MCG Log Law 11/26/2024

Last time:

τ_α preserves Teich disk $\Rightarrow$ $\delta_{\langle \tau_\alpha \rangle} = \frac{1}{2}$, pure exp growth

so Π multi-twist $\Rightarrow$ $\delta_\Pi \geq \frac{1}{2}$ $\leftarrow$ Π pure exp growth

? $\boxed{\delta_\Gamma > \delta_\Pi}$

Torelli map ?

$\mathbb{C}^g / \text{lattice}$

multi-twist case

$\Pi = \langle \tau_{\alpha_1}^{n_1}, \dots, \tau_{\alpha_k}^{n_k} \rangle$ α_i, α_j disjoint

does not preserves Teichmüller geodesic – if it did, Π preserves a hyperbolic space and is abelian. This bounds the rank ...

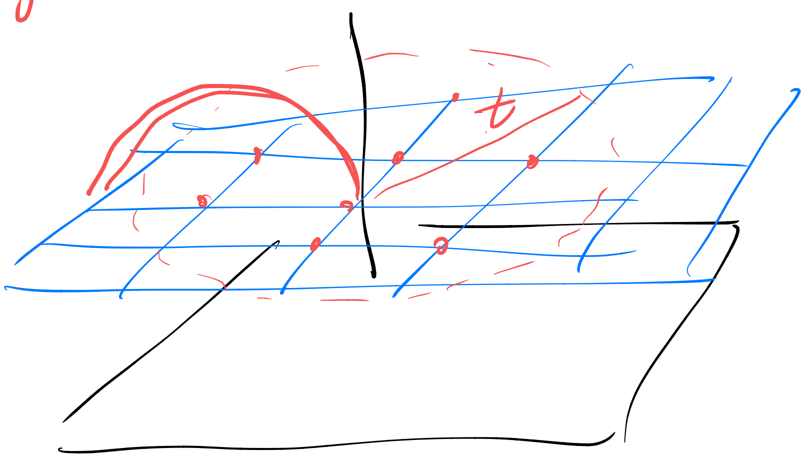
(?) can we get growth from knowing growth of generators

$\mathbb{Z}^2 \curvearrowright \mathbb{H}^3$ $\qquad \langle p_1, p_2 \rangle$ $\qquad \delta_{\mathbb{Z}^2} = 1$

$\frac{1}{2} \nearrow \qquad \nwarrow \frac{1}{2}$

$$\sum_{\gamma \in \Pi} e^{-s\, d(o, \gamma o)}$$

growth $\sim e^t$



$$\delta_\Pi = \limsup \frac{1}{t} \log \#\{ d(o, po) \le t \}$$

$d(o, m+n) = \sqrt{m^2+n^2}$ b/c Eucl.

$\sim \max\{|m|, |n|\}$

$\mathbb{Z}^n$ free abelian $\curvearrowright X$ isometries, δ_X bndd above by eucl.

$\rightsquigarrow \quad \delta_{\mathbb{Z}^n} \geq \frac{n}{2}$

Π $\quad \exists\ h$ hyp s.t. $\boxed{\langle h\rangle * \Pi < \Gamma}$ ← is this all we need?

$hp_1, hp_2 \cdots hp_k \qquad p_i \in \Pi \qquad$ all distinct elts in Γ

$s>0$

$$P_\Gamma(0,s) \geq \sum_{k\geq 1} \sum_{\substack{p_1,\dots,p_k \\ \in \Pi}} e^{-s\, d(0,\, hp_1\cdots hp_k 0)}$$

$$\geq \sum_{k\geq 1} \sum_{p_1,\dots,p_k} e^{-s\left(d(0,h0)+\dots+d(0,p_k 0)\right)}$$

$$\geq \sum_{k\geq 1} \Big(e^{-s\, d(0,h0)} \underbrace{\sum_{p\in\Pi} e^{-s\, d(0,p0)}}\Big)^k$$

$\boxed{P_\Pi(0,s) \text{ diverges at } s=\delta_\Pi}$

if $s>\delta_\Pi$ find $s_0 > \delta_\Pi$ s.t.

$$e^{-s_0 d(0,h0)} P_\Pi(0,s_0) > 1 \Rightarrow \delta_\Gamma \geq s_0$$

Π multi-twist vs elementary

~~calculating δ_Π?~~ don't need this

- $P_\Pi(o, \delta_\Pi) = \infty$
- $\delta_\Gamma > \delta_\Pi$
- examples Γ divergent type
- Π has mixed exponential growth

calculating δ_Π could be helpful

- read Gekhtman-Ma shadow lemma

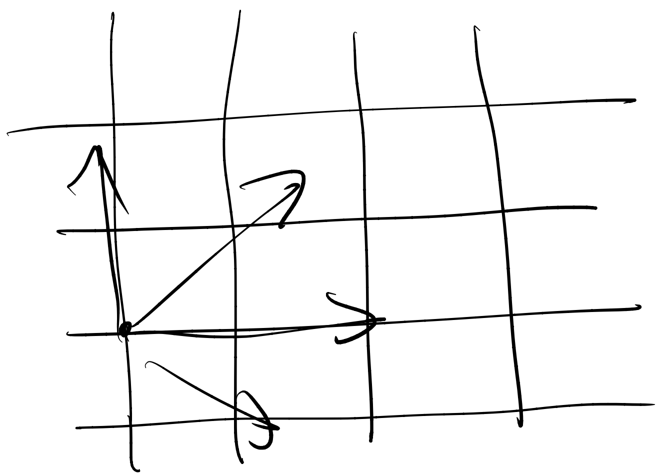
$$P_{\Pi}(0,1) = \sum e^{-d(0,\gamma 0)}$$

$$\geq \sum e^{-d_{Eucl}(0,\gamma 0)} \quad \text{finite}$$

$d_{hyp} \leq d_{eucl}$

$\mathbb{Z}^2 \curvearrowright \mathbb{R}^2$

linear



τ_α, τ_β commute $\quad \alpha \cap \beta = \emptyset$

$$\underbrace{D_\alpha \qquad D_\beta}$$

the ones coming from previous construction

are disjoint