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Physics 3020 – Methods of Computational Physics
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22 September 2015

Homework Set 2

3.1 This problem concerns an electrical circuit with n loops.

(a) Using Kirchoff's Laws we find n linearly independent equations that will determine the n currents I_i. Due to conservation of charge, the current going into a node must equal the current leaving a node. Therefore,

$$\begin{aligned} I_1 &= I_2 + I_3 + ... + I_n \\ I_1 - I_2 - I_3 - ... - I_n &= 0 \end{aligned} \tag{1}$$

Additionally, due to the conservation of energy, the voltage around a closed loop must be 0. We can write $n-1$ equations for $n-1$ loops:

$$\begin{aligned} I_2R_2 + I_1R_1 &= V_1 + V_2 \\ I_3R_3 + I_1R_1 &= V_1 + V_3 \\ &\vdots \\ I_nR_n + I_1R_1 &= V_1 + V_n \end{aligned} \tag{2}$$

Together, (1) and (2) form n linearly independent equations.

(b) We can write these equations in matrix form:

$$Ax = b \tag{3}$$

We form an n by n matrix from these equations with equation (1) forming the first row of matrix A and the $n-1$ equations in (2) forming the rest of the matrix. The b vector in the equation is just the constants on the right sides of the equations. We can now solve for the vector x which contains the currents.

We now implement this process in the function "currentSolve()". The program prompts the user for their desired number of loops, allocates the A and b matrices, prompts the user for voltages and resistances, and fills the matrices. Using "LUdmp", we create an upper and lower matrix in order to solve large matrices. Implementing the method alu.solve(), we solve for x to obtain each current I_i.

(c) Using 3 loops, and setting $V_1 = 2V$, $V_2 = -4V$, $V_3 = 3V$, $R_1 = 3\Omega$, $R_2 = 2\Omega$, $R_3 = 1\Omega$, our code determines currents

$$\begin{aligned} I_1 &= 0.727273 \\ I_2 &= -2.09091 \\ I_3 &= 2.81818 \end{aligned} \tag{4}$$

(d) For any n number of loops, we consider $R_i = 1\Omega$, $V_1 = 1V$, and all other $V_i = 0$. We can solve for the current I_1 analytically as a function of n. We use equation (2) to determine:

$$\begin{aligned} I_1 + I_2 &= 1 \\ I_1 + I_3 &= 1 \\ &\vdots \\ I_1 + I_n &= 1 \\ \implies I_n &= 1 - I_1 \end{aligned} \tag{5}$$

This implies that all currents other than I_1 are equal.

$$I_2 = I_3 = ... = I_n \tag{6}$$

Using equations (1) and (6) , we now determine:

$$I_1 = (n-1)I_n \tag{7}$$

Using equations (5) and (7), we solve for I_1

$$\begin{aligned} I_1 &= (n-1)(1-I_n) \\ I_1 &= \frac{n-1}{n} \end{aligned} \tag{8}$$

This result matches my code's results when I test for values of n such as 100.

(e) We now consider the case $V_i = 1V$ and $R_i = i\Omega$. When $n = 100$, we determine $I_1 = 0.99$. In order to make a plot of I_1 as a function of n, we create a table of values by iterating over values of n from $n = 1$ to $n = 100$ and find I_1.