main

March 16, 2021

Training a Neural Network to Recognize Digits

Import Dependencies

[1]:
```
import os

import numpy as np
import pandas as pd

import digits as dg
import net as nn
```

Loading Digits from Files

[2]:
```
DATA_PATH = os.path.join('..', 'data', 'cache')

allDigits = dg.getDigits()
trainDigits = dg.getDigits(
    kinds={'normal', 'normal-klein', 'digital', 'digital-klein'})
testDigits = dg.getDigits(kinds={'evag'})

DIGITS = {
    'all': dg.extractInputAndOutput(allDigits),
    'training': dg.extractInputAndOutput(trainDigits),
    'test': dg.extractInputAndOutput(testDigits),
}
```

Initialize Networks with Different Number of Hidden Neurons

[3]:
```
# This calculation might take a while.
# See below, how to load the results from cache instead

numOfHiddens = {5, 10, 15, 20, 25, 30, 35, 40, 50, 60, 70, 80, 90}
numOfNets = 100

input = DIGITS['all']['input']
output = DIGITS['all']['output']

initNets = {
```

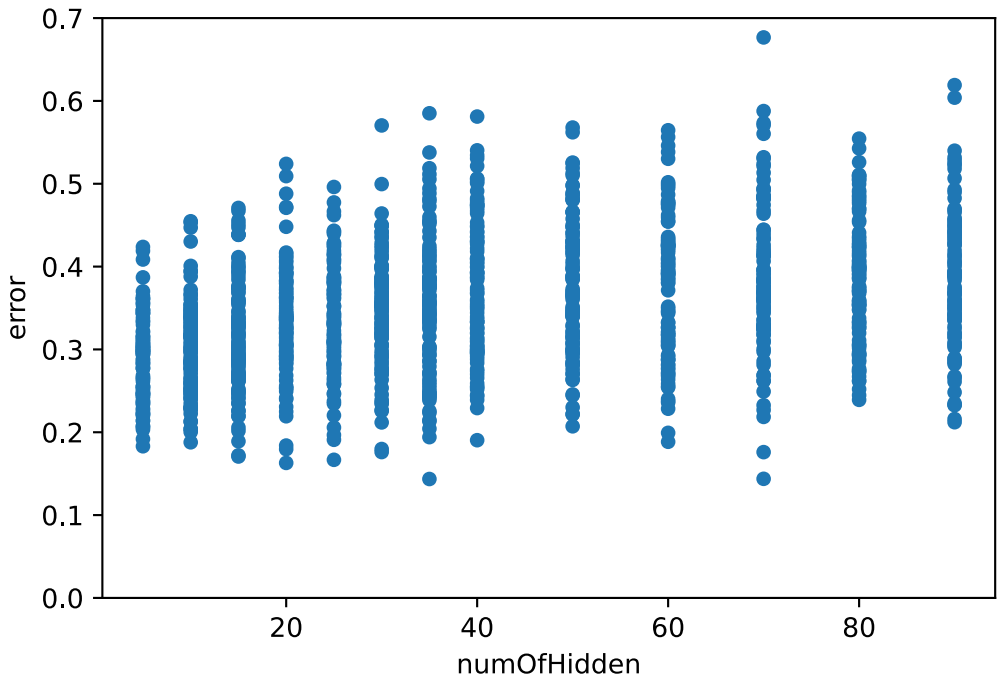
```
    'net': [],
    'numOfHidden': [],
    'error': []
}

for numOfHidden in numOfHiddens:
    np.random.seed(0)  # to create reproduceable results

    for _ in range(numOfNets):
        net = nn.init(35, numOfHidden, 10)
        error = nn.calcBatchError(net, input, output)

        initNets['net'].append(net)
        initNets['numOfHidden'].append(numOfHidden)
        initNets['error'].append(error)

INIT_NETS = pd.DataFrame(initNets)
```

[4]:
```
# store initial networks to cache
INIT_NETS.to_pickle(os.path.join(DATA_PATH, 'init-nets.pkl'))
```

[5]:
```
# load initial networks from cache
INIT_NETS = pd.read_pickle(os.path.join(DATA_PATH, 'init-nets.pkl'))
```

[6]:
```
INIT_NETS.plot.scatter('numOfHidden', 'error', ylim=(0, 0.7))
```

[6]: <AxesSubplot:xlabel='numOfHidden', ylabel='error'>



[7]:
```
INIT_NETS.groupby('numOfHidden').min('error').sort_values('error')
```

[7]:

numOfHidden	error
35	0.143571
70	0.143792
20	0.162996
25	0.166742
15	0.170721
30	0.176098
5	0.183089
10	0.187878
60	0.188517
40	0.190416
50	0.207182
90	0.212117
80	0.239134

Training Promising Networks

[8]:
```
# This calculation might take a while.
# See below, how to load the results from cache instead

numOfHiddens = {35, 70, 20, 25, 15, 30}
epoches = 1000
learnRate = 0.1

inputTrain = DIGITS['training']['input']
outputTrain = DIGITS['training']['output']

inputTest = DIGITS['test']['input']
outputTest = DIGITS['test']['output']

trainHistory = {
    'net': [],
    'numOfHidden': [],
    'errorTrain': [],
    'errorTest': [],
}

for numOfHidden in numOfHiddens:
    net = INIT_NETS[INIT_NETS.numOfHidden == numOfHidden].sort_values(
        'error').iloc[0].net

    for i in range(epoches):
```

```
        net = nn.trainBatch(net, inputTrain, outputTrain, learnRate)
        errorTrain = nn.calcBatchError(net, inputTrain, outputTrain)
        errorTest = nn.calcBatchError(net, inputTest, outputTest)

        trainHistory['net'].append(net)
        trainHistory['numOfHidden'].append(numOfHidden)
        trainHistory['errorTrain'].append(errorTrain)
        trainHistory['errorTest'].append(errorTest)

TRAIN_HIDDEN = pd.DataFrame(trainHistory)
```

[9]:
```
# store trained promising networks to cache
TRAIN_HIDDEN.to_pickle(os.path.join(DATA_PATH, 'train-hidden.pkl'))
```

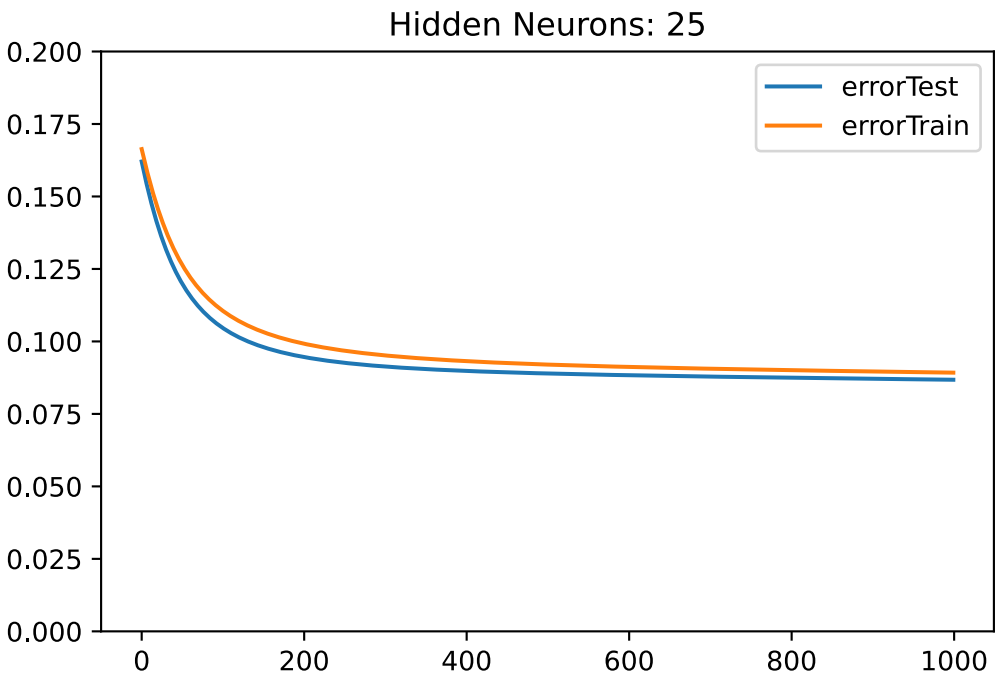
[10]:
```
# load trained promising networks from cache
TRAIN_HIDDEN = pd.read_pickle(os.path.join(DATA_PATH, 'train-hidden.pkl'))
```

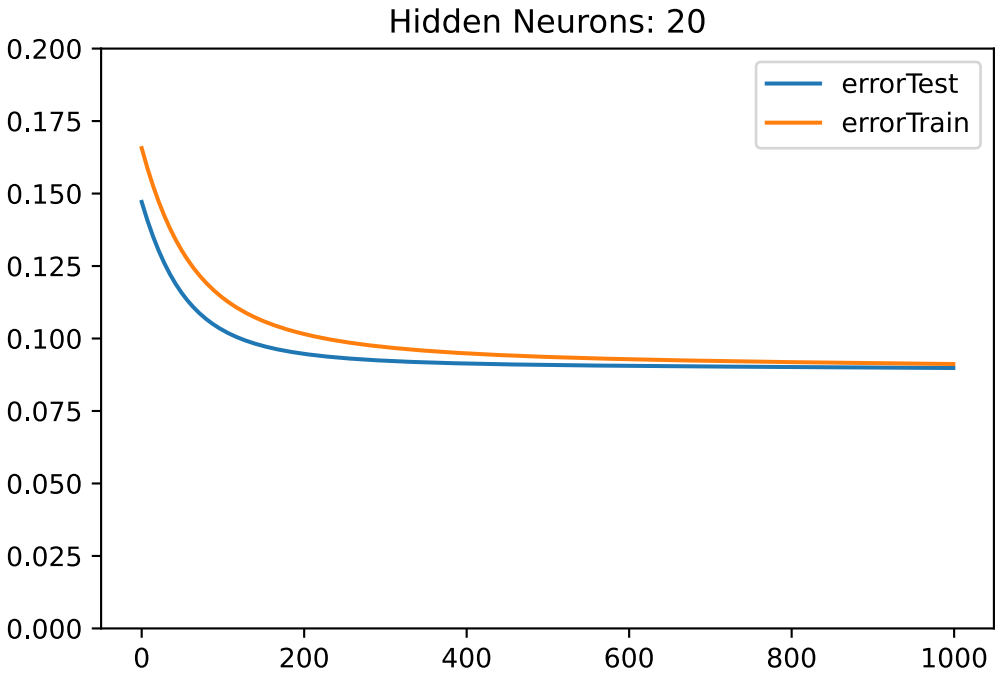
[11]:
```
end = TRAIN_HIDDEN.groupby('numOfHidden').min(
    'errorTrain').sort_values('numOfHidden')
start = INIT_NETS.groupby('numOfHidden').min(
    'error').sort_values('numOfHidden')

diff = end.join(start)
diff.rename(columns={'error': 'errorStart'}, inplace=True)
diff['diff'] = diff.errorStart - diff.errorTrain

diff.sort_values('diff', ascending=False)
```

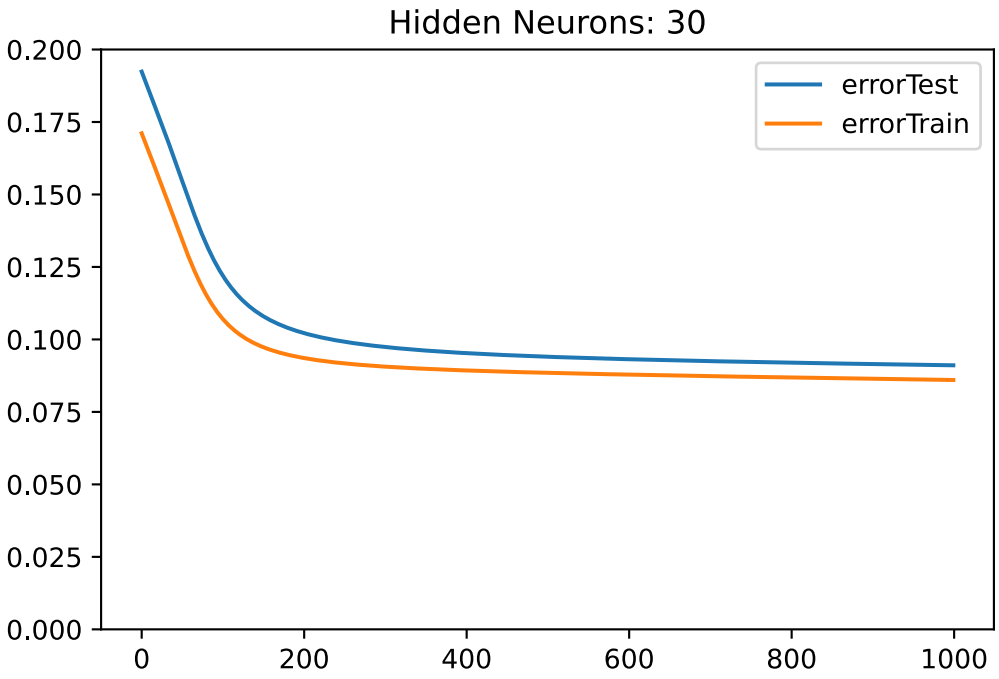
[11]:

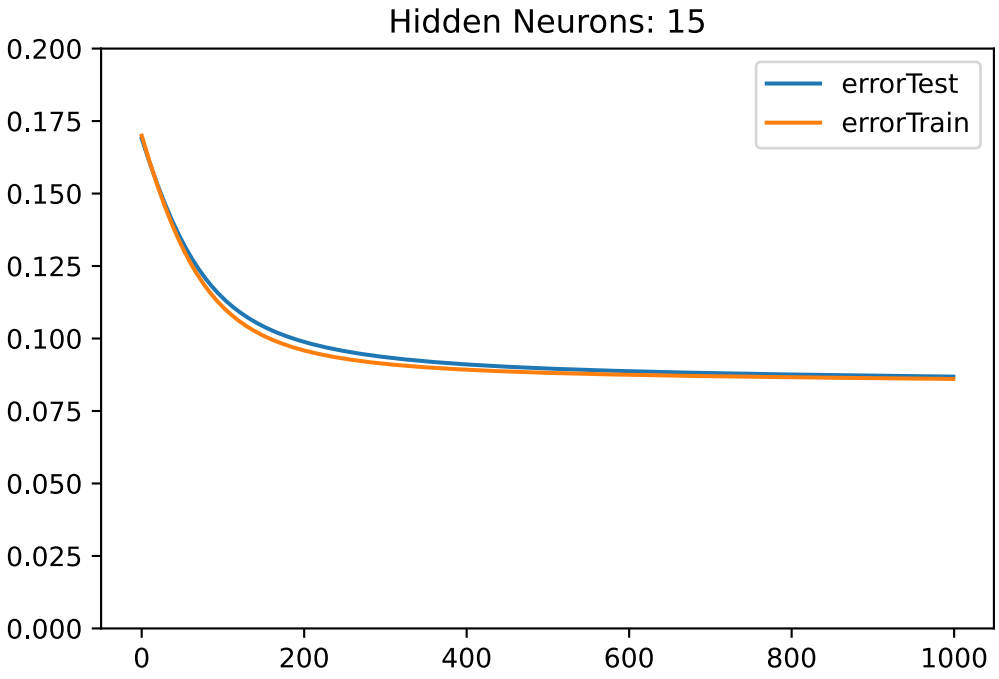
numOfHidden	errorTrain	errorTest	errorStart	diff
30	0.086010	0.091076	0.176098	0.090088
15	0.086051	0.086862	0.170721	0.084670
25	0.089231	0.086783	0.166742	0.077511
20	0.091201	0.089823	0.162996	0.071795
35	0.084060	0.086994	0.143571	0.059512
70	0.088598	0.091938	0.143792	0.055195

[12]:
```
for numOfHidden in {15, 20, 25, 30}:
    TRAIN_HIDDEN[TRAIN_HIDDEN.numOfHidden == numOfHidden].plot.line(
        title=f'Hidden Neurons: {numOfHidden}',
        y={'errorTrain', 'errorTest'}, ylim=(0, 0.2),
        use_index=False,
    )
```


Hidden Neurons: 25
errorTest
errorTrain
0.200
0.175
0.150
0.125
0.100
0.075
0.050
0.025
0.000
0
200
400
600
800
1000


Hidden Neurons: 20
errorTest
errorTrain
0.200
0.175
0.150
0.125
0.100
0.075
0.050
0.025
0.000
0
200
400
600
800
1000





Finding the Optimal Learning Rate

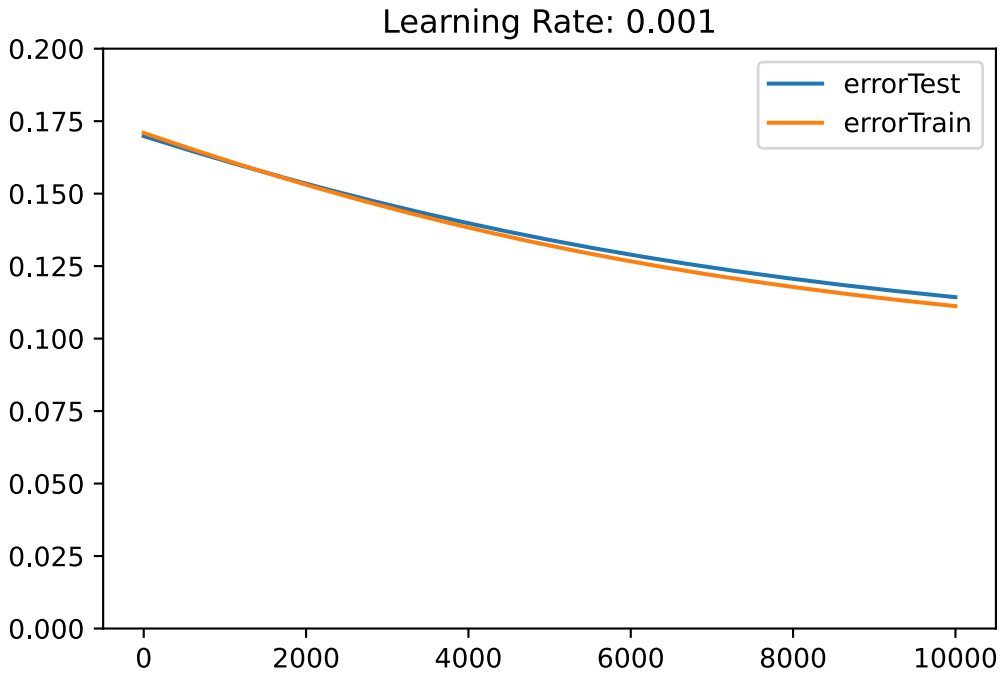
[13]:
```
# This calculation might take a while.
# See below, how to load the results from cache instead
numOfHidden = 15
learnRates = {1, 0.1, 0.01, 0.001}
epoches = 10000

startNet = INIT_NETS[INIT_NETS.numOfHidden == numOfHidden].sort_values(
    'error').iloc[0].net

inputTrain = DIGITS['training']['input']
outputTrain = DIGITS['training']['output']

inputTest = DIGITS['test']['input']
outputTest = DIGITS['test']['output']

trainHistory = {
    'net': [],
    'learnRate': [],
    'errorTrain': [],
    'errorTest': [],
}

for learnRate in learnRates:
    net = startNet

    for i in range(epoches):
        net = nn.trainBatch(net, inputTrain, outputTrain, learnRate)
        errorTrain = nn.calcBatchError(net, inputTrain, outputTrain)
        errorTest = nn.calcBatchError(net, inputTest, outputTest)

        trainHistory['net'].append(net)
        trainHistory['learnRate'].append(learnRate)
        trainHistory['errorTrain'].append(errorTrain)
        trainHistory['errorTest'].append(errorTest)

TRAIN_LR = pd.DataFrame(trainHistory)
```

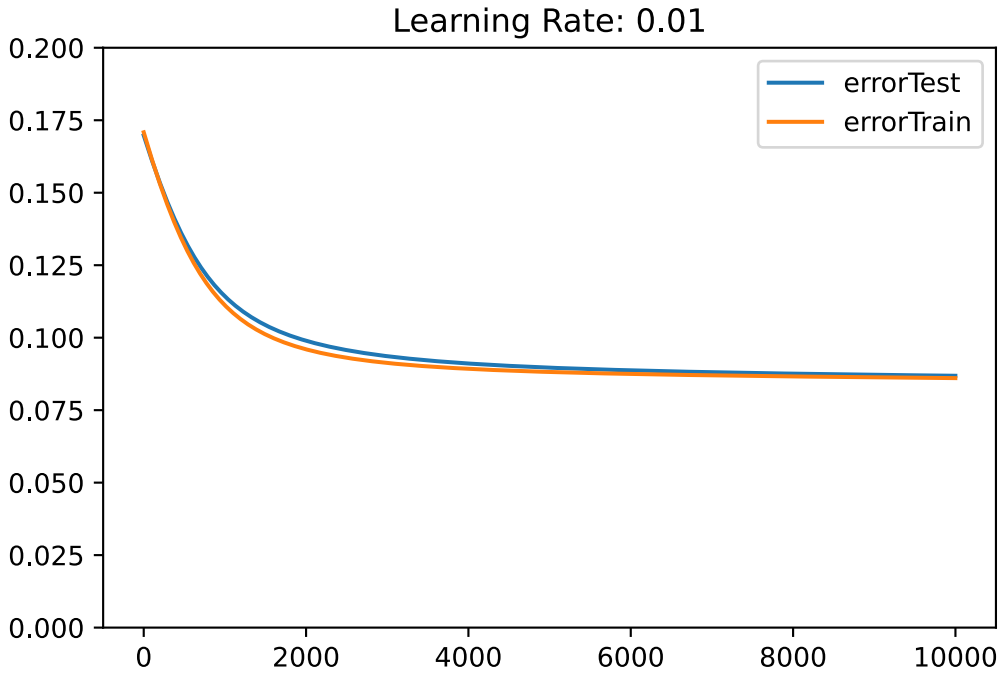
[14]:
```
# store training with different learning rates to cache
TRAIN_LR.to_pickle(os.path.join(DATA_PATH, 'train-learn-rate.pkl'))
```

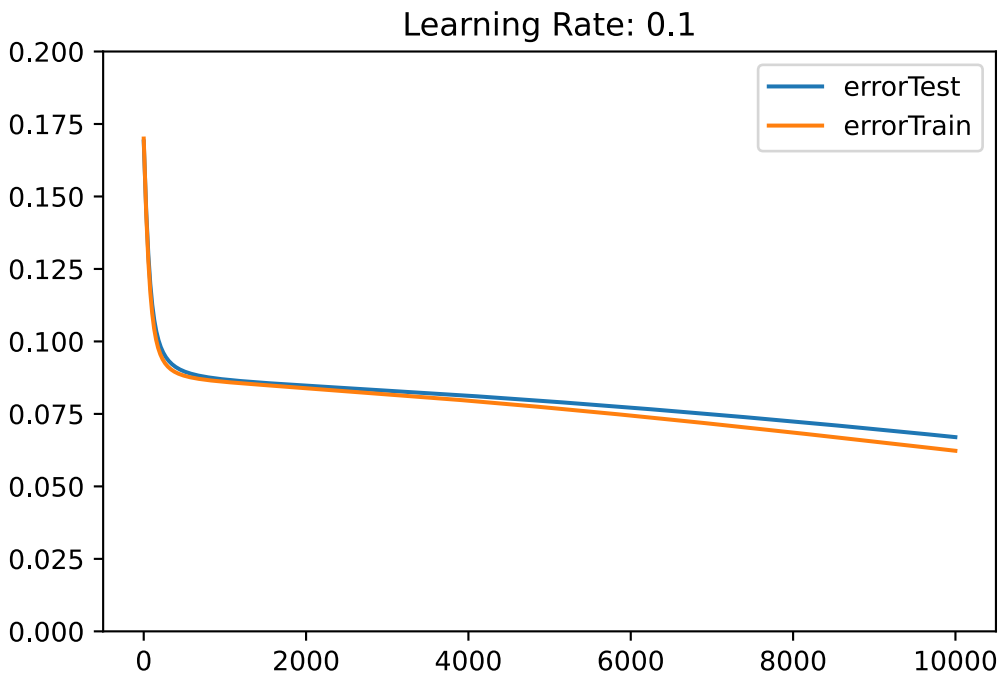
[15]:
```
# load training with different learning rates from cache
TRAIN_LR = pd.read_pickle(os.path.join(DATA_PATH, 'train-learn-rate.pkl'))
```

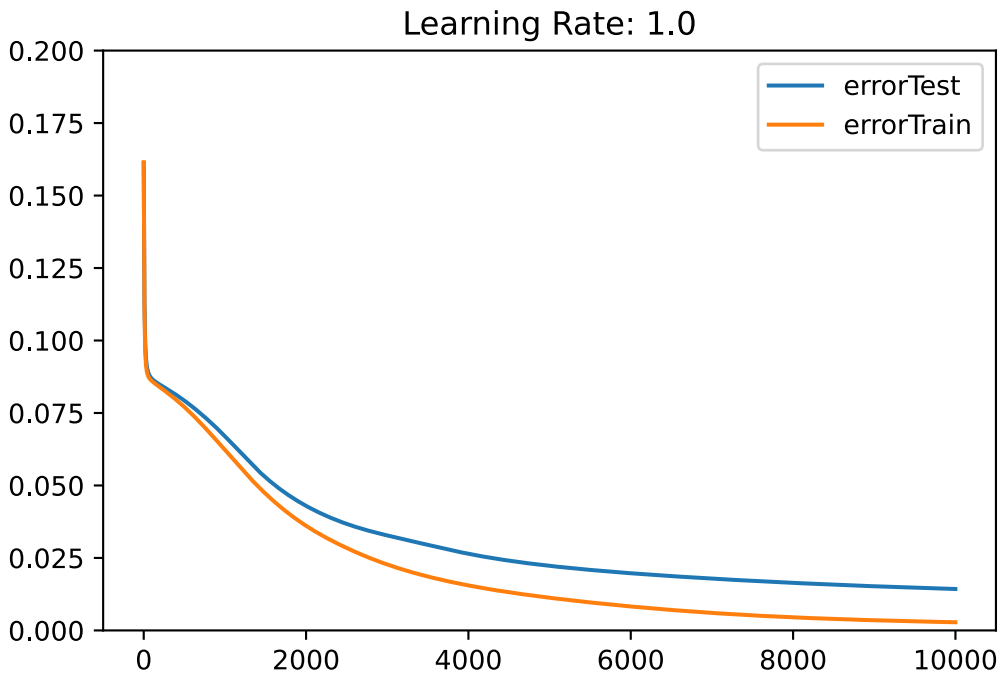
[16]:
```
for learnRate, df in TRAIN_LR.groupby('learnRate'):
    df.plot.line(
        title=f'Learning Rate: {learnRate}',
        y={'errorTrain', 'errorTest'}, ylim=(0, 0.2),
```

```
        use_index=False,
    )
```









[17]:
```
TRAIN_LR.groupby('learnRate').min('errorTrain').sort_values('errorTrain')
```

[17]:
```
            errorTrain  errorTest
learnRate
1.000         0.002821   0.014294
0.100         0.062284   0.066992
0.010         0.086051   0.086864
0.001         0.111204   0.114295
```

Final Training of the Network

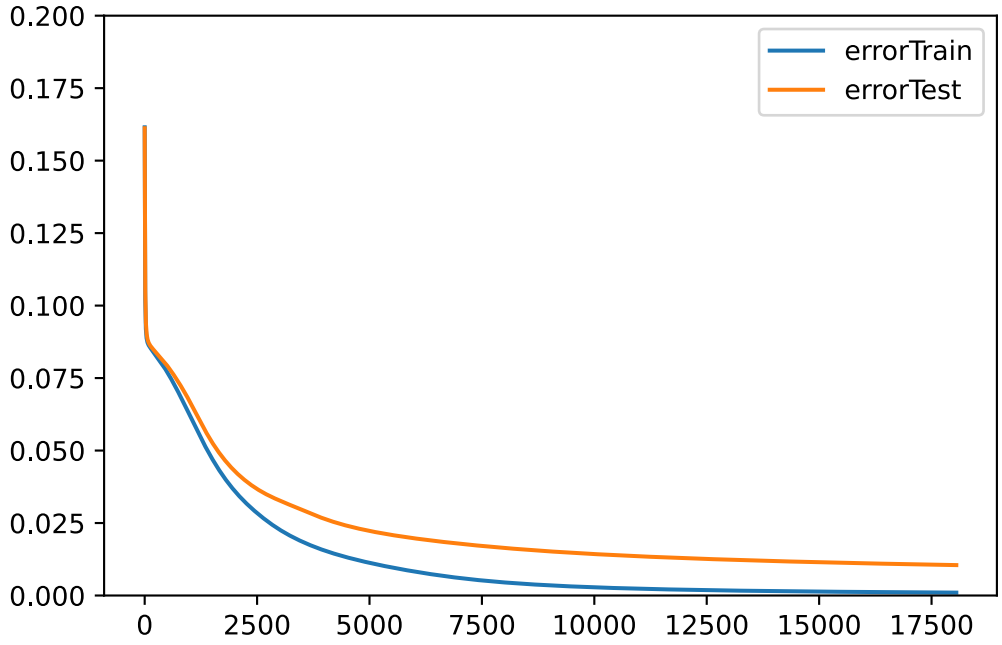
[18]:
```
# This calculation might take a while.
# See below, how to load the results from cache instead

learnRate = 1
targetError = 0.001

inputTrain = DIGITS['training']['input']
outputTrain = DIGITS['training']['output']

inputTest = DIGITS['test']['input']
outputTest = DIGITS['test']['output']

trainHistory = TRAIN_LR[TRAIN_LR.learnRate == learnRate].drop('learnRate', 1)
net = trainHistory.iloc[-1].net
error = nn.calcBatchError(net, inputTrain, outputTrain)

while error > targetError:
    net = nn.trainBatch(net, inputTrain, outputTrain, learnRate)
    errorTrain = nn.calcBatchError(net, inputTrain, outputTrain)
    errorTest = nn.calcBatchError(net, inputTest, outputTest)

    trainHistory = trainHistory.append({
        'net': net,
        'errorTrain': errorTrain,
        'errorTest': errorTest,
    }, ignore_index=True)

    error = errorTrain

TRAIN_FINAL = trainHistory
```

[19]:
```
# store final training round to cache
TRAIN_FINAL.to_pickle(os.path.join(DATA_PATH, 'train-final.pkl'))
```

[20]:
```
# load final training round from cache
TRAIN_FINAL = pd.read_pickle(os.path.join(DATA_PATH, 'train-final.pkl'))
```

[21]:
```
TRAIN_FINAL.plot.line(ylim=(0, 0.2))
```

[21]: <AxesSubplot:>



[22]:
```
TRAIN_FINAL.describe()
```

[22]:

	errorTrain	errorTest
count	18052.000000	18052.000000
mean	0.012627	0.022609
std	0.019830	0.017714
min	0.001000	0.010490
25%	0.001604	0.012087
50%	0.003481	0.015205
75%	0.013109	0.024004
max	0.161496	0.161170

Analyzing the Final Neural Network

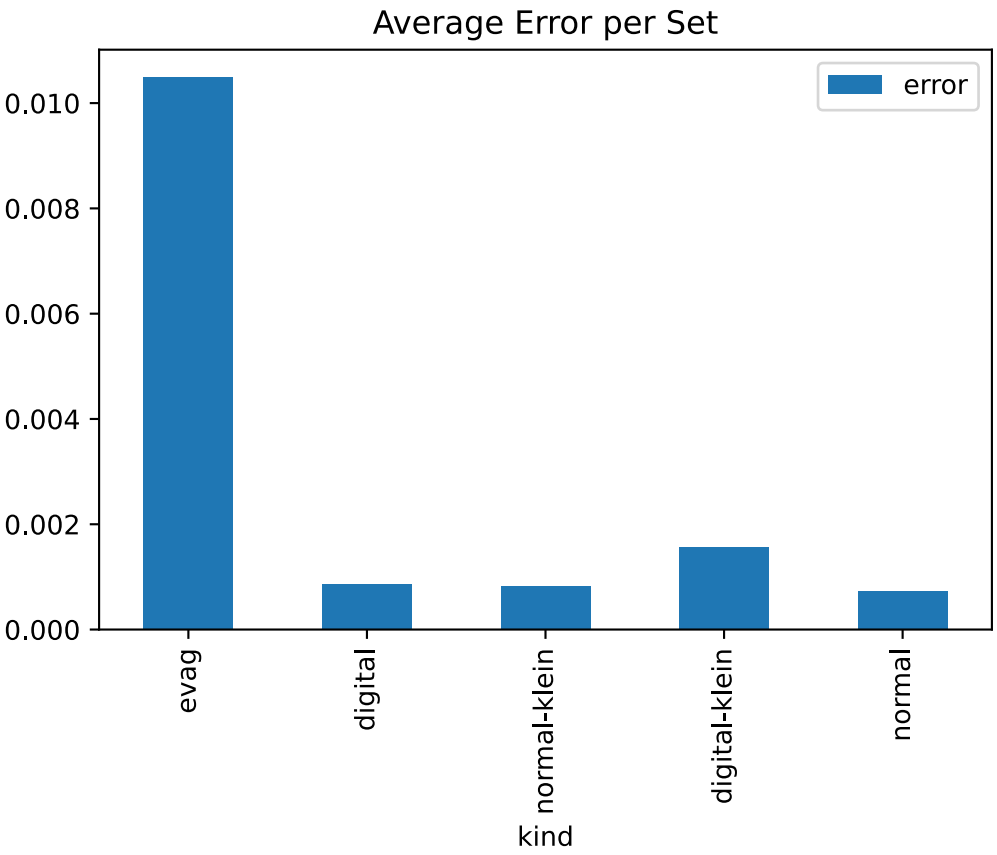
[23]:
```
FINAL_NET = TRAIN_FINAL.iloc[-1].net
```

[24]:
```
# store final network to cache
nn.save(FINAL_NET, os.path.join(DATA_PATH, 'final-net.pkl'))
```

[25]:
```
# load final network from cache
FINAL_NET = nn.load(os.path.join(DATA_PATH, 'final-net.pkl'))
```

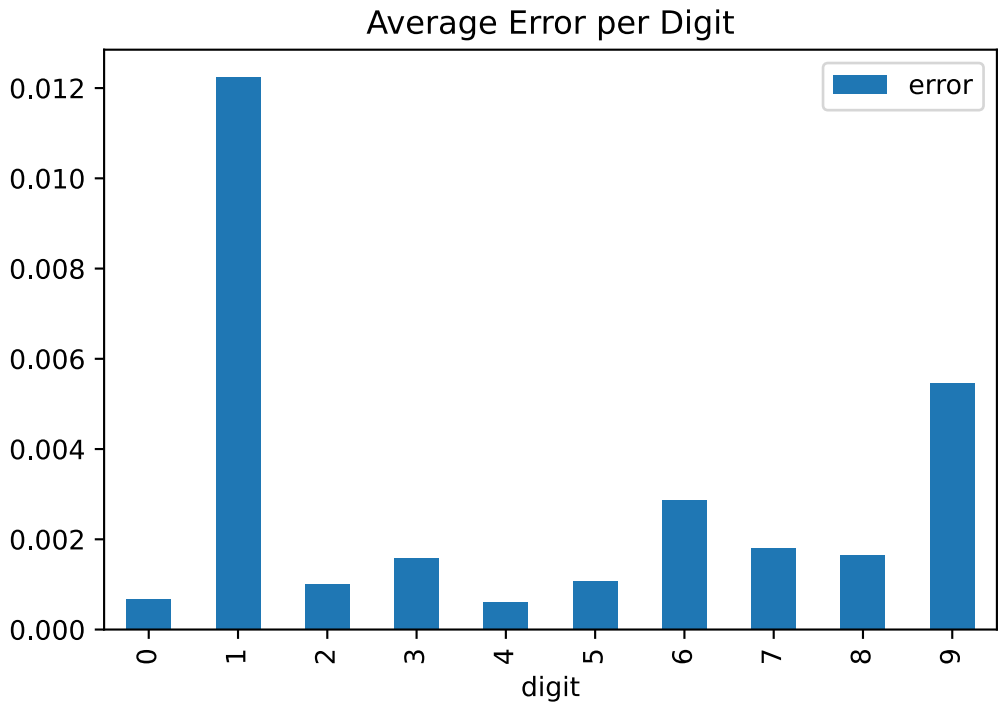
[26]:
```
net = FINAL_NET
result = {
    'kind': [],
    'error': [],
}

for kind in dg.ALL_KINDS:
    digits = dg.getDigits(kinds={kind})
    inOutputs = dg.extractInputAndOutput(digits)
    error = nn.calcBatchError(net, inOutputs['input'], inOutputs['output'])

    result['kind'].append(kind)
    result['error'].append(error)

pd.DataFrame(result).plot.bar(x='kind', title='Average Error per Set')
```

[26]:
```
<AxesSubplot:title={'center':'Average Error per Set'}, xlabel='kind'>
```



[27]:
```
net = FINAL_NET
result = {
    'digit': [],
    'error': [],
}

for digit in dg.ALL_DIGITS:
    digits = dg.getDigits(digits={digit})
    inOutputs = dg.extractInputAndOutput(digits)
    error = nn.calcBatchError(net, inOutputs['input'], inOutputs['output'])

    result['digit'].append(digit)
    result['error'].append(error)

pd.DataFrame(result).plot.bar(x='digit', title='Average Error per Digit')
```

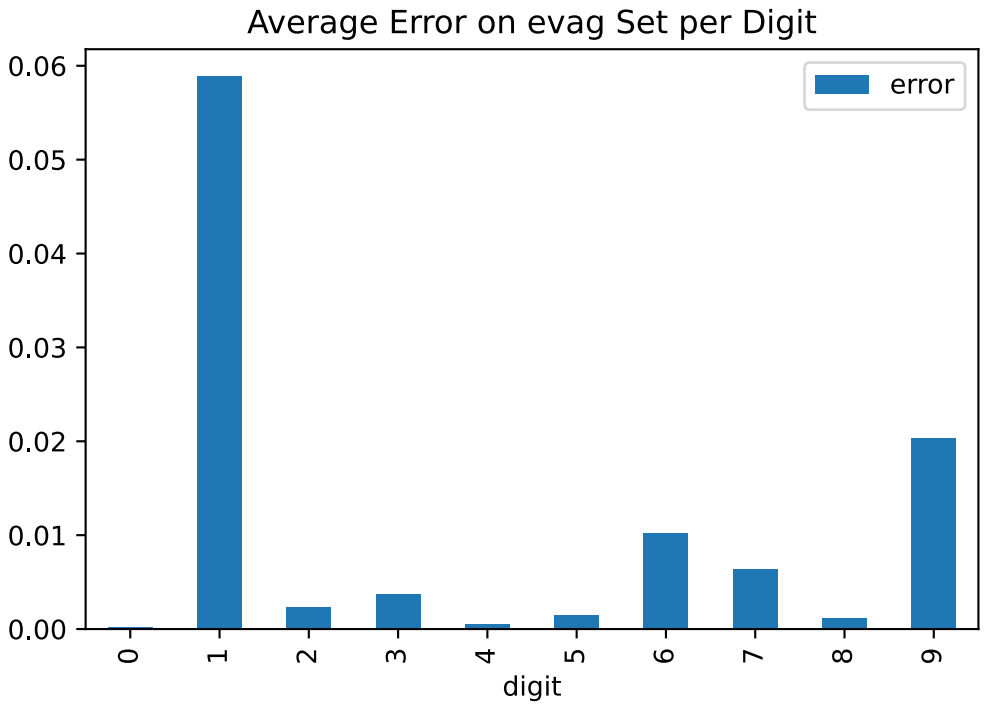
[27]:
```
<AxesSubplot:title={'center':'Average Error per Digit'}, xlabel='digit'>
```



[28]:
```
net = FINAL_NET
result = {
    'digit': [],
    'error': [],
}
```

```
for digit in dg.ALL_DIGITS:
    digits = dg.getDigits(digits={digit}, kinds={'evag'})
    inOutputs = dg.extractInputAndOutput(digits)
    error = nn.calcBatchError(net, inOutputs['input'], inOutputs['output'])

    result['digit'].append(digit)
    result['error'].append(error)

pd.DataFrame(result).plot.bar(x='digit', title='Average Error on evag Set per␣
↪Digit')
```

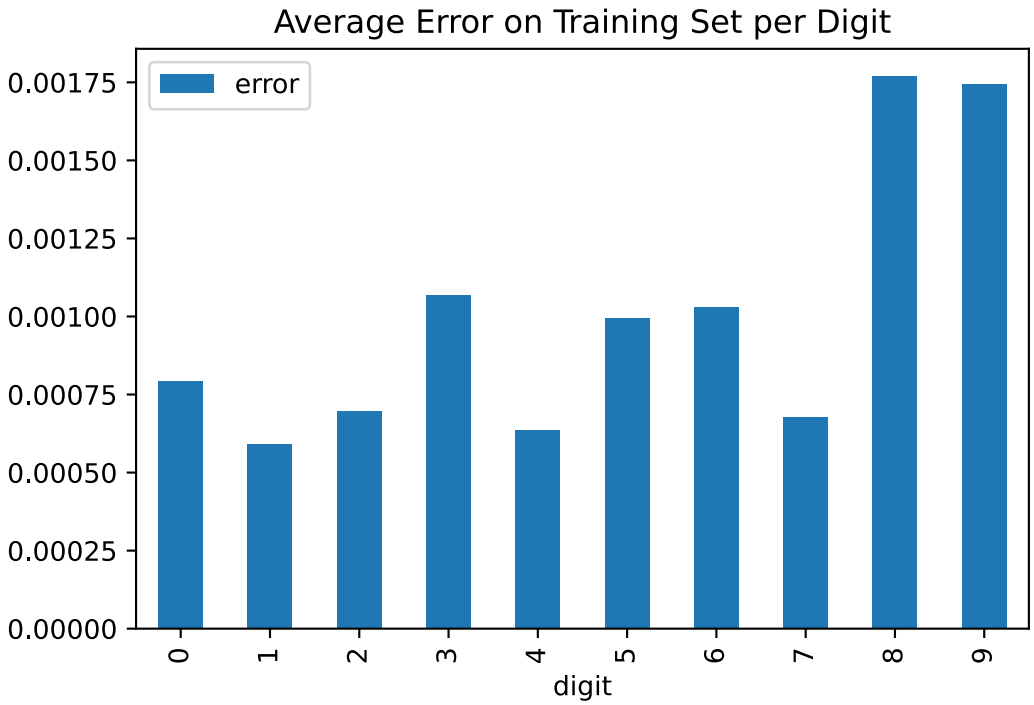
[28]:
```
<AxesSubplot:title={'center':'Average Error on evag Set per Digit'},
xlabel='digit'>
```



[29]:
```
net = FINAL_NET
result = {
    'digit': [],
    'error': [],
}

for digit in dg.ALL_DIGITS:
    digits = dg.getDigits(digits={digit}, kinds={'normal', 'normal-klein',␣
↪'digital', 'digital-klein'})
    inOutputs = dg.extractInputAndOutput(digits)
```

```
    error = nn.calcBatchError(net, inOutputs['input'], inOutputs['output'])

    result['digit'].append(digit)
    result['error'].append(error)

pd.DataFrame(result).plot.bar(x='digit', title='Average Error on Training Set␣
 ↪per Digit')
```

[29]:
```
<AxesSubplot:title={'center':'Average Error on Training Set per Digit'},
xlabel='digit'>
```



[ ]: