# Probability & Statistics CIS 2033: Hw 1

Hai Dang

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2.4. Among employees of a certain firm, 70% know C/C++, 60% know Fortran, and 50% know both languages. What portion of programmers

(a) does not know Fortran?

$$P(not\ Fortran) = P(\overline{F}) = 1 - P(F) = 1 - 0.6 = 0.4$$

(b) does not know Fortran and does not know C/C++?

$$\begin{aligned} &P(not\ Fortran\ \&\ not\ C/C\ {+}{+}) \\ &= P(\Omega) - [P((\overline{F} \cup (\overline{C/C\ {+}{+}}))/(both))] \\ &= P(\Omega) - [P(Fortran) + (P(C/C\ {+}{+}) - P(both)] \\ &= 1 - (0.7 + 0.6 - 0.5) = 1 - 0.8 = 0.2 \end{aligned}$$

<u>R</u>: Or is the question asking P(does not know both)?

(c) knows C/C++ but not Fortran?

$$P(C/C\ {+}{+}\ \ /Fortran) = P(C/C + +) - P(both) = 0.7 - 0.5 = 0.2$$

(d) knows Fortran but not C/C++?

$$P(Fortran\ /\ C/C+) = P(Fortran) - P(both) = 0.6 - 0.5 = 0.1$$

(e) If someone knows Fortran, what is the probability that he/she knows C/C++ too?

$$P(C/C\ {+}{+}\ |Fortran) = \frac{P(C/C\ {+}{+}\ \cap\ Fortran)}{P(Fortran)} = \frac{0.5}{0.6} = \frac{5}{6}$$

(f) If someone knows C/C++, what is the probability that he/she knows Fortran too?

$$P(Fortran|\ C/C\ {+}{+}) = \frac{P(Fortran\ \cap\ C/C\ {+}{+})}{P(C/C\ {+}{+})} = \frac{0.5}{0.7} = \frac{5}{7}$$

2.7. A system may become infected by some spyware through the internet or e-mail. Seventy percent of the time the spyware arrives via the internet, thirty percent of the time via email. If it enters via the internet, the system detects it immediately with probability 0.6. If via e-mail, it is detected with probability 0.8. What percentage of times is this spyware detected?

$$\begin{aligned} &Let\ I := spyware\ via\ internet,\ E := spyware\ via\ email, \\ &\quad D := spywaredetectedbysystem \end{aligned}$$

$Givens : P(I) = 0.7,\ P(E) = 0.3, P(D|I) = 0.6, P(D|E) = 0.8$

$$P(D|I) = \frac{P(D \cap I)}{P(I)} \rightarrow P(D \cap I) = P(I)P(D|I) = 0.6 * 0.7 = 0.42$$
$$P(D|E) = \frac{P(D \cap E)}{P(E)} \rightarrow P(D \cap E) = P(E)P(D|I) = 0.8 * 0.3 = 0.24$$
$$P(D) = P(I)P(D|I) + P(D \cap E)) = 0.66$$

2.11. A computer program is tested by 5 independent tests. If there is an error, these tests will discover it with probabilities 0.1, 0.2, 0.3, 0.4, and 0.5, respectively. Suppose that the program contains an error. What is the probability that it will be found

Given: T1 = 0.1, T2 = 0.2, T3 = 0.3, T4 = 0.4, T5 = 0.5

(a) by at least one test?

$$P(at\ least\ one\ test) = P(T1 \cup T2 \cup T3 \cup T4 \cup T5) = 1 - P(\overline{T1 \cup T2 \cup T3 \cup T4 \cup T5})$$
$$= 1 - P(\overline{T1} \cap \overline{T2} \cap \overline{T3} \cap \overline{T4} \cap \overline{T5} \cap) = 1 - P(\overline{T1})P(\overline{T2})P(\overline{T3})P(\overline{T4})P(\overline{T5})$$
$$= 1 - P(1 - T1)P(1 - T2)P(1 - T3)P(1 - T4)P(1 - T5)$$
$$= 1 - (1 - 0.1)(1 - 0.2)(1 - 0.3)(1 - 0.4)(1 - 0.5) = 0.8488$$

(b) by at least two tests?

$$P(at\ least\ two\ tests) = 1 - P(\overline{T1 \cup T2 \cup T3 \cup T4 \cup T5}) - P(T1 \cap \overline{T2} \cap \overline{T3} \cap \overline{T4} \cap \overline{T5})$$
$$- P(\overline{T1} \cap T2 \cap \overline{T3} \cap \overline{T4} \cap \overline{T5}) - P(\overline{T1} \cap \overline{T2} \cap T3 \cap \overline{T4} \cap \overline{T5}) - P(\overline{T1} \cap \overline{T2} \cap \overline{T3} \cap T4 \cap \overline{T5})$$
$$- P(\overline{T1} \cap \overline{T2} \cap \overline{T3} \cap \overline{T4} \cap T5)$$
$$= P(at\ least\ one\ ) - P(T1)P(\overline{T2})P(\overline{T3})P(\overline{T4})P(\overline{T5}) - ... - P(\overline{T1})P(\overline{T2})P(\overline{T3})P(\overline{T4})P(T5)$$
$$= 0.8488 - 0.0168 - 0.0378 - 0.0648 - 0.1008 - 0.1512 = 0.4774$$

(c) by all five tests?

$$P(all\ five\ tests) = (T1 \cap T2 \cap T3 \cap T4 \cap T5) = P(T1)P(T2)P(T3)P(T4)P(T5)$$
$$= (0.1)(0.2)(0.3)(0.4)(0.5) = 0.0012$$

2.13. An important module is tested by three independent teams of inspectors. Each team detects a problem in a defective module with probability 0.8. What is the probability that at least one team of inspectors detects a problem in a defective module?

Let T1 = team 1, T2 = team 2, T3 = team 3 where P(T1) = P(T2) = P(T3) = 0.8

$$P(at\ least\ one\ team\ detects\ a\ problem\ in\ defective\ module)$$
$$= P(T1 \cup T2\ T3) = 1 - P(\overline{T1} \cap \overline{T2} \cap \overline{T3})$$
$$= 1 - P(\overline{T1})^3 = 1 - (1 - T1)^3 = 1 - (1 - 0.8)^3 = 1 - (.2)^3 = 0.992$$

2.16. A computer maker receives parts from three suppliers, S1, S2, and S3. Fifty percent come from S1, twenty percent from S2, and thirty percent from S3. Among all the parts supplied by S1, 5% are defective. For S2 and S3, the portion of defective parts is 3% and 6%, respectively.

Givens: P(S1) = 0.5, P(S2) = 0.2, P(S3) = 0.3;
P(D | S1) = 0.05, P(D| S2) = 0.03, P(D| S3) = 0.06

(a) What portion of all the parts is defective?

$$P(D) = P(S1)P(D|S1) + P(S2)P(D|S2) + P(S3)P(D|S3) = 0.5(0.05) + 0.2(0.03) + 0.3(0.06) = 0.049$$

(b) A customer complains that a certain part in her recently purchased computer is defective. What is the probability that it was supplied by S1?

$$P(D|S1) = \frac{P(S1|D)P(S1)}{P(D)} = \frac{(0.05)(.5)}{0.049} = 0.510$$