DISCRETE MATHEMATICS

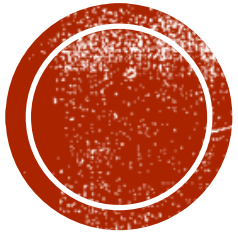
IN COMPUTER SCIENCE

Hsien-Chih Chang

January 14, 2022

Administrivia

- Homework 1 due today
- Homework 2 out today
- No work session next Monday (MLK day)



Various Proof Techniques

Proof by If and Only If

To prove P if and only if Q:

(P implies Q) and (Q implies P)

(P implies Q) and (¬P implies ¬Q)

Reductio ad absurdum, John Pettie, 1884

Exercise

A set of points X in the plane are in convex position if and only if no 3 points form a triangle that contains another point.

Proof by Cases

Split P into cases that are exhaustive:

P if and only if $P_1, P_2, \ldots, P_k$

($P_1, P_2, \ldots, P_k$ is TRUE)

implies (P is TRUE)

Reductio ad absurdum, John Pettie, 1884

Every 5 points in the plane must contain a convex quadrilateral/4-gon.

Happy Ending Problem

Proof by Counting

Constructs sets and compare their sizes.

- Sum principle:
  - $|P_1 \sqcup P_2 \sqcup \ldots \sqcup P_k| = |P_1| + |P_2| + \ldots + |P_k|$
- Product principle:
  - $|P_1 \times P_2 \times \ldots \times P_k| = |P_1| \times |P_2| \times \ldots \times |P_k|$

Reductio ad absurdum, John Pettie, 1884

Exercise

BubbleSort correctly sorts an array A[0..n-1] in at most n^2 steps.

```
BubbleSort(A[0..n-1]):
    for i from 1 to n-1:
        for j from 1 to n-1:
            if A[j-1] > A[j]:
                swap A[j-1] and A[j]
    return A[]
```

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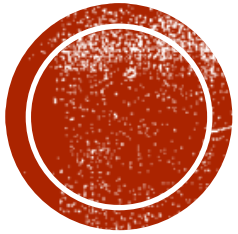
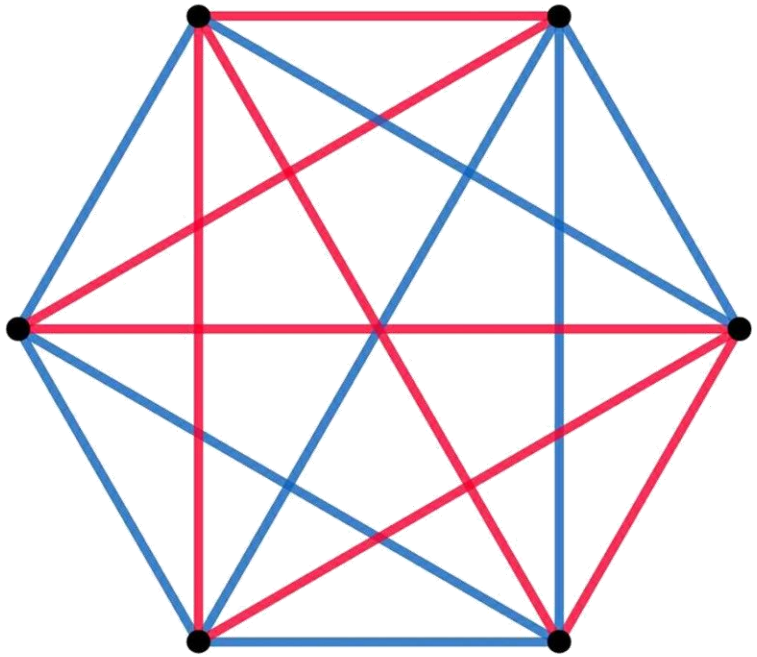


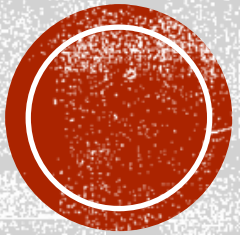
Application



Friendship Theorem.

Every party of 6 must have either 3 people who are friends, or 3 people who are strangers with each other

How many people do you need to guarantee a group of mutual friends/strangers of 4?

Next time.

Lifeline of a Computer Scientist: Induction.