LING 505 Sample Exam Questions

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1. Consider the sentence "The red panda is cuter than the giant panda."

   (a) Describe the relation "is cuter than" mathematically in terms of symmetry, transitivity, and reflexivity, applying the strongest possible properties (e.g. antisymmetry is a stronger property than non-symmetry). Assume one panda cannot be exactly as cute as another.

   (b) Can the "is cuter than" relation be used as an equivalence relation? Can it be used as an ordering, and if so is this ordering weak or strong?

   (c) How can the meaning of the sentence be changed when said in the presence of Figure 1? Hint: consider the specific pandas which are referenced in each usage of the sentence.

   (d) Bonus: Explain why in the problem statement of 1a "i.e." could not be used in place of "e.g."

Figure 1: Cute pandas.[1]

2. For finite alphabet Σ, let $L \subseteq \Sigma^*$ be a language such that each word in L contains an even number of every letter in Σ. Prove that $|L| = |\Sigma^*|$.

[1] Left by Mathias Appel, right by Cesar Aguilar, available at `www.pexels.com`

Answer Key

1. (a)
   - Irreflexive: It is impossible for a panda to be cuter than itself.
   - Asymmetric: If a panda is cuter than another then it is impossible for the other to be cuter than it.
   - Transitive: If a panda is cuter than another and that panda is cuter than a third, then the first panda is cuter than the third.

   (b) The relation cannot be used as an equivalence relation because it is not reflexive. However, it is a strong ordering, similar to the greater than operation.

   (c) In the context of Figure 1, the sentence would likely refer to the two pandas specifically but in a general context it would likely be generic singular, referring to each species of panda as a whole.

   (d) The parenthetical note was giving an example of strong properties, not explaining the strength of properties in a different way as would be done if "i.e." had been used.

2. *Proof.* Because $L \subseteq \Sigma^*$, $|L| \leq |\Sigma^*|$. Let $f : \Sigma^* \to L$ such that $f(w)$ is w with each character repeated exactly once. Because there are two instances of each character, there is an even number of each letter of Σ so $f(w) \in L$. For $x, y \in \Sigma^*$ if $f(x) = f(y)$, each even indexed character of $f(x)$ must make up x and y so $x = y$. Note that each odd indexed character of $f(x)$ is equal to the character preceding it. Thus f is injective, meaning $|\Sigma^*| \leq |L|$. Conclude $|L| = |\Sigma^*|$. $\square$