

Alexandria LRSP Scenario Models

A7.1 Standard parameters

Income limits

The models use HUD's FY 2023 Multifamily Tax Subsidy Projects (MTSP) Income Limits for Washington-Arlington-Alexandria, DC-VA-MD HUD Metro FMR Area.¹ These are the official income limits used to determine eligibility for LIHTC projects and other affordable multifamily properties financed by tax-exempt bonds.

While the MTSP limits differ slightly from the standard income limits used for Housing Choice Vouchers, public housing, and other HUD-supported assistance programs, they are used here because they publish limits for a greater range of AMI levels, including 40% AMI and 60% AMI. The City of Alexandria generally uses MTSP limits for its housing programs.

Table 1: FY 2023 MTSP Income Limits for Washington-Arlington-Alexandria, DC-VA-MD HUD Metro FMR Area

AMI	1 person	2 person	3 person	4 person	5 person
20% AMI	\$21,100	\$24,120	\$27,140	\$30,140	\$32,460
30% AMI	\$31,650	\$36,180	\$40,710	\$45,210	\$48,840
40% AMI	\$42,200	\$48,240	\$54,280	\$60,280	\$65,210
50% AMI	\$52,750	\$60,300	\$67,850	\$75,350	\$81,400
60% AMI	\$63,300	\$72,660	\$81,420	\$90,240	\$97,460
70% AMI	\$73,850	\$84,240	\$94,990	\$105,490	\$113,960
80% AMI	\$84,400	\$96,480	\$108,560	\$120,560	\$130,240

¹ [FY 2023 MTSP Income Limits](#) (Accessed 2024-01-19)

Fair Market Rents

Models where the rent subsidy is calculated based on Fair Market Rents (FMR) use the current Small Area Fair Market Rents (SAFMR) adopted by the Arlington Redevelopment and Housing Authority for 2023. SAFMRs are provided by ZIP code.

While actual subsidy amounts will depend on the ZIP code where the tenant lives, models will use the average values (by unit size) across all ZIP codes. This is a simplification to avoid making assumptions about the geographic distribution of participating households.

Table 2: ARHA 2023 Payment Standards

ZIP code	Studio	1 bedroom	2 bedroom	3 bedroom
22301	\$2,013	\$2,046	\$2,332	\$2,915
22302	\$1,980	\$2,013	\$2,288	\$2,860
22304	\$1,914	\$1,947	\$2,211	\$2,761
22305	\$1,859	\$1,892	\$2,156	\$2,695
22311	\$1,936	\$1,969	\$2,244	\$2,805
22312	\$1,848	\$1,870	\$2,134	\$2,673
22313	\$1,782	\$1,815	\$2,068	\$2,585
22314	\$2,563	\$2,607	\$2,970	\$3,718
Average	\$1,986.88	\$2,019.88	\$2,300.38	\$2,876.50

A7.2 Scenario A - Reduce Cost Burden for 30% to 50% AMI Households

This scenario outlines a LRSP with a total annual allocation of \$500,000. The primary goal of the program is to reduce housing cost burden among households with incomes between 30% and 50% AMI. The model uses the following inputs to estimate the number of households served.

Variable	Input
<i>Total program budget</i>	\$500,000
<i>Eligibility</i>	Household income between 30% and 50% AMI
<i>Subsidy amount</i>	Difference between the affordable monthly rent at 60% AMI and the affordable monthly rent at 40% AMI
<i>Distribution of unit sizes among participants</i>	25% - Studios 25% - 1-bedroom 25% - 2-bedroom 25% - 4-bedroom
<i>Distribution of incomes among participants</i>	25% - 30% AMI 50% - 40% AMI 25% - 30% AMI
<i>Administrative overhead</i>	15% of total program budget

Notes:

- No other eligibility conditions apply.
- “Affordable monthly rent” is 30% of gross household income.
- The subsidy calculated for each household is respective to their household size.
- The administrative overhead includes housing-specific case management.

Inputs

Assign budget (dollars) and overhead costs (percent) variables:

```
# Budget allocation
sA_budget <- 500000

# Overhead percentage
sA_overhead <- 0.15
```

Calculate affordable rents at 40% AMI and 60% AMI for households with 1 to 5 persons (hh_size) to determine monthly subsidy amounts (subsidy):

```
# Monthly subsidy about by household size
sA_subsidy <- hud_ami |>
  filter(
    AMI %in% c("40% AMI", "60% AMI"), # 40% and 60% AMI only
    str_detect(hh_size, "[12345]")    # 1-5 person households only
  ) |>
  mutate(
    aff_rent = income/12 * 0.3          # 30% of monthly income
  ) |>
  select(-3) |>
  pivot_wider(
    names_from = AMI,
    values_from = aff_rent
  ) |>
  mutate(
    subsidy = `60% AMI` - `40% AMI`    # Calculate subsidy
  ) |>
  select(1, 4)
```

hh_size	subsidy
person1	527.50
person2	610.50
person3	678.50
person4	749.00
person5	806.25

Assign household distributions by AMI and unit size (number of bedrooms):

```
# Distribution of households by AMI
sA_ami <- c(`30% AMI` = 0.25, `40% AMI` = 0.50, `50% AMI` = 0.25)

# Distribution of households by unit size
sA_unit <- c(
  `bedroom0` = 0.25, `bedroom1` = 0.25,
  `bedroom2` = 0.25, `bedroom3` = 0.25
)
```

Subsidy amounts are determined by household size, but the model assumptions only provide a distribution of unit sizes. Therefore, we need to make reasonable assumptions about the distribution of different household sizes among each unit size, as defined by number of bedrooms (bedrooms). This breakdown is shown in the code below:

- Studio: 100% 1-person
- 1-bedroom: 50% 1-person, 50% 2-person
- 2-bedroom: 30% 2-person, 40% 3-person, 30% 4-person
- 3-bedroom: 30% 3-person, 40% 4-person, 30% 3-person

```
# Distribution of household sizes by unit size
sA_person <- list(
  `bedroom0` = c(`person1` = 1),
  `bedroom1` = c(`person1` = 0.5, `person2` = 0.5),
  `bedroom2` = c(`person2` = 0.3, `person3` = 0.4, `person4` = 0.3),
  `bedroom3` = c(`person3` = 0.3, `person4` = 0.4, `person5` = 0.3)
)
```

bedrooms	person1	person2	person3	person4	person5
bedroom0	1	-	-	-	-
bedroom1	0.5	0.5	-	-	-
bedroom2	-	0.3	0.4	0.3	-
bedroom3	-	-	0.3	0.4	0.3

Calculations

Create a function that tabulates the respective household distributions by AMI, household size, and unit type:

```
sA_dist_fn <- function() {  
  
  # Build data frame with AMI and unit size distributions  
  dist <- expand.grid(  
    AMI = names(sA_ami),  
    bedrooms = names(sA_unit)  
  ) |>  
  # Calculate unique shares for both AMI and unit size  
  mutate(households = sA_ami[AMI] * sA_unit[bedrooms])  
  
  # Initialize an empty data frame for the final distribution  
  final_distribution <- data.frame()  
  
  # Iterate distribution by household size  
  for (bedroom in names(sA_person)) {  
    current_dist <- subset(dist, bedrooms == bedroom)  
  
    for (person_count in names(sA_person[[bedroom]])) {  
      current_dist$hh_size <- as.character(person_count)  
  
      current_dist$pct <- current_dist$households *  
        sA_person[[bedroom]][person_count]  
  
      final_distribution <- rbind(  
        final_distribution,  
        current_dist[, c("AMI", "bedrooms", "hh_size", "pct")]  
      )  
    }  
  }  
  
  final_distribution  
}  
  
sA_dist <- sA_dist_fn()
```

AMI	bedrooms	hh_size	pct
30% AMI	bedroom0	person1	0.06250
	bedroom1	person1	0.03125
		person2	0.03125
	bedroom2	person2	0.01875
		person3	0.02500
		person4	0.01875
	bedroom3	person3	0.01875
		person4	0.02500
		person5	0.01875
40% AMI	bedroom0	person1	0.12500
	bedroom1	person1	0.06250
		person2	0.06250
	bedroom2	person2	0.03750
		person3	0.05000
		person4	0.03750
	bedroom3	person3	0.03750
		person4	0.05000
		person5	0.03750
50% AMI	bedroom0	person1	0.06250
	bedroom1	person1	0.03125
		person2	0.03125
	bedroom2	person2	0.01875
		person3	0.02500
		person4	0.01875
	bedroom3	person3	0.01875
		person4	0.02500
		person5	0.01875

Because the subsidy amounts only vary by household size, unit size and AMI are no longer necessary. Sum the individual household shares by household size:

```
# Household share subtotals by household size
sA_hhsize <- sA_dist |>
  summarise(pct = sum(pct), .by = hh_size)
```


hh_size	pct
person1	0.375
person2	0.200
person3	0.175
person4	0.175
person5	0.075

Join the monthly subsidy amounts by household size (subsidy) and calculate annual subsidy per household (subsidy_annual):

```
# Annual subsidy per household size
sA_subsidy_annual <- sA_hhsize |>
  left_join(sA_subsidy) |>
  mutate(subsidy_annual = subsidy * 12)
```

hh_size	pct	subsidy	subsidy_annual
person1	0.375	527.50	6330
person2	0.200	610.50	7326
person3	0.175	678.50	8142
person4	0.175	749.00	8988
person5	0.075	806.25	9675

Determine each share of the total annual program budget (minus overhead costs) by household size (budget), then divide by the annual subsidy per household to calculate the estimates number of households served per household size (hh_served):

```
# Estimated budget and households served
sA_served <- sA_subsidy_annual |>
  mutate(
    budget = (sA_budget * (1 - sA_overhead)) * pct,
    hh_served = budget / subsidy_annual
  )
```

hh_size	pct	subsidy	subsidy_annual	budget	hh_served
person1	0.375	527.50	6330	159375	25.177725
person2	0.200	610.50	7326	85000	11.602512
person3	0.175	678.50	8142	74375	9.134733
person4	0.175	749.00	8988	74375	8.274922
person5	0.075	806.25	9675	31875	3.294574

Model results

Round each estimate down to the nearest whole number and determine total:

```
# Rounded estimates with grad total
sA_estimate <- sA_served |>
  select(1, 5, 6) |>
  mutate(
    hh_served = floor(hh_served),
    hh_size = case_match(
      hh_size,
      "person1" ~ "1 person",
      "person2" ~ "2 person",
      "person3" ~ "3 person",
      "person4" ~ "4 person",
      "person5" ~ "5 person",
    )
  ) |>
  adorn_totals()
```

Table 4: Scenario A - Estimated households served by household size

Household size	Budget	Households served
1 person	\$159,375	25
2 person	\$85,000	11
3 person	\$74,375	9
4 person	\$74,375	8
5 person	\$31,875	3
Total	\$425,000	56

Scenario A results

Under Scenario A, a total program budget of \$500,000 with a 15% administrative overhead leaves \$425,000 to fund rental assistance. Given the assumed household distributions by AMI, unit size, and household size, the total number of households served is 56. Over half of these would be persons living on their own, or with one other person.

A7.3 Scenario B - Stabilize Unhoused Persons

This scenario outlines a LRSP serving a total of 150 households experiencing housing insecurity. The primary goal of the program is to provide deep rental assistance to help these households achieve housing stability and avoid homelessness. The model uses the following inputs to estimate the annual program cost required to serve 150 households.

Variable	Input
<i>Total households served</i>	150
<i>Eligibility</i>	Household/individual determined to be homeless in City's annual Point-in-Time count
<i>Subsidy amount</i>	Difference between the affordable monthly rent at 60% AMI and the households' current affordable monthly rent
<i>Distribution of household types among participants</i>	2/3 - Single-person 1/3 - Household with children
<i>Distribution of unit sizes among participants</i>	2/3 - Studios 1/3 - 2-bedroom
<i>Distribution of incomes among participants</i>	50% - SSI income 50% - \$1,500 per month
<i>Administrative overhead</i>	20% of total program budget

Notes:

- No other eligibility conditions apply.
- "Affordable monthly rent" is 30% of gross household income.
- The subsidy calculated for each household is respective to their household size.
- The administrative overhead is higher than Scenario A to accommodate more intensive case management requirements for persons experiencing homelessness.

Inputs

Assign households served and overhead costs (percent) variables:

```
# Total households served
sB_hh_served <- 150

# Overhead percentage
sB_overhead <- 0.20
```

We can reasonably assume that all single-person households will live in studios, while all households with children will live in 2-bedroom units. Therefore, we do not need separate distribution shares for each. However, we do need to determine more specific household sizes.

For this model, among households with children, we will assume the following breakdown, as shown in the code below:

- 1/2 are 2-person (adult and child)
- 1/4 are 3-person (adult and two children, or two adults and child)
- 1/4 are 4-person (adult and three children, or two adults and two children)

```
# Distribution of households by size
sB_person <- c(
  `person1` = 0.667, # 2/3
  `person2` = 0.167, # 1/2 of 1/3
  `person3` = 0.083, # 1/4 of 1/3
  `person4` = 0.083 # 1/4 of 1/3
)
```

Assign household distribution by income and calculate :

```
# Distribution of households by income
sB_income <- c(`ssi` = 0.5, `1500` = 0.5)
```

Because SSI amounts depend on whether the beneficiary lives alone or is married, their affordable rents will vary. However, due to the relatively small share of households in this model that could include two married adults, we will assume that any persons enrolled in SSI receive benefits for an individual. The current monthly SSI amount for an eligible individual is \$943.²

²[SSI Federal Payment Amounts for 2024](#) (Accessed 2024-01-19)

```
# Monthly SSI income for eligible individual
sB_ssi <- 943
```

Calculations

Calculate affordable rents for SSI income and \$1,500/month (`income_source`) for households with 1 to 4 persons (`hh_size`) to determine monthly subsidy amounts (`subsidy`):

```
# Monthly subsidy about by household size
sB_subsidy <- hud_ami |>
  filter(
    AMI %in% c("60% AMI"),          # 60% AMI only
    str_detect(hh_size, "[1234]") # 1-4 person households only
  ) |>
  mutate(
    aff_rent_60ami = income/12 * 0.3 # 30% of monthly income
  ) |>
  select(2, 4) |>
  mutate(
    `SSI` = sB_ssi * 0.3, # 30% of SSI
    `$1,500/mo` = 1500 * 0.3 # 30% of $1,500
  ) |>
  pivot_longer(
    3:4,
    names_to = "income_source",
    values_to = "aff_rent"
  ) |>
  mutate(
    subsidy = aff_rent_60ami - aff_rent # Calculate subsidy
  ) |>
  select(3, 1, 5)
```

income_source	hh_size	subsidy
SSI	person1	1299.6
	person2	1533.6
	person3	1752.6
	person4	1973.1
\$1,500/mo	person1	1132.5
	person2	1366.5
	person3	1585.5
	person4	1806.0

A7.4 Scenario C – Reduce Severe Cost burden for Lower-Income Working Families

This scenario outlines a LRSP with a total annual allocation of \$500,000. The primary goal of the program is to reduce housing cost burden among households with incomes between 30% and 50% AMI. Households must have one working adult and one or more dependent. The model uses the following inputs to estimate the number of households served.

Variable	Input
<i>Total program budget</i>	\$500,000
<i>Eligibility</i>	Household income between 30% and 50% AMI At least one working adult At least one dependent
<i>Subsidy amount</i>	Difference between the SAFMR and 40% AMI of gross household income
<i>Distribution of unit sizes among participants</i>	1/3 - Studios 1/3 - 1-bedroom 1/3 - 2-bedroom
<i>Distribution of incomes among participants</i>	25% - 30% AMI 50% - 40% AMI 25% - 30% AMI
<i>Administrative overhead</i>	15% of total program budget

Notes:

- SAFMR refers to the Small Area Fair Market Rent as adopted by Alexandria Redevelopment and Housing Authority (ARHA) for 2023. (See table below.)
- The affordable monthly rent is 40% of gross household income, not the standard 30%.
- The subsidy calculated for each household is respective to their household size.
- The administrative overhead includes housing-specific case management.