1] $X = \max(X_1, 2X_2)$

For continuous random variable,

$$E(X) = \int_{-\infty}^{\infty} x f(x)\, dx$$

where $f(x)$ is PDF & x is the values x can take.

Now,
PDF is derivative of CDF

$$\underbrace{F_X(x)}_{CDF} = P(X \leq x) \quad \text{for all } x \in R$$

Now, $F_X(x) = P(\max(X_1, 2X_2) \leq x)$

$= P(X_1 \leq x) \times P(2X_2 \leq x)$ —— ① (since X_1 & X_2 are independent rv)

Now, Since X_1 & X_2 are uniformly distribut
$P(X_1 \le x)$ has following values

$$
\begin{cases}
0 & \text{if } x < 0 \\
x & \text{if } \cancel{x \ge 0 \text{ and}}\ 0 \le x \le 1 \\
1 & \text{if } x > 1
\end{cases}
$$

$P(2X_2 \le x)$ has following values
ie $P(X_2 \le \frac{x}{2})$

$$
\begin{cases}
0 & \text{if } x < 0 \\
x/2 & \text{if } \cancel{x \ge 0 \text{ and}}\ 0 \le x \le 2 \\
1 & \text{if } x > 2
\end{cases}
$$

From eq. (1),

$$
\begin{aligned}
F_X(x) &= 0 && \text{if } x < 0 \\
&= x^2/2 && \text{if } 0 \le x \le 1 \\
&= x/2 && \text{if } 1 < x \le 2 \\
&= 1 && \text{if } x > 2
\end{aligned}
$$

Since PDF is derivative of CDF

$$f(x) = \frac{d}{dx} F_x(x).$$

$$\therefore f(x) = \begin{cases} 0 & \text{if } x < 0 \\ x & \text{if } 0 \le x \le 1 \\ 1/2 & \text{if } 1 < x \le 2 \\ 1 & \text{if } x > 2 \end{cases}$$

(a) $E(X) = \int_{-\infty}^{\infty} x\, f(x)\, dx$

$$= \int_0^1 x(x)\, dx + \int_1^2 x\left(\frac{1}{2}\right) dx$$

$$= \int_0^1 x^2\, dx + \frac{1}{2}\int_1^2 x\, dx$$

$$= \frac{1}{3} + \frac{1}{4}(4-1) = 1 + \frac{1}{12} = \frac{13}{12}$$

$$= \boxed{1.0833}$$

(b) $Var(x) = E(x^2) - (E(x))^2$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx$$

$$= \int_0^1 x^2 \times (x)\, dx + \int_1^2 x^2 \times \left(\frac{1}{2}\right)$$

$$= \frac{x^4}{4}\Big|_0^1 + \frac{x^3}{6}\Big|_1^2$$

$$= \frac{1}{4} + \frac{7}{6} = 1.416$$

~~$\therefore Var(x) = \frac{1}{4} - \left(\frac{1}{3}\right)^2 = \boxed{\frac{5}{36}}$~~

$$\therefore Var(x) = 1.416 - (1.0833)^2$$

$$\boxed{0.2431}$$

$$\max(X_1, 2X_2) = X_1 \quad \text{if} \quad X_2 \le \frac{X_1}{2}$$
$$= X_2 \quad \text{if} \quad X_2 > \frac{X_1}{2}$$

$$E(X * X_1) = \int_{x_1=0}^{1} \int_{x_2=0}^{1} X_1 \times \max(X_1, 2X_2)\, dx_2\, dx_1$$

$$\int_0^1 \left(\int_{x_2=0}^{x_1/2} X_1 * X_1\, dx_2 \right) dx_1 + \int_0^1 \left(\int_{\frac{x_1}{2}}^{1} x_1\, 2x_2\, dx_2 \right) dx_1$$

$$= \int_0^1 x_1^2 \left(\int_0^{x_1/2} dx_2 \right) dx_1 + 2\int_0^1 x_1 \left(\int_{\frac{x_1}{2}}^{1} x_2\, dx_2 \right) dx_1$$

$$= \int_0^1 x_1^2 \times \frac{x_1}{2}\, dx_1 + 2\int_0^1 x_1 \times \left.\frac{x_2^2}{2}\right|_{\frac{x_1}{2}}^{1} dx_1$$

$$= \int_0^1 \frac{x_1^3}{2} + 2\int_0^1 x_1 \times \left(\frac{1}{2} - \frac{x_1^2}{8} \right) dx_1$$

$$= \int_0^1 \frac{x_1^3}{2} + 2\int_0^1 \frac{x_1}{2} - \frac{x_1^3}{8}\, dx_1$$

$$= \frac{1}{8} + 2\left(\frac{1}{4} - \frac{1}{32} \right) = \boxed{\frac{9}{16}}$$

Now,

$$E(x) = 1.0833 = \left(\frac{13}{12}\right)$$

$$E(X_1) = \int_{-\infty}^{\infty} x\, f(x)\, dx$$

$$= \int_0^1 x\, dx$$

$$= \frac{1}{2}$$

$$\therefore \operatorname{Cov}(X X_1) = E(X X_1) - E(x)\, E(X_1)$$

$$= \frac{9}{16} - \left(\frac{13}{12}\right)\left(\frac{1}{2}\right)$$

$$= \boxed{\frac{1}{48}}$$