DISCRETE MATHEMATICS

IN COMPUTER SCIENCE

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Logistics

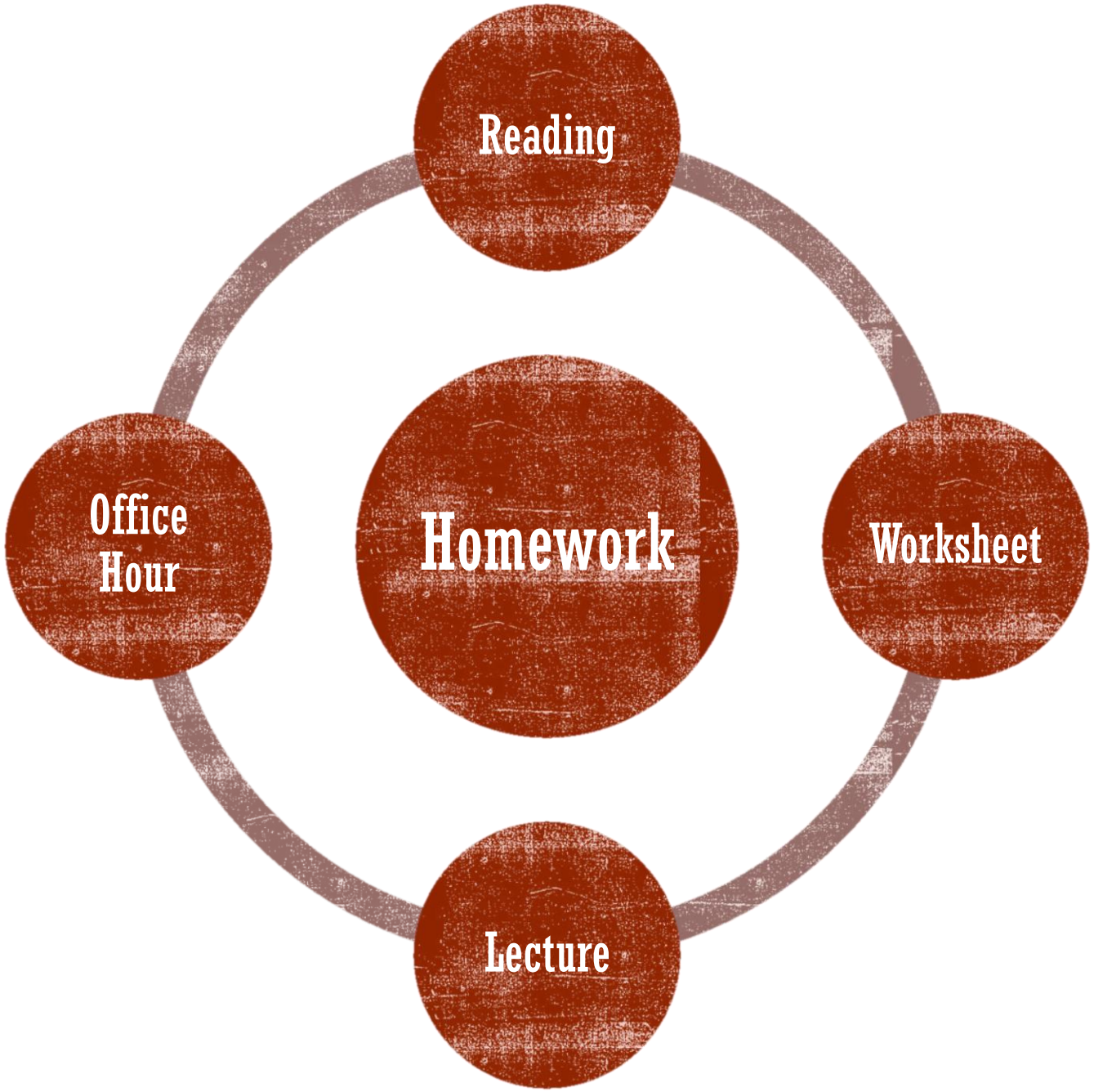
- When in doubt, search the syllabus
- Ask questions on Slack

- Homework 1 will be out today

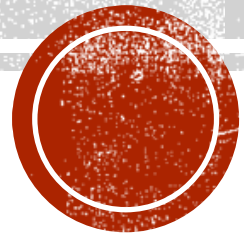
Study Units

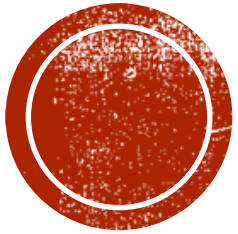
- 17 units in 3 modules
  - Proofs
  - Graphs
  - Combinatorics

- Open everything
- Collaboration encouraged

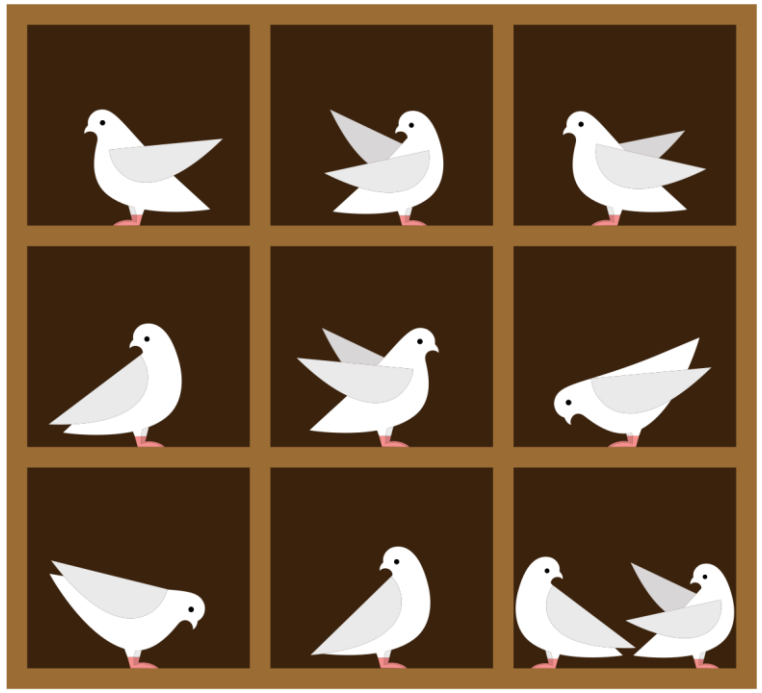


STOP ME IF YOU ARE LOST





Jargons and Pigeonhole Principle



Pigeonhole Principle

- For any integers c and k
- If there are more than $c \cdot k$ pigeons occupying k holes
- Then at least c+1 pigeons must be in the same hole

At least 3 people in this room born on the same day of the week

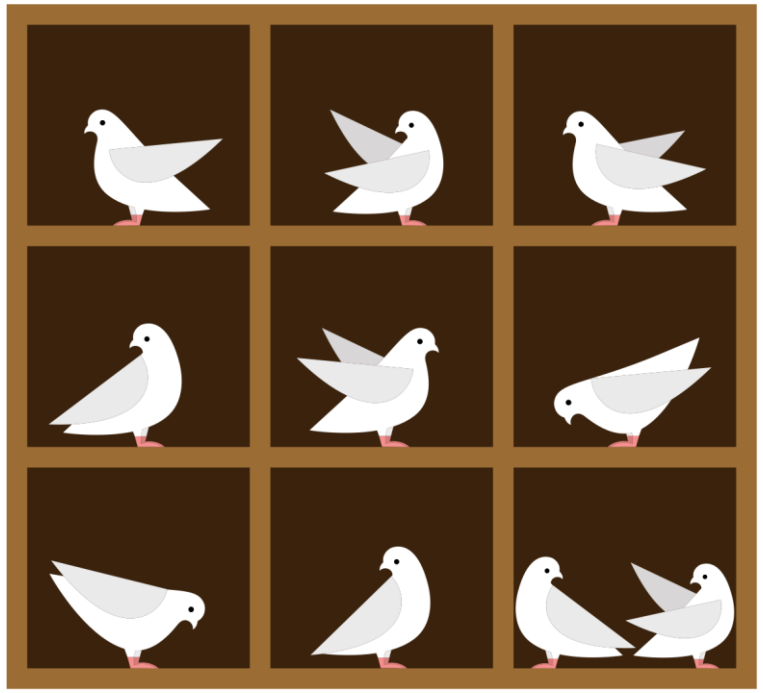
Jargon

set / element
roster/builder notation
belong / subset
cardinality

function
domain / codomain
range

Pick 16 numbers from 1 to 30.
Prove that there is at least a pair that adds up to 31.

Exercise



Pigeonhole Principle

- For any sets P, H, and integer c
- If $|P| > c \cdot |H|$
- Then any function from P to H maps at least c+1 elements in P to the same element in H

Type Checking

$$S = \{a, b, \{b\}\}$$

- $a \in S$ or $a \subseteq S$?
- $b \in S$ or $b \subseteq S$?
- $\{b\} \in S$ or $\{b\} \subseteq S$?
- $\{\} \in S$ or $\{\} \subseteq S$?

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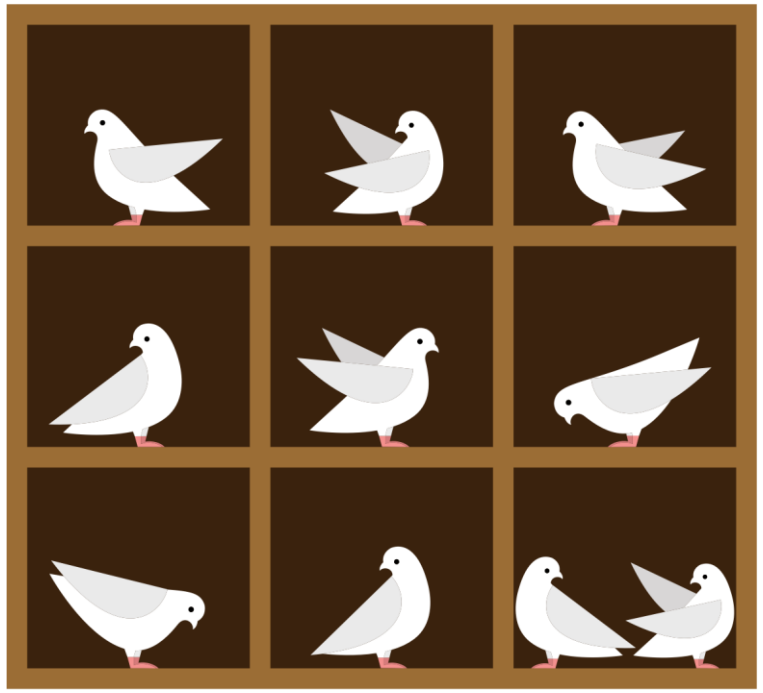
Deadly Sin!

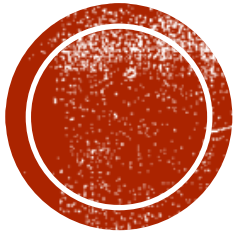
More Jargon

- Injection / Injective
- Surjection / Surjective
- Bijection / bijective



Pigeonhole Principle

- For any sets P and H
- If $|P| > |H|$
- Then any function from P to H is not injective



Application: Lossless Compression

Sequences and Strings

- Jargon
- product of sets
  union / intersection
- sequences
  alphabet
  strings

No All-Purpose Lossless Compression

- Compression scheme that shortens any input text doesn't exist.

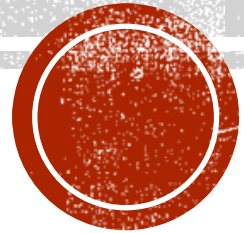
- Wait that's easy.

No All-Purpose Lossless Compression

- Every compression scheme that shortens some input text must lengthen some other input text.

Every compression scheme that shortens some input text must lengthen some other input text.

Yes this is a Language course



Next time.

How to make a statement?

Propositions and Logic.