Stochastic volatility models with application

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January 2018

Problem

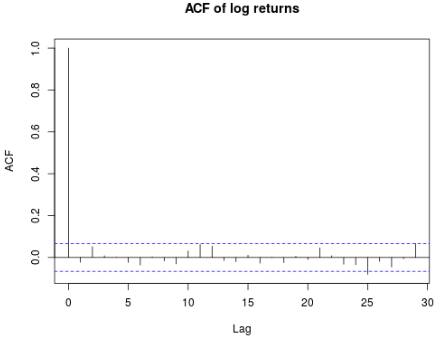
Investing in Siemens AG (SIE.DE) in 2008

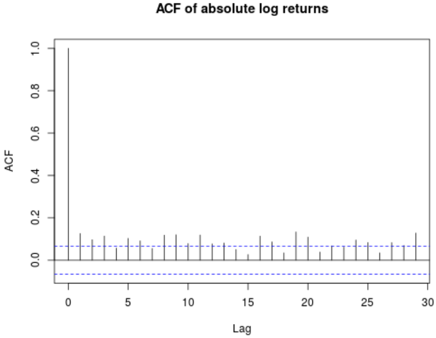
- Model the price movement (returns)
- Make predictions

What model to use?

Empirical facts about log returns

- No autocorrelation
- Autocorrelation in the absolute values





Model

Empirical facts about log returns

- No autocorrelation
- Autocorrelation in the absolute values

$$
\begin{aligned}
y_t &= \varepsilon_t \sqrt{e^{h_t}} \\
h_{t+1} &= \mu + \phi(h_t - \mu) + \sigma \eta_t \\
\forall t : \varepsilon_t, \eta_t &\sim \text{i.i.d. } \mathcal{N}(0, 1)
\end{aligned}
$$

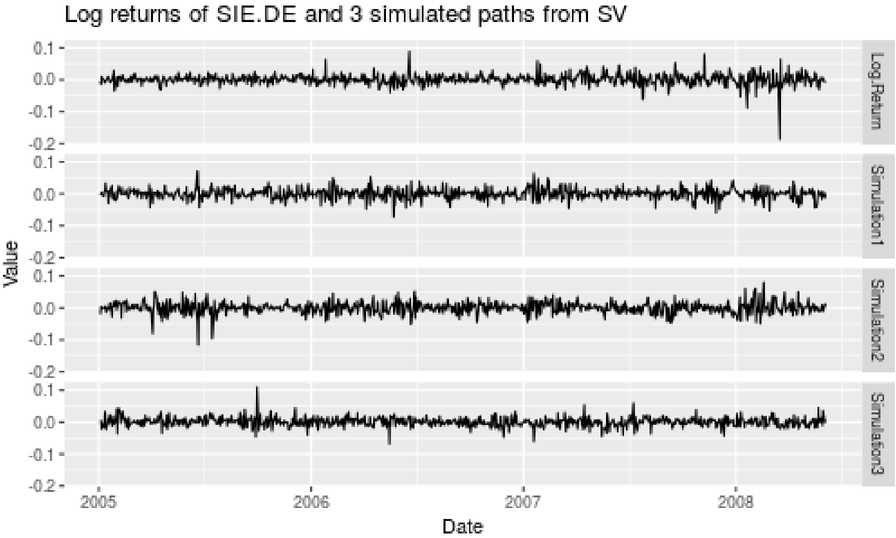
SV vs. EGARCH

$$
\begin{aligned}
y_t &= \varepsilon_t \sqrt{e^{h_t}} \\
h_{t+1} &= \mu + \phi(h_t - \mu) + \sigma \quad g(\varepsilon_t) \\
\forall t : \varepsilon_t, \eta_t &\sim \text{i.i.d. } \mathcal{N}(0, 1)
\end{aligned}
$$

Model

$$
\begin{aligned}
y_t &= \varepsilon_t \sqrt{e^{h_t}} \\
h_{t+1} &= \mu + \phi(h_t - \mu) + \sigma \eta_t \\
\forall t : \varepsilon_t, \eta_t &\sim \text{i.i.d. } \mathcal{N}(0,1)
\end{aligned}
$$

SV model simulations



Parameters

$$y_t = \varepsilon_t e^{h_t/2}$$
$$h_{t+1} = \mu + \phi(h_t - \mu) + \sigma\eta_t$$
$$\forall t : \varepsilon_t, \eta_t \sim \text{i.i.d. } \mathcal{N}(0,1)$$

	Support	Prior sensitivity
ϕ	$(-1,1)$	High
σ	$\mathbb{R}^+$	Medium
μ	$\mathbb{R}$	Low

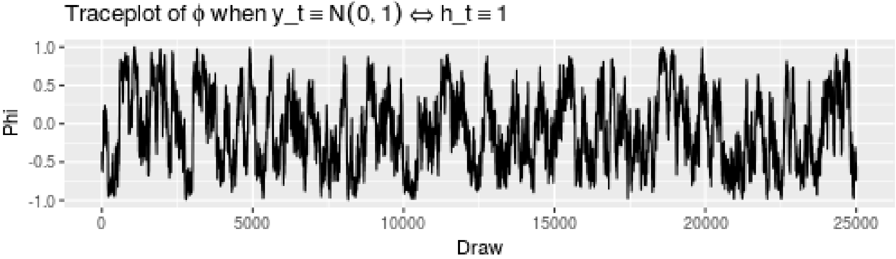
Table: Support and sensitivity of parameters

Identifiability of ϕ

Keep possible issues in mind!

$$y_t = \varepsilon_t e^{h_t/2}$$
$$h_{t+1} = \mu + \phi(h_t - \mu) + \sigma\eta_t$$

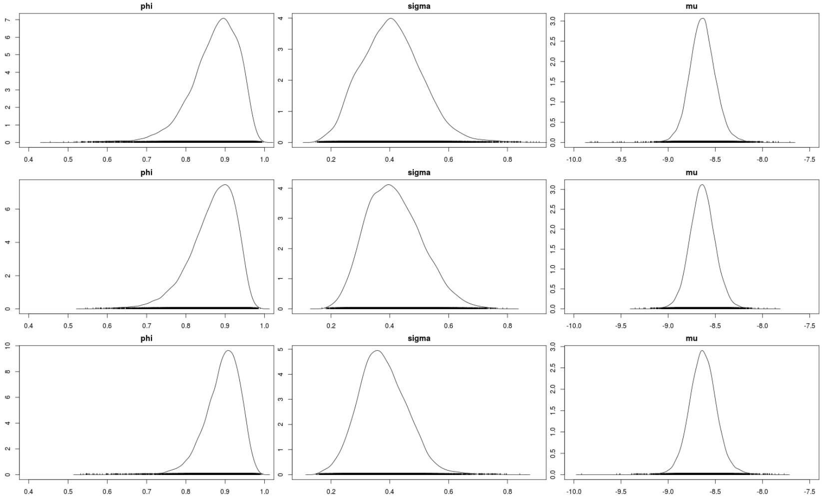
E.g. h_t (almost) constant $\implies$ no information about ϕ in the data.



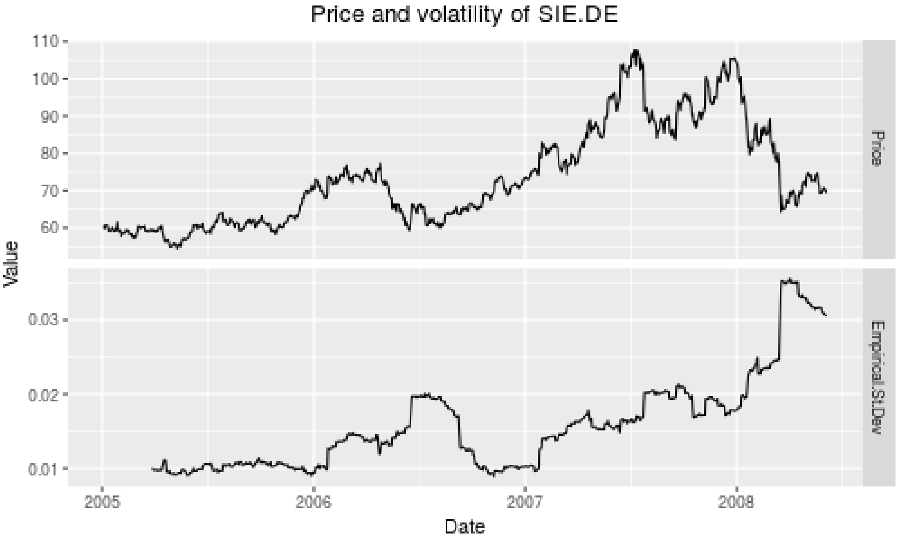
Conclusion: informative prior is needed for ϕ

Results

Increasing "informativeness" on ϕ:



Another phenomenon

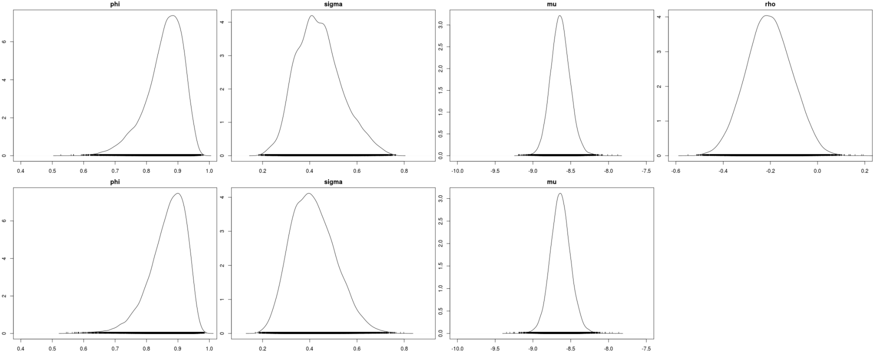


SV with leverage

$$
\begin{aligned}
y_t &= \varepsilon_t e^{h_t/2} \\
h_{t+1} &= \mu + \phi(h_t - \mu) + \sigma\eta_t \\
\forall t: \varepsilon_t &\sim \text{i.i.d. } \mathcal{N}(0,1) \\
\forall t: \eta_t &\sim \text{i.i.d. } \mathcal{N}(0,1) \\
\text{cor}(\varepsilon_t, \eta_t) &= \rho
\end{aligned}
$$

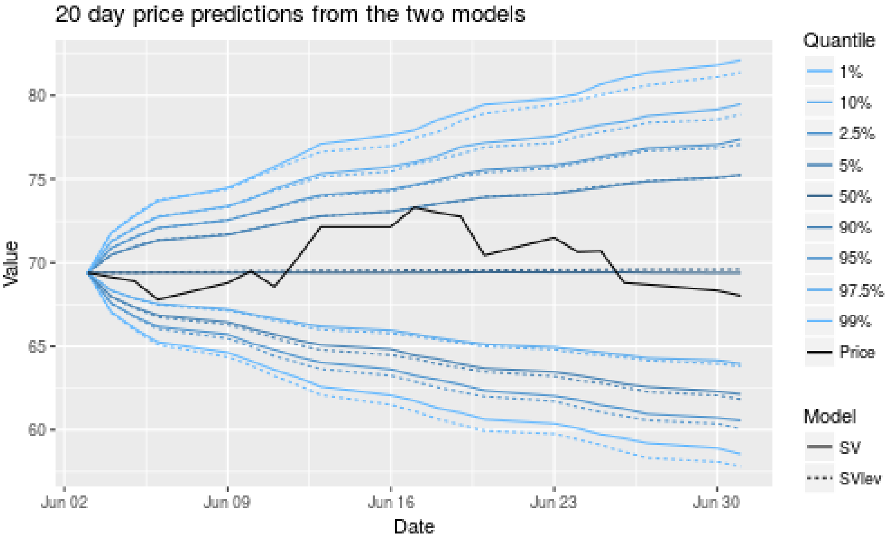
Comparison of results

SV with leverage



SV without leverage

Prediction



20 day price predictions from the two models
Value
80
75
70
65
60
Jun 02
Jun 09
Jun 16
Jun 23
Jun 30
Date
Quantile
1%
10%
2.5%
5%
50%
90%
95%
97.5%
99%
Price
Model
SV
SVlev