MCG Log Law 12/3/2024

Last time:

- $P_{\Pi}(0, \delta_{\Pi}) = \infty$? *

  $\Rightarrow \delta_{\Gamma} > \delta_{\Pi}$ if $\exists h$ s.t. $h * \Pi < \Gamma$

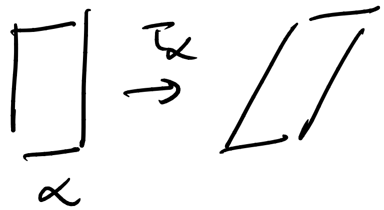
- examples Γ divergent type?

- Π has mixed exponential growth?

calculating δ_{Π} could be helpful

- read Gekhtman-Ma shadow lemma

T_α cylinder surface



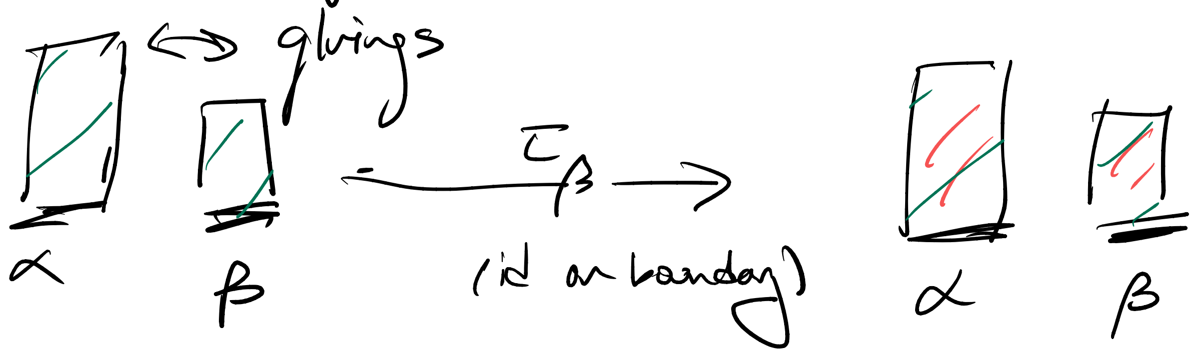
$$d_{Th}(S_1, S_2) = \log K \qquad K = \text{max dilatation } S_1 \text{ to } S_2$$

$$= d_{Teich}(S_1, S_2)$$

$$\overline{\lim} \ \frac{1}{t} \log \# \{ \gamma \in \Pi : d(o, \gamma o) \le t \}$$

S

T_α, T_β multicylinder translation surface



different markings (homeo btwn cylinder surf. and top. surf)

same gluings

$d_{\text{Teich}} = \log$ dilatation

Conj: $\delta_{\mathbb{Z}^n} = \frac{n-1}{2}$

? $\#\{ d(S, \gamma S) \le t \}$

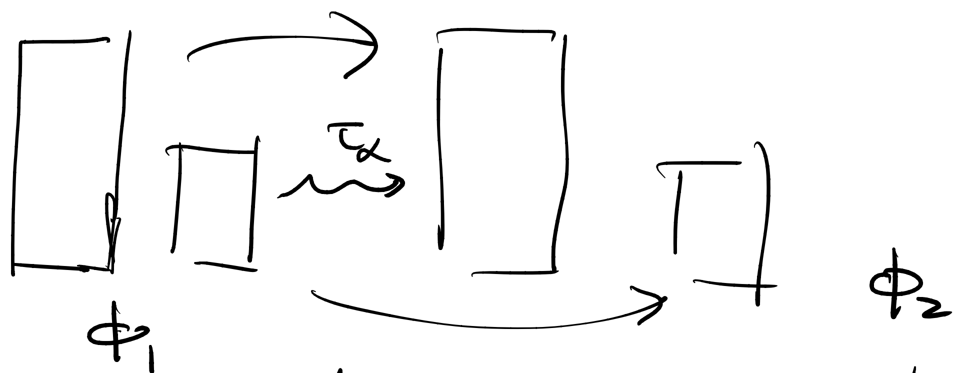
$\#\{ d(S, \tau_\alpha^k S) \le t \}$ $\#\{ d(S, \tau_\beta^l S) \le t \}$

Clm: $d(S, \tau_\alpha^k \tau_\beta^l S) \le \cancel{c} \max \{ d(S, \tau_\alpha^k S), d(S, \tau_\beta^l S) \}$

$\Rightarrow \delta_{\langle \tau_\alpha, \tau_\beta \rangle} \ge \delta_{\langle \tau_\alpha \rangle} + \delta_{\langle \tau_\beta \rangle} = 1$

clm: $d(S, \tau_\alpha^k \tau_\beta^\ell S) \leq c \max \{ d(S, \tau_\alpha^k S), d(S, \tau_\beta^\ell S) \}$

need



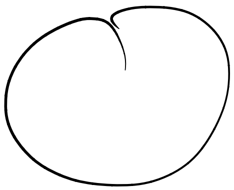
ψ has stretch factor close to Thurston map, fixes boundary

$$\psi(F, \phi_1) = (F, \phi_2)$$

Let ψ be affine $\longrightarrow$ should $=$ or be almost Thurston map

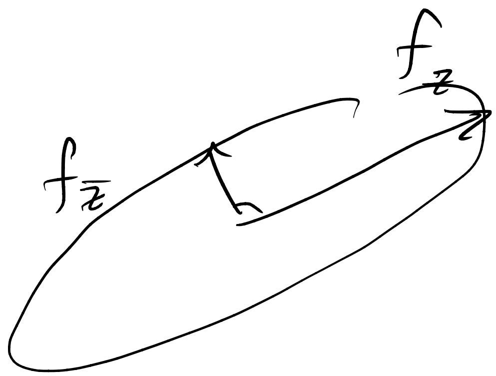
? affine on 1 cylinder $=$ Thurston map

$$K_f(p) = \frac{|f_z(p)| + |f_{\bar z}(p)|}{|f_z(p)| - |f_{\bar z}(p)|} = \frac{1 + |\mu_f(p)|}{1 - |\mu_f(p)|}$$



$$K_f = \sup_p K_f(p)$$

$$\mu_f = f_{\bar z} / f_z$$



$$K_{f \circ g} \leq K_f K_g$$

? affine on 1 cylinder = Thurston map

$d_{Teich} = \log \min K_f$ f homeo

Fact Thurston map f_{Th} ! realizes minimum

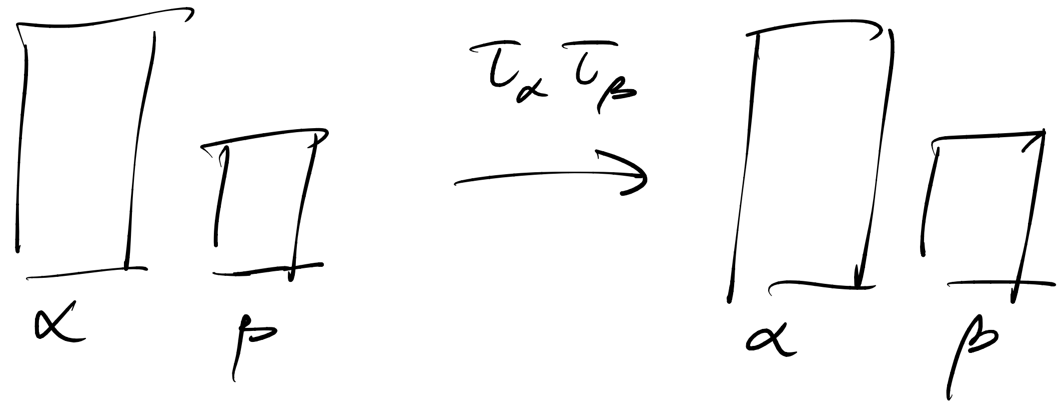
and ! has constant dilatation

Since affine maps have constant dilatation,

it is f_{Th}

$K_{\tau_\alpha} \leq K_{\tau_\beta}$

<u>define</u>



$$K_{\tau_\alpha \tau_\beta} = K_{\phi_{\alpha\beta}}$$

$$K_{\tau_\alpha} = K_{\phi_\alpha} \qquad \phi_\alpha \text{ affine}$$

$$K_{\tau_\alpha \tau_\beta} \leq \max\{K_{\tau_\alpha}, K_{\tau_\beta}\}$$
$$= K_{\tau_\beta}$$

$\phi = \begin{cases} \phi_\alpha & \text{on } C_\alpha \\ \phi_\beta & \text{on } C_\beta \end{cases}$

$[\phi] = \tau_\alpha \tau_\beta$

since $\phi_{\alpha\beta}$ min,

$K_{\phi_{\alpha\beta}} \leq K_\phi$

$= \max$

by <u>definition</u>

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$$d(S, \tau_\alpha^k \tau_\beta^l S) \leq c \max \{ d(S, \tau_\alpha^k S), d(S, \tau_\beta^l S) \}$$

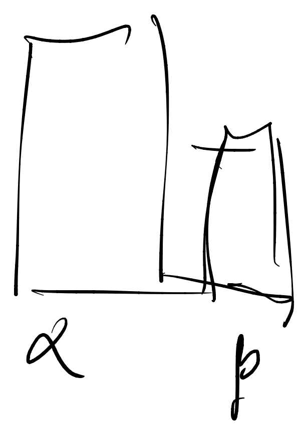
$\rightsquigarrow$ lower bound on $\delta_{\langle \tau_\alpha, \tau_\beta \rangle}$

conj: $\delta = \delta + \delta$

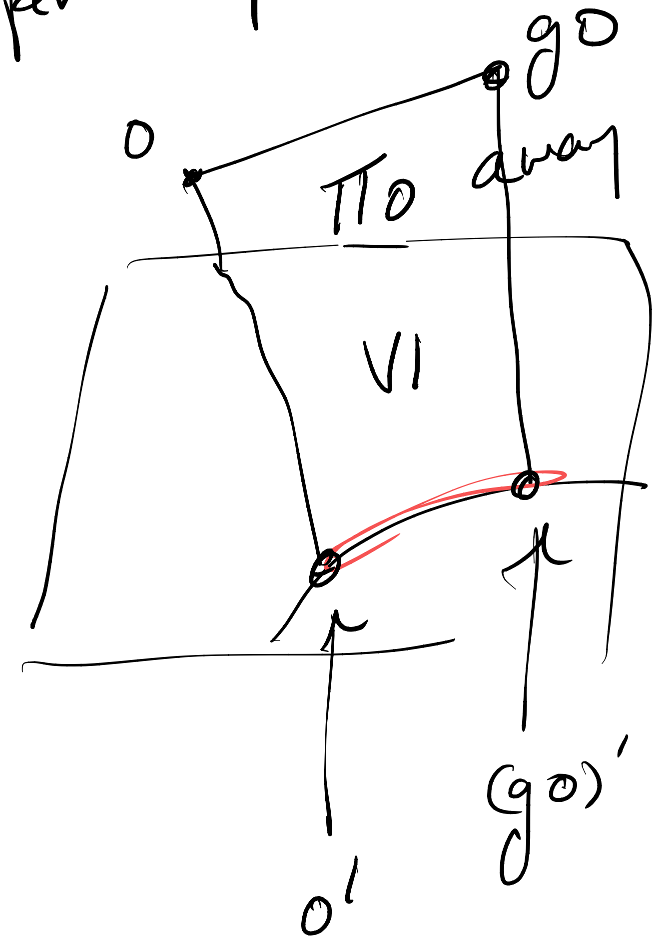
upper bound

$\tau_\alpha^k \tau_\beta^l$

flat structure so that twisting is affine



$\rightsquigarrow$ hyperbolic plane $\mathbb{H}^2$ in $\mathcal{T}(S)$



$\mathbb{H}^2$ = span (tangent vector determined by )

if $d(o, go) \geq d(o', (go)')$ then it works

BGIT of MM and subsurface projections

Rafi: hyperbolicity in Teich

does BGIT for Teich metric