM
 - Moore Hall B13 Filene Auditorium
- **Conflict Midterm 1**
 - Feb 1 (Tue) 5–8PM
 - Place to be updated
- **SAS/Conflict Conflict**
 - Come talk to me
- **Closed-book written exam**
- **Scope: Module P on proofs**
- **One-page two-sided cheatsheet**
 - Must be hand-written



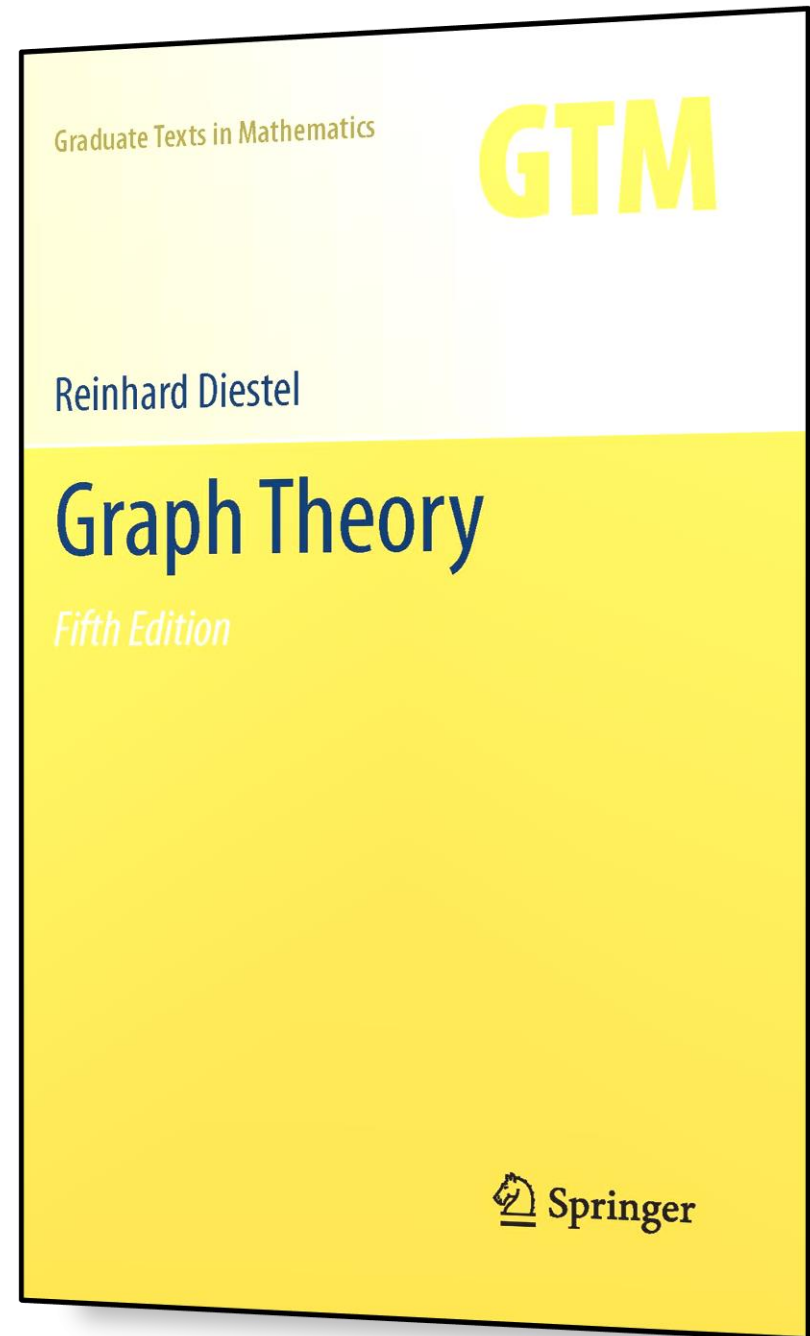
BOOKS

- **Introduction to Graph Theory**
by Douglas B. West
 - Pros: Clear writing style, lots of exercises
 - Cons: Expansive



BOOKS

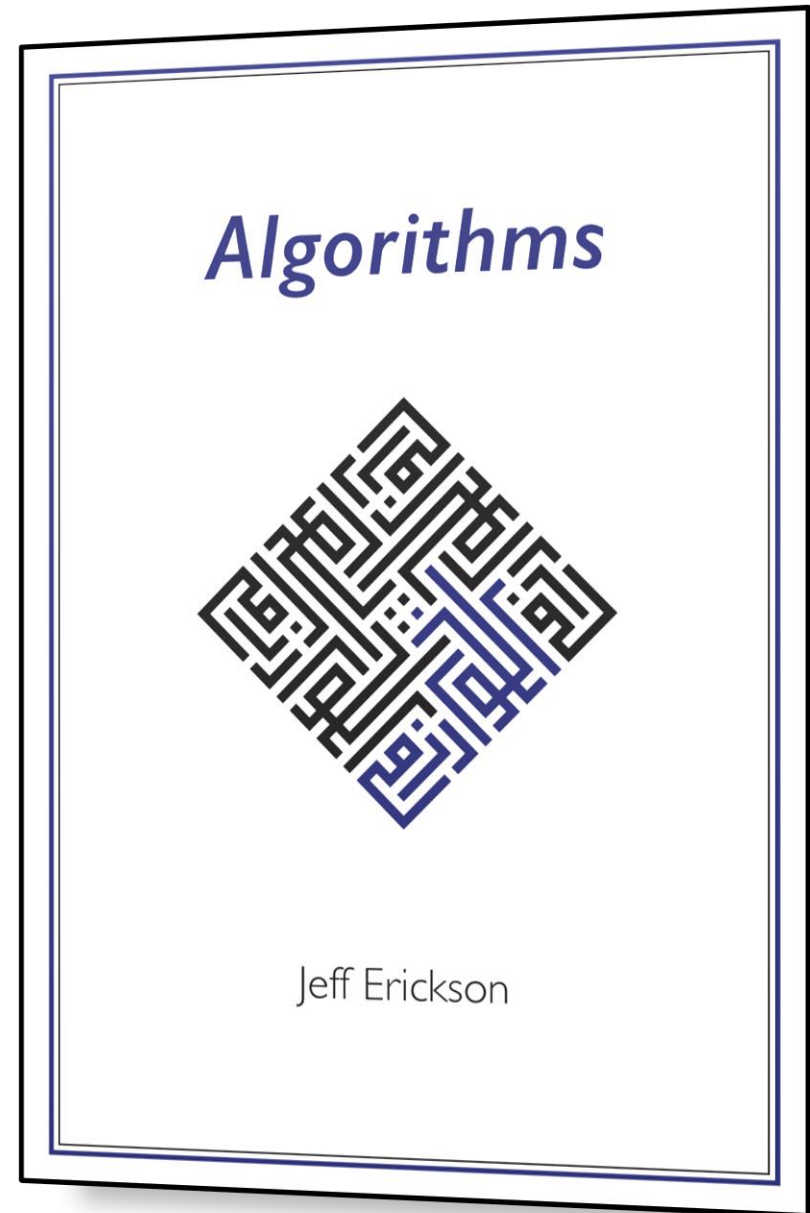
- **Graph Theory**
by Reinhard Diestel
 - Pros: Free, modern in topics
 - Cons: A bit dense, more advanced

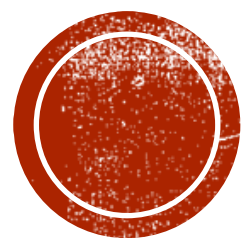


BOOKS

■ **Algorithms** by Jeff Erickson

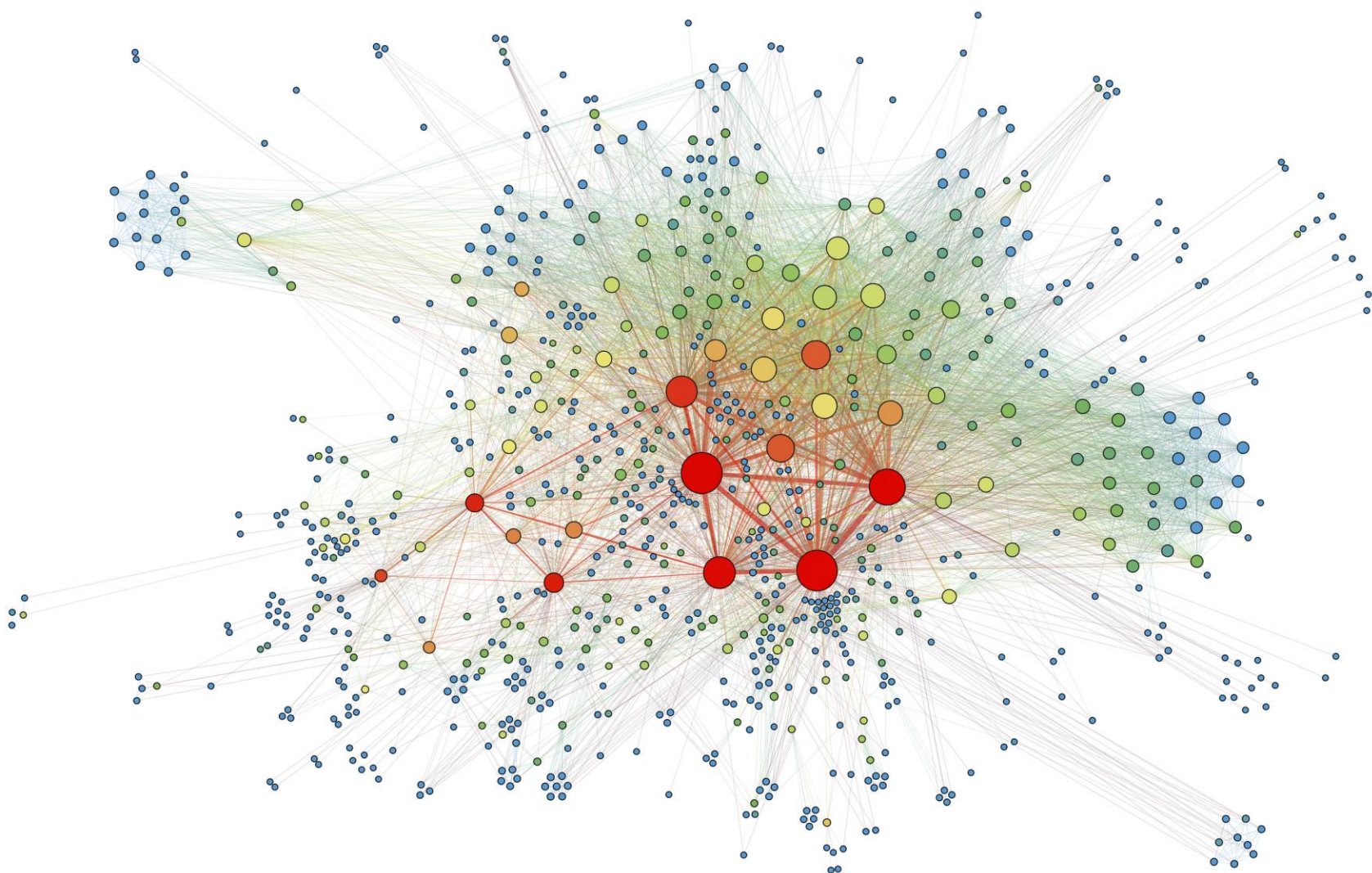
- Pros: Free, chapters on graphs and recursion
- Cons: Really a textbook for COSC 31





GRAPHS





Jargon

graph

vertex / node

edge / arc

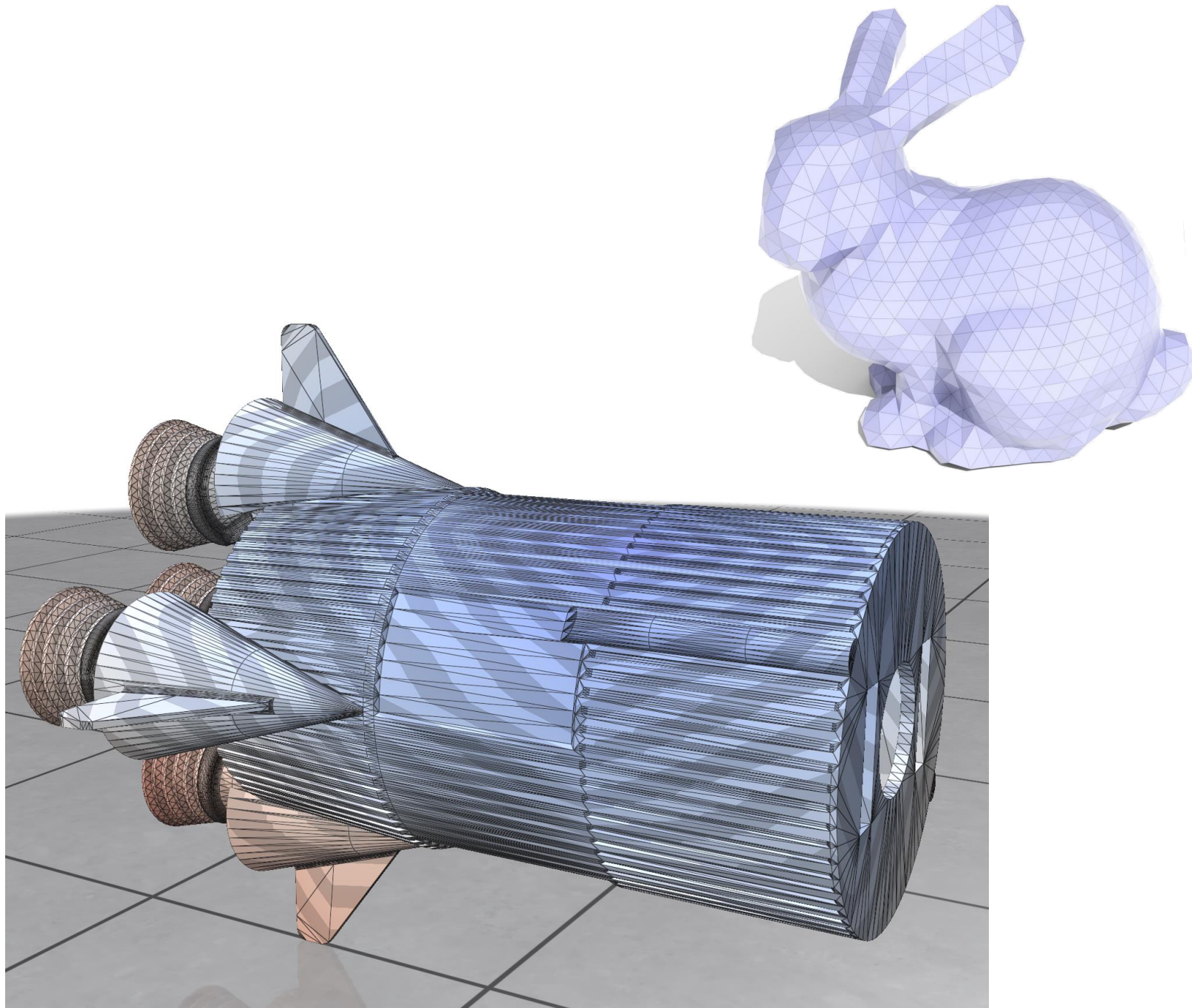
undirected / directed

simple graph

loop

multi-edges

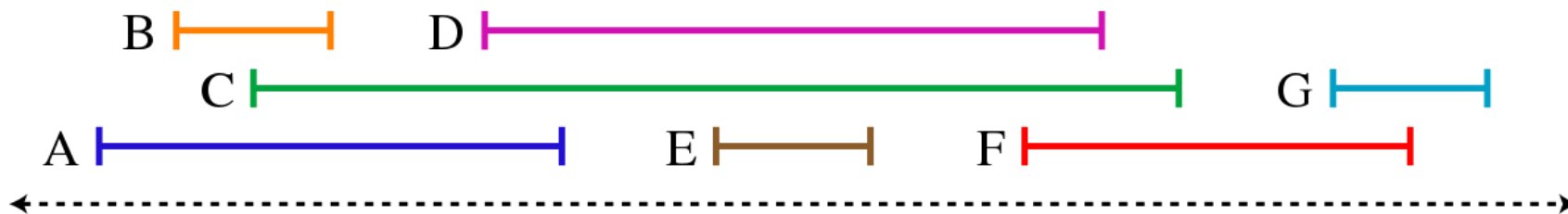
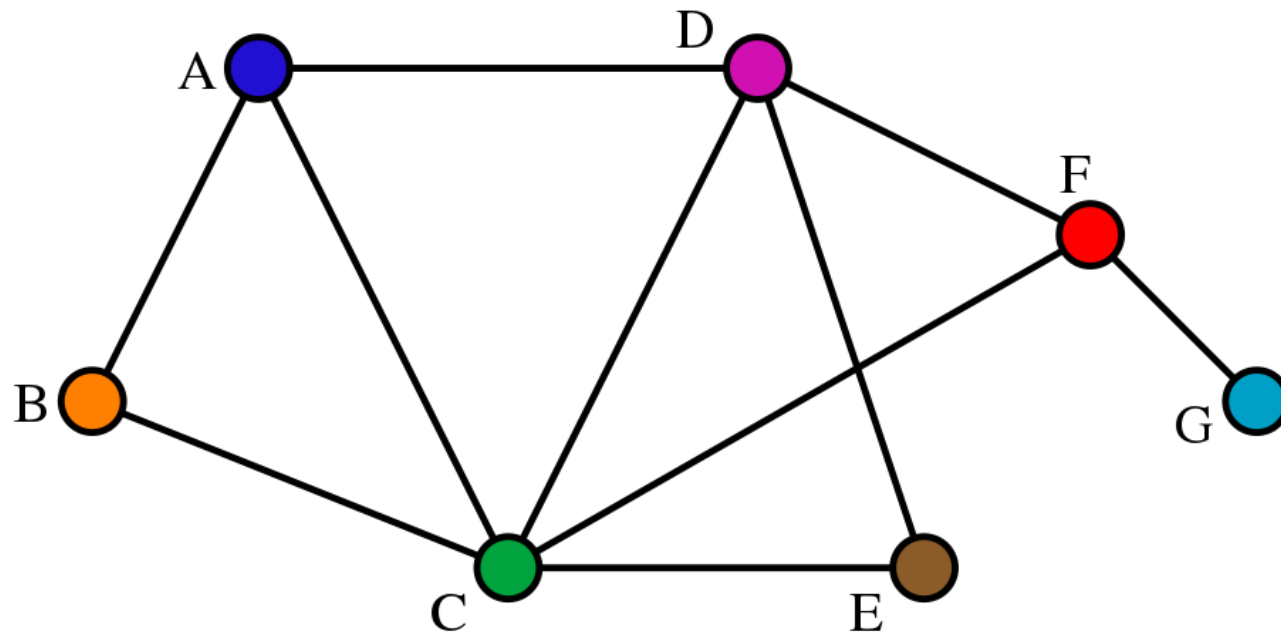




EXAMPLE

Meshes

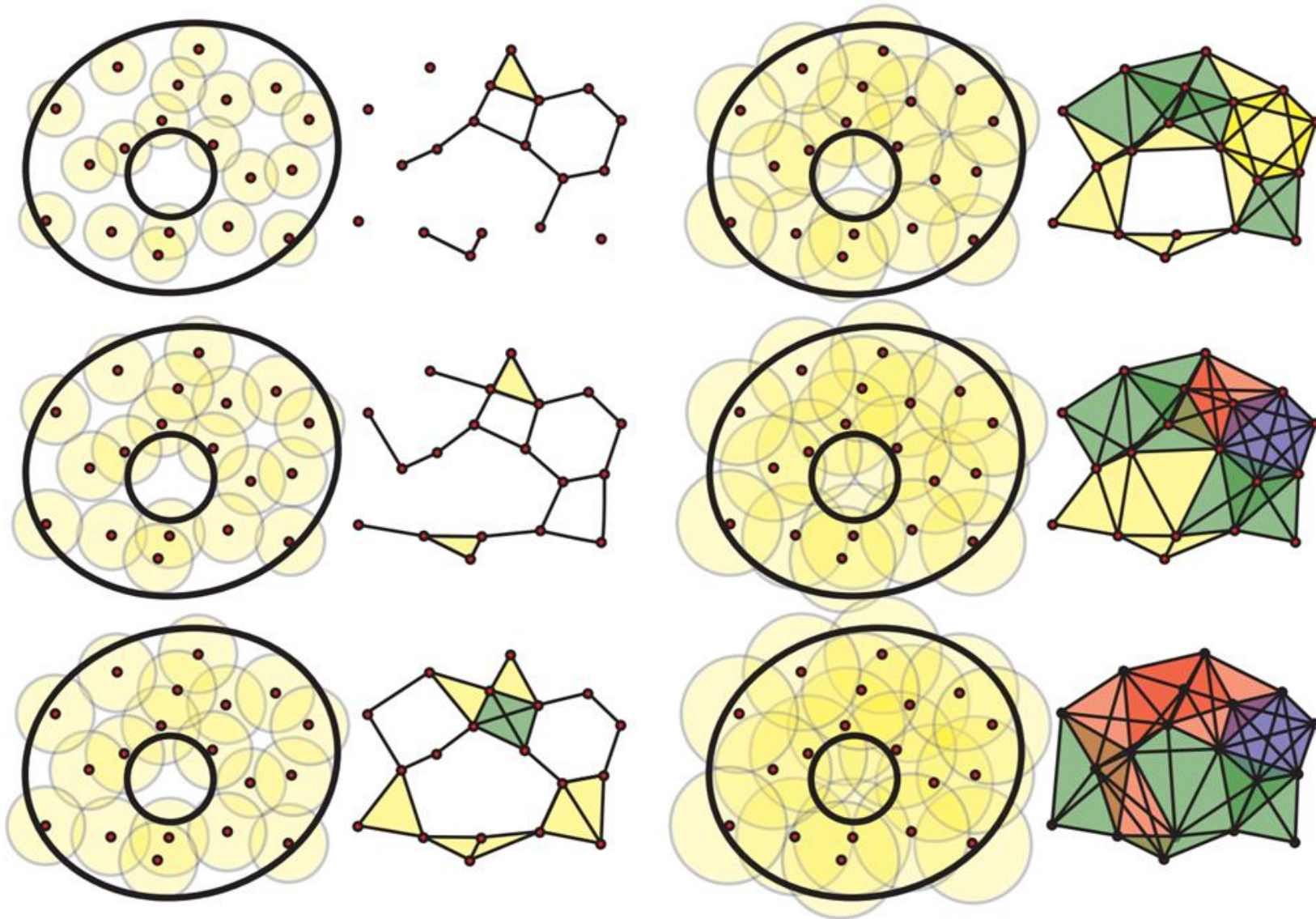




EXAMPLE

Scheduling problem as
interval graph

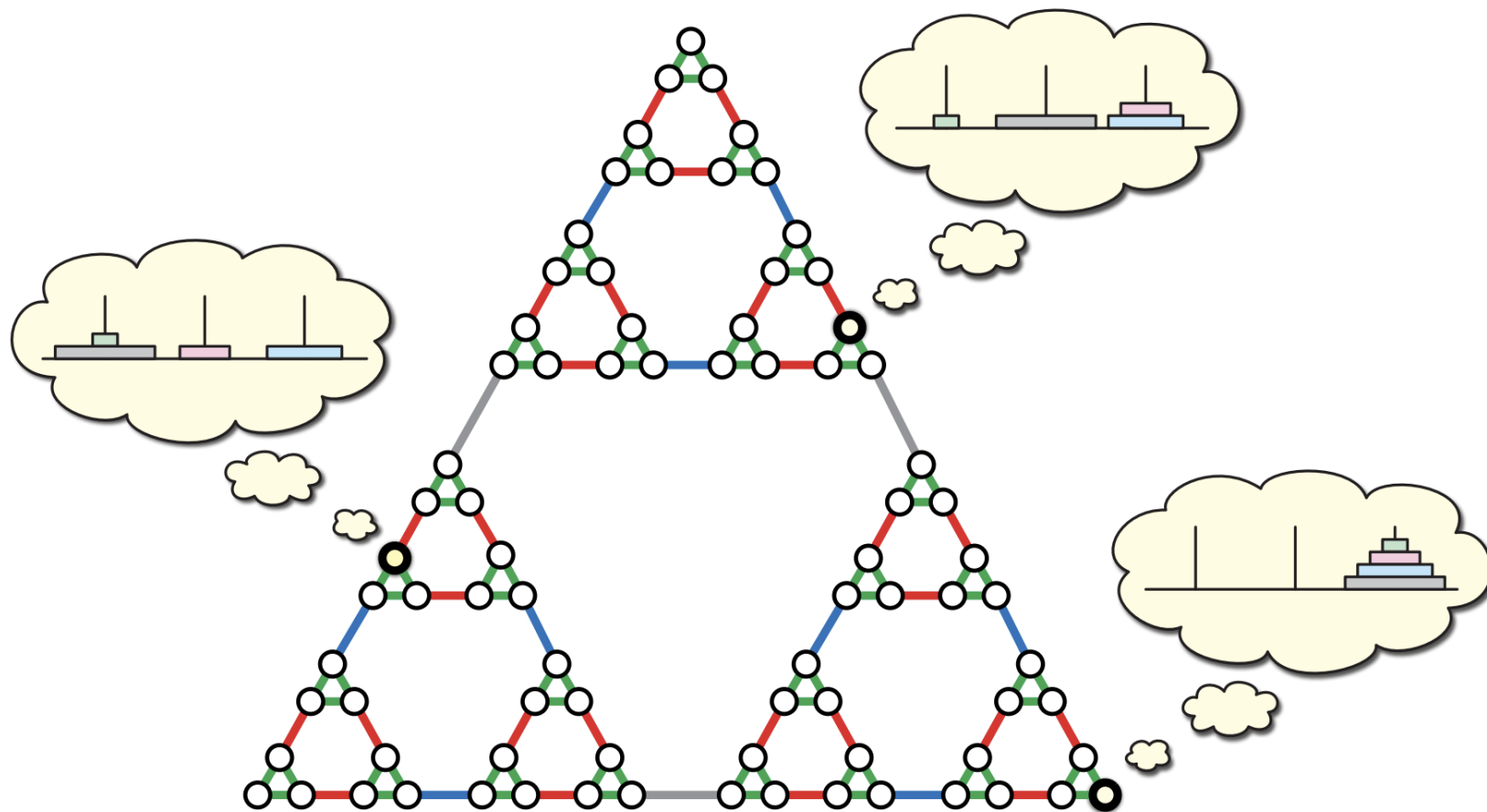




EXAMPLE

Reconstructing underlying
space through sampling

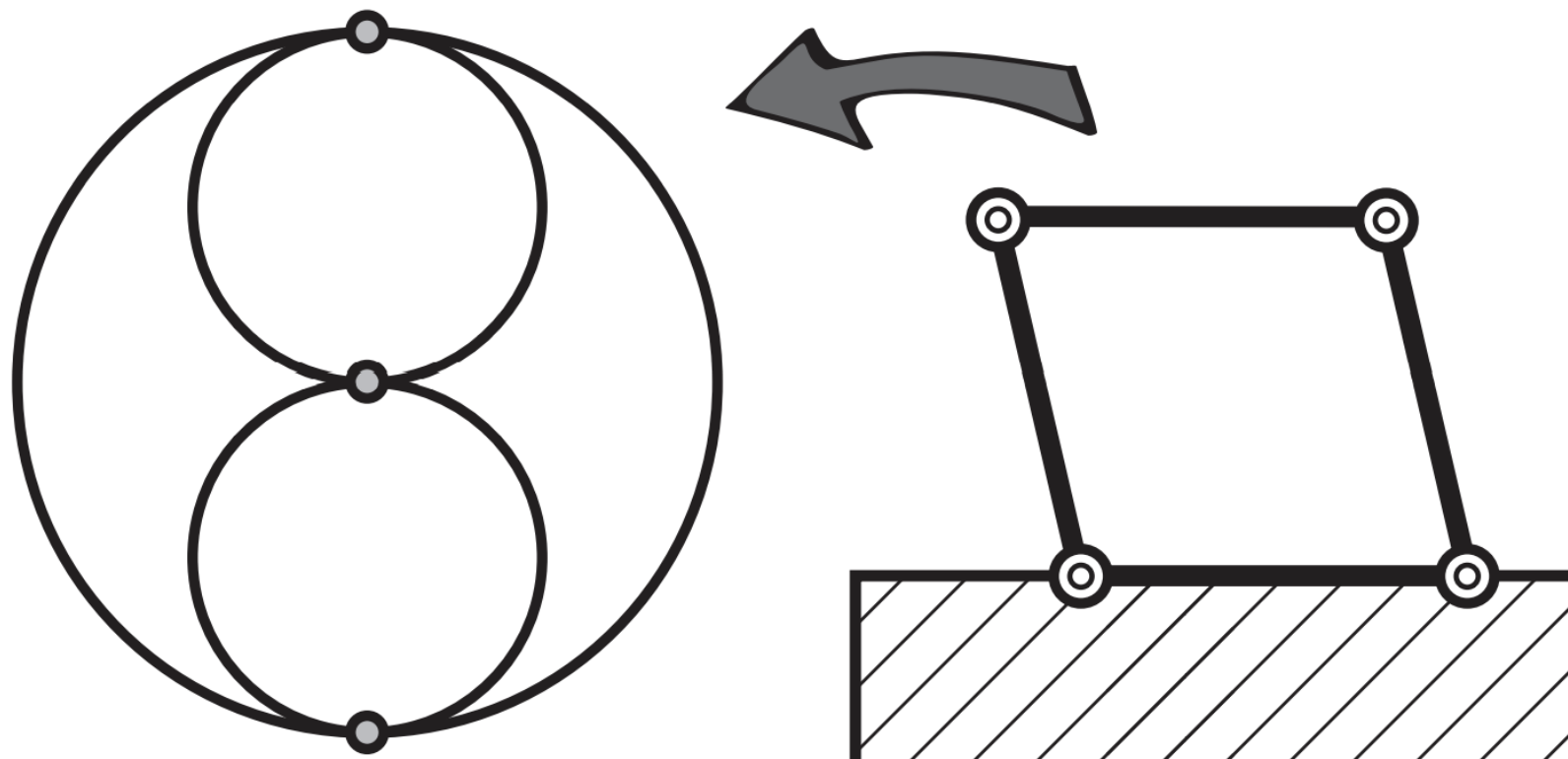




EXAMPLE

State diagram for
Tower of Hanoi

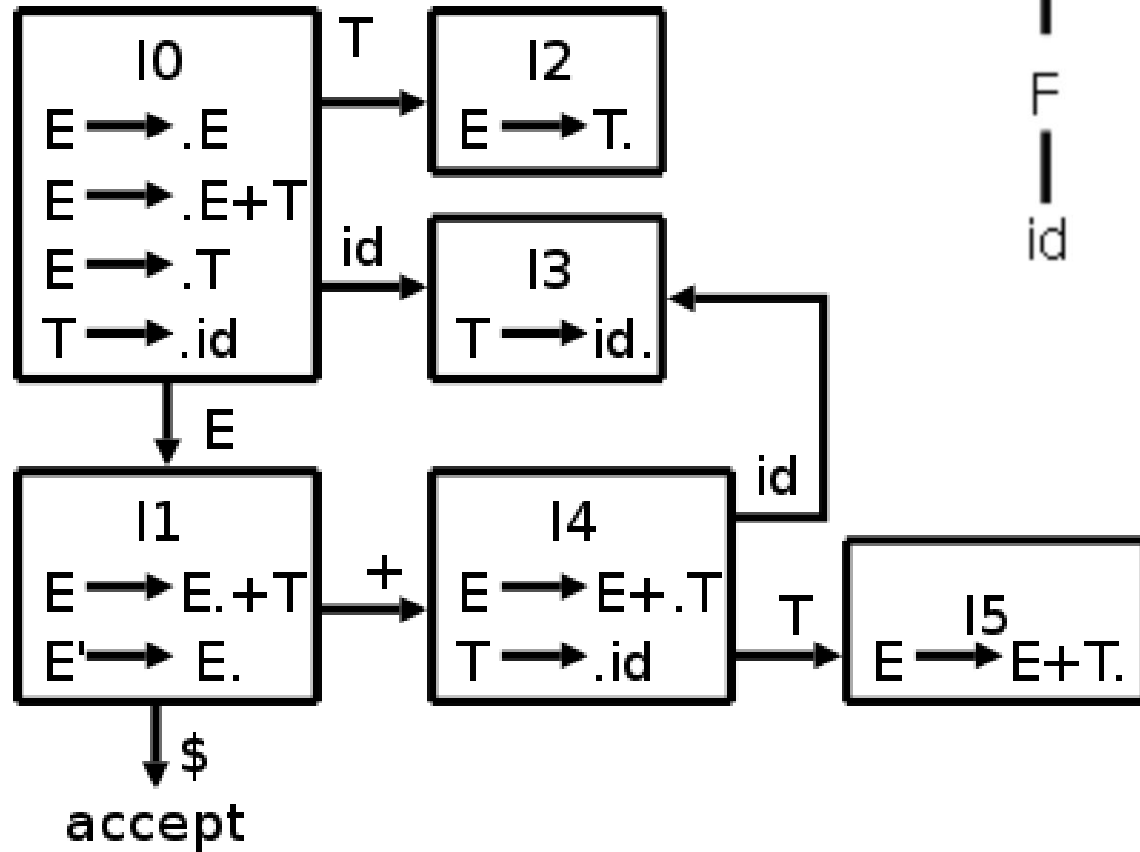




EXAMPLE

Configuration of
4-bar mechanism



$$\begin{aligned}
 E &\rightarrow E + T \mid T \\
 T &\rightarrow T * F \mid F \\
 F &\rightarrow (E) \mid id
 \end{aligned}$$


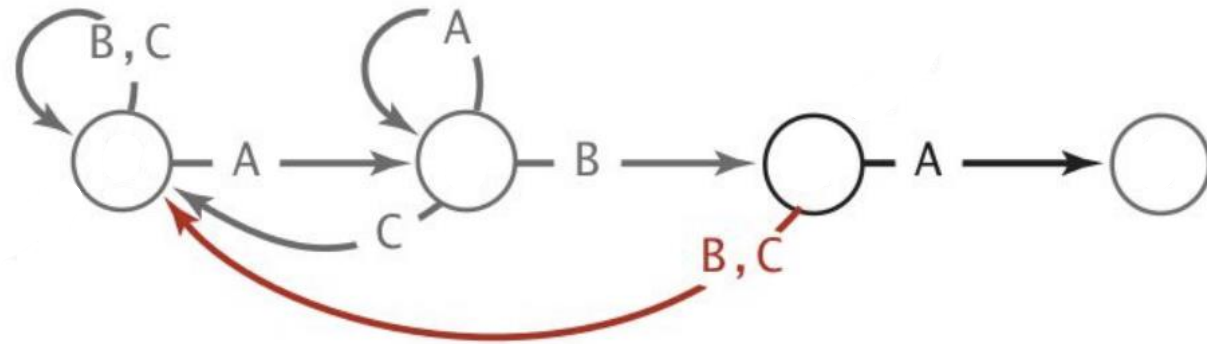
EXAMPLE

Parsing grammar



String = ACAABBABA

Pattern = ABA



EXAMPLE

Knuth-Morris-Pratt
substring-matching



More Jargon

neighbor / neighborhood

incident / adjacent

endpoint

vertex degree

isolated

regular

subgraph

induced subgraph



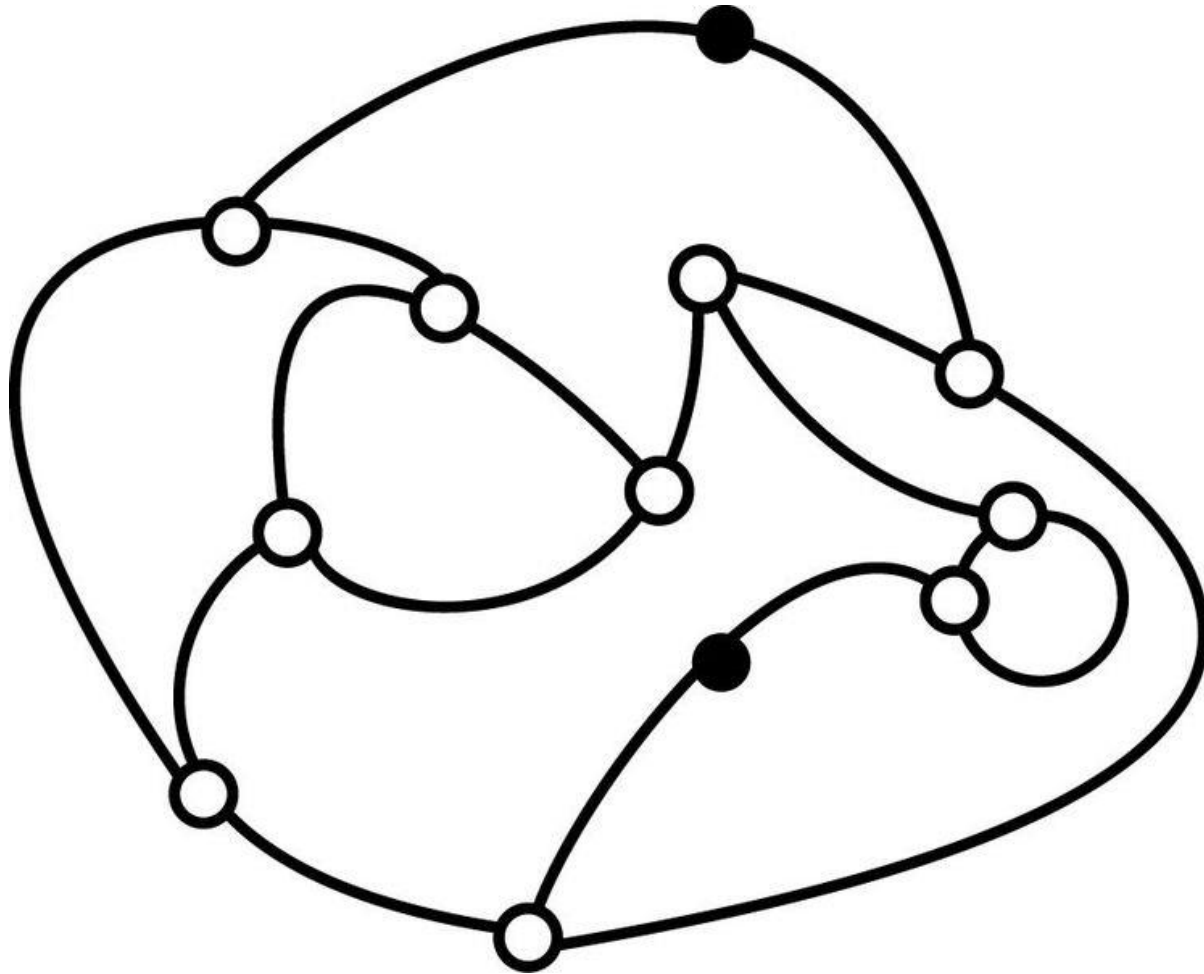
HANDSHAKE LEMMA

For any graph G ,

$$\sum_{v \text{ in } V(G)} \deg_G v = 2 |E(G)|$$

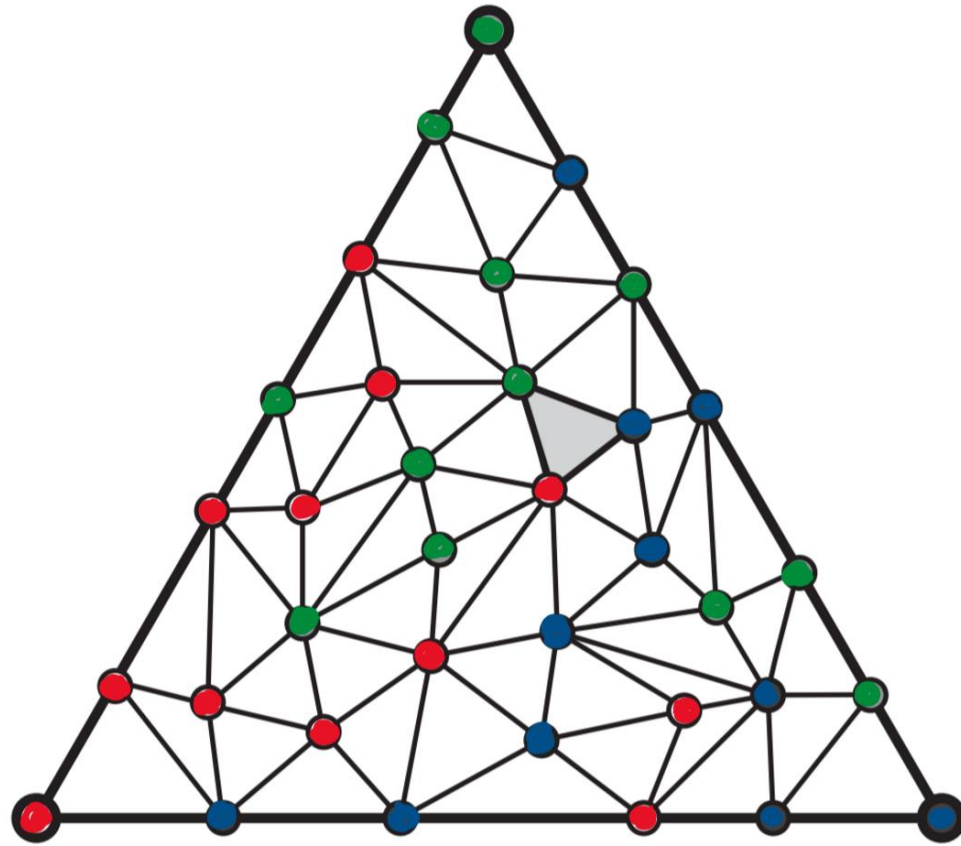


THERE ARE EVEN NUMBER OF ODD-DEGREE VERTICES.



COROLLARY

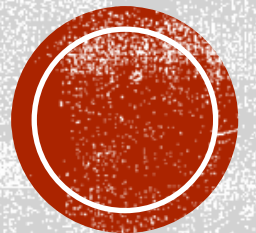




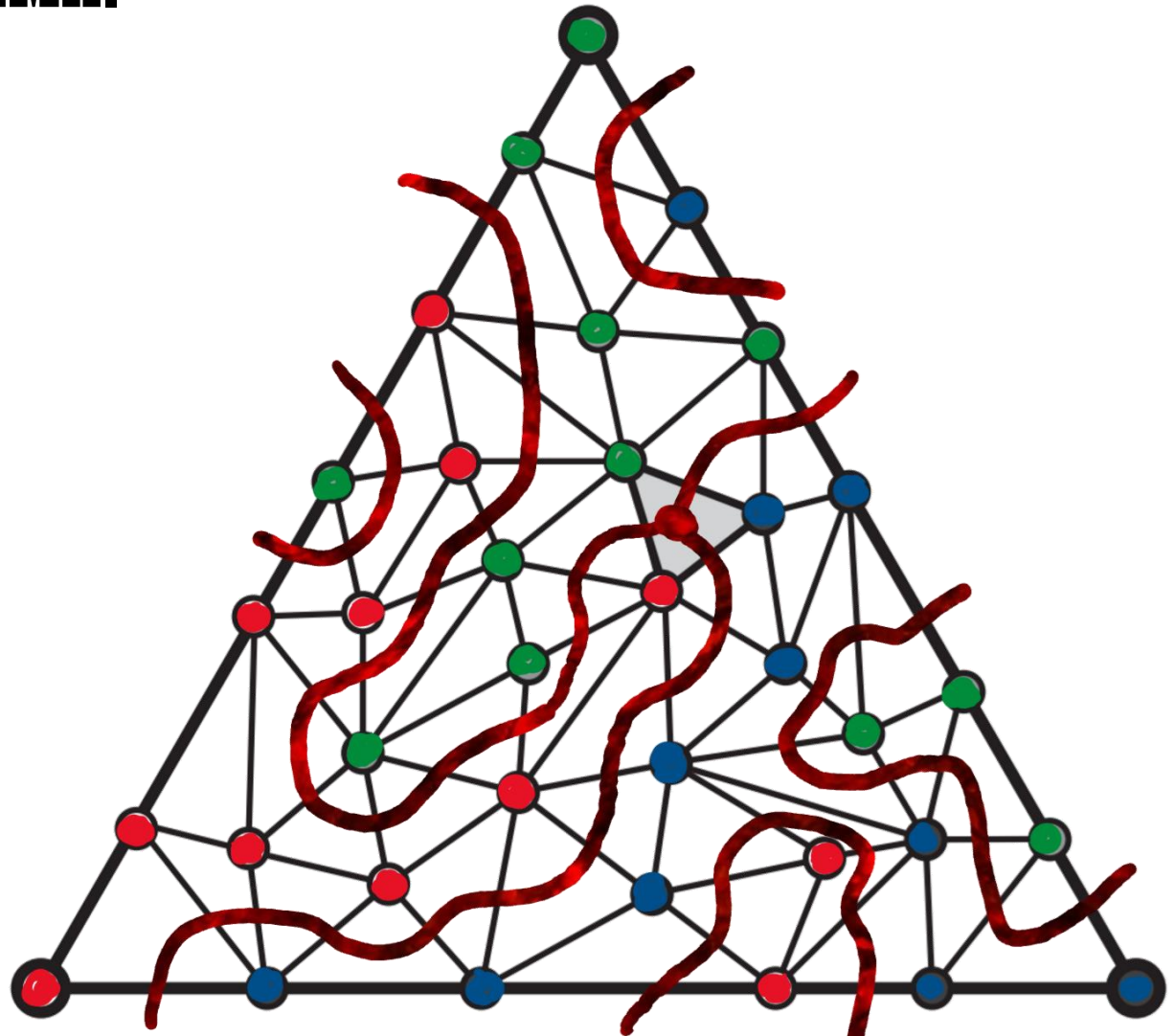
SPERNER'S LEMMA

[Sperner 1928] [Knaster–Kuratowski–Mazurkiewicz 1929]

Every properly 3-colored triangulation of Δ
contains a triangle with all the colors



PROOF OF SPERNER'S LEMMA.



SOME VERTEX HAS DEGREE AT MOST $2|E|/|V|$.

COROLLARY



My wife and I recently attended a party with four other married couples. Various handshakes took place. No one shook hands with one's own spouse, and no two people shook hands more than once. After all the handshakes were over, I asked everyone how many times they had shaken hands. To my surprise, each person gave a different answer. How many times did my wife shake hands?

NEXT TIME.
WALKS AND PATHS, WHATEVER-FIRST SEARCH

