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Innovative Applications of O.R.

A multi-criteria Police Districting Problem for the efficient and effective design of patrol sector

M. Camacho-Collados [a,c], F. Liberatore [b,*], J. M. Angulo [c]

[a] Deputy Directorate for Operations, Spanish National Police Corps, Madrid, Spain
[b] AIDA, Computer Science Department, Autonomous University of Madrid, Madrid, Spain
[c] Statistics and Operations Research Department, University of Granada, Granada, Spain

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ABSTRACT

The Police Districting Problem (PDP) concerns the efficient and effective design of patrol sectors in terms of performance attributes such as workload, response time, etc. A balanced definition of the patrol sector is desirable as it results in crime reduction and in better service. In this paper, a multi-criteria Police Districting Problem defined in collaboration with the Spanish National Police Corps is presented. This is the first model for the PDP that considers the attributes of area, risk, compactness, and mutual support. The decision-maker can specify his/her preferences on the attributes, on workload balance, and efficiency. The model is solved by means of a heuristic algorithm that is empirically tested on a case study of the Central District of Madrid. The solutions identified by the model are compared to patrol sector configurations currently in use and their quality is evaluated by public safety service coordinators. The model and the algorithm produce designs that significantly improve on the current ones.

1. Introduction

For most of the 20th century, police districts have been drawn by police officers on a road map with a marker, just by following the major streets in the area, without making too much of an effort to accomplish geographic or workload balance (Bruce, 2009). Since the seminal paper by Mitchell (1972), a number of mathematical optimization models have been proposed and the Police Districting Problem (PDP) was born. The PDP aims at partitioning the territory under the jurisdiction of a Police Department in the best possible way, with respect to several time, cost, performance, and topological attributes. Only after recent advances in Geographic Information Systems (GIS) and computer technology, which have allowed reasonable computational times and ease of representation and manipulation, have automatic methodologies for the definition of police districts gained popularity among practitioners (Wang, 2012). However, studies integrating GIS and sophisticated mathematical modeling for police districting remain a rarity (Bruce, 2009), and the “map-and-marker method” is still one of the most widely used redistricting procedures. Nevertheless, the importance of a balanced definition of the police districts is unquestioned and the implementation of tools for aiding in making the decisions about the allocation of police resources has proven to be extremely beneficial, as shown by the substantial academic literature on this topic in the last decades (D'Amico, Wang, Batta, & Rump, 2002). In fact, all the works report a dramatic improvement in workload distribution compared to hand-made districts which, in turn, results in enhanced performance and efficiency.

In Spain, the security of towns is the responsibility of the Spanish National Police Corps (SNPC), usually sharing territory with other local security forces. The SNPC is an armed Institute of a civil nature, dependent on the Ministry of Home Affairs. Among its duties are: keeping and restoring order and public safety and preventing the commission of criminal acts. The SNPC is one of the country's most valued institutions and is located at the global forefront of the fight against crime, with the aim of constant innovation. The socio-economic context in recent years in Spain has been that of a serious crisis, which has reduced the resources and the number of police officers available to the SNPC. In order to continue providing the same level of security, the SNPC is taking cutting-edge steps to increase its competitiveness. Under the current system, the distribution of patrols is the responsibility of the inspectors who, under normal conditions, locate the agents according to the neighborhood borders defined more than 50 years ago. To improve the effectiveness of patrolling operations and increase the efficiency in the use of scarce resources, the SNPC has started to develop a Decision Support System (DSS) comprising tools and models to assist in various public security

* Corresponding author. Tel.: +34 914888002.
E-mail addresses: mcamacho0007@policia.es (M. Camacho-Collados), federico.liberatore@urjc.es (F. Liberatore), jmangulo@ugr.es (J.M. Angulo).

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tasks (Camacho-Collados & Liberatore, 2015). One of the main objectives of the system is the implementation of a predictive patrolling policy to increase the presence of agents in the areas where they are most needed, to reduce the probability of the occurrence of crime. To this end, the authors have developed, in collaboration with professionals from the SNPC, an optimization model for the definition of patrolling sector configurations, tailored to suit the requirements of the SNPC. As the model is to be included in the DSS and, therefore, should be sufficiently interactive, the authors implemented a heuristic algorithm that provides good solutions quickly. By combining the proposed algorithm with a crime risk forecasting model (Perry, McInnis, Price, Smith, & Hollywood, 2013; Short, Bertozzi, & Brantingham, 2010), a predictive patrolling system is obtained. For the SNPC, the implementation of a predictive patrolling system also represents a paradigm shift, from detention to prevention, resulting in reductions in the costs of detention and in an improvement in the actual, subjective, and social level of safety.

The contributions of this article are the following. An extensive literature review on the PDP is presented. This review includes the identification of the main aspects revised in related reports and a categorized description of methodological approaches. In summary, a broad range of references is classified to identify lacks in the literature. The main contribution of this paper lies in the optimization model for the PDP designed in collaboration with the SNPC. The model is multi-criteria in nature as it includes in the optimization process four different attributes. Also, the model allows the decision maker to define her preference between global optimality and workload balance among the patrol districts. The model is solved by means of a fast local search algorithm, to comply with the strict time requirements given by its inclusion as a tool in a DSS. The model and the algorithm are tested on a case study on the Central District of Madrid. We show empirically that the optimization methodology proposed generates solutions that outperform the current patrolling configurations adopted by the SNPC. Finally, concluding remarks and research guidelines are given.

The rest of the paper is organized as follows. In Section 2 we briefly introduce the generic districting problem and we review more in detail the literature on the PDP. In Section 3 we present the proposed multi-criteria PDP model and the algorithm devised to solve it. Next, we test the algorithm on a case study of the Central District of Madrid and compare the quality of the solutions with the patrolling configurations currently used in the district. We conclude with some insights and guidelines for future research.

2. Literature review

This section presents the problem of defining the districts, and contextualizes it in the framework of police resource allocation. A conceptual classification of previous research according to the attributes considered and methodologies adopted is presented, and then some insights will be provided.

2.1. The districting problem

District design can be seen as the problem of grouping elementary units (or atoms) of a given territory into larger districts (or clusters), according to relevant attributes (or criteria). Depending on the problem faced, the attributes considered might belong to different contexts, including economic, demographic, geographic, etc. In the last decades, the districting problem has been applied to a broad number of fields, including:

- Electric power districting (Bergey, Ragsdale, & Hoskote, 2003a; 2003b).
- Emergency service districting (Iannoni, Morabito, & Saydam, 2009; Larson, 1974).
- Internet networking (Park, Lee, Park, & Lee, 2000).
- Health information systems (Braa & Hedberg, 2002).
- Police patrol districting.
- Political districting for the definition of electoral areas (Bozcaya, Erkut, & Laporte, 2003; Cirincione, Darling, & O'Rourke, 2000; Mehrotra, Johnson, & Nemhauser, 1998).
- Public transportation network districting (Tavares-Pereira, Rui Figueira, Mousseaus, & Roy, 2007; 2009).
- Sales and service districting (Blais, Lapierre, & Laporte, 2003; Galvão, Novaes, Souza de Cursi, & Souza, 2006).
- School districting (Caro, Shirabe, Guignard, & Weintraub, 2004; Schoepfle & Church, 1991).
- Social facilities districting (Minciardi & Zoppoli, 1981).
- Solid waste disposal districting (Hanafi & Freville, 1999).
- Winter service districting (Muyldermans, 2003; Muyldermans, Cattrysse, Oudheusden, & Lotan, 2002).

A unified territorial design model that allows the formulation and solution of districting problems in a variety of applications is the subject of Kalcsics and Schröeder (2005). The authors also review the existing literature in territorial design, highlighting application fields, criteria, and solution methodologies for solving these types of problems.

2.2. The Police Districting Problem

In the United States, police departments partition the territory under their jurisdiction according to a hierarchical structure: command districts (or precincts), patrol sectors (or beats), and reporting districts (or r-districts). Each command district hosts a headquarters where the commanding officer supervises the operations. A command district is subdivided into patrol sectors, each having at least one car assigned to patrol the area and attend to the calls originating from it. Finally, r-districts constitute the atomic element in the hierarchy: the smallest geographical unit for which statistics are kept. As reported in Sarac, Batta, Bhadury, and Rump (1999), r-districts can coincide with census block groups. In Europe, the territorial organizational structure of police departments depends on the country or the region considered. Nevertheless, a hierarchal structure similar to the one adopted in the United States is predominant.

The PDP concerns the optimal grouping of r-districts into externally "homogeneous" patrol sectors. In fact, the car assigned to the patrol sector should attend to all the incidents taking place in the area. Normally, if the car is busy responding to a call when another incident happens, a car from a neighboring area has to attend to it. As Mayer (2009) points out, this generally leads to a domino effect, where cars are pulled from their area to another, leaving the patrol sector unattended and, therefore, more susceptible to criminal incidents. In the light of this scenario, a balanced workload among the districts and the enforcement of a maximal response time become of primary importance.

The first paper on the PDP is presented by Mitchell (1972), which proposes a clustering heuristic for the redesign of patrol beats in Anaheim, California. The author considers the total expected weighted distance to incidents, as well as a workload measure defined as the sum of the expected service time and the expected travel time. Bodily (1978) adopts a utility theory model that incorporates the preferences of three interest groups, namely, the citizens, the administrators, and the service personnel. A simple local search algorithm swaps patrol beats from one sector to another to improve the value of the utility function. Benveniste (1985) was the first author to include workload equalization in the optimization process, solving a non-linear stochastic model by means of an approximation algorithm. D'Amico et al. (2002) solve a police districting problem subject to constraints of contiguity, compactness, convexity, and equal size. The novelty of the model lies in the incorporation of queuing measures to compute patrol office workloads and response times to calls for service, computed by external software, PCAM (Chaiken & Dormont, 1978a; 1978b). PCAM optimizes a queuing model for the deployment of police resources, providing the optimal number of cars per district. The authors solve the problem by means of a simulated annealing algorithm that iteratively calls the PCAM routine. At each step, the neighborhood is determined by a simple exchange procedure that takes into account the following constraints: the average response time per district is bounded from above; the ratio of the size of the largest and smallest districts is bounded from above; districts must be connected; the ratio of the longest Euclidean path and the square root of the area in each district is bounded from above to preserve compactness; districts must be convex. The algorithm is applied to a real-world case for the Buffalo Police Department, NY. The following objectives were considered: minimize the maximum workload (by decremental bounding constraining) and minimize the maximum average response time. A different approach is proposed by Curtin, Qui, Hayslett-McCall, and Bray (2005), who apply a covering model to determine the police patrol sectors. The covering model defines the centers of the police patrol sectors in such a way that the maximum number of (weighted) incidents is covered. An incident is considered to be covered if it lies within an acceptable service distance from the center of a patrol sector. The model is integrated in a GIS and applied to a case study involving the City of Dallas, TX. In a subsequent article, Curtin, Hayslett-McCall, and Qiu (2010) extend their covering model to include backup coverage (e.g., multiple coverage of high priority locations). The resulting model is bi-objective in nature. The authors propose a single objective model that maximizes the priority weighted coverage (i.e., a location is counted separately each time it is covered), while ensuring a minimum covering level in terms of the priority-weighted number of locations covered (each covered location counted only once). The model is tested on the Dallas data and refinements of the model are proposed (e.g., maximum workload per patrol sector). Zhang and Brown (2013) propose a parametrized redistricting procedure for police patrols. The methodology consists of a heuristic algorithm that generates alternative districting plans. Next, the plans are evaluated in terms of the average response time and workload. With this aim, an agent-based simulation model was implemented in a GIS. The location and times of the incidents taking place in each district were modeled by an empirical distribution based on real incident data. Finally, the procedure identifies the set of non-dominated solutions. The methodology has been tested on a case study based on the Charlottesville Police Department, VA.

2.2.1. *Attributes*

While analyzing the existing literature on the PDP, certain basic features common to all the contributions emerged. In fact, all the applications considered include measures for workload, response time, and the geometrical properties of the districts. Nevertheless, the implementations vary considerably. Unlike Kalcsics and Schröeder (2005), the term "attributes" has been adopted instead of "criteria", with the aim of providing a more generic framework that classifies all the relevant characteristics of a PDP solution, regardless of whether they are optimized in the objective function, or expressed as constraints.

Workload. Given the complex nature of police procedures and operations, and the great variability of the tasks that an agent can undertake, defining the workload could be complicated. In Bruce (2009), there are provided a number of questions that can help clarify what to consider as part of the workload. Albeit difficult, an accurate definition of workload is desirable, as it ensures homogeneity in terms of the quality and speed of service, and equalizes the burden on police officers (Bodily, 1978).

In the literature on the PDP, different definitions of workload have been adopted. In Mitchell (1972), the workload is computed as the sum of the total expected service time and the total expected travel time. Curtin et al. (2005, 2010) use the number (or frequency) of calls (or incidents) occurring at each district as a proxy for the workload. As different calls can have different service times, some authors consider this measure to be too naïve, as it might produce unbalanced patrol districts. In Bodily (1978) and D'Amico et al. (2002), workload is defined as the fraction of working time that an agent spends attending to calls. An equivalent measure is proposed by Benveniste (1985). Given the stochastic nature of her model, workload is measured in terms of the probability of a patrol car's being busy. Once the probability is known, the time spent attending and answering calls can be easily calculated. More recently, workload has been defined as a combination of different characteristics. In Sarac et al. (1999), the authors aggregate population and call volume. Kistler (2009) makes use of the convex combination of total hours worked (i.e., from dispatch to call clearance), number of calls, and population. Finally, Zhang and Brown (2013) consider both the average travel time and the response time.

Response time. Response time is an important performance measure: it is the time between the arrival of a call for service and the arrival of a unit at the location of the incident. According to Bodily (1978), a reduction in the response time results in a number of beneficial effects, such as:

- Increased likelihood of intercepting a crime in progress.
- Deterrent effect on criminals.
- Increased confidence in the police.

Generally speaking, most authors only take into consideration the travel times (Bodily, 1978; Kistler, 2009; Mitchell, 1972; Zhang & Brown, 2013) or travel distances (Benveniste, 1985; Curtin et al., 2005, 2010). The only study considering the queuing effect is D'Amico et al. (2002), where the authors apply an external model, PCAM (Chaiken & Dormont, 1978a; 1978b), to compute the total response time including the queuing time of the calls and the travel time to the location of the incident.

Geometry. In 1812, Albright Gerry, the Governor of the Commonwealth of Massachusetts at the time, manipulated the division of his state and proposed a salamander-shaped district to gain an electoral advantage, leading to the expression "gerrymandering" (resulting from merging "Gerry" and "salamander"). Since then, designing electoral districts having certain geometric properties has been of primary importance to ensure neutrality and prevent political interference in the districting process.

In the context of the PDP, geometric attributes are still relevant for efficiency (e.g., establishing boundaries that would be easy to patrol and would not frustrate the patrol officers) and for administrative reasons (e.g., coordinating with other agencies). To the best of our knowledge, only three works have explicitly included the geometric properties in the design process, such as the properties of compactness (D'Amico et al., 2002; Kistler, 2009; Sarac et al., 1999), contiguity (D'Amico et al., 2002; Sarac et al., 1999), and convexity (D'Amico et al., 2002), which are generally obtained as a consequence of optimizing the travel distance or the travel time. Also, the district area is considered in all the mentioned works. Additionally, in Kistler (2009), the total length of the streets in a district is included.

Other attributes. Recently, a number of attributes that do not fall into any of the previous categories have been introduced. These attributes generally try to capture complex real-world requirements. The Buffalo Police Department needed to redesign the r-districts in such a way that the existing district boundaries would be respected, and the access to demographic data as well as their use by other agencies would be easy (Sarac et al., 1999). The Tucson Police Department needed to consider the boundaries of gang territories, city council wards, neighborhood associations, and the Davis–Monthan Air Force Base (Kistler, 2009). Finally, in Curtin et al. (2010), backup coverage (i.e., multiple coverage) of incident locations is introduced.

2.2.2. Methodologies and approaches

Many districting approaches have appeared in the literature. In this subsection, the contributions are categorized according to the methodology adopted, and their main characteristics are presented.

Optimization models. According to Kalcsics and Schröeder (2005), the first mathematical program for the districting problem was proposed by Hess, Weaver, Siegfeldt, Whelan, and Zitlau (1965), and considered a neutral definition of the political districts. Since then, a large number of models have been proposed, mostly in the context of location analysis. Similarly, in Curtin et al. (2005, 2010), maximal covering models are proposed. On the other hand, Mitchell (1972) presents a set partitioning model that considers minimizing the expected distance inside of each subset and equalizing the workload of all the subsets. A different perspective is adopted by Benveniste (1985) and D'Amico et al. (2002), where patrol cars and agents are modeled as servers in a stochastic model. Benveniste (1985) proposes a Stochastic Optimization model, while D'Amico et al. (2002) include a queuing model inside of a simulated annealing algorithm to compute response times that incorporate queuing effects.

Geographic information systems (GIS). Kistler (2009) uses a GIS to redesign the Tucson Police Department districts. Most commercial GIS can be extended to integrate optimization routines. In Curtin et al. (2005) and Curtin et al. (2010), GIS are used in conjunction with a maximal covering model. Wang (2012) presents the main application areas of GIS in police practice. Among the various applications, Wang mentions the possibility of using GIS as a police force planning tool. Namely, he refers to hot-spot policing and police districting. Concerning the latter, Wang identifies three main objectives: meeting a response time threshold, minimizing the cost of operations, and balancing workload across districts. The author mentions that future research in this area should explore other goals, such as minimizing the total cost (response time), minimizing the number of districts (dispatch centers), maximizing equal accessibility, or a combination of several goals. Finally, Zhang and Brown (2013) implement an agent-based simulation inside of a GIS.

Other methods. Two studies have adopted approaches that do not fall into any of the other categories. Bodily (1978) devises a decision model based on utility theory to achieve the best solution in terms of the surrogate utility of three interest groups. The work by Sarac et al. (1999) is an example of the proverbial expression "simpler is better." After attempting to redesign r-districts by using a multi-criteria set partitioning model, the authors realized that census blocks satisfied all the requirements. It is important to notice that their approach is successful because of the specific requirements the Buffalo PD imposed on the r-district configuration (e.g., easy access to demographic data, suitable for use by other agencies).

3. A multi-criteria Police Districting Problem

This section illustrates the PDP developed in collaboration with the SNPC. The goal of the model is to partition into patrol sectors the territory under the jurisdiction of a district in the best possible way. The criteria for evaluating the goodness of the configurations of the patrol sectors were identified after interviewing several service coordinators and a number of agents involved in public safety operations. The result is a mathematical optimization model which finds an efficient configuration in terms of prevention service and attention to calls, distributing the workload equitably between the agents.

During the interviews with the public servants involved in public safety operations, several desirable characteristics were identified in order to find a "good" territory partition.

- Compact areas: A compact area allows better control of the territory by the agents, as travel times from one point to another within the area are minimal. Therefore, the more compact an area is, the faster the response of agents who are in the area to emergency calls.
- Homogeneity in terms of workload: Generating patrol sectors that are similar in terms of workload is quite useful for two main reasons. First, it ensures a more efficient distribution of work and, therefore, better service. Second, greater equality in the workplace increases the satisfaction of the agents.
- Mutual support: It is desirable that agents be able to count on the support of agents assigned to other patrol sectors in case of need and emergency.

Our model differs from those proposed so far in the literature in a number of relevant aspects. In general, our focus is on crime prevention. For this reason, the purpose of our model is to increase the effectiveness of the deterrent effect of the agents' presence on the territory, by concentrating the agents in the areas with a higher risk of crime. On the other hand, previous approaches such as D'Amico et al. (2002) and Zhang and Brown (2013) focus on reaction to crime incidents and aim at optimizing the response to emergency calls and, hence, to crimes that have already happened. Additionally, we present the first model for the PDP that optimizes at the same time attributes of area, crime risk, compactness, and support. Specifically, mutual support is an attribute that has not been included in any previous model. Mutual support differs from backup coverage (Curtin et al., 2010) in that the former regards the possibility of receiving backing in any point of the patrol sector from any other agent in the district, while the latter only concerns the overlapping areas between patrol sectors. Furthermore, our model allows the decision-maker to explicitly and easily include his/her preferences in the optimization process by means of weights associated to the attributes. In the formulation proposed by D'Amico et al. (2002), the user can specify his/her preferences only by adjusting the righthand side coefficients in the constraints, while in Curtin et al. (2005) and Curtin et al. (2010), no user preference is considered. Finally, all the approaches previously presented in the literature require specific data and information, such as the time, location, and service time of incidents and emergency calls, which might not be available. This requirement makes these models inapplicable in any context where this information is not available. Also, these methodologies do not take into consideration, and hence they cannot be extended to, all the non-violent crimes that are not reported by emergency calls, such as, pickpockets, theft of vehicles, or property damage.

In the next section, the structure of the optimization mathematical model incorporating these properties is explained.

3.1. Input data

Without loss of generality, the territory under the jurisdiction of the district is assumed to be divided into a square grid, G, of n rows and m columns, having elements indexed by $(i, j) \in G$. Following this structure, we define two data matrices both having n rows and m columns:

- The crime risk matrix, R. Its entries, $r_{ij} \in R$, are non-negative real numbers specifying the crime risk associated with the corresponding locations. The risk of criminal activity can be estimated with past data or using a predictive policing model (Perry et al., 2013).
- The area matrix, A. Its entries, $a_{ij} \in A$, are non-negative real numbers specifying the total street length at each tile of the grid. This data can be easily obtained using a GIS.

Finally, the number of patrol sectors, p, is required. The model uses this information to define the number of areas into which to partition the territory.

3.2. Notes on taxicab geometry

The representation of the territory as a grid necessarily involves certain simplifications when considering geometric properties such as continuity and distance. Given the loss of information on the urban fabric of streets and roads resulting from using a grid as a model, it is natural and necessary to apply a taxicab geometry. In this geometry, the distance between two points, also called the Manhattan distance, is the sum of the (absolute) differences of their coordinates. Therefore, the distance between the points $a = (i, j)$ and $b = (k, l)$ is calculated as

$$dist(a, b) = |i - k| + |j - l|. \tag{1}$$

Following this definition, two points are considered adjacent if and only if their distance is equal to 1. A subset of points s is defined to be connected if between any pair of points (belonging to s) there is a path of adjacent points (belonging to s) connecting them. Within a connected subset s, the minimum distance between any pair of points is defined as the length of the shortest path connecting them formed by points belonging to s. If this path does not exist, then the subset is not connected. The matrix of the shortest paths between pairs of points belonging to s, F^s, can be calculated efficiently using the Floyd–Warshall algorithm (Floyd, 1962; Warshall, 1962). We refer to its elements as $F^s_{a,b}$, where $a, b \in s$; $F^s_{a,b} = \infty$ when there is no path connecting points a and b. The connectivity condition can be expressed as

$$0 \le F^s_{a,b} < \infty, \quad \forall a, b \in s \Longleftrightarrow s \text{ is connected.} \tag{2}$$

Finally, we present the property of convexity. In taxicab geometry, the definition of convexity is related to the notion of the orthogonal convex hull of a subset. In this paper, we exploit the following property: a subset of points s is convex if, and only if, for all pairs of points belonging to s, the shortest path distance (inside of the subset) is equal to the Manhattan distance between them:

$$F^s_{a,b} = dist(a, b), \quad \forall a, b \in s \Longleftrightarrow s \text{ is convex.} \tag{3}$$

3.3. Constraints

We now present the model constraints. As explained in the previous sections, the model must generate a patrol sector configuration. The districts can not overlap and they must cover the whole territory. Mathematically, a partition is a family of non-empty subsets completely covering the initial set and in which each pair of these subsets are disjoint. Thus, the first condition that any solution has to satisfy is to define a partition, P, of the territory considered. This translates to a definition of the subsets over the matrices A and R. Each subset $s \in P$ contains some of the matrix entries and represents a patrol sector. From now on, the terms subset and (patrol) sector will refer to the same concept. The second restriction concerns the cardinality of the partition. The number of subsets in the partition must be exactly p. The third condition regards the subsets' geometry. Only connected subsets are feasible. This condition implies that an agent cannot be assigned to a patrol district composed of two or more separate areas of the city. Furthermore, all the subsets are required to be convex. When a subset is convex, it is also optimally efficient in terms of distances between its points. In fact, in a convex subset, there is a minimal shortest path connecting any pair of points. Therefore, this condition allows the generation of patrol sectors that are more efficient in terms of movement within the area. The resulting PDP can be characterized by the following mathematical program, adapted from King, Jacobson, Sewell, and Cho (2012).

$$\text{opt} \quad obj(P) \tag{4}$$

$$\text{s.t.} \quad \exists s \in P | (i, j) \in s \quad \forall (i, j) \in G \tag{5}$$

$$Empty_s(P) = 0 \quad \forall s \in P \tag{6}$$

$$|P| = p \tag{7}$$

$$Conn_s(P) = 1 \quad \forall s \in P \tag{8}$$

$$Conv_s(P) = 1 \quad \forall s \in P \tag{9}$$

In the model, $obj(P)$ is an objective reflecting the goals of the decision maker. The constraints (5) require that all the points of the grid must belong to a subset. $Empty_s(P)$ is an indicator function that equals 1 when s is empty (i.e., no points have been assigned to it) and zero otherwise. The cardinality constraint (7) forces the number of subsets to be exactly p. Finally, $Conn_s(P)$ is an indicator function that equals 1 when s is connected and zero otherwise, and $Conv_s(P)$ is an indicator function that equals 1 when s is convex and zero otherwise.

3.4. Attributes

To find the best possible partition, a methodology is needed that allows the comparison of the different solutions in terms of "goodness." To evaluate this, we need to define some unambiguous criteria. More specifically, we consider the following attributes for each subset $s \in P$:

- **Area**, a^s. This attribute identifies the size of the territory that an agent should patrol. It is calculated as

$$a^s = \sum_{(i,j) \in s} a_{ij}. \tag{10}$$

- **Support received**, b^s. Two districts support each other if the distance between their geometric medians is less than or equal to a defined constant, K. We recommend defining K as

$$K = \left\lceil \frac{\max\{m, n\}}{\sqrt{p}} \right\rceil. \tag{11}$$

  The geometric median, o^s, of a subset s is the point minimizing the sum of the distances to the elements of the subset:

$$o^s = \arg\min_{a \in s} \left\{ \sum_{b \in s} F^s_{a,b} \right\}. \tag{12}$$

  Finally, the support received by a subset can be calculated as follows:

$$b^s = |\{s' \in P | dist(o^s, o^{s'}) \le K, \ s \ne s'\}|. \tag{13}$$

- **Demand**, c^s. The demand is defined as the total risk of the subset, i.e., the sum of the risks associated to the points belonging to the subset:

$$c^s = \sum_{(i,j) \in s} r_{ij}. \tag{14}$$

  It is important to remember that the r_{ij} identify the crime risk associated to a point. Therefore, the demand c^s identifies how "dangerous" the subset is in terms of the expected crime risk.
- **Diameter**, d^s. The diameter of a subset is defined as the maximum distance between any pair of points belonging to the subset:

$$d^s = \max_{a,b \in s} \left\{ F^s_{a,b} \right\}. \tag{15}$$

  The diameter is an efficiency measure. In fact, compact districts have small diameters. Moreover, the diameter can be interpreted as the maximum distance that the agent associated to the district should travel in case of an emergency call. Therefore, a small diameter results in a low response time.

The attributes defined are not comparable, as they are associated to different dimensions. To make comparisons between them, we need to convert the attributes into dimensionless ratios:

- **Area ratio**, α^s. This is the ratio of the subset area to the whole area:

$$\alpha^s = \frac{a^s}{\sum_{(i,j)\in G} a_{ij}}. \tag{16}$$

- **Isolation ratio**, β^s. To express all the ratios as quantities to be minimized, we consider the isolation of a subset as the complement of the support received:

$$\beta^s = \frac{p-1-b^s}{p-1}. \tag{17}$$

- **Demand ratio**, γ^s. This is the ratio of the subset demand to the whole demand:

$$\gamma^s = \frac{c^s}{\sum_{(i,j)\in G} r_{ij}}. \tag{18}$$

- **Diameter ratio**, δ^s. This is the ratio of the subset diameter to the maximum diameter possible. We estimate this quantity as the maximum Manhattan distance between two points in the grid:

$$\delta^s = \frac{d^s}{\max_{a,b\in G}\{dist(a,b)\}}. \tag{19}$$

Now that all the attributes have been expressed in a dimensionless fashion, it is necessary to define the relative importance of each ratio. The decision maker can express preferences by associating weights to the attributes: w_α, w_β, w_γ, and w_δ. A larger weight assigns more importance to the minimization of the attribute. We can now define the workload W^s of a subset s as the sum of the products of weights with the ratios:

$$W^s = w_\alpha \cdot \alpha^s + w_\beta \cdot \beta^s + w_\gamma \cdot \gamma^s + w_\delta \cdot \delta^s. \tag{20}$$

3.5. Objective function

After analyzing the information provided by the professionals of the SNPC, we identified two primary necessities that our model should take into account:

- The model should define districts that are as efficient as possible, in terms of the attributes considered and the weights specified.
- The model should define districts that are as homogeneous as possible, in terms of the attributes considered and the weights specified.

Unfortunately, there might be a trade-off between these requirements. As an example, an increase in the homogeneity of the districts could reduce the global efficiency, and vice-versa. Therefore, we define a multi-criteria objective function that takes into consideration the preferences of the decision maker with respect to these factors:

$$\min\ obj(P) = \lambda \cdot \max_{s\in P}\{W^s\} + (1-\lambda)\cdot \frac{\sum_{s\in P} W^s}{p}, \tag{21}$$

where $0 \le \lambda \le 1$. The term $\max_{s\in P}\{W^s\}$ represents the worst workload, while the term $\frac{\sum_{s\in P} W^s}{p}$ is the average workload[1]. The objective function defined, inspired by the extended goal programming paradigm introduced by Romero (2001, 2004), allows the decision maker to examine the trade-off between optimization and balance by a parametric analysis. In fact, by varying λ, the model gives a range from optimization ($\lambda = 0$) to balance ($\lambda = 1$).

[1] The average workload term of the objective function includes constant terms, such as $\Sigma\alpha^s = 1$ and $\Sigma\gamma^s = 1$. We decided to include them so that the worst workload and the average workload could have the same magnitude and, therefore, be comparable.

3.6. The optimization algorithm

The model resulting from (4)–(9) is extremely complex. In fact, Drexl and Haase (1999) showed that subset contiguity can be enforced by using a number of inequalities, similar to the sub-tour elimination constraints in vehicle routing, that increases exponentially with the number of subsets, making it intractable in large problems. Shirabe (2005, 2009) proposed a fluid flow approach to contiguity, yielding a mixed-integer program formulation that avoids this exponential increase by adding continuous decision variables measuring a flow volume. Nevertheless, this formulation is also intractable in large problems. Also, to the best of the authors' knowledge, no linear formulation for the convexity condition has been presented in the literature. Additionally, for the purposes of this research, computational time is critical since the model is to be included in an integrated DSS and, therefore, the user would expect a solution within a reasonable time. Therefore, the presented model is solved by means of a heuristic algorithm. Namely, we adopt a random search algorithm that, on each iteration, generates a new solution using a randomized greedy heuristic and then improves it using a local search algorithm. Additionally, the random search algorithm can be initialized by a solution provided by the user, which is then optimized by means of local search.

Random greedy algorithm. This algorithm generates an initial solution by randomly choosing the first element of each subset and then expanding the subsets in a greedy fashion while preserving their connectivity and convexity. Initially, the partition subsets are empty. In the first phase of the algorithm, each subset is initialized with a randomly chosen point. At each iteration of the second phase, the algorithm extends the initial solution by assigning a point to a subset. The algorithm chooses the combination of point and subset that results in the best feasible solution. The algorithm ends when all the points have been assigned to subsets. Due to the convexity condition, it is possible that the algorithm cannot assign all the points to subsets. In this case, the algorithm returns an empty set.

Algorithm 1 Random greedy algorithm.

procedure *GreedyHeuristic*(A, R, p){Phase 1 – Random initialization of the subsets.}
  $C \leftarrow (i,j) \in G$; {Initialize points.}
  for all $s \in P$ **do**
    $c \leftarrow rand(C)$; {Randomly choose a point from C.}
    $C \leftarrow C\backslash c$; {Remove c from C.}
    $s \leftarrow c$; {Assign c to s.}
  end for
  {Phase 2 – Subset expansion.}
  while $C \neq \emptyset$ **do**
    $P^\star \leftarrow \emptyset$;
    for all $\{s \in P\}$ **and** $\{c | c \in Neighborhood(s) \wedge c \in C\}$ **do**
      $s \leftarrow s \cup c$; {Assign c to s.}
      if $Conv_s(P) = 1$ **and** $obj(P) < obj(P^\star)$ **then**
        $P^\star \leftarrow P$; {Save the best solution found so far.}
        $c^\star \leftarrow c$; {Save the last point added to a subset.}
      end if
      $s \leftarrow s\backslash c$; {Remove c from s.}
    end for
    $P \leftarrow P^\star$; {Update the current solution with the best solution found so far.}
    $C \leftarrow C\backslash c^\star$; {Remove $c^\star$ from C.}
  end while
  return $P^\star$;
end procedure

The procedure $rand()$ randomly chooses an element from the input set. The set $Neighborhood(s)$ returns the neighboring points, i.e., the set of feasible points that do not belong to s and whose distance from at least one of the points in s is exactly one:

$$Neighborhood(s) = \{a = (i, j) \in G \backslash s | \exists b \in s | dist(a, b) = 1\}. \quad (22)$$

The neighboring set of points can be efficiently calculated by keeping a list for each subset that is updated every time a point is added to or removed from the subset. Subsets can be checked for convexity ($Conv_s(P) = 1$) by applying condition (3), having a complexity equal to $O(|s|^2)$. King et al. (2012); King, Jacobson, and Sewell (2014) propose data structures specifically designed for the efficient implementation of contiguity and hole constraints in local search algorithms for planar graph partitioning. Nevertheless, implementing such sophisticated data structures in our algorithm is unnecessary, as no real benefit would result from reducing the complexity of the convexity test. In fact, the complexity for running the convexity test is dominated by that of the Floyd–Warshall algorithm ($O(|s|^3)$ to compute the shortest-path distance matrix, on which the convexity test is based.

Local search algorithm. The local search algorithm improves the solution generated by the greedy algorithm by reassigning the points located at the subsets' borders. At each step of the algorithm, all the feasible reassignments of a point are considered. The algorithm chooses the reassignment that results in the best partition. If the solution found is better than the previous one, then it is taken as the starting point of the next iteration.

Algorithm 2 Local search algorithm.

procedure $LocalSearch(A, R, p, P)$
  improved $\leftarrow$ **true**;
  while improved **do**
    improved $\leftarrow$ **false**;
    $P^\star \leftarrow P$; {Initialize the best solution found with the current one.}
    for all $\{s^A \in P\}$ **and** $\{c \in s^A\}$ **and** $\{s^B \in P | c \in Neighborhood(s^B)\}$ **do**
      $s^A \leftarrow s^A \backslash c$; {Remove c from s^A.}
      $s^B \leftarrow s^B \cup c$; {Assign c to s^B.}
      if $\forall s \in P,\ Empty_s(P) = 0$ **and** $Conn_s(P) = 1$ **and** $Conv_s(P) = 1$ **and** $obj(P) < obj(P^\star)$ **then**
        $P^\star \leftarrow P$; {Save the best solution found so far.}
        improved $\leftarrow$ **true**; {The solution improved.}
      end if
      $s^B \leftarrow s^B \backslash c$; {Remove c from s^B.}
      $s^A \leftarrow s^A \cup c$; {Assign c to s^A.}
    end for
    $P \leftarrow P^\star$; {Update the current solution with the best solution found so far.}
  end while
  return $P^\star$;
end procedure

Subsets can be checked for connectivity ($Conn_s(P) = 1$) by applying condition (2), having a complexity equal to $O(|s|^2)$. Also the connectivity test requires the shortest-path distance matrix computed using the Floyd–Warshall algorithm.

Random search algorithm. Initially, if no initial solution $\hat{P}$ is provided by the user, the best solution is initialized to empty. Otherwise, the best solution is initialized by optimizing $\hat{P}$ by means of local search. At each iteration, the random search algorithm generates a new solution by calling *GreedyHeuristic* and *LocalSearch*. The new solution is compared with the best solution found. The algorithm iterates according to a certain looping condition, *Loop*. In our implementation, the algorithm runs for a fixed amount of computational time.

Algorithm 3 Random search algorithm..

procedure $RandomSearch(A, R, p, N, \hat{P})$
  if $\hat{P} \neq \emptyset$ **then**
    $P^\star \leftarrow LocalSearch(A, R, p, \hat{P})$; {Initialization by user provided solution.}
  else
    $P^\star \leftarrow \emptyset$; {Initialize the best solution found to empty set.}
  end if
  $n \leftarrow 0$; {Initialize the number of iterations to zero.}
  while $Loop()$ **do**
    $P \leftarrow GreedyHeuristic(A, R, p)$; {Generate a new solution.}
    $P \leftarrow LocalSearch(A, R, p, P)$; {Improve the current solution.}
    if $obj(P) < obj(P^\star)$ **then**
      $P^\star \leftarrow P$; {Save the best solution found so far.}
    end if
    $n \leftarrow n + 1$; {Increase the iteration counter.}
  end while
  return $P^\star$;
end procedure

4. Case study: the Central District of Madrid

The algorithm has been applied and tested on a case study of the Central District of Madrid. The solutions identified by the optimization algorithm have been analyzed and compared to the standard patrolling configurations currently adopted by inspectors of the SNPC.

4.1. The Central District of Madrid

Madrid is the capital of Spain and the most populous city in the country, with 3,207,247 inhabitants as of 2013. In the metropolitan area as a whole, the population is 6,543,031. The Central District of Madrid, on which we focus our work, has an area of more than two square miles and comprises six neighborhoods: Palacio, Embajadores, Cortes, Justicia, Universidad, and Sol. Its population is approximately 150,000 inhabitants.

4.1.1. Datasets

To determine the best grid size, we can take advantage of the results of Gorr, Olligschlaeger, and Thompson (2003). In fact, the authors show that the average monthly crime counts for each cell of the grid needs to be on the order of 30 or more to achieve good forecast accuracy. The resulting grid for the Central District of Madrid has nine rows and nine columns, and can be seen in Fig. 1. Crime analysts from the SNPC stated that the grid is sufficiently precise for the determination of patrol districts.

In this case study, we consider the thefts committed during the month of October, 2011. Theft is the most frequent type of crime committed in Spain and one of the main priorities for the SNPC is its reduction. The month of October has been chosen, as it is an "average" month in terms of population and activity and it has only one holiday. More specifically, we consider the following working shifts:

- SATT3: Saturday, 10/15/2011, night shift (10 PM–8 AM).
- SUNT1: Sunday, 10/16/2011, morning shift (8 AM–3 PM).
- MONT2: Monday, 10/17/2011, afternoon shift (3 PM–10 PM).

These three shifts have been chosen for their representativeness, as crime activity varies by time of day, day of the week, and by sector, and exhibits seasonal effects (Cohen, 2006). Fig. 1 illustrates the distribution of thefts in the three shifts considered. SATT3 is characterized by a high level of nightlife, with people coming from other districts of Madrid as well as other cities. In the picture it can be seen

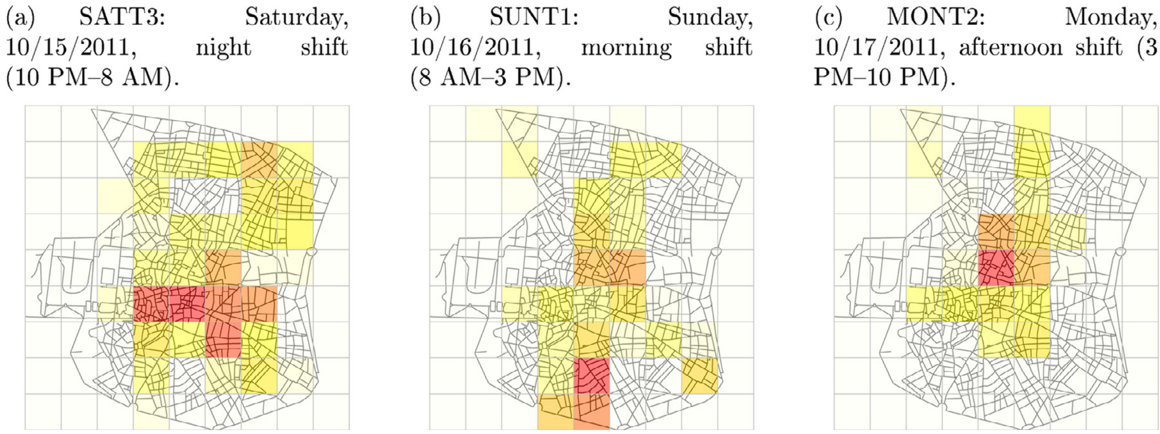


Fig. 1. Number of thefts in the Central District of Madrid. Red represents a high crime level, while white represents no criminal activity. (For the interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

that thefts are committed in almost all the territory, with the highest levels concentrated around Plaza Mayor, the central plaza of the city. SUNT1 has a moderate level of criminality, mostly concentrated in the south of the district where a very popular flea market (El Rastro) is held every Sunday morning. Finally, MONT2 presents the characteristics of a normal business day, with low levels of criminal activity, mostly concentrated in the commercial area.

4.2. Current patrolling configurations analysis

During an interview, a service coordinator in charge of the patrolling operations of the Central District of Madrid stated that, on a “normal day,” one of the following patrol sector configurations is applied:

- CONF2: The district is divided into two big sectors by the Gran Via, the main artery in the territory, and the agents are free to patrol the assigned area *ad libitum*. The northern sector includes two neighborhoods (i.e., Universidad and Justicia) while the southern sector includes four neighborhoods (i.e., Palacio, Sol, Embajadores, and Cortes).
- CONF6: The district is partitioned according to its six neighborhoods.

To be able to compare the performance of these configurations with those identified by the optimization algorithm, we represented CONF6 and CONF2 using the same grid structure adopted by the optimization algorithm, as illustrated in Figs. 2a and 3a. The cells of the grid shared by more than one sector have been assigned to the sector occupying the most of its area. It should be noticed that both configurations present one sector that is not convex, i.e., the green sector in CONF6 and the light blue sector in CONF2). Therefore, the configuration currently adopted by the SNPC would be infeasible according to the optimization model proposed. This might result in better attribute values for these solutions than those achievable with a feasible solution. In the following, we use these configurations as a comparative basis for the quality of the solutions identified by the optimization algorithm.

4.3. Analysis of the optimization model solutions

We now analyze the quality of the solutions found by the optimization algorithm, by comparing them to the patrolling configurations currently adopted by the SNPC. The optimization algorithm was implemented in C++. The experiments were run on a computer with an Intel Core i5-2500K CPU having four cores at 3.30 gigahertz and 4 gigabytes RAM memory. The program was run on only one core and the measured RAM memory use is less than 2 megabytes. Given that the police district optimizer should be part of a DSS and, therefore, be sufficiently interactive, the computational time limit for each test was set to 60 seconds. Concerning the parameters, we asked a service coordinator in charge of the patrolling operations of the Central District to define her preferences among the criteria and the values for the weights and the parameter lambda. The parameters values adopted in the experiments are the following:

- Dataset: {SATT3, SUNT1, MONT2}.
- Number of patrol sectors, p: {2, 6}.
- Preference weights, $(w_\alpha, w_\beta, w_\gamma, w_\delta)$: {(0.45, 0.05, 0.45, 0.05)}.
- Balance coefficient, λ: {0.1}.

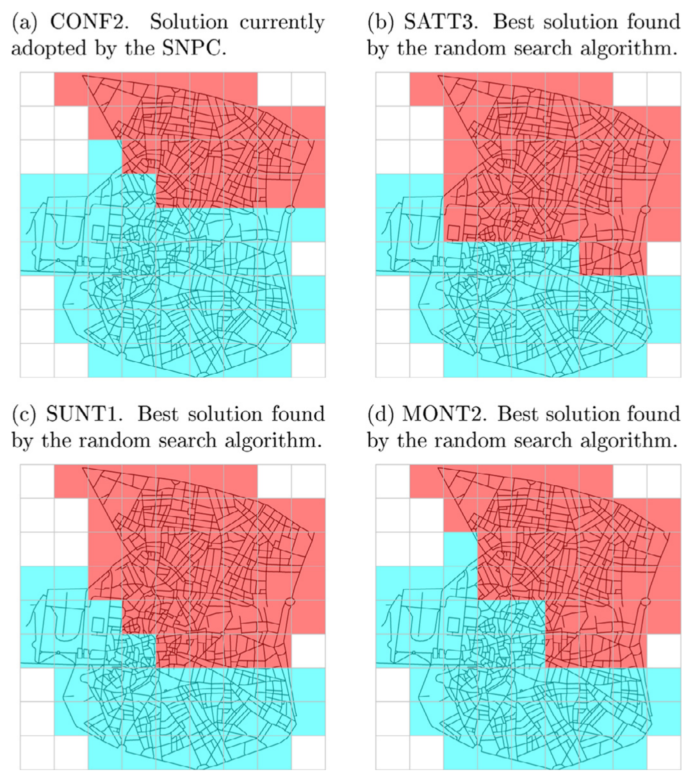
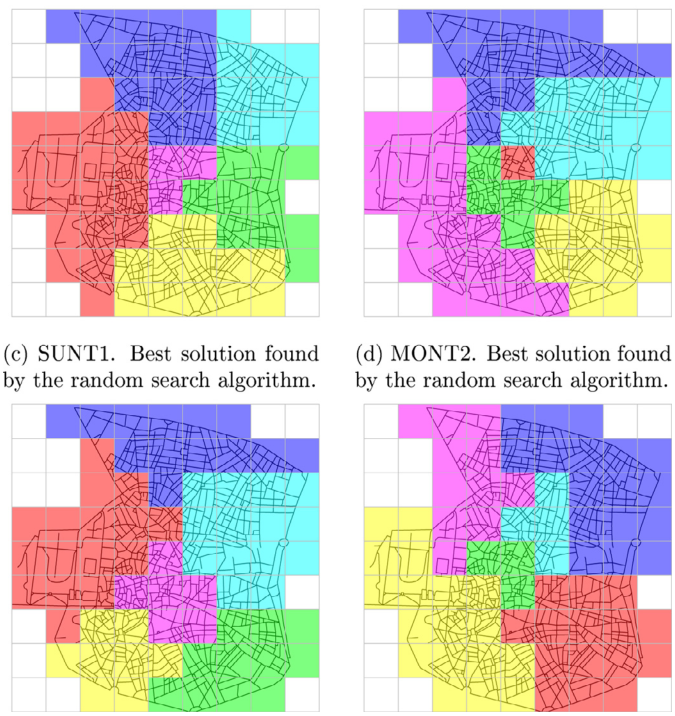


Fig. 2. Comparison of the patrolling configurations currently adopted by the SNPC with those generated by the optimization algorithm. Scenario with two patrol sectors. Each sector is represented in a different color. (For the interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)



(a) CONF6. Solution currently adopted by the SNPC.

(b) SATT3. Best solution found by the random search algorithm.

(c) SUNT1. Best solution found by the random search algorithm.

(d) MONT2. Best solution found by the random search algorithm.

Fig. 3. Comparison of the patrolling configurations currently adopted by the SNPC with those generated by the optimization algorithm. Scenario with six patrol sectors. Each sector is represented in a different color. (For the interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.).

In any event, the algorithm can be run for any feasible combination of the parameters. In the following, we compare the solutions found by the proposed algorithm and the patrolling configurations currently adopted by the SNPC.

4.3.1. Scenario with two patrol sectors

As the optimization algorithm is random in nature, we ran each configuration 50 times. The best solutions found by the algorithm are displayed in Fig. 2. According to the figures, the optimization algorithms assigns a greater area to the northern sector than the current solution of the SNPC does. Also, we can see that the northern sector slowly decreases in size as we move from the Saturday night shift to the Monday afternoon shift, to adapt to the changes in the crime activity level and distribution.

The solution and the attribute values are illustrated and compared in Table 1a. The first three columns report the dataset, the methodology, and the objective function value. Then, for each attribute, the average and the worst value are given. The area and demand averages are not shown, as they are constant. For the algorithm, we show the 95 percent confidence interval computed over the 50 runs. Also, to simplify the interpretation of the differences in the attributes values, we show the percentage improvement of our solutions over the current solution adopted by the SNPC. The improvement was calculated as $100 \cdot (1 - \frac{Z_{\text{ALG}}}{Z_{\text{SNPC}}})$, except for the average and min support, that was calculated as $100 \cdot (\frac{Z_{\text{ALG}}}{Z_{\text{SNPC}}} - 1)$, where Z_{ALG} is the value of the solution computed by the optimization algorithm and Z_{SNPC} is the value for the current patrolling configuration in use by the SNPC. In the instances considered, the proposed algorithm produces patrolling configurations that are always better than the current one in terms of the objective function, with an average improvement of 11.97 percent. Also, we can see that all the attributes experience a significant improvement, with the exception of the diameter, which worsens by 4.55 percent on average.

4.3.2. Scenario with six patrol sectors

The best solutions found by the algorithm are shown in Fig. 3. We can see that there are significant differences between them and the current patrolling configuration. From the observation of the configurations with six subsets, one can see the importance of designing patrolling districts tailored for the specific characteristics of each shift. In fact, we can see that the size and location of the sectors changes to adapt to the crime distribution in each shift. For the Saturday night shift (Fig. 3b), the focus is on the center of the district, where most of the nightlife takes place. On Sunday morning (Fig. 3c), as expected, we can see that most of the agents should be located on the southern part of the district, where the flea market takes place. On the other hand, on Monday afternoon (Fig. 3d), patrolling in the southern part of the district can be reduced (only two sectors), in favor of a greater control of the central and the northern parts of the district, where the commercial activities are located.

The solution and the attribute values are illustrated and compared in Table 1b. Also in the scenario with six patrol sectors the algorithm generates better partitions than those currently in use in the SNPC, with an average improvement of 10.40 percent. In fact, it can be seen that the objective function value of the current configuration is always larger that of the configurations generated by the optimization algorithm. Also, the optimization algorithm improves notably the average and minimum support, while keeping the max area and the max demand below the values of the current solutions. The only exception is for dataset SATT3, where the max demand is much higher than that of the current solution.

5. Conclusions

The purpose of this paper was to introduce a model for the optimization of patrolling sectors, specifically tailored to suit the requirements of the Spanish National Police Corps (SNPC). This model will be part of a Decision Support System (DSS) for the implementation of a predictive patrolling policing. The model proposed is multi-criteria in nature. Given the non-linear nature of its restrictions, we propose a local search heuristic algorithm for its solution. A case study of the Central District of Madrid was presented and the performance of the algorithm was assessed. We showed empirically that the algorithm rapidly generates patrolling configurations that are more efficient than those currently adopted by the SNPC. However, this research is just a scratch on the surface and several lines of research can be pursued, as explained in the following.

- The model could be improved to increase its realism. As suggested by Sarac et al. (1999), census cell blocks could substitute for the current grid structure. Unfortunately, adopting this structure would increase the complexity of the model and the time necessary for its solution. In fact, the convexity restriction on the subsets' geometry would require the implementation of specific data structures, such as those proposed by King et al. (2012, 2014). In this case, finding a balance between realism and solvability would be imperative, as the model should be solved in real-time.
- The approximation introduced by the current area measure could be improved by considering other more realistic measures. A previous implementation of the model computed the minimum length Hamiltonian Cycle. However, initial computational experiments showed that that was computationally inefficient. Further research could focus on its time-efficient implementation, or on alternative representative measures (Opasanon & Miller-Hooks, 2006; Pal & Bose, 2009).
- The effectiveness of other heuristic and metaheuristic algorithms such as tabu search, ant colonies, and genetic algorithms could be investigated.

Table 1
Comparison of the patrolling configurations currently adopted by the SNPC with those generated by the optimization algorithm. The tables show the solution values, the attribute values, and the percentage improvement of the algorithm solutions over the current patrolling configuration.

(a) Scenario with two patrol sectors

Dataset	Method	Obj($\overline{P}$)	Max area	Avg support	Min support	Max demand	Avg diameter	Max diameter
SATT3	CONF2	0.56	83588	0	0	33.4	11	12
	Algorithm 3	[0.49, 0.49]	[66247, 66247]	[1, 1]	[1, 1]	[24.59, 24.59]	[11.5, 11.5]	[12, 12]
	Improvement	[12.5 percent, 12.5 percent]	[20.75 percent, 20.75 percent]	∞	∞	[26.38 percent, 26.38 percent]	[−4.55 percent, −4.55 percent]	[0, 0]
SUNT1	CONF2	0.56	83588	0	0	19.64	11	12
	Algorithm 3	[0.49, 0.49]	[70728, 70728]	[1, 1]	[1, 1]	[13.76, 13.76]	[11.5, 11.5]	[12, 12]
	Improvement	[12.5 percent, 12.5 percent]	[15.38 percent, 15.38 percent]	∞	∞	[29.94 percent, 29.94 percent]	[−4.55 percent, −4.55 percent]	[0, 0]
MONT2	CONF2	0.55	83588	0	0	12.80	11	12
	Algorithm 3	[0.49, 0.49]	[68002, 68002]	[1, 1]	[1, 1]	[11.42, 11.42]	[11.5, 11.5]	[12, 12]
	Improvement	[10.91 percent, 10.91 percent]	[18.65 percent, 18.65 percent]	∞	∞	[10.78 percent, 10.78 percent]	[−4.55 percent, −4.55 percent]	[0, 0]

(b) Scenario with six patrol sectors.

Dataset	Method	$Obj(\overline{P})$	Max area	Avg support	Min support	Max demand	Avg diameter	Max diameter
SATT3	CONF2	0.20	33065	2.33	1	8.85	5.17	7
	Algorithm 3	[0.19, 0.19]	[28281.22, 30050.94]	[3.66, 3.75]	[2.65, 2.87]	[13.49, 14.06]	[5.43, 5.62]	[8.74, 9.42]
	Improvement	[5 percent, 5 percent]	[9.12 percent, 14.47 percent]	[57.08 percent, 60.94 percent]	[165 percent, 187 percent]	[−58.87, −52.43]	[−8.70 percent, −5.03 percent]	[−24.86 percent, −34.57 percent]
SUNT1	CONF2	0.21	33065	2.33	1	11.21	5.17	7
	Algorithm 3	[0.18, 0.19]	[27851, 29331.08]	[3.64, 3.69]	[2.88, 3]	[6.44, 6.88]	[5.25, 5.35]	[7.29, 7.79]
	Improvement	[9.52 percent, 14.29 percent]	[11.29 percent, 15.77 percent]	[56.22 percent, 58.37 percent]	[188 percent, 200 percent]	[38.63, 42.55]	[−3.48 percent, −1.55 percent]	[−11.29 percent, −4.14 percent]
MONT2	CONF2	0.21	33065	2.33	1	7.48	5.17	7
	Algorithm 3	[0.18, 0.18]	[32122.76, 33319.8]	[3.81, 3.89]	[2.68, 2.88]	[6.71, 7.16]	[5.24, 5.36]	[9.15, 9.61]
	Improvement	[14.29 percent, 14.29 percent]	[−0.77 percent, 2.85]	[63.52 percent, 66.95 percent]	[168 percent, 188 percent]	[10.29 percent, 4.28 percent]	[−3.68 percent, −1.35 percent]	[−37.29 percent, −30.71 percent]

- Although the model is intrinsically non-linear, decomposition methods such as Column Generation or Benders' Decomposition could be applied to solve the problem to optimality. Also, these methodologies could still be used to generate good heuristic solutions should the solution process take longer than the allowed computational time.
- Recent papers have analyzed the statistical effect of law enforcement actions on crime patterns (Jones, Brantingham, & Chayes, 2010). By including these effects in an optimization problem it would be possible to formulate a model for the design of patrol configurations that result in a reduction of the future level of criminality. The model would be similar in nature to theoretical games (Hohzaki & Maehara, 2010) and to fortification/interdiction problems used to hedge against intentional attacks (Kress, Royset, & Rozen, 2012; Scaparra & Church, 2008; Zoroa, Fernandez-Saez, & Zoroa, 2012) and natural disasters (Liberatore, Scaparra, & Daskin, 2012).
- A service coordinator in charge of the patrolling operations in the Central District of Madrid pointed out that an important component is ensuring that the agents' job is "entertaining," as opposed to dull and boring. It could be an interesting challenge for modelers to come up with an "entertainment" attribute to be included during the optimization process.
- Finally, the model and algorithm presented in this work will be included in an integrated DSS for the implementation of a smart patrolling policing that we are currently developing in collaboration with the SNPC. Certainly, this research will open new opportunities for the application of OR methods and models in the police sector.

We hope that this paper will be a useful source of ideas for future research on policing models and will contribute further to the development and solution of more complex models for the PDP.

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