wanted to determine the following

1) δ_Γ when Γ cvx cpt, CLM RAAG, Dehn Twist group

2) is $H \curvearrowright \Lambda_\rho \setminus$ fps cocompact when H is a multitwist group

3) $\mu(\text{fps}) = 0 \leftrightarrow$ ergodicity? when is μ ergodic?

4) is Λ_ρ well-defined?

5) Morse properties?

Next questions

6) Mixed exponential growth

Gekhtman-Ma:

3) Poincaré series diverges at δ_G $\Longleftrightarrow$ conical pts full PS-measure

$\Downarrow$

no atoms, full support

prove for CLM RAAGs?

4) $\Lambda_G = \overline{\{\text{fps pA}\}}$ by defn

$\cancel{\Lambda_G = \overline{G_0}}$ ∂G_0

6): purely exponential growth

$$\lim_{R\to\infty} \frac{\#\{d(x,gy)\le R\}}{e^{\delta_G R}} = c \quad \Rightarrow \quad \boxed{\#\{d(x,gy)\le R\} \asymp e^{\delta_R R}}$$

$G < \mathrm{Mod}(S)$ nonelementary

let $c > \varepsilon > 0$. then

$$-\varepsilon < \frac{\#\{d(x,gy)\le R\}}{e^{\delta_G R}} - c < \varepsilon$$

$$(c-\varepsilon)e^{\delta_G R} < \#\{d(x,gy)\le R\} < (c+\varepsilon)e^{\delta_G R}$$

To discuss

$\delta_\Pi < \delta_\Gamma$

- $\Gamma = \langle \tau_\alpha \rangle$: $\delta_\Gamma = ?$
  $\Gamma \curvearrowright \partial \mathrm{Teich} \setminus f_p$ cocompact

S = punctured torus $\quad MOD(S) = SL(2,\mathbb{Z}) \curvearrowright \mathrm{Teich} \cong \mathbb{H}^2$

what are the Dehn twists?

- can we get $\boxed{\text{CLM RAAGS}}$ or Veech groups
  with ~~$\Lambda_\Gamma = \partial \mathrm{Teich}$?~~

$\Lambda_\Gamma = S^1 \subseteq \partial \mathrm{Teich}$

- ~~Start trying to prove for Veech groups?~~ for free! no!

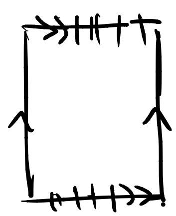
Global shadow lemma?

$\delta_\Gamma \qquad \Gamma = \langle T_\alpha \rangle$

$T_\alpha \, \mathbb{H}^2 \not\subseteq \mathbb{H}^2$

Fact: every Teichmüller disk corresponds to a quadratic diff'l (tangent vector)

clm: T_α does not send a quatratic diff'l to affine translate of quadratic diff'l (not clear if it is always possible or not)



transl surf = cylinder, T_α gen. by cyl is affine so preserves Teich disk

mult. by affine matrix

$G = \langle \tau_{\alpha_1}, \tau_{\alpha_2}, \tau_{\alpha_3} \rangle$

clm: τ_α preserves Teich dist

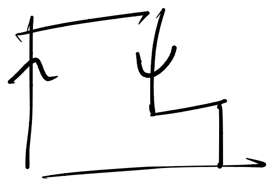
idea: α is ① sep. or ② non-sep.

① only finitely many genus for the separated halves
Mod-orbit determined by genus of 2 sides

② non-sep: all in the same orbit of Mod(S) so suffices to prove for only one

$\rightsquigarrow$ suffices to check fin. many curves.
then hopefully arrange into a cylinder as per previous page

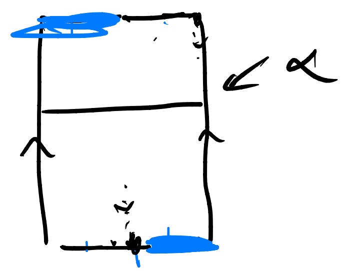
half transl surface



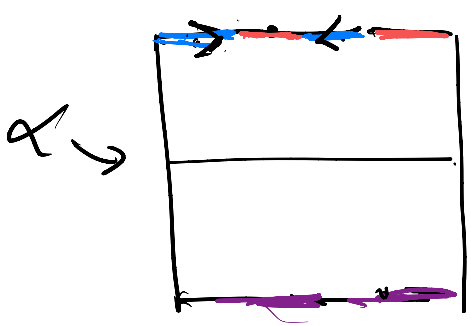
Fact: $\forall$ curve α, can arrange the transl surface so that α is horizontal like this and the surf. is a cylinder.

(chenxi's paper)

nonsep.



sep all horizontal segments are identified only horizontally



apply T_α to these: is it affine?

$\begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix}$ yes clearly

Prop: τ_α preserves a Teichmüller disk

$$\delta_{\langle \tau_\alpha \rangle} = \frac{1}{2} > 0$$

?

$G = \langle \tau_{\alpha_1}, \tau_{\alpha_2}, \tau_{\alpha_3} \rangle$ is $\delta_G > \frac{1}{2}$?

Lemma: yes by ping-pong b/c G contains pAs

$G \cong * \langle \tau_{\alpha_i} \rangle$

Π (multi-twist grp harder)
Dehn twist grp

$$\delta_{\Pi} \geq \delta_{\langle \tau_\alpha \rangle} = \frac{1}{2} > 0$$

$\delta_{\Gamma} > \delta_{\Pi}$ harder ... CLM RAAGs?

still need

$$\# \{ g \in \Pi : d(x, gx) \leq R \} \asymp e^{\delta_{\Pi} R} R^{a_\Pi}$$

Dehn twist grp $\tau_\alpha \curvearrowright$ Teich Disk

✓ $\asymp e^{\delta_{\Pi} R}$

intermediate example more complicated than Veech groups

? what are possible convex cocompact subgroups in genus 2?

Torelli map: Teich $\longrightarrow$ {marked $\mathbb{C}^g$/lattice}

$(S,g) \longmapsto \mathrm{Jac}_g$

it is almost a surjection when $g = 2, 3$ by dimension count

Teich Disk $D/\mathrm{Veech} \longleftrightarrow \mathbb{C}$/lattice & marking $\mathbb{Z}^2 \xrightarrow{\sim}$ lattice

try instead $\mathbb{C}^g$/lattice $\simeq \mathbb{Z}^g$

this could be a good example to try to still do homogeneous dyns, generalize Masur.

McMullen paper dyn of $SL(2,\mathbb{R}) \curvearrowright \mathrm{Teich}(S_2)$
section 3 & 4)

Harry: Shadow Lemma in Gekhtman-Ma
setup Github
add some papers