DISCRETE MATHEMATICS

IN COMPUTER SCIENCE

HSIEN-CHIH CHANG

FEBRUARY 16, 2022

Administrivia

- Midterm 2
  - Feb 21 (Mon) 6–9PM
  - Carpenter 013 Herb West Lecture Hall
- Conflict Midterm 2
  - Feb 22 (Tue) 6–9PM
- SAS/Conflict Conflict/COVID
  - Come talk to me

- Closed-book written exam
- Scope: Module G on graphs
- One-page two-sided cheatsheet
  - Must be hand-written

New Module: Counting

Proof by Counting

Constructs sets and compare their sizes.

- Sum principle:
  - $|P_1 \sqcup P_2 \sqcup \ldots \sqcup P_k| = |P_1|+|P_2|+ \ldots +|P_k|$
- Product principle:
  - $|P_1 \times P_2 \times \ldots \times P_k| = |P_1|\times|P_2|\times \ldots \times|P_k|$

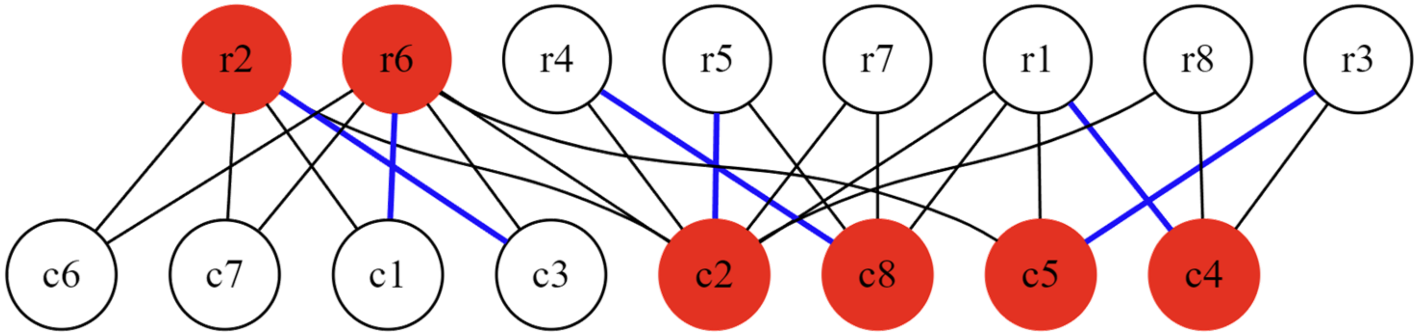
Reductio ad absurdum, John Pettie, 1884

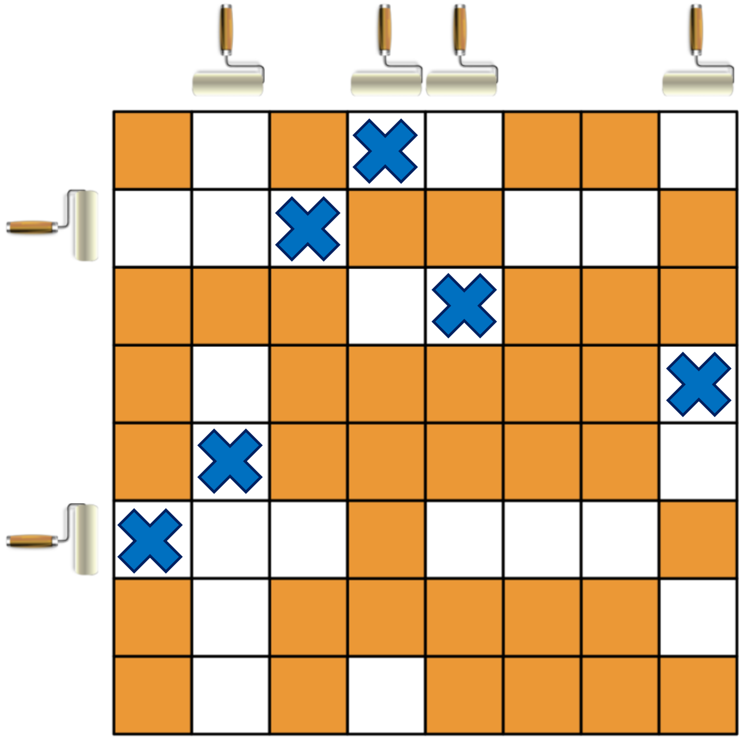
Example: Sequences

How many 9-bit binary strings are there whose first two bits are the same?

Example: Permutations

How many permutations of $\{1, 2, \ldots, 9\}$ are there?





Combinatorial Equivalence

If there is a bijection between set A and B, then

$$|A| = |B|$$

How many subsets of odd sizes does a size-n set have?

Combinatorial Equivalence

Combinatorial Equivalence

How many subsets of odd sizes does a size-n set have?

How many binary strings of length-n are there that have odd number of 1s?

Division Principle

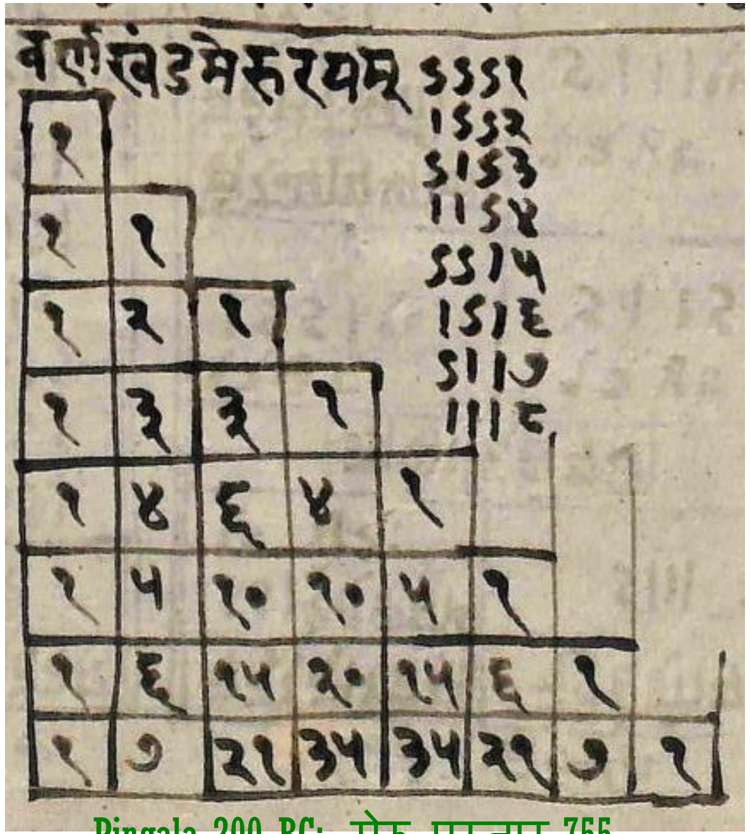
- If there is a k-to-1 mapping from S to P,
  - $|S|/k = |P|$

Reductio ad absurdum, John Pettie, 1884

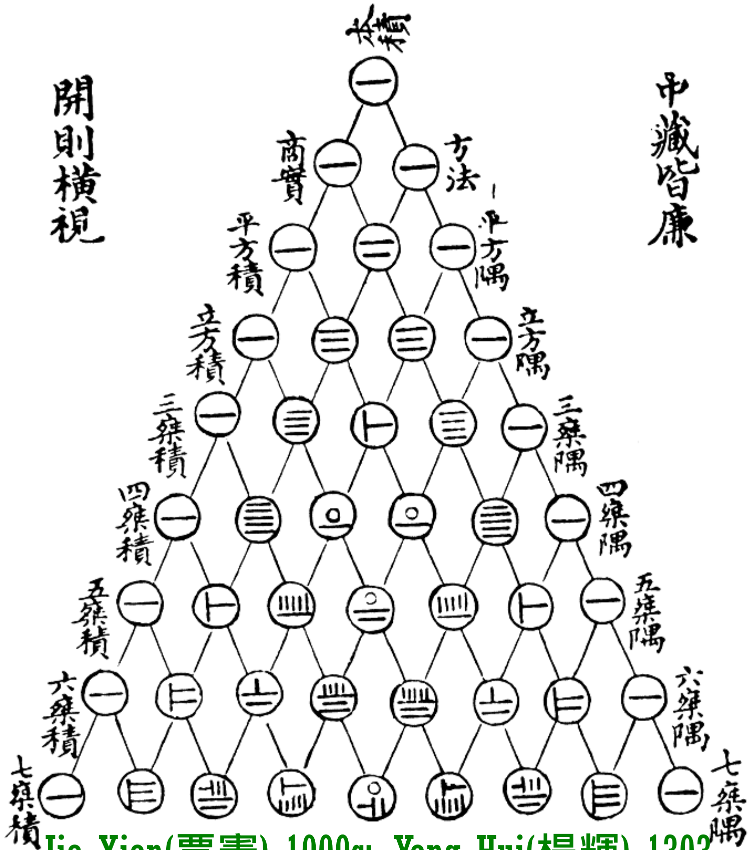
Example: Balls

How many ways to arrange 5 red balls, 4 blue balls, and 3 green balls in a sequence?

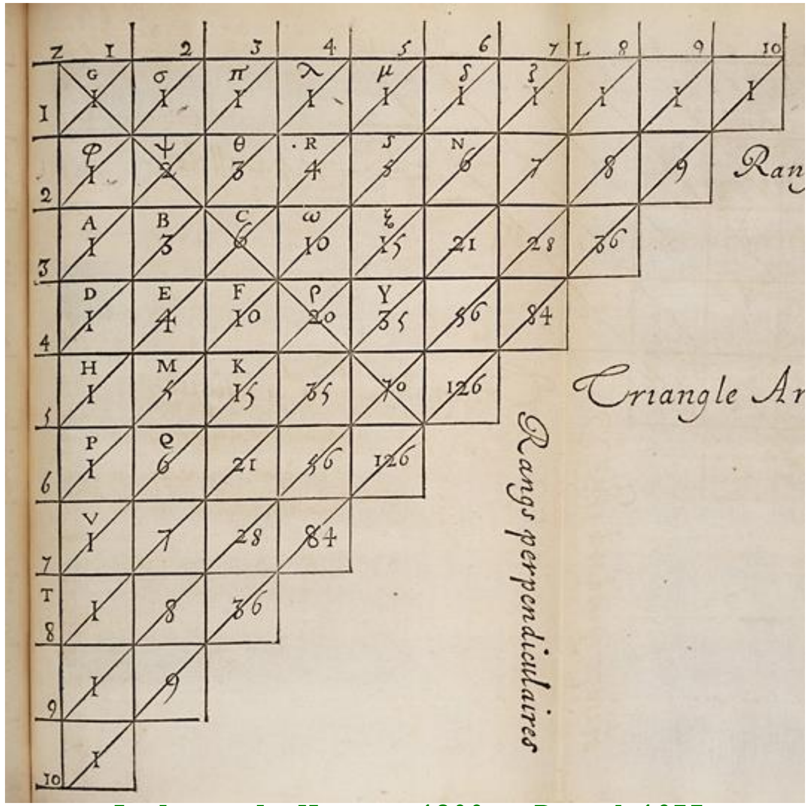
Binomial Coefficient



Pingala 200 BC; मेरु प्रस्तार 755



Jia Xian(賈憲) 1000s; Yang Hui(楊輝) 1303



Jordanus de Nemore 1200s; Pascal 1655

Binomial Coefficient

$\binom{n}{k}$:= #ways to choose size-k subset from {1, ..., n}

How many ways to choose size-k subset from {1, ..., n}?

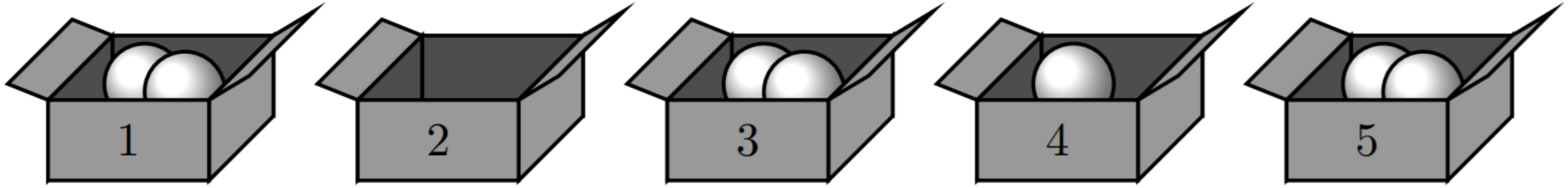
Example: Subsets

Example: Strings

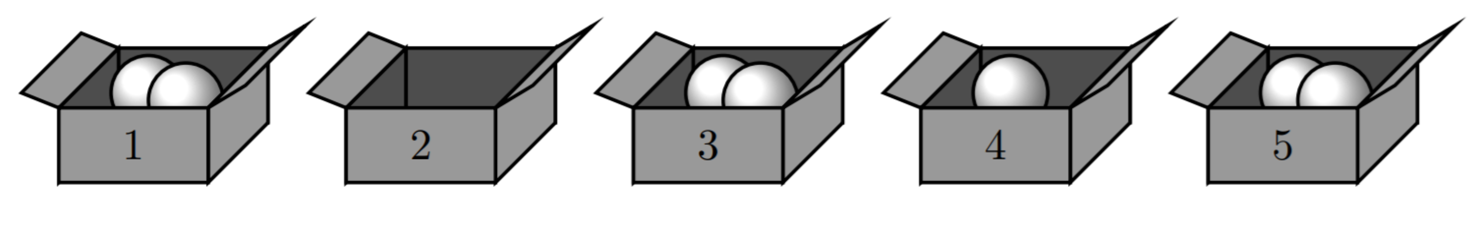
How many 9-bit binary strings are there with at least 2 ones?

Example: Balls and Boxes

How many ways to put 7 unlabeled balls into 5 distinct boxes?



Four-Fold Formula



- To put k things into n distinct and ordered boxes:

	Labeled	Not Labeled
Repetition	n^k	$\binom{n+k-1}{k}$
No Repetition	$\frac{n!}{(n-k)!}$	$\binom{n}{k}$

Tips: Don't over-/under-count!

Next time.

Counting smartly.