Project 3: Lesst Squares and Curve Fittings

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1 Introduction

Project 3 deals with finding the best function for non trusted data by using least square method. Unlike previous spline interpolation, the data points in this projects include noise. In order to filter out those noise, we propose a potential curve through the data point then take the difference. If the difference is big, then the error is big. We want to minimize the error. We sum the error from initial data point to the end. The curve that have the least error is our curve of best fit.

2 Methods

2.1 Non-weighted Least Square Fittings

Given a set of data points, we perform an error minimization process to produce the best curve of different orders. For the first order linear curve, we know the format of the solution: $\hat{y} = ax + b$. The error between the proposed curve and the data points is the difference between them; therefore, we have

$$Error = \hat{y} - y = (ax + b) - y$$

We do not want negative values for error, so we will square the equation.

$$I = Error^2 = (ax + b - y)^2$$

Since our job is to minimize the error, we want to know what values of the coefficients (a and b) gives the lowest error. From calculus, we know the minima of an equation occurs when the first derivative is 0.

$$\frac{\delta I}{\delta a} = 0, \frac{\delta I}{\delta b} = 0$$

From there, we have 2 equations and 2 unknowns. We can solve for a and b using Cramer's Rule. We repeat the procedures for quadratic and cubic curves. The size of the coefficient matrix depends on how many coefficient the curve function we choose has. For example, the quadratic curve has 3 coefficients, so we have a 3x3 matrix to solve. Since the maximum size matrix we have for this project is 4x4, we can use Cramer's Rule. However, for larger size matrix, we will use LU decomposition because it would be computationally intensive to calculate the determinant of larger size matrices for Cramer's Rule.

$$I_{quad} = (a + bx + cx^2) - y$$
$$\frac{\delta I}{\delta a} = 0$$
$$\frac{\delta I}{\delta b} = 0$$
$$\frac{\delta I}{\delta c} = 0$$

$$I_{quad} = (a + bx + cx^2 + dx^3) - y$$
$$\frac{\delta I}{\delta a} = 0$$
$$\frac{\delta I}{\delta b} = 0$$
$$\frac{\delta I}{\delta c} = 0$$
$$\frac{\delta I}{\delta d} = 0$$

2.2 Weighted Least Square Fittings

Similar to non-weighted least square fittings, we still aiming to minimize the error through the first partial derivatives of proposed curve. However, before we solve the matrix system $Ax = b$, we will multiply the system by a weighted matrix. The weighted factor penalizes the outlier data points that affects the average.

3 Programming

The least square coefficient matrix depends on 3 values, which are x,y, and n. x and y are given data points; n is the number of data points. Thus, we read x,y,and n. Construct a coefficient matrix from the error minimization equations (first partial derivatives). Solve for the unknown coefficients using Cramer's Rule. After obtaining the unknown coefficients, we substitute them back into the proposed function. Since Cramer's Rule require determinant calculations, we will also need a subroutine to calculate determinant.

However, we can also use LU decomposition to solve for the matrix system. LU decomposition method utilize Gaussian elimination, so it is not limited by the size of the coefficient matrix.

4 Results

The higher the order of proposed curve, the closer it gets to the exact curve. There is no difference between Cramer's Rule and LU decomposition methods. There is also no significant difference between the weighted and non-weighted fittings.

5 Discussion

The lack of visible difference between the weighted and non-weighted fitting could be due to the fact that the data points are too clean with no outliers. Although Cramer's Rule and LU decomposition produce no difference, LU decomposition requires less subroutines and can handle bigger size matrices. Therefore, the coding for LU decomposition has less lines than the Cramer's Rule counterpart (for matrices bigger than 4x4).