Definition. A matrix represent a collection of numbers in rows and cols.

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

Order of matrix $\#rows \times \#cols$

$$A_{3\times 3} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

Note: by matrix we consider the square matrix
Operations on Matrices
Addition/ Subtraction: $A = (a_{i,j})_{m\times n}$ and $B = (b_{i,j})_{m\times n} \implies A \pm B = (a_{i,j} \pm b_{i,j})_{m\times n}$
Properties of Addition:

- Commutative : $A + B = B + A$
- Associative : $(A + B) + C = A + (B + C)$

Multiplication: iff $\#cols(A) = \#rows(B) \implies AB_{rows(A)\times cols(B)}$
In general, $A = [a_{i,j}]_{m\times n}$ and $B = [b_{i,j}]_{n\times p}$ then $AB = [c_{i,k}]_{m\times p}$ where, $c_{i,k} = \sum_{j=1}^{n} a_{ij}b_{jk}$
Properties of Multiplication:

- Not Commutative : $AB \neq BA$
- Associative : $(AB)C = A(BC)$

Scalar Multiplication multiplying each elements with a real number. Let $A = [a_{i,j}]$ and let $k \in \mathbb{R}$ then $kA = [ka_{i,j}]$.
Transpose of Matrix: interchange of rows and cols, denoted as A^T
Types of Matrices
Square Matrix: matrix of same order i.e $A_{n\times n}$
Upper triangular Matrix: $A_{n\times n}$ s.t all the elements below the main diagonal are 0 i.e $A = [a_{ij}] \impliedby a_{ij} = 0\, \forall i > j$
Lower triangular Matrix: $A_{n\times n}$ s.t all the elements above the main diagonal are 0 i.e $A = [a_{ij}] \impliedby a_{ij} = 0\, \forall i < j$
Symmetric Matrix: $A^T = A$ (above/below diagonal elements are same)
Skew Symmetric Matrix (Anti-symmetric): $A^T = -A$
Diagonal Matrix: $A = [a_{ij}]_{n\times n}$ where, $a_{ij} = 0\, \forall i \neq j$
Identity or Unit Matrix: $I = [a_{ij}]_{n\times n}$ where,

$$a_{ij} = \begin{cases} 1 & \text{if i = j} \\ 0 & \text{Otherwise} \end{cases}$$

Orthogonal Matrix: $A^T A = AA^T = I$

Idempotent Matrix: $A^2 = A$
Idempotent Matrix: $A^2 = I$
Singular Matrix: $|A| = 0$
Non-Singular Matrix: $|A| \neq 0$

Minor of a_{ij} denoted as M_{ij} is obtained by deleting i^{th} rows and j^{th}cols

$$\delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \implies M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$$

Co-factor of Matrix: $a_{ij} = (-1)^{i+j} M_{ij}$
Adjoint of Matrix matrix obtained by taking the transpose of cofactor matrix of a given matrix.
Inverse of Matrix: $A^{-1} = \frac{adj(A)}{|A|}$ where, $|A| \neq 0$

Theorem 1. $Aadj(A) = adj(A)A = |A|.I$

Theorem 2. A is said to be invertible iff $AA^{-1} = A^{-1}A = I$

Theorem 3. Invertible matrix has an unique inverse

Theorem 4. If A and B are invertible then AB also invertible s.t $(AB)^{-1} = B^{-1}A^{-1}$

Theorem 5. If A is invertible then A^T is also invertible.

Theorem 6. If A is invertible symmetric then A^{-1} also symmetric

Theorem 7. If A and B are nonsingular then $adj(AB) = adj(B)adj(A)$

Theorem 8. If $|A| \neq 0 \implies |adj(A)| = |A|^{n-1}$

Theorem 9. If $|A| \neq 0 \implies adj(adj(A)) = |A|^{n-2}.A$