Appendix 7: Scenario Models Methodology

A7.1 Setup

This document provides a step-by-step methodology to model three different Local Rent Supplement Program (LRSP) scenarios for the City of Alexandria. Data is created, transformed, and visualized using the R coding language. The R libraries used for this analysis are listed below.

```
library(tidyverse)
library(scales)
library(kableExtra)
library(formattable)
library(gt)
library(ggtext)
library(janitor)
```

A7.2 Standard parameters

Income limits

The models use HUD's FY 2023 Multifamily Tax Subsidy Projects (MTSP) Income Limits for Washington-Arlington-Alexandria, DC-VA-MD HUD Metro FMR Area.[1] These are the official income limits used to determine eligibility for LIHTC projects and other affordable multifamily properties financed by tax-exempt bonds.

While the MTSP limits differ slightly from the standard income limits used for Housing Choice Vouchers, public housing, and other HUD-supported assistance programs, they are used here because they publish limits for a greater range of AMI levels, including 40% AMI and 60% AMI. The City of Alexandria generally uses MTSP limits for its housing programs.

Table 1: FY 2023 MTSP Income Limits for Washington-Arlington-Alexandria, DC-VA-MD HUD Metro FMR Area

AMI	1 person	2 person	3 person	4 person	5 person	6 person
20% AMI	$21,100	$24,120	$27,140	$30,140	$32,460	$34,980
30% AMI	$31,650	$36,180	$40,710	$45,210	$48,840	$52,470
40% AMI	$42,200	$48,240	$54,280	$60,280	$65,210	$69,960
50% AMI	$52,750	$60,300	$67,850	$75,350	$81,400	$87,450
60% AMI	$63,300	$72,660	$81,420	$90,240	$97,460	$104,940
70% AMI	$73,850	$84,240	$94,990	$105,490	$113,960	$122,430
80% AMI	$84,400	$96,480	$108,560	$120,560	$130,240	$139,920

[1] FY 2023 MTSP Income Limits (Accessed 2024-01-19)

Fair Market Rents

Models where the rent subsidy is calculated based on Fair Market Rents (FMR) use the current Small Area Fair Market Rents (SAFMR) adopted by the Arlington Redevelopment and Housing Authority for 2023. SAFMRs are provided by ZIP code.

While actual subsidy amounts will depend on the ZIP code where the tenant lives, models will use the average values (by unit size) across all ZIP codes. This is a simplification to avoid making assumptions about the geographic distribution of participating households.

Table 2: ARHA 2023 Payment Standards

ZIP code	Studio	1 bedroom	2 bedroom	3 bedroom
22301	$2,013	$2,046	$2,332	$2,915
22302	$1,980	$2,013	$2,288	$2,860
22304	$1,914	$1,947	$2,211	$2,761
22305	$1,859	$1,892	$2,156	$2,695
22311	$1,936	$1,969	$2,244	$2,805
22312	$1,848	$1,870	$2,134	$2,673
22313	$1,782	$1,815	$2,068	$2,585
22314	$2,563	$2,607	$2,970	$3,718
Average	**$1,986.88**	**$2,019.88**	**$2,300.38**	**$2,876.50**

A7.3 Scenario A - Reduce Cost Burden for 30% to 50% AMI Households

This scenario outlines a LRSP with a total annual allocation of $500,000. The primary goal of the program is to reduce housing cost burden among households with incomes between 30% and 50% AMI. The model uses the following inputs to estimate the number of households served.

Variable	**Input**
Total program budget	$500,000
Eligibility	Household income between 30% and 50% AMI
Subsidy amount	Difference between the affordable monthly rent at 60% AMI and the affordable monthly rent at 40% AMI
Distribution of household sizes among participants	15% - 1-person 15% - 2-person 20% - 3-person 20% - 4-person 20% - 5-person 10% - 6-person
Administrative overhead	15% of total program budget

Notes:

- No other eligibility conditions apply.
- "Affordable monthly rent" is 30% of gross household income.
- The subsidy calculated for each household is respective to their household size. No assumed breakdown of households by AMI is needed.
- The administrative overhead includes housing-specific case management.

Inputs

Assign budget (dollars) and overhead costs (percent) variables:

```
# Budget allocation
sA_budget <- 500000

# Overhead percentage
sA_overhead <- 0.15
```

Assign household distributions by household size (number of persons):

```
# Distribution of households by household size
sA_person <- tibble(
  hh_size = paste0("person", 1:6),
  pct = c(0.15, 0.15, 0.20, 0.20, 0.20, 0.10)
)
```

Calculations

Calculate affordable rents at 40% AMI and 60% AMI for households with 1 to 6 persons (`hh_size`) to determine monthly subsidy amounts (`subsidy`):

```
# Monthly subsidy about by household size
sA_subsidy <- hud_ami |>
  filter(
    AMI %in% c("40% AMI", "60% AMI"), # 40% and 60% AMI only
    str_detect(hh_size, "[123456]")   # 1-6 person households only
    ) |>
  mutate(
    aff_rent = income/12 * 0.3        # 30% of monthly income
  ) |>
  select(-3) |>
  pivot_wider(
    names_from = AMI,
    values_from = aff_rent
  ) |>
  mutate(
    subsidy = `60% AMI` - `40% AMI`   # Calculate subsidy
  ) |>
  select(1, 4)
```

hh_size	subsidy
person1	527.50
person2	610.50
person3	678.50
person4	749.00
person5	806.25
person6	874.50

Join the monthly subsidy amounts by household size (`subsidy`) and calculate annual subsidy per household (`subsidy_annual`):

```
# Annual subsidy per household size
sA_subsidy_annual <- sA_person |>
  left_join(sA_subsidy) |>
  mutate(subsidy_annual = subsidy * 12)
```

hh_size	pct	subsidy	subsidy_annual
person1	0.15	527.50	6330
person2	0.15	610.50	7326
person3	0.20	678.50	8142
person4	0.20	749.00	8988
person5	0.20	806.25	9675
person6	0.10	874.50	10494

Calculate the theoretical share of subsidy allocated for each household size (`subsidy_share`). Determine the number of households served (`hh_served`) by normalizing `subsidy_share` to the known budget, then calculate the budget share (`budget`) for each household size:

```
# Annual subsidy per household type
sA_served <- sA_subsidy_annual |>
  mutate(
    subsidy_share = subsidy_annual * pct, # Subsidy per HH type
    hh_served = pct*(sA_budget * (1 - sA_overhead))/sum(subsidy_share) # Adjust to known budget
  ) |>
  mutate(budget = hh_served * subsidy_annual)
```

hh_size	subsidy_annual	subsidy_share	budget	hh_served
person1	6330	949.50	47706.23	7.537
person2	7326	1098.90	55212.62	7.537
person3	8142	1628.40	81816.57	10.049
person4	8988	1797.60	90317.78	10.049
person5	9675	1935.00	97221.24	10.049
person6	10494	1049.40	52725.56	5.024
Total	-	-	425000.00	50.244

Model results

Round each estimate to the nearest whole number and determine total:

```
# Rounded estimates with grand total
sA_estimate <- sA_served |>
  select(1, 7, 6) |>
  mutate(
    hh_served = round(hh_served),
    hh_size = case_match(
      hh_size,
      "person1" ~ "1 person",
      "person2" ~ "2 person",
      "person3" ~ "3 person",
      "person4" ~ "4 person",
      "person5" ~ "5 person",
      "person6" ~ "6 person"
    )
  ) |>
  adorn_totals()
```

Table 4: Scenario A - Estimated Households Served by Household Size

Household size	Annual cost	Households served
1 person	$47,706	8
2 person	$55,213	8
3 person	$81,817	10
4 person	$90,318	10
5 person	$97,221	10
6 person	$52,726	5
Total	**$425,000**	**51**

Scenario A results

Average annual program cost per household: $9,803.92

Under Scenario A, a total program budget of $500,000 with a 15% administrative overhead leaves $425,000 to fund rental assistance. Given the assumed household distribution by household size, the total number of households served is 51.

A7.4 Scenario B - Stabilize Unhoused Persons

This scenario outlines a LRSP serving a total of 150 households experiencing housing insecurity. The primary goal of the program is to provide deep rental assistance to help these households achieve housing stability and avoid homelessness. The model uses the following inputs to estimate the annual program cost required to serve 150 households.

Variable	**Input**
Total households served	150
Eligibility	Household/individual determined to be homeless in City's annual Point-in-Time count
Subsidy amount	Difference between the affordable monthly rent at 60% AMI and the households' current affordable monthly rent
Distribution of household types among participants	2/3 - Single-person 1/3 - Household with children
Distribution of unit sizes among participants	2/3 - Studios 1/3 - 2-bedroom
Distribution of incomes among participants	50% - SSI income 50% - $1,500 per month
Administrative overhead	20% of total program budget

Notes:

- No other eligibility conditions apply.
- "Affordable monthly rent" is 30% of gross household income.
- The subsidy calculated for each household is respective to their household size.
- The administrative overhead is higher than Scenario A to accommodate more intensive case management requirements for persons experiencing homelessness.

Inputs

Assign households served and overhead costs (percent) variables:

```
# Total households served
sB_hh_served <- 150

# Overhead percentage
sB_overhead <- 0.20
```

We can reasonably assume that all single-person households will live in studios, while all households with children will live in 2-bedroom units. Therefore, we do not need separate distribution shares for each. However, we do need to determine more specific household sizes.

For this model, among households with children, we will assume the following breakdown, as shown in the code below:

- 1/2 are 2-person (adult and child)
- 1/4 are 3-person (adult and two children, or two adults and child)
- 1/4 are 4-person (adult and three children, or two adults and two children)

```
# Distribution of households by size
sB_person <- c(
  `person1` = 0.667, # 2/3
  `person2` = 0.167, # 1/2 of 1/3
  `person3` = 0.083, # 1/4 of 1/3
  `person4` = 0.083  # 1/4 of 1/3
  )
```

Assign household distribution by income and calculate :

```
# Distribution of households by income
sB_income <- c(`ssi` = 0.5, `1500` = 0.5)
```

Because SSI amounts depend on whether the beneficiary lives alone or is married, their affordable rents will vary. However, due to the relatively small share of households in this model that could include two married adults, we will assume that any persons enrolled in SSI receive benefits for an individual. The current monthly SSI amount for an eligible individual is $943.[2]

```
# Monthly SSI income for eligible individual
sB_ssi <- 943
```

[2] SSI Federal Payment Amounts for 2024 (Accessed 2024-01-19)

Calculations

Calculate affordable rents for SSI income and $1,500/month (`income_source`) for households with 1 to 4 persons (`hh_size`) to determine monthly subsidy amounts (`subsidy`):

```
# Monthly subsidy about by household size
sB_subsidy <- hud_ami |>
  filter(
    AMI %in% c("60% AMI"),         # 60% AMI only
    str_detect(hh_size, "[1234]") # 1-4 person households only
    ) |>
  mutate(
    aff_rent_60ami = income/12 * 0.3 # 30% of monthly income
  ) |>
  select(2, 4) |>
  mutate(
    `ssi` = sB_ssi * 0.3,  # 30% of SSI
    `1500` = 1500 * 0.3 # 30% of $1,500
  ) |>
  pivot_longer(
    3:4,
    names_to = "income_source",
    values_to = "aff_rent"
  ) |>
  mutate(
    subsidy = aff_rent_60ami - aff_rent # Calculate subsidy
  ) |>
  select(3, 1, 5)
```

income_source	hh_size	subsidy
ssi	person1	1299.6
	person2	1533.6
	person3	1752.6
	person4	1973.1
1500	person1	1132.5
	person2	1366.5
	person3	1585.5
	person4	1806.0

Tabulate unique shares for both income source and household size:

```
sB_dist <- expand.grid(
  income_source = names(sB_income),
  hh_size = names(sB_person)
  ) |>
  mutate(pct = sB_income[income_source] * sB_person[hh_size])
```

income_source	hh_size	pct
ssi	person1	0.3335
	person2	0.0835
	person3	0.0415
	person4	0.0415
1500	person1	0.3335
	person2	0.0835
	person3	0.0415
	person4	0.0415

Multiply each household share by the total number of households served (150) to determine the respective number served for each group (`hh_served`), rounded to the nearest whole number:

```
sB_served <- sB_dist |>
  mutate(hh_served = round(pct * sB_hh_served))
```

Model results

Join the monthly subsidy amounts(`subsidy`) by income source and household size and calculate annual subsidy per household (`subsidy_annual`). Multiply that figure by the number of households served to calculate the rental assistance required (`budget_rent`), then re-total to account for overhead costs (`budget_total`):

```
sB_budget <- sB_served |>
  left_join(sB_subsidy, join_by(income_source, hh_size)) |>
  mutate(
    subsidy_annual = subsidy * 12,
    budget_rent = subsidy_annual * hh_served,
    budget_total = budget_rent/(1 - sB_overhead)
    ) |>
  select(1, 2, 4, 7, 8) |>
  arrange(desc(income_source)) |>
  adorn_totals()
```

Table 6: Scenario B - Estimated annual budget by income source and household size

Income source	Household size	Households served	Estimated budget	
			Rental assistance	Overhead included
SSI	1 person	50	$779,760.00	$974,700.00
	2 person	13	$239,241.60	$299,052.00
	3 person	6	$126,187.20	$157,734.00
	4 person	6	$142,063.20	$177,579.00
$1,500/month	1 person	50	$679,500.00	$849,375.00
	2 person	13	$213,174.00	$266,467.50
	3 person	6	$114,156.00	$142,695.00
	4 person	6	$130,032.00	$162,540.00
Total	**-**	**150**	**$2,424,114.00**	**$3,030,142.50**

Scenario B results

Average annual program cost per household: $20,200.95

Under Scenario B, a total of 150 households experiencing housing insecurity are served. Although two-thirds are individuals who have lower housing costs than households with children, the average cost per household is over twice that of Scenario A, due to the deep level of subsidy provided. The estimated annual cost for the rental assistance alone is $2,424,114.00. Coupled with a higher administrative overhead for expanded case management (20%), the total projected funding required is $3,030,142.50.

A7.5 Scenario C – Reduce Severe Cost Burden for Lower-Income Working Families

This scenario outlines a LRSP with a total annual allocation of $500,000. The primary goal of the program is to reduce housing cost burden among households with incomes between 30% and 50% AMI. Households must have one working adult and one or more dependent. The model uses the following inputs to estimate the number of households served.

Variable	**Input**
Total program budget	$500,000
Eligibility	Household income between 30% and 50% AMI At least one working adult At least one dependent
Subsidy amount	Difference between the SAFMR and 40% AMI of gross household income
Distribution of household sizes among participants	15% - 2-person 25% - 3-person 25% - 4-person 25% - 5-person 10% - 6-person
Distribution of incomes among participants	25% - 30% AMI 50% - 40% AMI 25% - 50% AMI
Administrative overhead	15% of total program budget

Notes:

- SAFMR refers to the Small Area Fair Market Rent as adopted by Alexandria Redevelopment and Housing Authority (ARHA) for 2023.
- The affordable monthly rent is 40% of gross household income, not the standard 30%.
- The subsidy calculated for each household is respective to their household size.
- Families will only occupy 1-bedroom, 2-bedroom, or 3-bedroom units.
- The administrative overhead includes housing-specific case management.

Inputs

Assign budget (dollars) and overhead costs (percent) variables:

```
# Budget allocation
sC_budget <- 500000

# Overhead percentage
sC_overhead <- 0.15
```

Assign household distributions by AMI and household size (number of persons):

```
# Distribution of households by AMI
sC_ami <- c(`30% AMI` = 0.25, `40% AMI` = 0.50, `50% AMI` = 0.25)

# Distribution of households by household size
sC_person <- tibble(
  hh_size = paste0("person", 2:6),
  pct = c(0.15, 0.25, 0.25, 0.25, 0.10)
)
```

Because subsidy amounts will be calculated using SAFMR, we also need to estimate household shares across units by size (number of bedrooms). The model will use the following assumptions:

- 2-person: 100% 1-bedroom
- 3-person: 10% 1-bedroom, 90% 2-bedroom
- 4-person: 80% 2-bedroom, 20% 3-bedroom
- 5-person: 100% 3-bedroom
- 6-person: 100% 3-bedroom

```
# Distribution of household sizes by unit size
sC_unit <- list(
    `person2` = c(`bedroom1` = 1),
    `person3` = c(`bedroom1` = 0.1, `bedroom2` = 0.9),
    `person4` = c(`bedroom2` = 0.8, `bedroom3` = 0.2),
    `person5` = c(`bedroom3` = 1),
    `person6` = c(`bedroom3` = 1)
  )
```

persons	bedroom1	bedroom2	bedroom3
person2	1	-	-
person3	0.1	0.9	-
person4	-	0.8	0.2
person5	-	-	1
person6	-	-	1

Create data frame with all permutations for income, unit size, and household sizes. Exclude non-valid combinations of unit and household size:

```
sC_hh_type <- expand.grid(
  AMI = names(sC_ami),
  bedrooms = unlist(lapply(names(sC_unit), function(unit) names(sC_unit[[unit]]))),
  hh_size = sC_person$hh_size
  ) |>
  distinct() |>
  filter(
    !(bedrooms == "bedroom1" & !hh_size %in% c("person2", "person3")),
    !(bedrooms == "bedroom2" & hh_size %in% c("person2", "person5", "person6")),
    !(bedrooms == "bedroom3" & hh_size %in% c("person2", "person3"))
  )
```

Calculations

Create a function that tabulates the respective household distributions by AMI, household size, and unit type:

```
sC_dist_fn <- function() {

  # Build data frame with AMI and household size distributions
  dist <- expand.grid(
    AMI = names(sC_ami),
    hh_size = sC_person$hh_size
  ) |>
    mutate(
      # Match hh_size with sC_person$hh_size to fetch the correct percentage
      households = sC_ami[AMI] * sC_person$pct[match(hh_size, sC_person$hh_size)]
    )

  # Initialize an empty data frame for the final distribution
  final_distribution <- data.frame()

  # Iterate distribution by unit size
  for (person in unique(sC_hh_type$hh_size)) {
    current_dist <- subset(dist, hh_size == person)

    for (bedroom_count in names(sC_unit[[person]])) {
      current_dist$bedrooms <- as.character(bedroom_count)

      current_dist$pct <- current_dist$households *
        sC_unit[[person]][bedroom_count]

      final_distribution <- rbind(
        final_distribution,
        current_dist[, c("AMI", "bedrooms", "hh_size", "pct")]
      )
    }
  }

  return(final_distribution)
}

sC_dist <- sC_dist_fn()
```

Calculate affordable rents (`aff_rent`) at 30% AMI, 40% AMI, and 50% AMI for households with 2 to 6 persons (`hh_size`):

```
# Monthly affordable rents by household size
sC_aff_rents <- hud_ami |>
  filter(
    AMI %in% c("30% AMI", "40% AMI", "50% AMI"), # 30%, 40%, and 50% AMI only
    str_detect(hh_size, "[23456]") # 2-6 person households only
    ) |>
  mutate(
    aff_rent = income/12 * 0.4 # 40% of monthly income
  ) |>
  select(-3)
```

AMI	hh_size	aff_rent
30% AMI	person2	1206.00
	person3	1357.00
	person4	1507.00
	person5	1628.00
	person6	1749.00
40% AMI	person2	1608.00
	person3	1809.33
	person4	2009.33
	person5	2173.67
	person6	2332.00
50% AMI	person2	2010.00
	person3	2261.67
	person4	2511.67
	person5	2713.33
	person6	2915.00

Join `sC_hh_type` with average FMR by unit size (`fmrs_avg`) and affordable rents (`sC_aff_rents`), then find difference between values to calculate the monthly subsidy (`subsidy`):

```
sC_subsidy <- sC_hh_type |>
  left_join(sC_aff_rents, join_by(AMI, hh_size)) |>
  left_join(fmrs_avg) |>
  mutate(subsidy = fmr_avg - aff_rent)
```

AMI	bedrooms	hh_size	aff_rent	fmr_avg	subsidy
30% AMI	bedroom1	person2	1206.00	2019.88	813.88
		person3	1357.00	2019.88	662.88
	bedroom2	person3	1357.00	2300.38	943.38
		person4	1507.00	2300.38	793.38
	bedroom3	person4	1507.00	2876.50	1369.50
		person5	1628.00	2876.50	1248.50
		person6	1749.00	2876.50	1127.50
40% AMI	bedroom1	person2	1608.00	2019.88	411.88
		person3	1809.33	2019.88	210.54
	bedroom2	person3	1809.33	2300.38	491.04
		person4	2009.33	2300.38	291.04
	bedroom3	person4	2009.33	2876.50	867.17
		person5	2173.67	2876.50	702.83
		person6	2332.00	2876.50	544.50
50% AMI	bedroom1	person2	2010.00	2019.88	9.88
		person3	2261.67	2019.88	-241.79
	bedroom2	person3	2261.67	2300.38	38.71
		person4	2511.67	2300.38	-211.29
	bedroom3	person4	2511.67	2876.50	364.83
		person5	2713.33	2876.50	163.17
		person6	2915.00	2876.50	-38.50

> ⚠ Some affordable rents almost equal to FMRs
>
> Note that the subsidies for 5 household types are negligible — under $50. (See red values.) These cases are the result of higher affordable rents among those earning 50% AMI or more, along with the higher 40% tenant contribution.

For the purposes of this model, these household types with no or very little subsidy need will be excluded. The sum of their respective shares will be redistributed to the remaining 16 household combinations.

Calculate total shares of excluded and remaining household types:

```
sC_excl <- sC_subsidy |>
  left_join(sC_dist, join_by(AMI, bedrooms, hh_size)) |>
  mutate(
    status = case_when(
      subsidy < 50 ~ "exclude",
      subsidy > 50 ~ "retain"
    ),
    .before = 7
  )
```

status	pct
retain	0.825
exclude	0.175

Evenly distributing this 0.175 across the remaining 16 combinations would not respect the original group distributions by AMI and household size. Therefore, this surplus share will be manually redistributed to each remaining combination to ensure the new AMI and household size group subtotals are as close to the original values as possible.

First, we determine the change in shares by each AMI and household size group resulting from excluding the 5 invalid combinations.

Change in shares grouped by AMI:

AMI	pct	pct_excl	diff
30% AMI	0.25	0.250	0.000
40% AMI	0.50	0.500	0.000
50% AMI	0.25	0.075	-0.175

Change in shares grouped by household size:

hh_size	pct	pct_excl	diff
person2	0.15	0.1125	-0.0375
person3	0.25	0.1875	-0.0625
person4	0.25	0.2000	-0.0500
person5	0.25	0.2500	0.0000
person6	0.10	0.0750	-0.0250

In redistributing these shares, we will need to:

- Significantly increase the remaining 50% AMI household types (4-person and 5-person in 3-bedroom units) to maintain overall balance across AMI groups
- Reduce the corresponding shares in both 30% AMI and 40% AMI groups to maintain overall balance across household sizes
- Slightly increase the 2-person, 3-person, and 6-person household types in both 30% AMI and 40% AMI groups to account for losses within 50% AMI category

To accomplish this, the excluded share is divided into 16 parts (p):

$$p = \frac{0.175}{16} = 0.0109375$$

We can increase or decrease each household combination by a multiple of p as long as the net increase across all households is $+16p$.

The following allocation reproduces the original AMI distribution, and a new household size distribution where each category is within ± 0.02 of the original share.

```
sC_redist <- sC_excl_grp |>
  filter(status == "retain") |>
  select(1, 2, 3, 5) |>
  arrange(AMI, hh_size, bedrooms) |>
  mutate(
    p_shares = case_when(
      AMI == "30% AMI" & bedrooms == "bedroom2" & hh_size == "person4" ~ 0,
      AMI == "30% AMI" & bedrooms == "bedroom3" & hh_size == "person4" ~ -1,
      AMI == "30% AMI" & bedrooms == "bedroom3" & hh_size == "person5" ~ -3,
      AMI == "40% AMI" & bedrooms == "bedroom3" & hh_size == "person4" ~ -2,
      AMI == "40% AMI" & bedrooms == "bedroom3" & hh_size == "person5" ~ -4,
      AMI == "40% AMI" & bedrooms == "bedroom3" & hh_size == "person6" ~ 2,
      AMI == "50% AMI" ~ 8,
      .default = 1
    ),
    pct_redist = pct + (p*p_shares)
  )
```

AMI	bedrooms	hh_size	pct	p_shares	pct_redist
30% AMI	bedroom1	person2	0.0375	1	0.0484
		person3	0.0063	1	0.0172
	bedroom2	person3	0.0562	1	0.0672
		person4	0.0500	0	0.0500
	bedroom3	person4	0.0125	-1	0.0016
		person5	0.0625	-3	0.0297
		person6	0.0250	1	0.0359
40% AMI	bedroom1	person2	0.0750	1	0.0859
		person3	0.0125	1	0.0234
	bedroom2	person3	0.1125	1	0.1234
		person4	0.1000	1	0.1109
	bedroom3	person4	0.0250	-2	0.0031
		person5	0.1250	-4	0.0813
		person6	0.0500	2	0.0719
50% AMI	bedroom3	person4	0.0125	8	0.1000
		person5	0.0625	8	0.1500

Original distribution of households by AMI retained:

```
sC_redist |>
  group_by(AMI) |>
  select(-2) |>
  summarise(pct_redist = sum(pct_redist)) |>
  kbl(booktabs = T, align = "lc", linesep = "") |>
  row_spec(0, bold = T) |>
  kable_styling(full_width = T)
```

AMI	pct_redist
30% AMI	0.25
40% AMI	0.50
50% AMI	0.25

New distribution of households by household size:

hh_size	pct_redist
person2	0.1344
person3	0.2313
person4	0.2656
person5	0.2609
person6	0.1078

Rejoin the redistributed households with the calculated subsidy amounts per household:

```
sC_redist_subsidy <- sC_subsidy |>
  right_join(sC_redist, join_by(AMI, bedrooms, hh_size)) |>
  select(1:3, 6, 9)
```

AMI	bedrooms	hh_size	subsidy	pct_redist
30% AMI	bedroom1	person2	813.88	0.0484
		person3	662.88	0.0172
	bedroom2	person3	943.38	0.0672
		person4	793.38	0.0500
	bedroom3	person4	1369.50	0.0016
		person5	1248.50	0.0297
		person6	1127.50	0.0359
40% AMI	bedroom1	person2	411.88	0.0859
		person3	210.54	0.0234
	bedroom2	person3	491.04	0.1234
		person4	291.04	0.1109
	bedroom3	person4	867.17	0.0031
		person5	702.83	0.0813
		person6	544.50	0.0719
50% AMI	bedroom3	person4	364.83	0.1000
		person5	163.17	0.1500

Calculate annual subsidy per household (`subsidy_annual`) and the theoretical share of subsidy allocated for each household (`subsidy_share`). Determine the number of households served (`hh_served`) by normalizing `subsidy_share` to the known budget:

```
# Annual subsidy per household type
sC_subsidy_annual <- sC_redist_subsidy |>
  mutate(subsidy_annual = subsidy * 12) |>
  select(AMI, bedrooms, hh_size, "pct" = pct_redist, subsidy_annual) |>
  mutate(
    subsidy_share = subsidy_annual * pct, # Subsidy per HH type
    hh_served = # Adjust to known budget
      pct*(
        sC_budget * (1 - sC_overhead)
        )/sum(subsidy_share)
  )
```

AMI	bedrooms	hh_size	subsidy_annual	subsidy_share	hh_served
30% AMI	bedroom1	person2	9766.50	473.06	3.257
		person3	7954.50	136.72	1.156
	bedroom2	person3	11320.50	760.60	4.518
		person4	9520.50	476.03	3.362
	bedroom3	person4	16434.00	25.68	0.105
		person5	14982.00	444.78	1.996
		person6	13530.00	486.23	2.416
40% AMI	bedroom1	person2	4942.50	424.75	5.778
		person3	2526.50	59.21	1.576
	bedroom2	person3	5892.50	727.36	8.300
		person4	3492.50	387.45	7.459
	bedroom3	person4	10406.00	32.52	0.210
		person5	8434.00	685.26	5.463
		person6	6534.00	469.63	4.833
50% AMI	bedroom3	person4	4378.00	437.80	6.724
		person5	1958.00	293.70	10.086

Calculate and summarize the estimated budget and households served by AMI and household size:

```
sC_served <- sC_subsidy_annual |>
  mutate(budget = hh_served * subsidy_annual) |>
  summarise(
    budget = sum(budget),
    hh_served = sum(hh_served),
    .by = c(AMI, hh_size)
  )
```

AMI	hh_size	budget	hh_served
30% AMI	person2	31808.22	3.257
	person3	60334.15	5.673
	person4	33733.82	3.467
	person5	29906.27	1.996
	person6	32693.73	2.416
40% AMI	person2	28559.34	5.778
	person3	52887.90	9.876
	person4	28238.06	7.669
	person5	46076.10	5.463
	person6	31577.35	4.833
50% AMI	person4	29437.07	6.724
	person5	19747.98	10.086
Total	-	425000.00	67.239

Model results

Round each estimate to the nearest whole number and determine total:

```
# Rounded estimates with grand total
sC_estimate <- sC_served |>
  arrange(AMI, hh_size) |>
  mutate(
    hh_served = round(hh_served),
    hh_size = case_match(
      hh_size,
      "person2" ~ "2 person",
      "person3" ~ "3 person",
      "person4" ~ "4 person",
      "person5" ~ "5 person",
      "person6" ~ "6 person"
    )
  ) |>
  adorn_totals()
```

Table 8: Scenario C - Estimated Households Served by Household Size

Income	Household size	Budget	Households served
30% AMI	2 person	$31,808	3
	3 person	$60,334	6
	4 person	$33,734	3
	5 person	$29,906	2
	6 person	$32,694	2
40% AMI	2 person	$28,559	6
	3 person	$52,888	10
	4 person	$28,238	8
	5 person	$46,076	5
	6 person	$31,577	5
50% AMI	4 person	$29,437	7
	5 person	$19,748	10
Total	**-**	**$425,000**	**67**

Scenario C results

Average annual program cost per household: $7,462.69

Under Scenario C, a total program budget of $500,000 with a 15% administrative overhead leaves $425,000 to fund rental assistance. Given the assumed household distributions by AMI, unit size, and household size, the total number of households served is 67.