main

March 22, 2021

Training a Neural Network to Recognize Digits

Import Dependencies

[1]:
```
import os

import numpy as np
import pandas as pd

import digits as dg
import net as nn
```

Loading Digits from Files

[2]:
```
DATA_PATH = os.path.join('..', 'data', 'cache')

allDigits = dg.getDigits()
trainDigits = dg.getDigits(
    kinds={'normal', 'normal-klein', 'digital', 'digital-klein'})
testDigits = dg.getDigits(kinds={'evag'})

DIGITS = {
    'all': dg.extractInputAndOutput(allDigits),
    'training': dg.extractInputAndOutput(trainDigits),
    'test': dg.extractInputAndOutput(testDigits),
}
```

Initialize Networks with Different Number of Hidden Neurons

[3]:
```
# This calculation might take a while.
# See below, how to load the results from cache instead

# parameters
numOfHiddens = {5, 10, 15, 20, 25, 30, 35, 40, 50, 60, 70, 80, 90}
numOfNets = 100

input = DIGITS['all']['input']
output = DIGITS['all']['output']
```

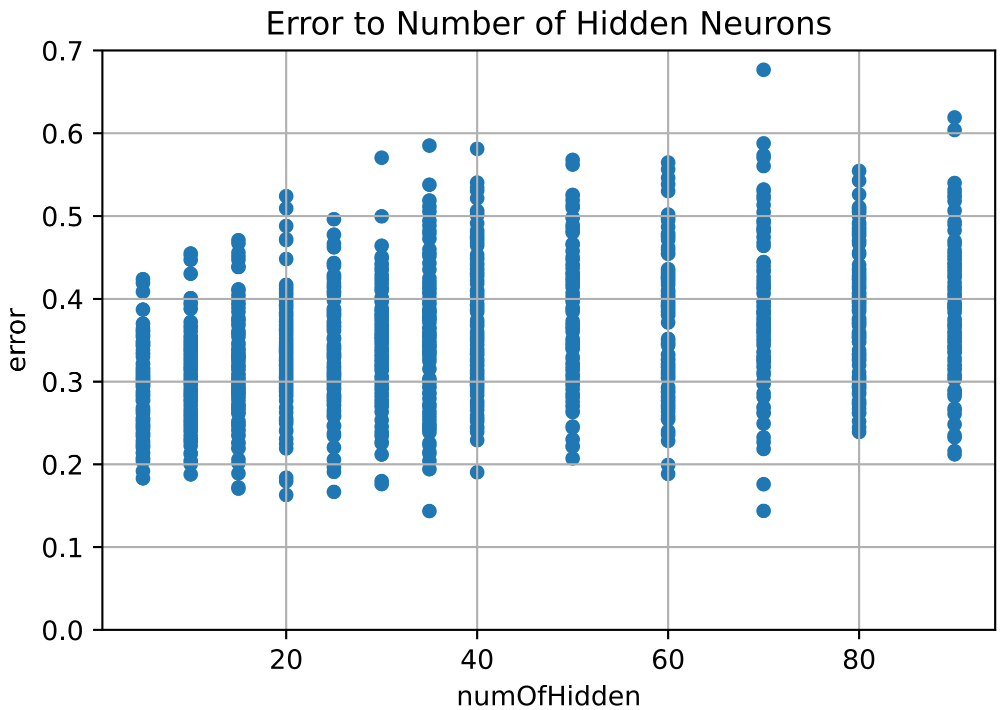
```python
initNets = {
    'net': [],
    'numOfHidden': [],
    'error': [],
}

for numOfHidden in numOfHiddens:  # generate nets for each number of hidden␣
↪neurons
    # seed the random number generator to create reproduceable results
    np.random.seed(0)

    for _ in range(numOfNets):  # generate several nets of the same topology
        net = nn.init(35, numOfHidden, 10)
        error = nn.calcBatchError(net, input, output)

        initNets['net'].append(net)
        initNets['numOfHidden'].append(numOfHidden)
        initNets['error'].append(error)

INIT_NETS = pd.DataFrame(initNets)
```

[4]:
```python
# store initial networks to cache
INIT_NETS.to_pickle(os.path.join(DATA_PATH, 'init-nets.pkl'))
```

[5]:
```python
# load initial networks from cache
INIT_NETS = pd.read_pickle(os.path.join(DATA_PATH, 'init-nets.pkl'))
```

[6]:
```python
# show error values for the initial networks by topology
INIT_NETS.plot.scatter(
    'numOfHidden', 'error',
    title='Error to Number of Hidden Neurons',
    ylim=(0, 0.7), grid=True,
)
```

[6]:
```
<AxesSubplot:title={'center':'Error to Number of Hidden Neurons'},
xlabel='numOfHidden', ylabel='error'>
```



[7]:
```
# show nets with smallest error value
INIT_NETS.groupby('numOfHidden').min('error').sort_values('error')
```

[7]:

	error
numOfHidden	
35	0.143571
70	0.143792
20	0.162996
25	0.166742
15	0.170721
30	0.176098
5	0.183089
10	0.187878
60	0.188517
40	0.190416
50	0.207182
90	0.212117
80	0.239134

Training Promising Networks

[8]:
```
# This calculation might take a while.
# See below, how to load the results from cache instead
```

```python
# parameters
numOfHiddens = {35, 70, 20, 25, 15, 30}
epoches = 1000
learnRate = 0.1

inputTrain = DIGITS['training']['input']
outputTrain = DIGITS['training']['output']

inputTest = DIGITS['test']['input']
outputTest = DIGITS['test']['output']

trainHistory = {
    'net': [],
    'numOfHidden': [],
    'errorTrain': [],
    'errorTest': [],
}

for numOfHidden in numOfHiddens:  # do training for all promising nets
    net = INIT_NETS[INIT_NETS.numOfHidden == numOfHidden].sort_values(
        'error').iloc[0].net

    for i in range(epoches):  # train for several epochs
        net = nn.trainBatch(net, inputTrain, outputTrain, learnRate)
        errorTrain = nn.calcBatchError(net, inputTrain, outputTrain)
        errorTest = nn.calcBatchError(net, inputTest, outputTest)

        trainHistory['net'].append(net)
        trainHistory['numOfHidden'].append(numOfHidden)
        trainHistory['errorTrain'].append(errorTrain)
        trainHistory['errorTest'].append(errorTest)

TRAIN_HIDDEN = pd.DataFrame(trainHistory)
```

[9]:
```python
# store trained promising networks to cache
TRAIN_HIDDEN.to_pickle(os.path.join(DATA_PATH, 'train-hidden.pkl'))
```

[10]:
```python
# load trained promising networks from cache
TRAIN_HIDDEN = pd.read_pickle(os.path.join(DATA_PATH, 'train-hidden.pkl'))
```

[11]:
```python
# calculate which net's error was reduced the most
end = TRAIN_HIDDEN.groupby('numOfHidden').min(
    'errorTrain').sort_values('numOfHidden')
start = INIT_NETS.groupby('numOfHidden').min(
    'error').sort_values('numOfHidden')

diff = end.join(start)
```

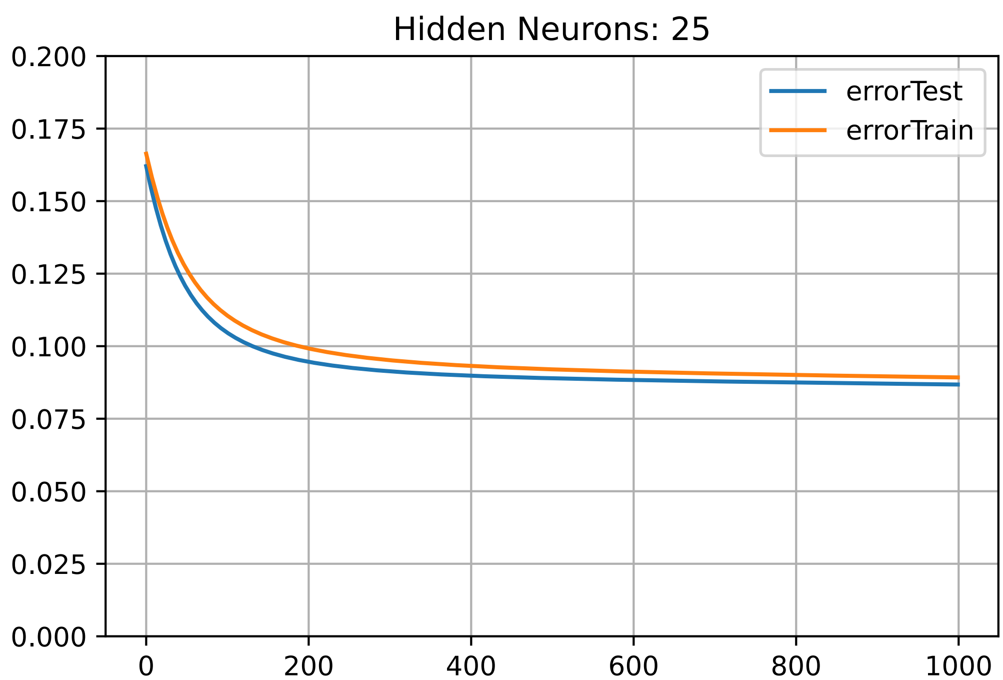
```
diff.rename(columns={'error': 'errorStart'}, inplace=True)
diff['diff'] = diff.errorStart - diff.errorTrain

diff.sort_values('diff', ascending=False)
```

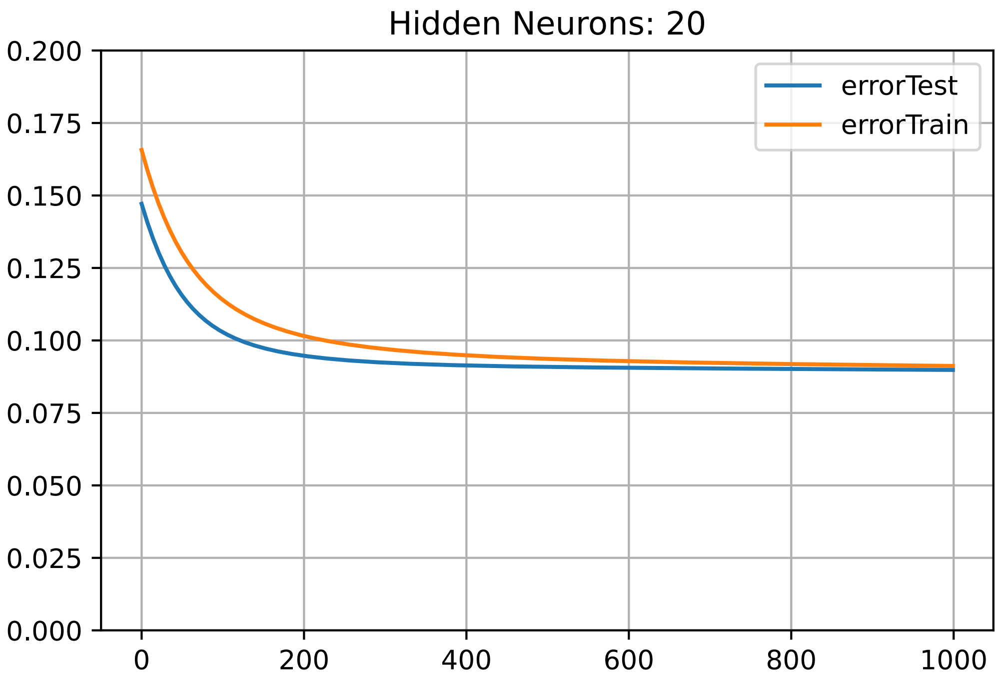
[11]:

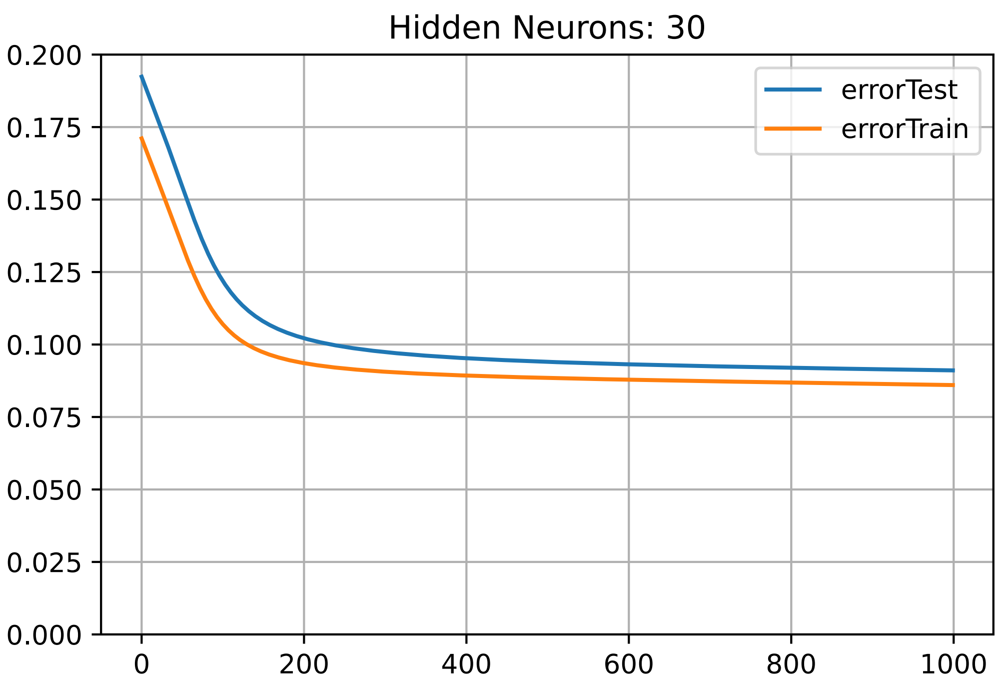
	errorTrain	errorTest	errorStart	diff
numOfHidden				
30	0.086010	0.091076	0.176098	0.090088
15	0.086051	0.086862	0.170721	0.084670
25	0.089231	0.086783	0.166742	0.077511
20	0.091201	0.089823	0.162996	0.071795
35	0.084060	0.086994	0.143571	0.059512
70	0.088598	0.091938	0.143792	0.055195

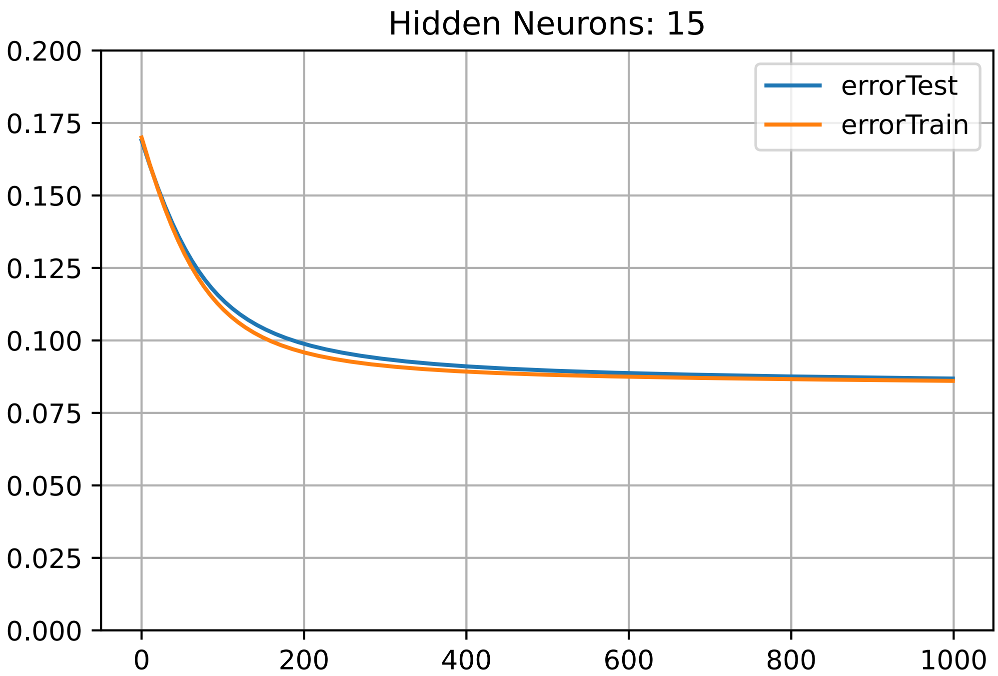
[12]:

```
# show the development of error values during training
for numOfHidden in {15, 20, 25, 30}:
    TRAIN_HIDDEN[TRAIN_HIDDEN.numOfHidden == numOfHidden].plot.line(
        title=f'Hidden Neurons: {numOfHidden}',
        y={'errorTrain', 'errorTest'}, ylim=(0, 0.2),
        use_index=False, grid=True,
    )
```




Hidden Neurons: 20
0.200
0.175
0.150
0.125
0.100
0.075
0.050
0.025
0.000
0
200
400
600
800
1000
errorTest
errorTrain


Hidden Neurons: 30
0.200
0.175
0.150
0.125
0.100
0.075
0.050
0.025
0.000
0
200
400
600
800
1000
errorTest
errorTrain



Finding the Optimal Learning Rate

[13]:
```
# This calculation might take a while.
# See below, how to load the results from cache instead

# parameters
numOfHidden = 15
learnRates = {1, 0.1, 0.01, 0.001}
epoches = 10000

startNet = INIT_NETS[INIT_NETS.numOfHidden == numOfHidden].sort_values(
    'error').iloc[0].net

inputTrain = DIGITS['training']['input']
outputTrain = DIGITS['training']['output']

inputTest = DIGITS['test']['input']
outputTest = DIGITS['test']['output']

trainHistory = {
    'net': [],
    'learnRate': [],
    'errorTrain': [],
    'errorTest': [],
}
```

```python
for learnRate in learnRates:  # do training for all learning rates
    net = startNet

    for i in range(epoches):  # do training for several epochs
        net = nn.trainBatch(net, inputTrain, outputTrain, learnRate)
        errorTrain = nn.calcBatchError(net, inputTrain, outputTrain)
        errorTest = nn.calcBatchError(net, inputTest, outputTest)

        trainHistory['net'].append(net)
        trainHistory['learnRate'].append(learnRate)
        trainHistory['errorTrain'].append(errorTrain)
        trainHistory['errorTest'].append(errorTest)

TRAIN_LR = pd.DataFrame(trainHistory)
```

[14]:

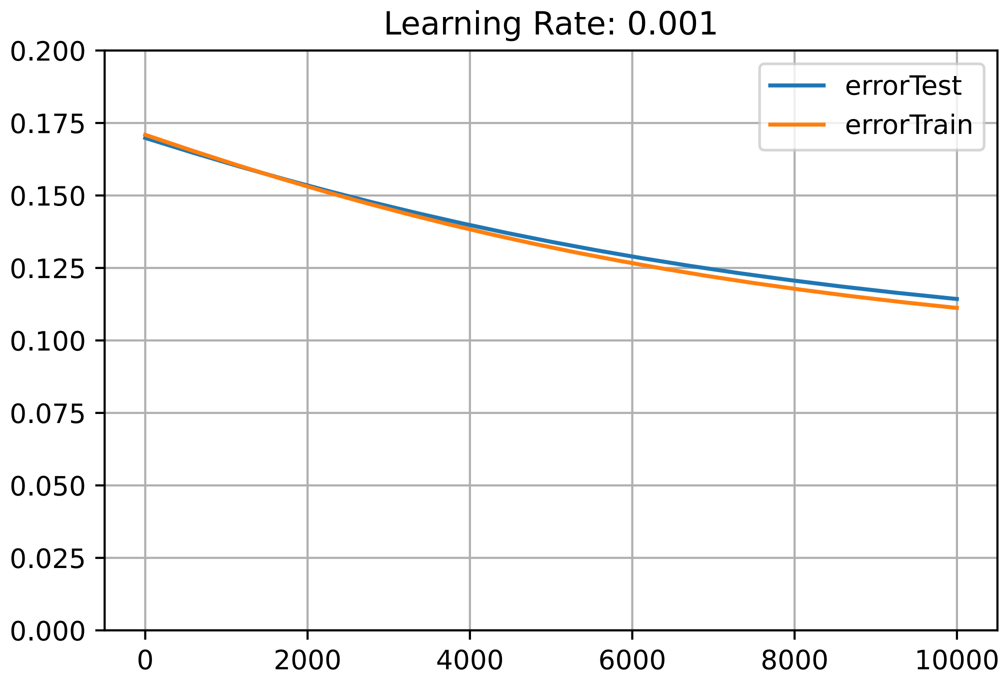
```python
# store training with different learning rates to cache
TRAIN_LR.to_pickle(os.path.join(DATA_PATH, 'train-learn-rate.pkl'))
```

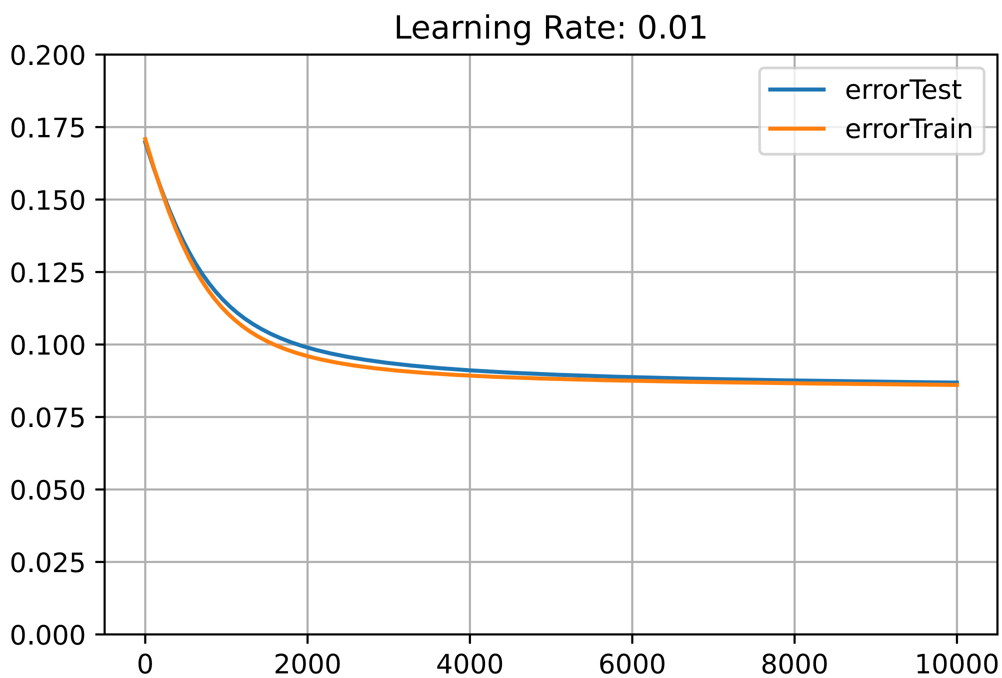
[15]:

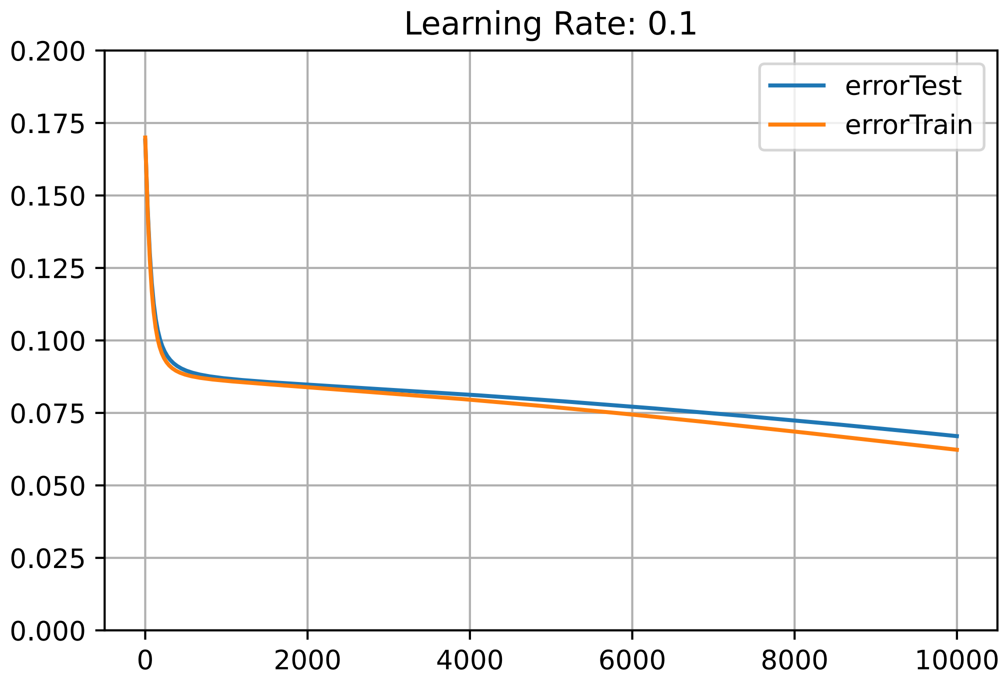
```python
# load training with different learning rates from cache
TRAIN_LR = pd.read_pickle(os.path.join(DATA_PATH, 'train-learn-rate.pkl'))
```

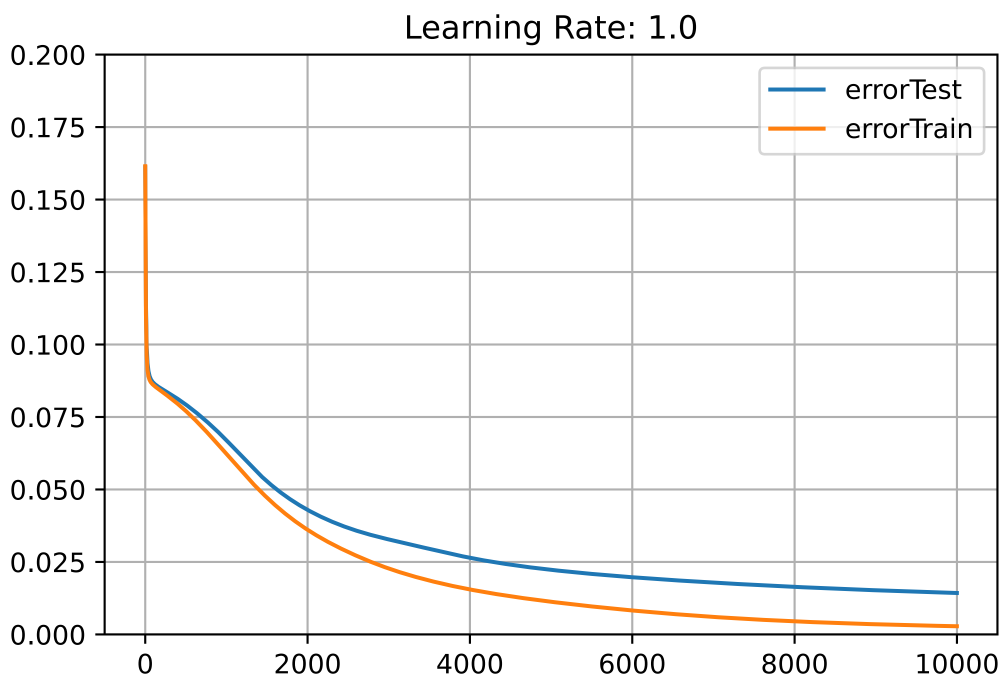
[16]:

```python
# show development of error during training for different learning rates
for learnRate, df in TRAIN_LR.groupby('learnRate'):
    df.plot.line(
        title=f'Learning Rate: {learnRate}',
        y={'errorTrain', 'errorTest'}, ylim=(0, 0.2),
        use_index=False, grid=True,
    )
```


Learning Rate: 0.001
errorTest
errorTrain
0.200
0.175
0.150
0.125
0.100
0.075
0.050
0.025
0.000
0
2000
4000
6000
8000
10000


Learning Rate: 0.01
errorTest
errorTrain
0.200
0.175
0.150
0.125
0.100
0.075
0.050
0.025
0.000
0
2000
4000
6000
8000
10000





[17]:
```
# show how high the remaining error values ares
TRAIN_LR.groupby('learnRate').min('errorTrain').sort_values('errorTrain')
```

[17]:

learnRate	errorTrain	errorTest
1.000	0.002821	0.014294
0.100	0.062284	0.066992
0.010	0.086051	0.086864
0.001	0.111204	0.114295

Final Training of the Network

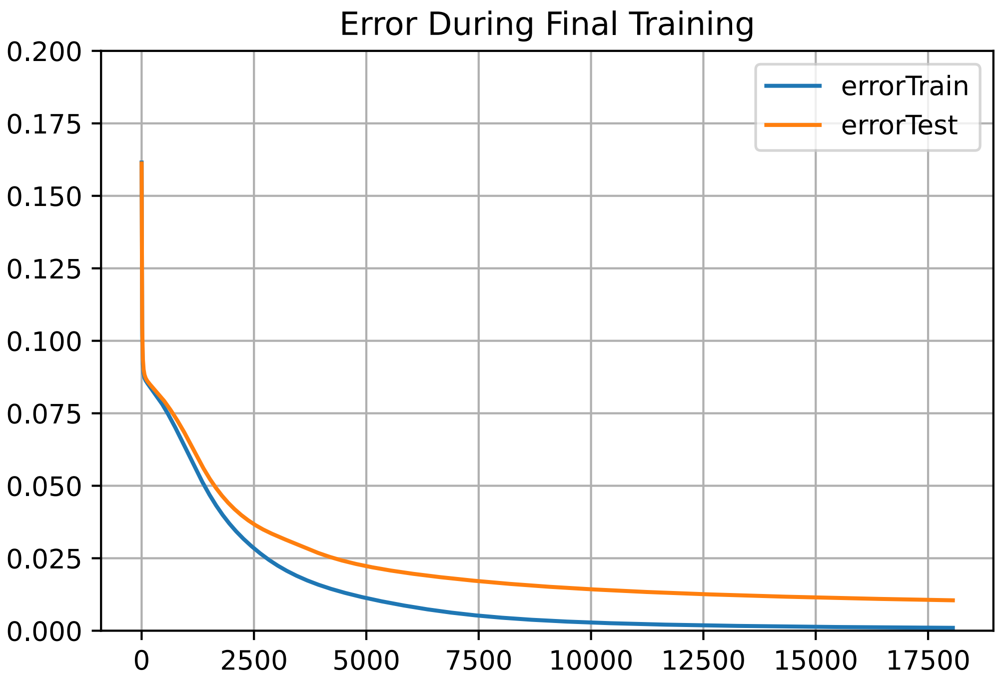
[18]:
```
# This calculation might take a while.
# See below, how to load the results from cache instead

# parameters
learnRate = 1
targetError = 0.001

inputTrain = DIGITS['training']['input']
outputTrain = DIGITS['training']['output']

inputTest = DIGITS['test']['input']
outputTest = DIGITS['test']['output']

# reuse all the training steps from before, when learning rate was determined
trainHistory = TRAIN_LR[TRAIN_LR.learnRate == learnRate].drop('learnRate', 1)
net = trainHistory.iloc[-1].net
error = nn.calcBatchError(net, inputTrain, outputTrain)

while error > targetError:  # train as long as the error is too high
    net = nn.trainBatch(net, inputTrain, outputTrain, learnRate)
    errorTrain = nn.calcBatchError(net, inputTrain, outputTrain)
    errorTest = nn.calcBatchError(net, inputTest, outputTest)

    trainHistory = trainHistory.append({
        'net': net,
        'errorTrain': errorTrain,
        'errorTest': errorTest,
    }, ignore_index=True)

    error = errorTrain

TRAIN_FINAL = trainHistory
```

[19]:
```
# store final training round to cache
TRAIN_FINAL.to_pickle(os.path.join(DATA_PATH, 'train-final.pkl'))
```

[20]:
```
# load final training round from cache
TRAIN_FINAL = pd.read_pickle(os.path.join(DATA_PATH, 'train-final.pkl'))
```

[21]:
```
# show the development of the error during training the final network
TRAIN_FINAL.plot.line(
    title='Error During Final Training',
    ylim=(0, 0.2), grid=True,
)
```

[21]:
```
<AxesSubplot:title={'center':'Error During Final Training'}>
```



[22]:
```
# show how many epochs there were in the end
TRAIN_FINAL.describe()
```

[22]:

	errorTrain	errorTest
count	18052.000000	18052.000000
mean	0.012627	0.022609
std	0.019830	0.017714
min	0.001000	0.010490
25%	0.001604	0.012087
50%	0.003481	0.015205
75%	0.013109	0.024004
max	0.161496	0.161170

Analyzing the Final Neural Network

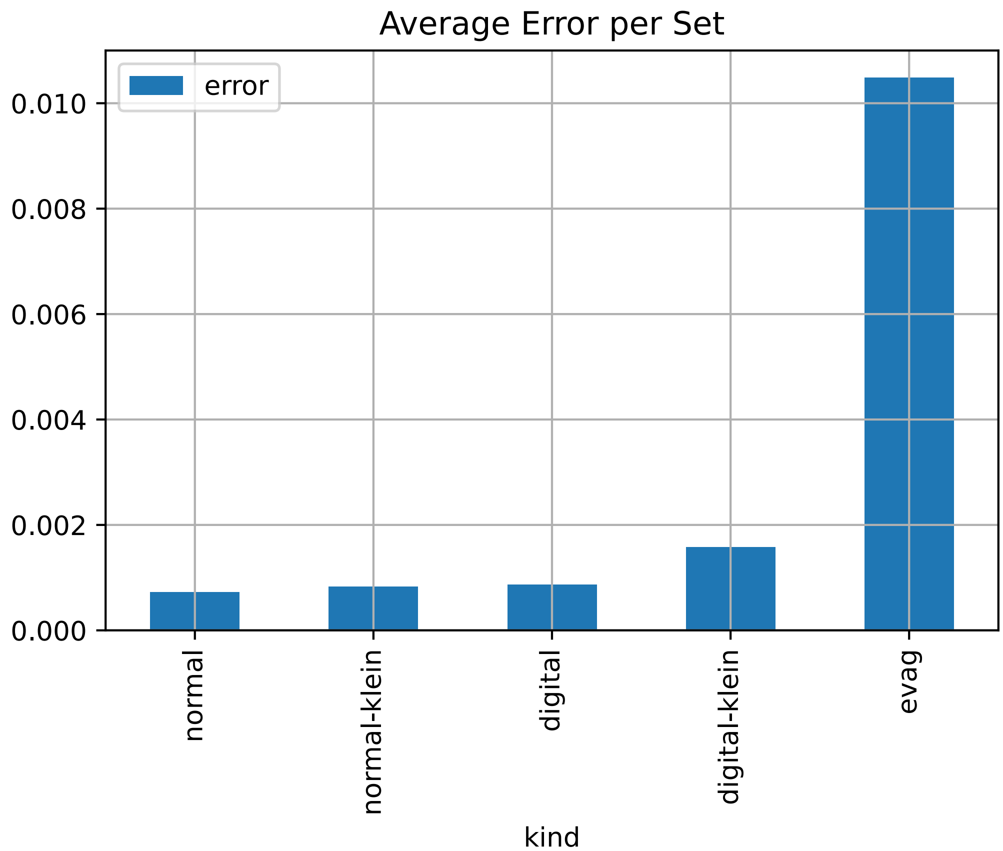
[23]:
```
FINAL_NET = TRAIN_FINAL.iloc[-1].net
```

[24]:
```python
# store final network to cache
nn.save(FINAL_NET, os.path.join(DATA_PATH, 'final-net.pkl'))
```

[25]:
```python
# load final network from cache
FINAL_NET = nn.load(os.path.join(DATA_PATH, 'final-net.pkl'))
```

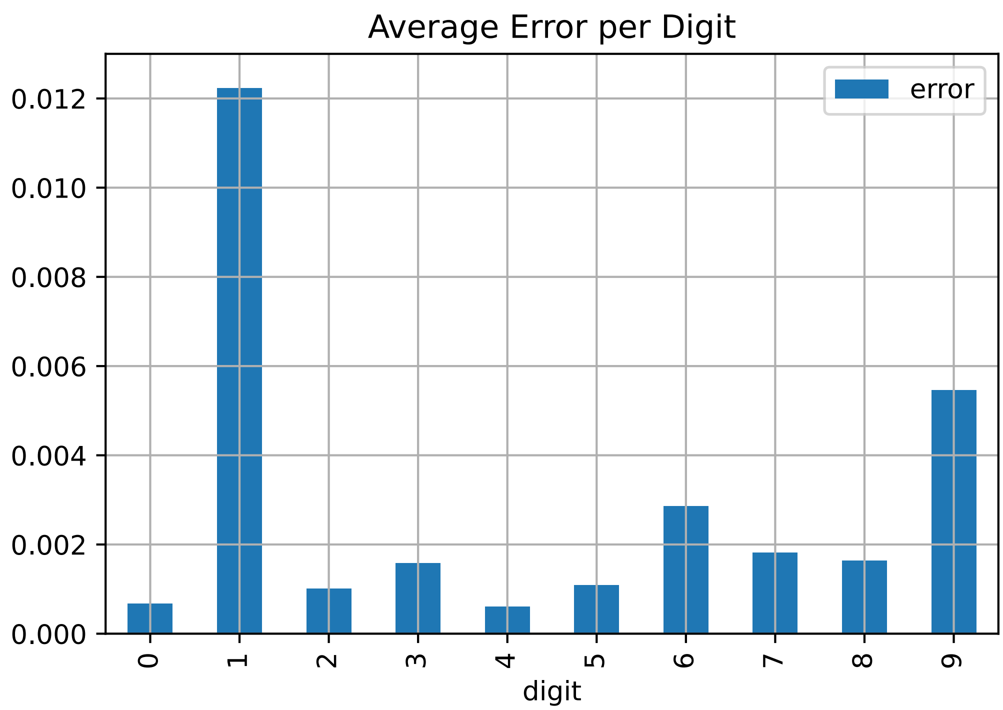
[26]:
```python
# show the average error per digit set

net = FINAL_NET
result = {
    'kind': [],
    'error': [],
}

for kind in dg.ALL_KINDS:
    digits = dg.getDigits(kinds={kind})
    inOutputs = dg.extractInputAndOutput(digits)
    error = nn.calcBatchError(net, inOutputs['input'], inOutputs['output'])

    result['kind'].append(kind)
    result['error'].append(error)

pd.DataFrame(result).sort_values('error').plot.bar(
    x='kind', title='Average Error per Set',
    grid=True, ylim=(0, 0.011),
)
```

[26]:
```
<AxesSubplot:title={'center':'Average Error per Set'}, xlabel='kind'>
```



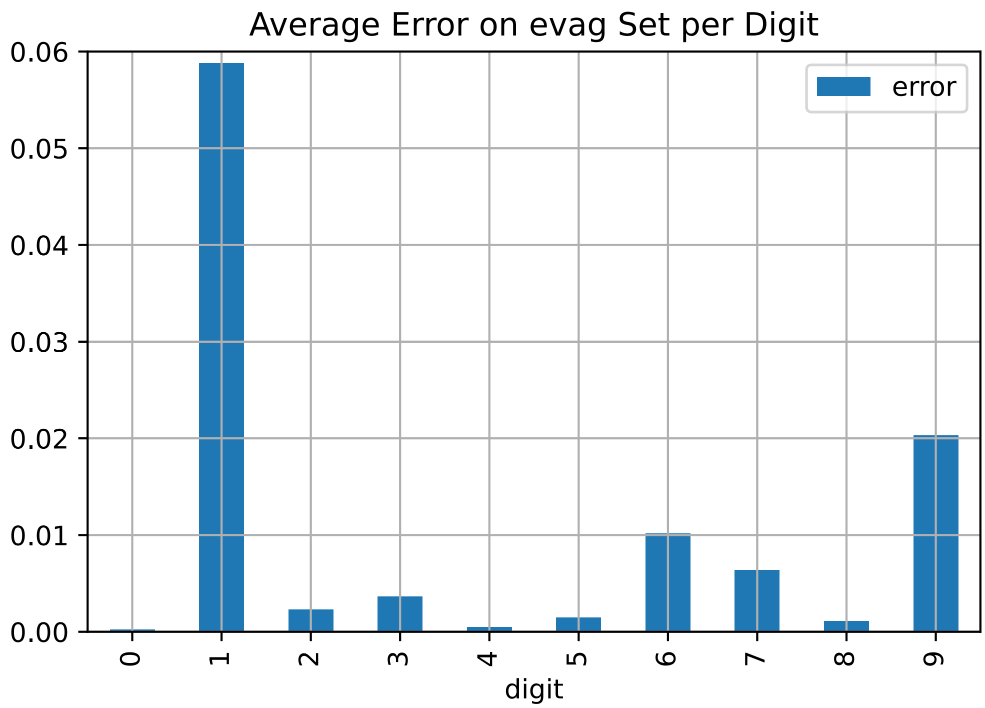
[27]:
```
# show the average error per digit (over all sets)

net = FINAL_NET
result = {
    'digit': [],
    'error': [],
}

for digit in dg.ALL_DIGITS:
    digits = dg.getDigits(digits={digit})
    inOutputs = dg.extractInputAndOutput(digits)
    error = nn.calcBatchError(net, inOutputs['input'], inOutputs['output'])

    result['digit'].append(digit)
    result['error'].append(error)

pd.DataFrame(result).plot.bar(
    x='digit', title='Average Error per Digit',
    grid=True,  ylim=(0, 0.013),
)
```

[27]: <AxesSubplot:title={'center':'Average Error per Digit'}, xlabel='digit'>



[28]:
```
# show the average error per digit for the test data

net = FINAL_NET
result = {
    'digit': [],
    'error': [],
}

for digit in dg.ALL_DIGITS:
    digits = dg.getDigits(digits={digit}, kinds={'evag'})
    inOutputs = dg.extractInputAndOutput(digits)
    error = nn.calcBatchError(net, inOutputs['input'], inOutputs['output'])

    result['digit'].append(digit)
    result['error'].append(error)

pd.DataFrame(result).plot.bar(
    x='digit', title='Average Error on evag Set per Digit',
    grid=True,  ylim=(0, 0.06),
)
```

[28]: <AxesSubplot:title={'center':'Average Error on evag Set per Digit'}, xlabel='digit'>



[29]:
```
# show the average error per digit for the training data

net = FINAL_NET
result = {
    'digit': [],
    'error': [],
}

for digit in dg.ALL_DIGITS:
    digits = dg.getDigits(digits={digit}, kinds={
                              'normal', 'normal-klein', 'digital', 'digital-klein'})
    inOutputs = dg.extractInputAndOutput(digits)
    error = nn.calcBatchError(net, inOutputs['input'], inOutputs['output'])

    result['digit'].append(digit)
    result['error'].append(error)

pd.DataFrame(result).plot.bar(
    x='digit', title='Average Error on Training Set per Digit',
    grid=True,  ylim=(0, 0.0018),
)
```

[29]: <AxesSubplot:title={'center':'Average Error on Training Set per Digit'}, xlabel='digit'>