DISCRETE MATHEMATICS

IN COMPUTER SCIENCE

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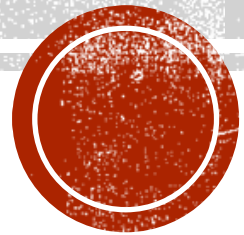
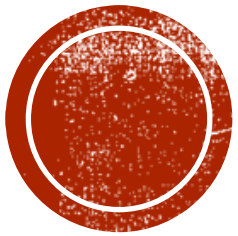
Logistics

- Homework 1 due this Friday
- No work session next Monday (MLK day)

STOP ME IF YOU ARE LOST





Logic

Jargon

proposition
all Boolean operators
implies
if and only if

predicate
for all / exists
negation of quantifiers

Poker Cards

"Implication" Paradox

- If [good thing] had happened, I would be happy.
- If [bad thing] had happened, I would be happy.

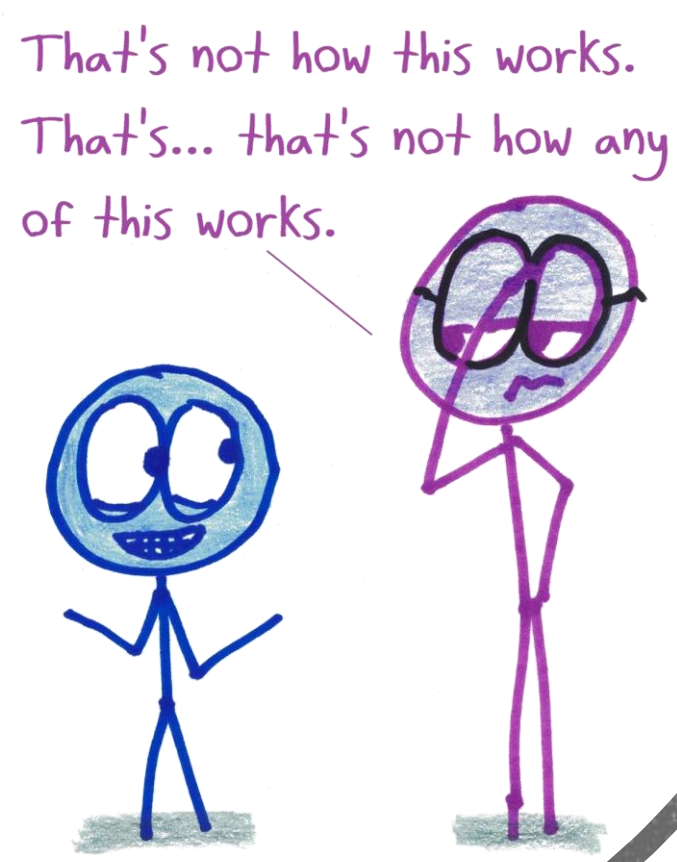
- This is useful when "for all" quantifier presents
  - For all even number n, n^2 is also an even number

suppose f(x) = y

INVERSE FUNCTION: g(y) = x

CONVERSE FUNCTION: f(y) = x

CONTRAPOSITIVE FUNCTION:
g(not y) = not x

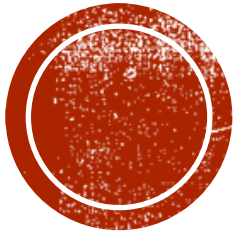


Contrapositives

P implies Q

is equivalent to

not Q implies not P



Proof by Contradiction

Assuming what you said is false, then …

Proof by Contradiction

To prove P implies Q:

Assume P is TRUE but Q is FALSE,
and derive a contradiction.

(P and not Q) implies FALSE

Reductio ad absurdum, John Pettie, 1884

If a sequence of parentheses is balanced, then it must start with '('.

Exercise

If real number x is not a fraction, then $\sqrt{x}$ is not a fraction.

Exercise

There are infinitely many primes.

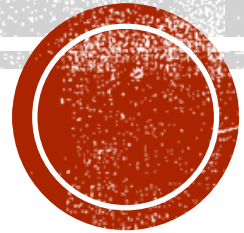
Exercise

There is no smallest positive fraction.

Exercise

Application: Limitation of Computers

Are there problems that Computers cannot solve?



Halting Problem

- Input: source-code ⟨C⟩ and test-input x
- Output: Yes if C(x) halts; No if C(x) runs forever

No All-Purpose Infinite-Loop Detector

Halting problem cannot be solved by any computer.

A chess player may offer the sacrifice of a pawn or even a piece, but a mathematician offers the game.

— G. H. Hardy, "A Mathematician's Apology", 1940

Next time.
More proof techniques.