THE DEPARTMENT OF BIOMEDICAL ENGINEERING

Education– Image Reconstruction I

3D Filtered Backprojection

Fundamentals, Practicalities, and Applications

Jeff Siewerdsen, PhD
Department of Biomedical Engineering
Johns Hopkins University

Johns Hopkins University
Schools of Medicine and Engineering

Acknowledgments

I-STAR Laboratory

Imaging for Surgery, Therapy, and Radiology
www.jhu.edu/istar

Hopkins Collaborators

JW Stayman, W Zbijewski (BME)
Y Otake, J Lee (Comp. Science)
R Taylor, G Hager (Comp. Science)
J Prince (Electrical Engineering)
D Reh (Head and Neck Surgery)
G Gallia (Neurosurgery)
J Khanna (Spine Surgery)
J Wong (Radiation Oncology)
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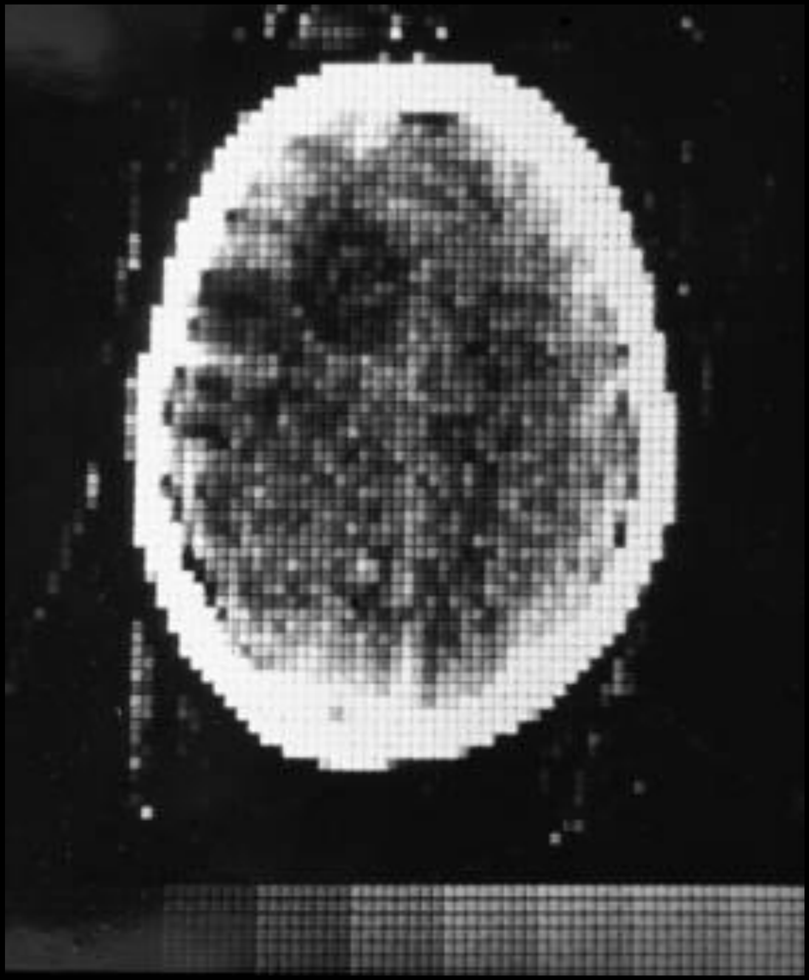
Elekta Oncology Systems

Evolution and Proliferation of CT

Sir Godfrey Hounsfield
Nobel Prize, 1979

Evolution and Proliferation of CT



c. 1975

c. 2011

Overview

Computed Tomography
Generations
Natural history and new technology

3D Image Reconstruction
Filtered backprojection
Basics and practicalities
Open-source resources

Image Quality
Artfiacts
Spatial Resolution
Contrast Resolution
Noise

Proliferation and Applications
Diagnostic imaging
Image-guided interventions

First-Generation CT

Scan and Rotate:

Linear scan of source and detector

Line integral measured

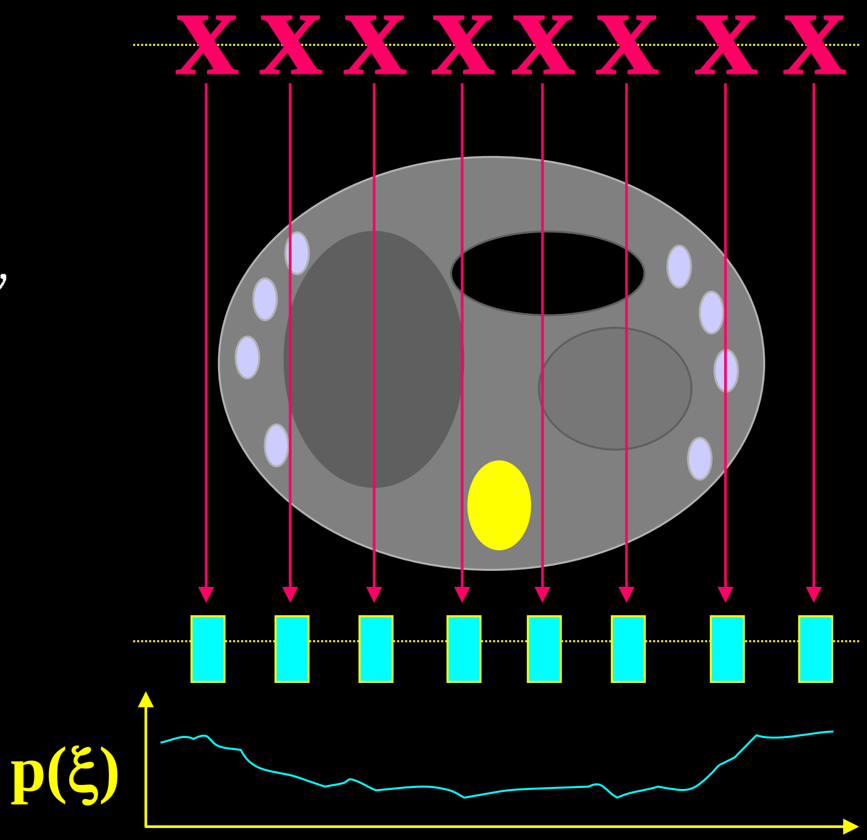
at each position: p(ξ)

$$p(\xi;\theta) = p_0 e^{-\int_0^{SDD} \mu(x,y,z)dy}$$

Rotate source-detector Δθ

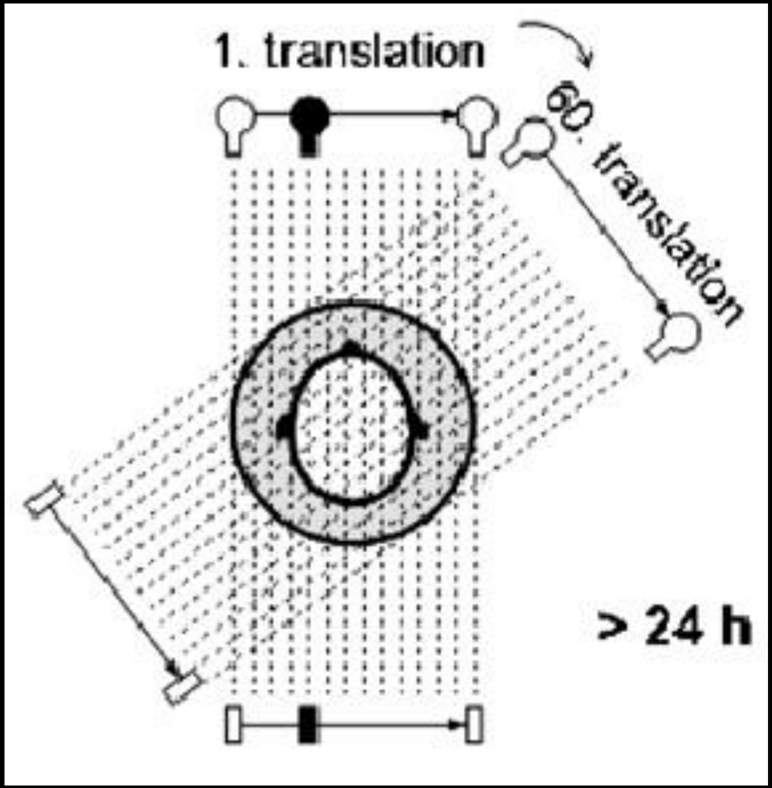
Repeat linear scan…

Projection data: p(ξ,θ)



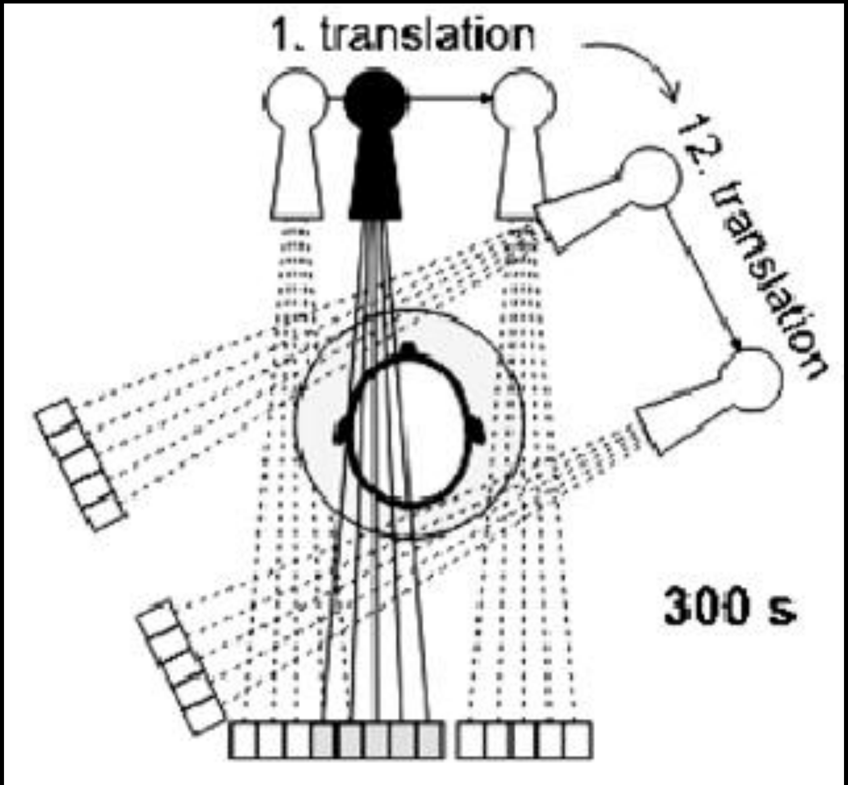
CT "Generations"

1st Generation (1970)



Pencil Beam
Translation / Rotation

2nd Generation (1972)

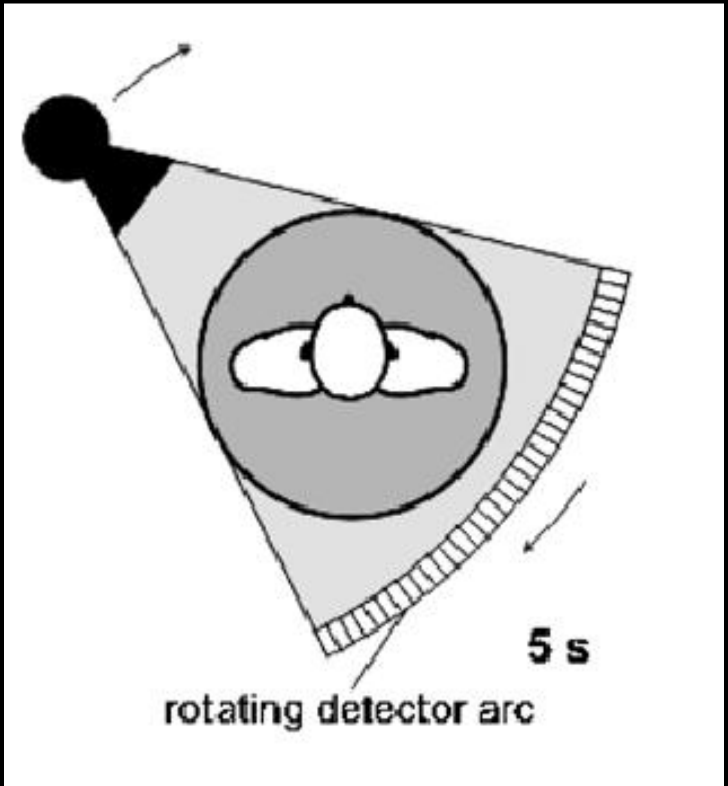


Fan Beam
Translation / Rotation

WA Kalender, Computed Tomography, 2nd Edition (2005)

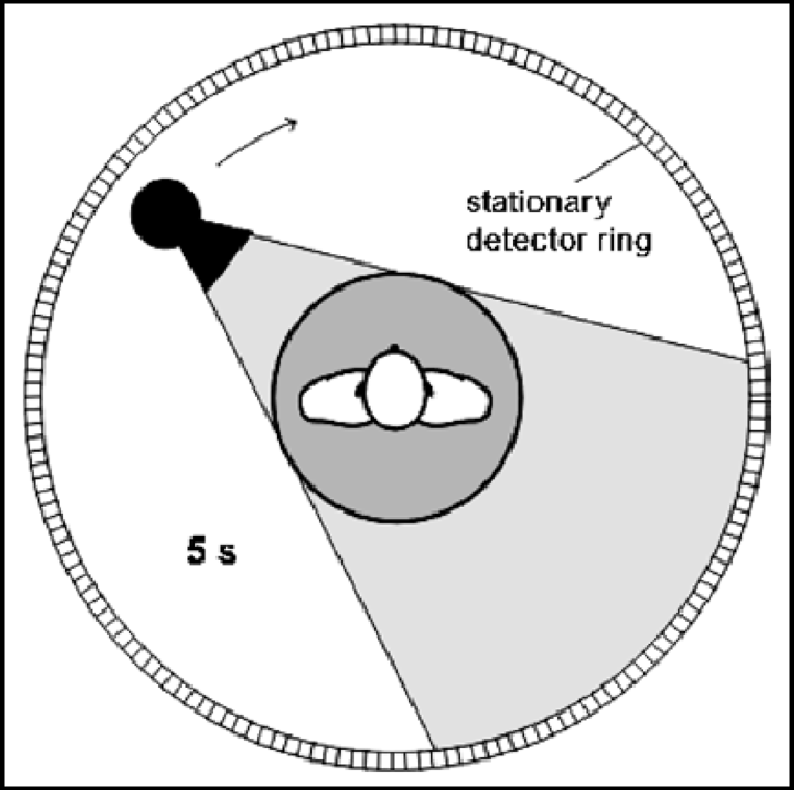
CT "Generations"

3rd Generation (1976)



Fan Beam
Continuous Rotation

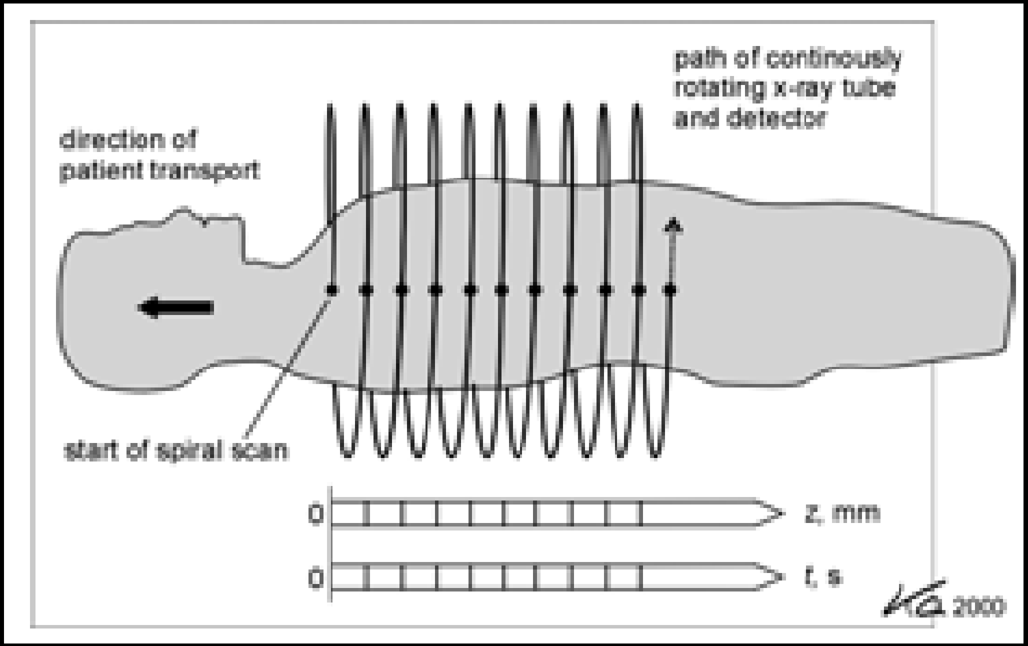
4th Generation (1978)



Fan Beam
Continuous Tube Rotation
Stationary Detector

WA Kalender, Computed Tomography, 2nd Edition (2005)

Helical (Spiral CT)



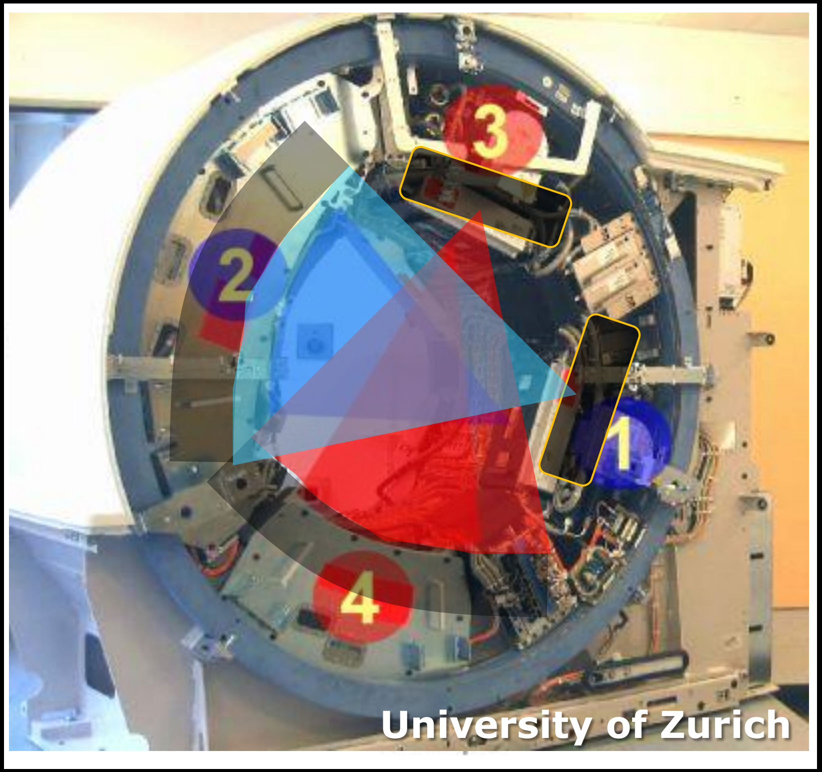
Pitch <1 :
Overlap
Higher z-resolution
Higher dose

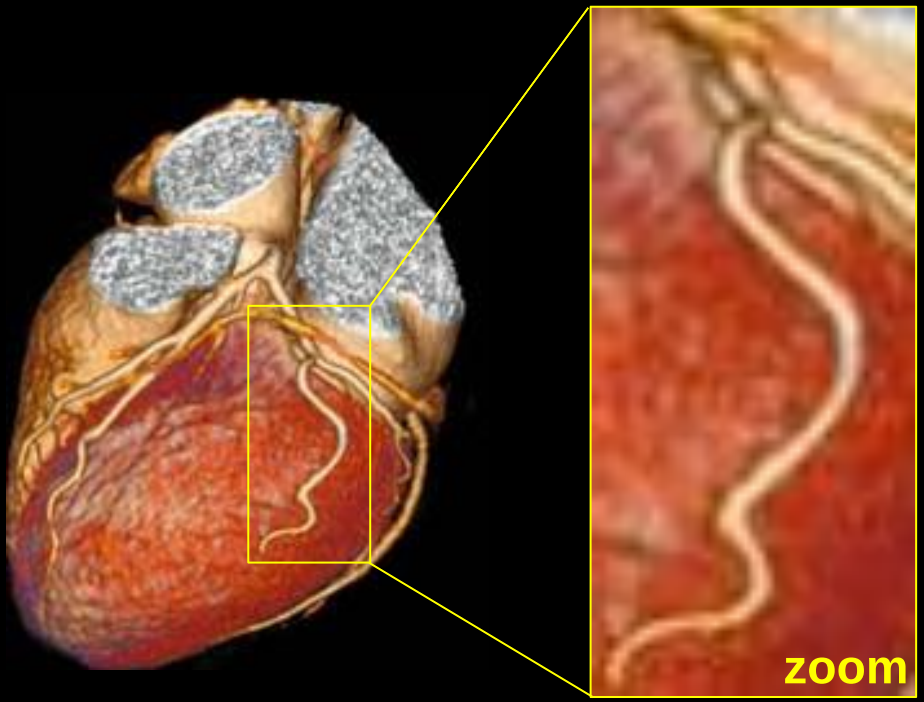
Pitch >1:
Non-overlap
Lower z-resolution
Lower patient dose

$$\text{Pitch} = \frac{\text{Table increment / rotation (mm)}}{\text{Beam collimation width (mm)}}$$

WA Kalender, Computed Tomography, 2nd Edition (2005)

Dual-Source CT



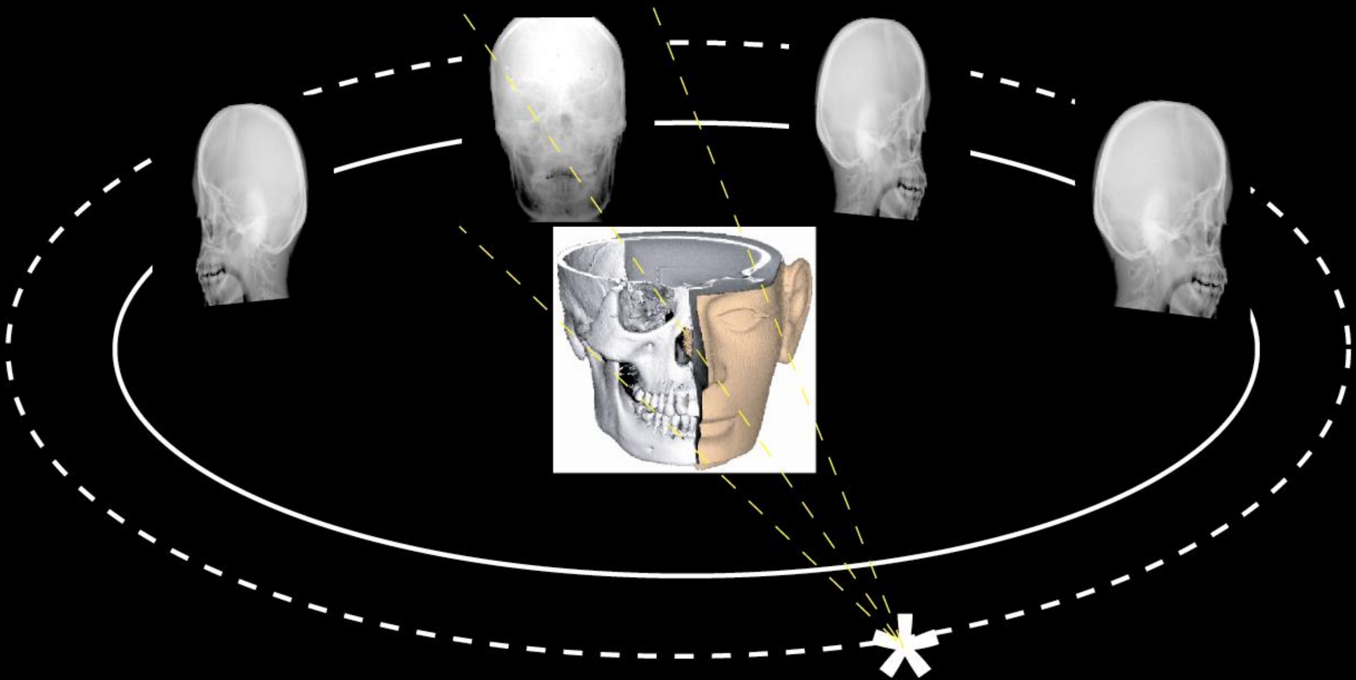


Two complete x-ray and data acquisition systems on one gantry.
330 ms rotation time
(effective 83 ms scan time)

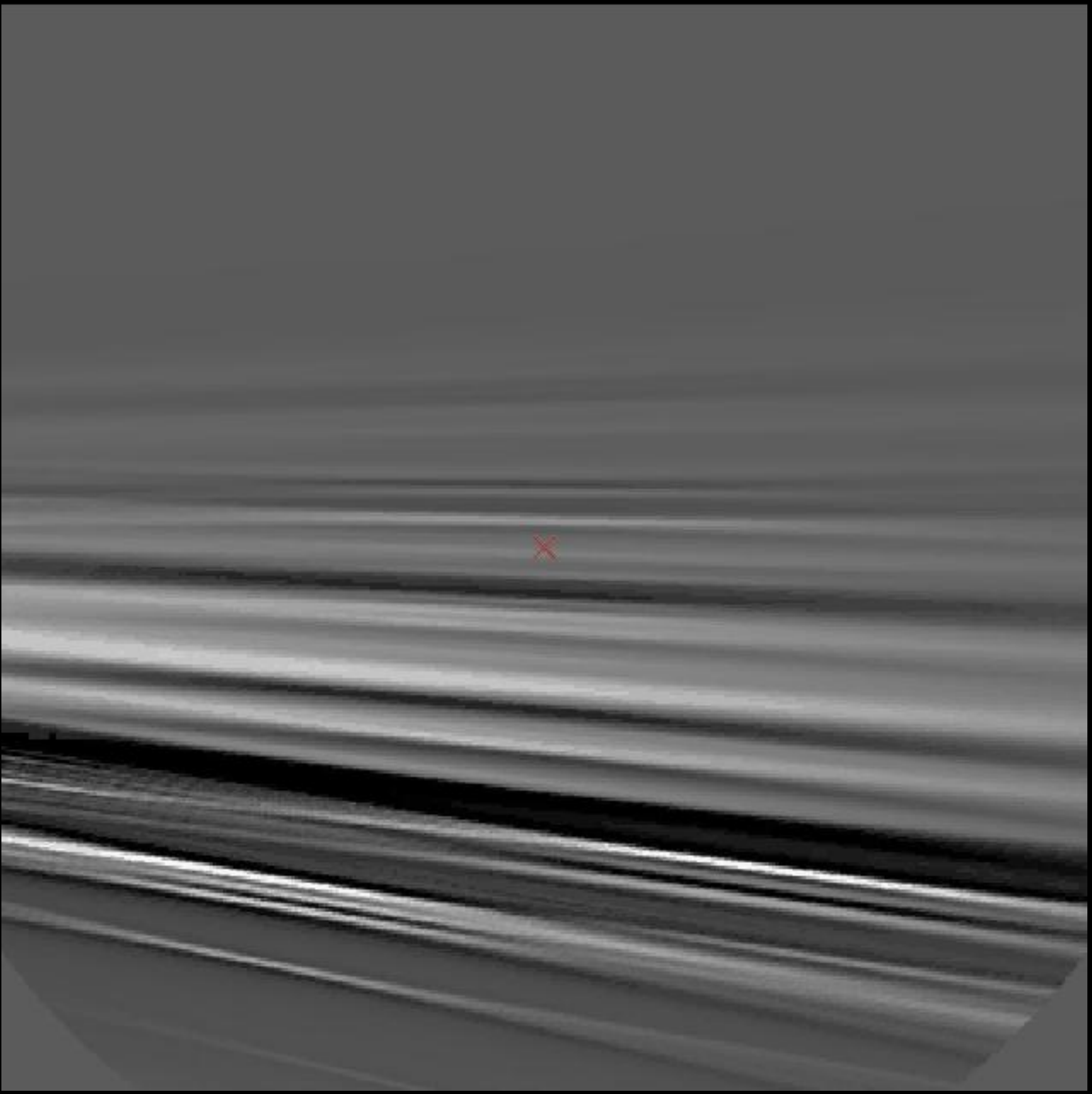
Siemens Healthcare– *Somatom Definition*

Recent Advances: *Cone-Beam* CT

Fully 3-D Volumetric CT



Conventional CT:	**Cone-Beam CT:**
~~Fan-Beam~~	Cone-Beam Collimation
~~1-D Detector Rows~~	Large-Area Detector
~~Slice Reconstruction~~	3-D Volume Images
~~Multiple Rotations~~	Single Rotation

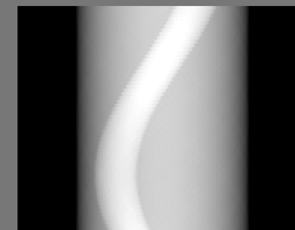


Filtered Backprojection: The Basics

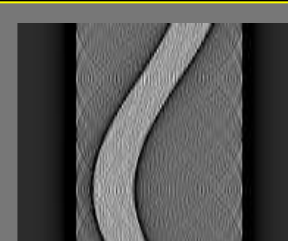
Implementation

Loop over all views (all θ)

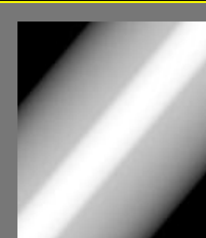
Projection at angle θ
p(ξ,θ)



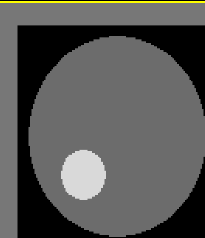
Filtered Projection
g(ξ,θ)



Backproject g(ξ,θ).
Add to image μ(x,y)



μ(x,y)



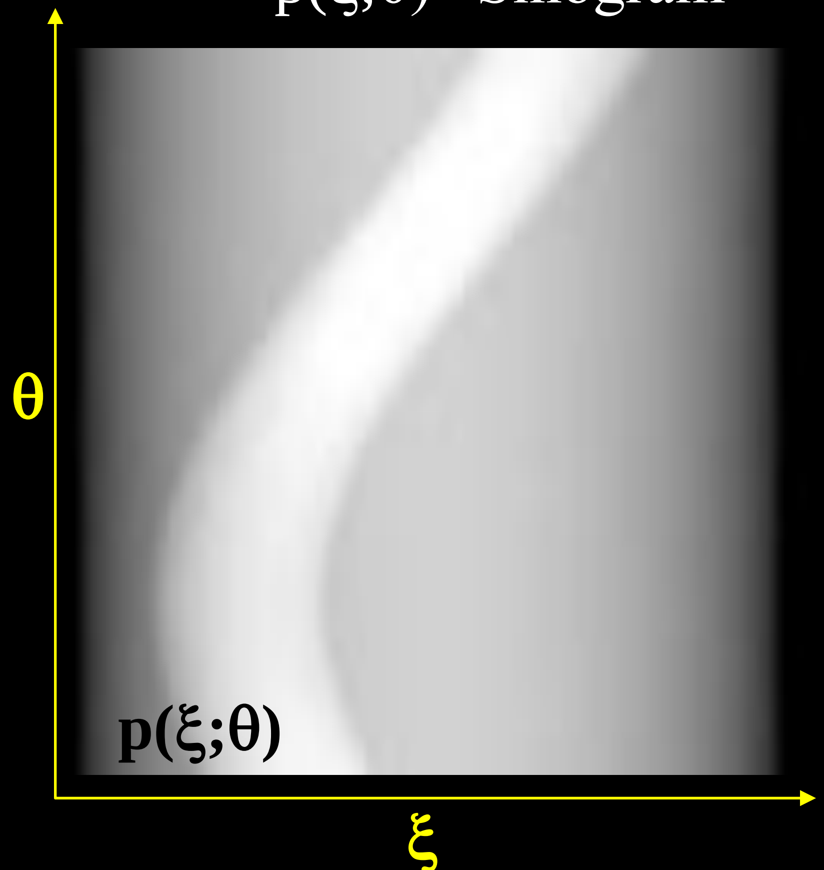
The Sinogram $p(\xi,\theta)$

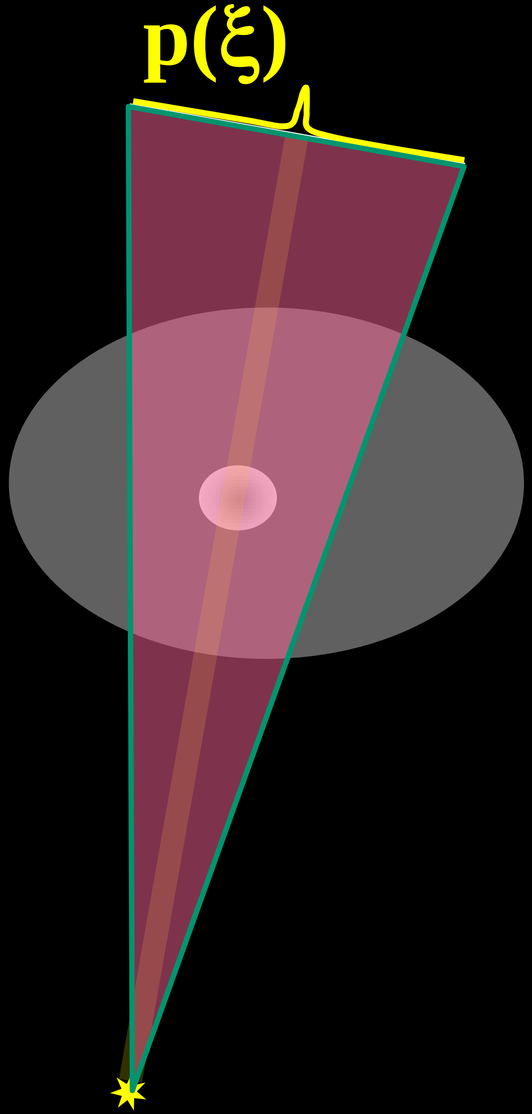
<u>The Sinogram:</u>

Line integral projection $p(\xi)$

… measured at each angle θ

→ $p(\xi;\theta)$ "Sinogram"





Filtered Backprojection

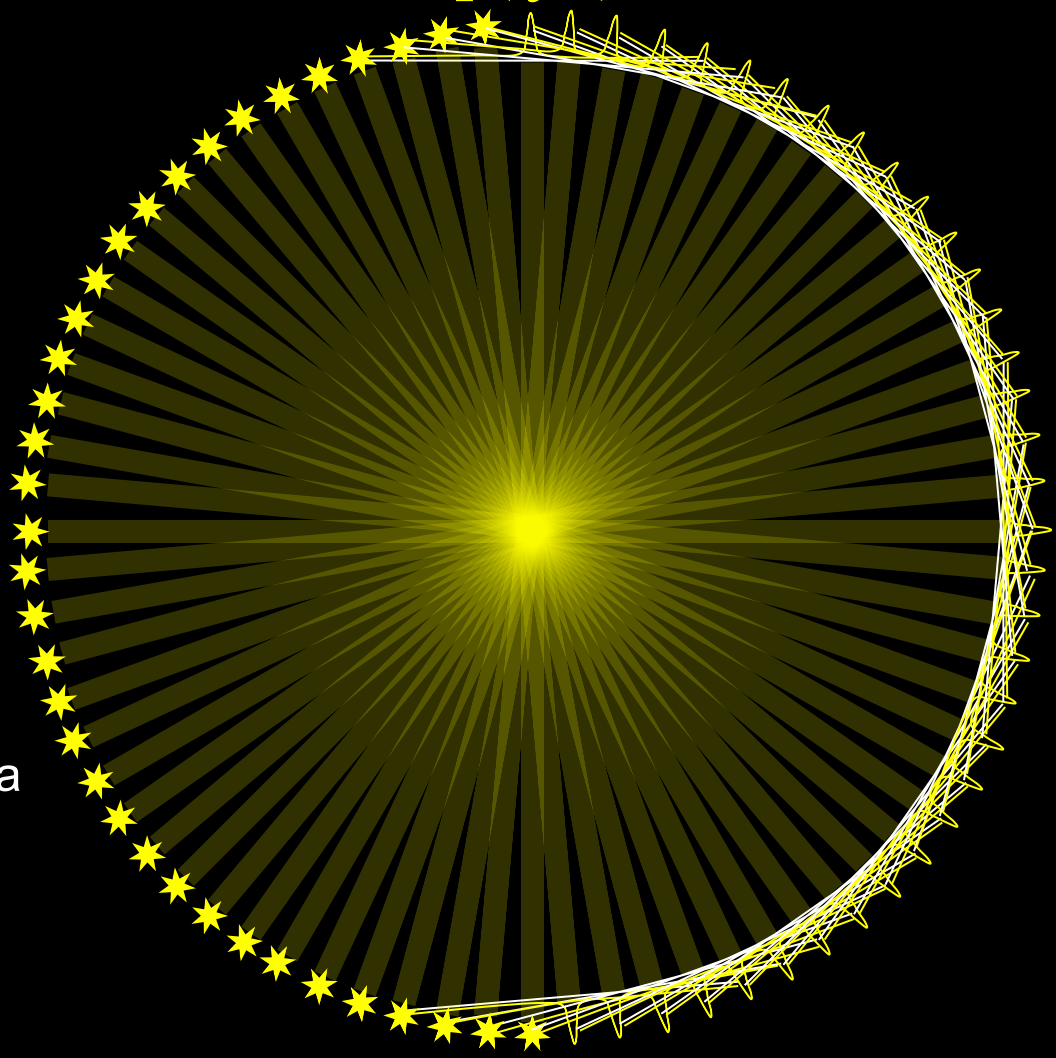
Simple Backprojection:

Trace projection data p(ξ;θ) through the reconstruction matrix from the detector (ξ) to the source

Simple backprojection yields radial density (1/r)

Therefore, a point-object is reconstructed as (1/r)

Solution: “Filter” the projection data by a “ramp filter” |r|



Filtered Backprojection

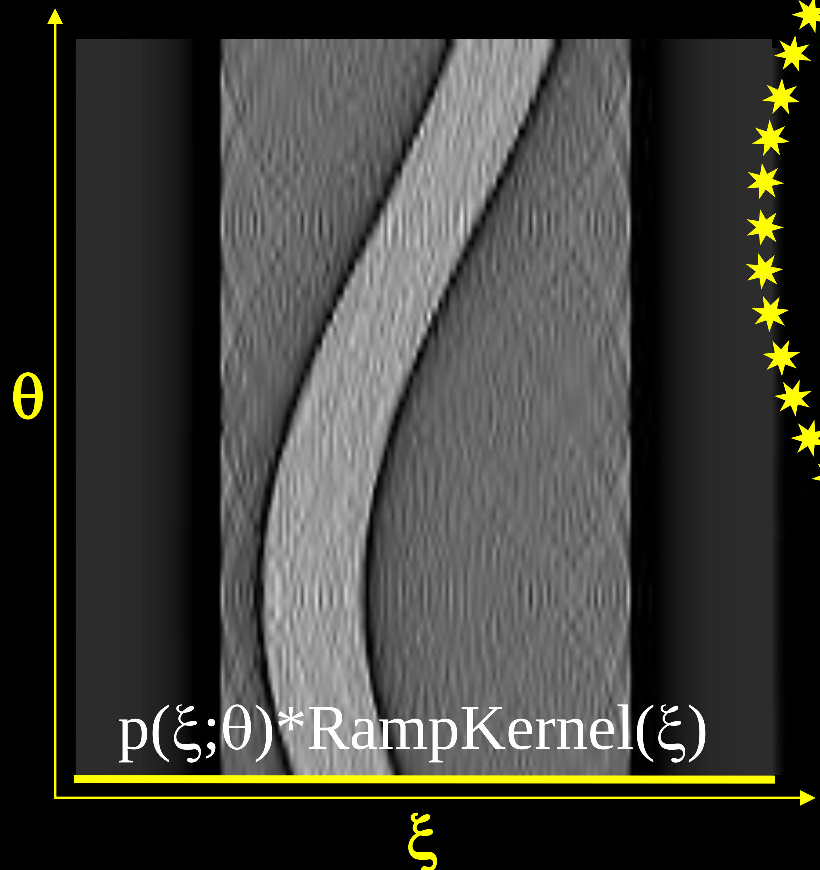
The Filtered Sinogram:

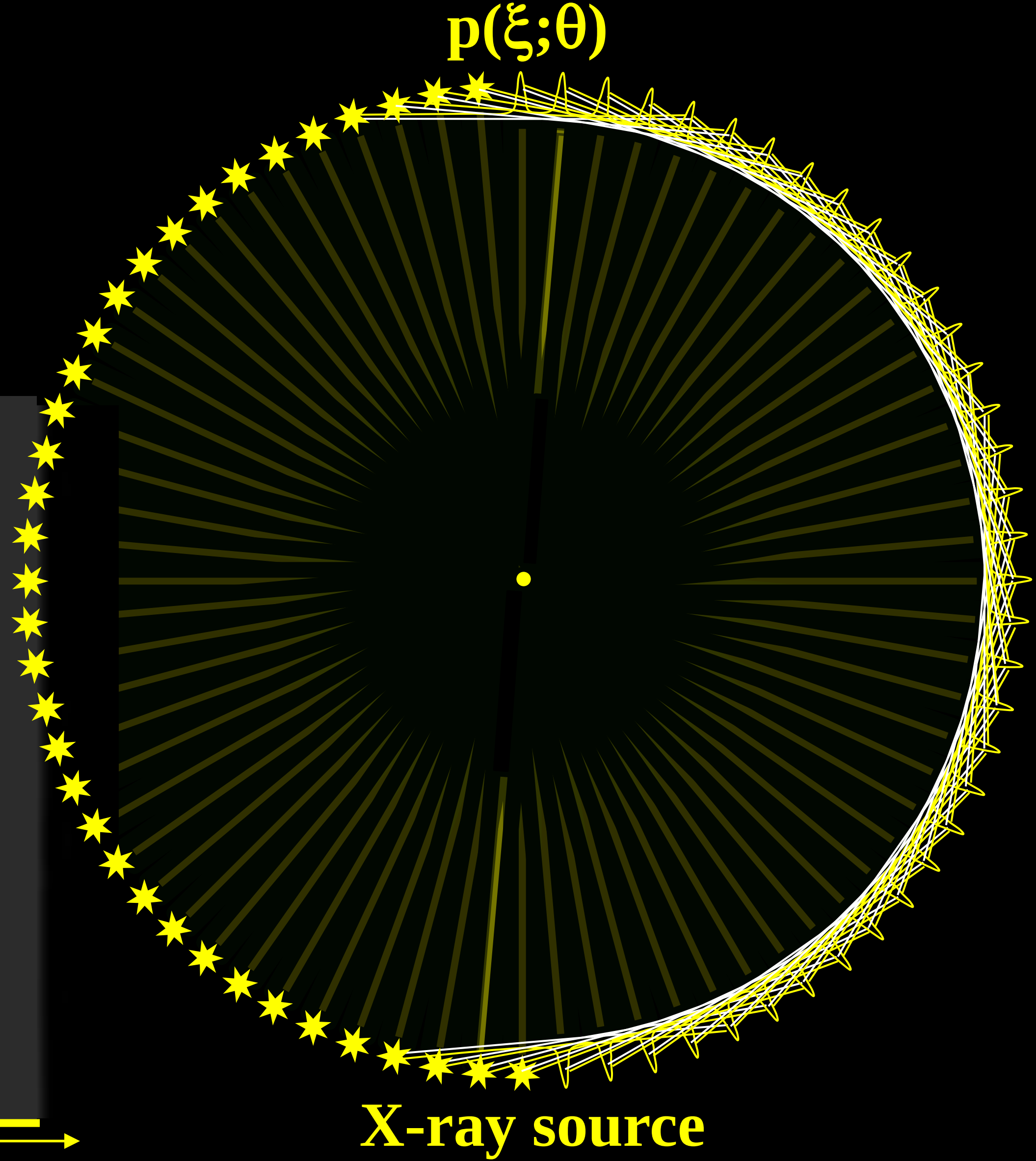
Convolve with RampKernel(ξ)

p(ξ)*RampKernel(ξ)

Equivalent to Fourier product

$M(f_x)|f_x|$

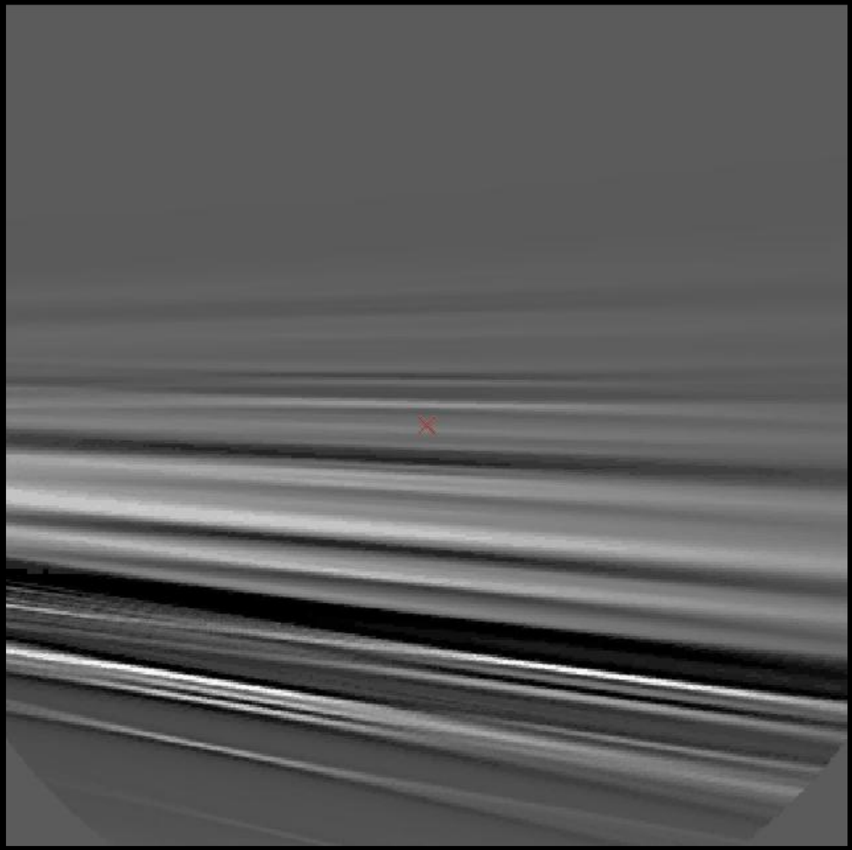




Filtered Backprojection

Projection Data

$p(\xi,\theta)$

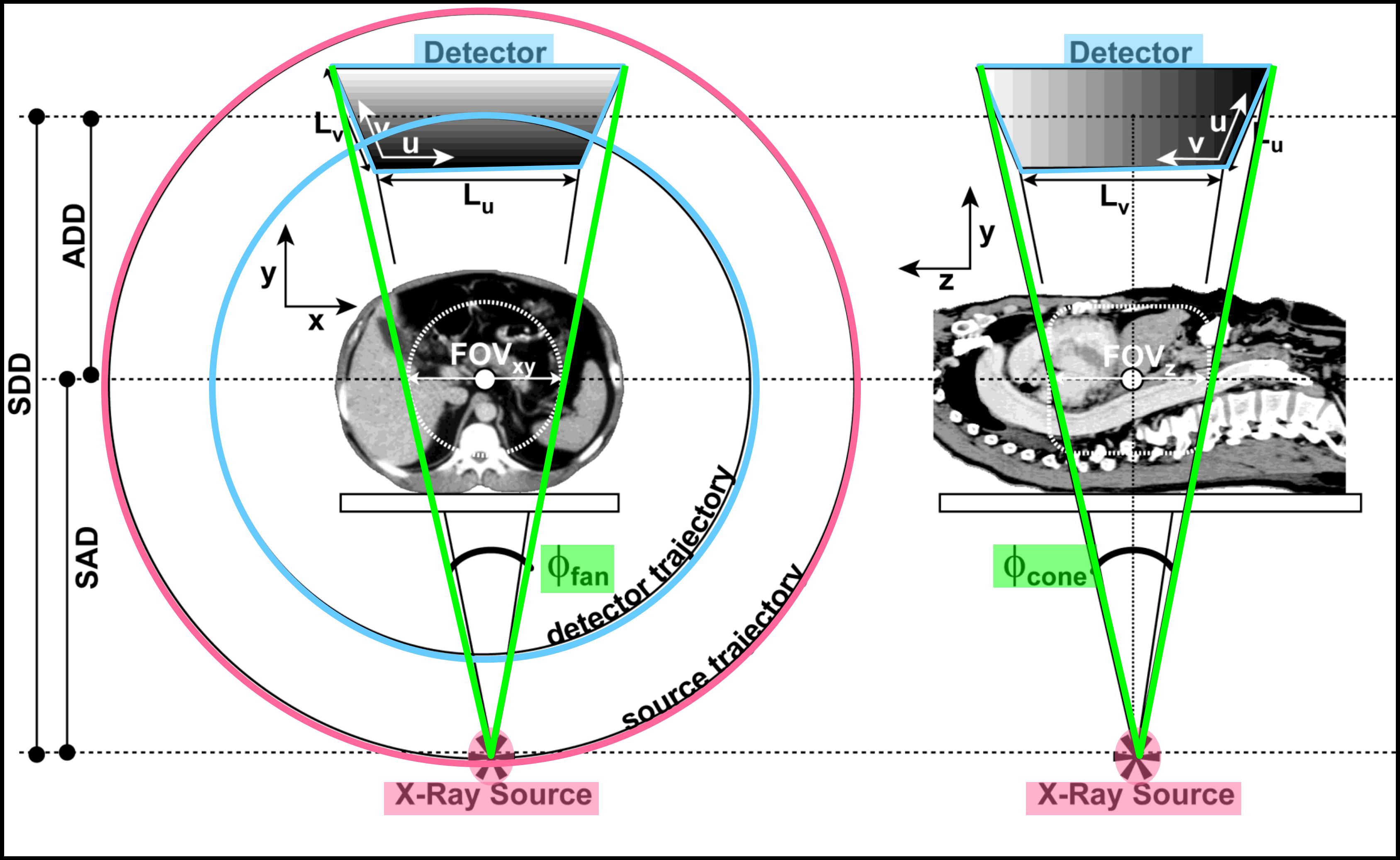


3D Reconstruction (Axial Slice)

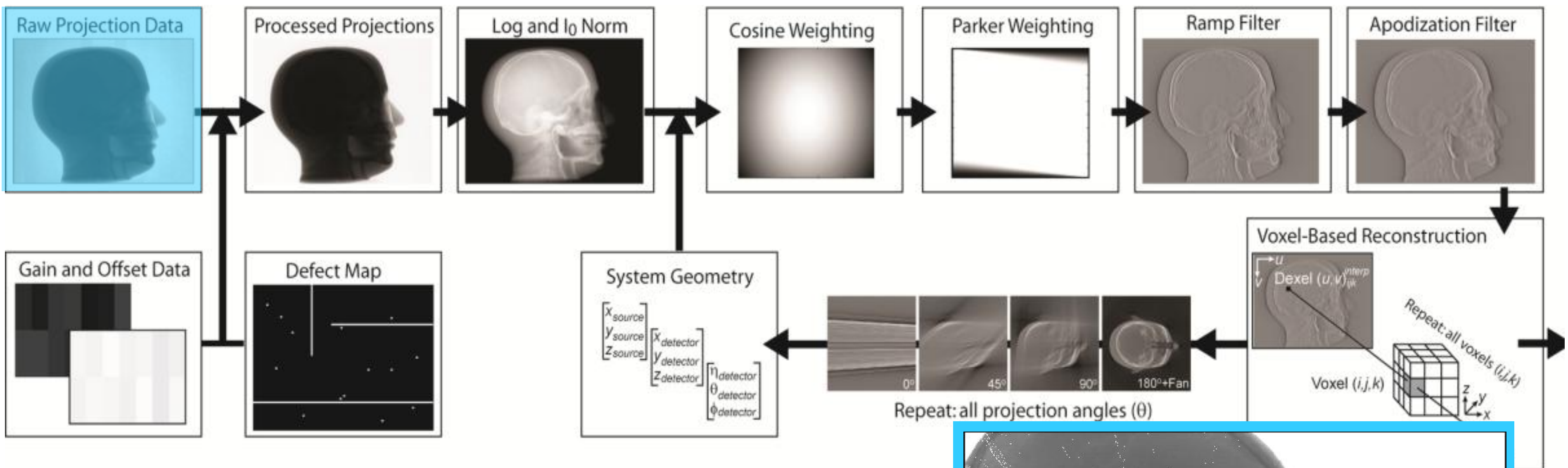
$\mu(x,y,z)$

Cone-Beam Geometry

Definitions and Coordinate Systems

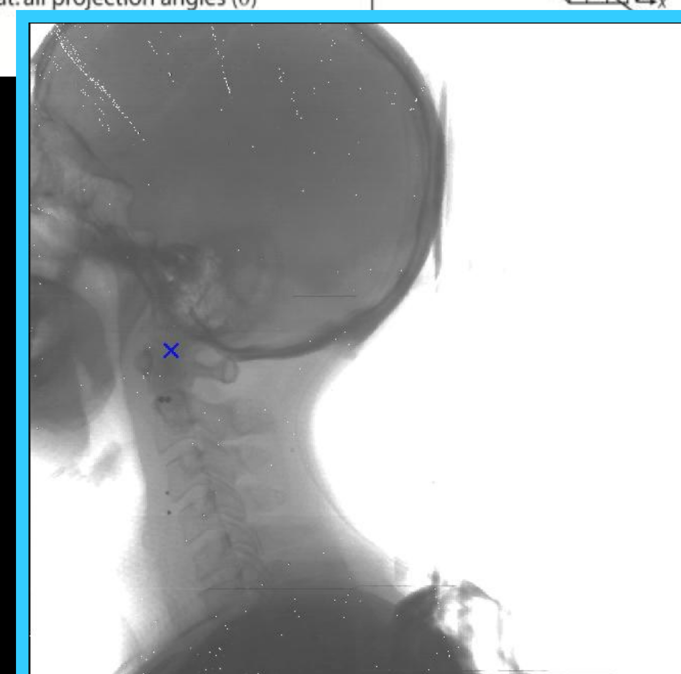


3D Filtered Backprojection

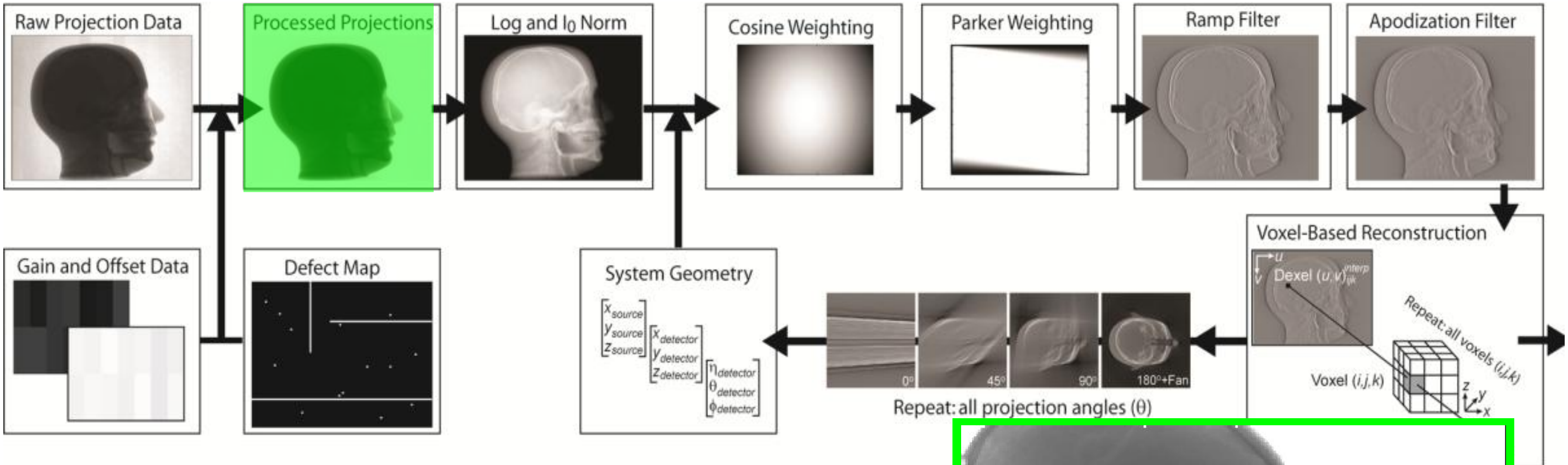


Raw Projection Data

$$p(u,v;\theta) = p_0 e^{-\int_0^{SDD} \mu(x,y,z)dy}$$

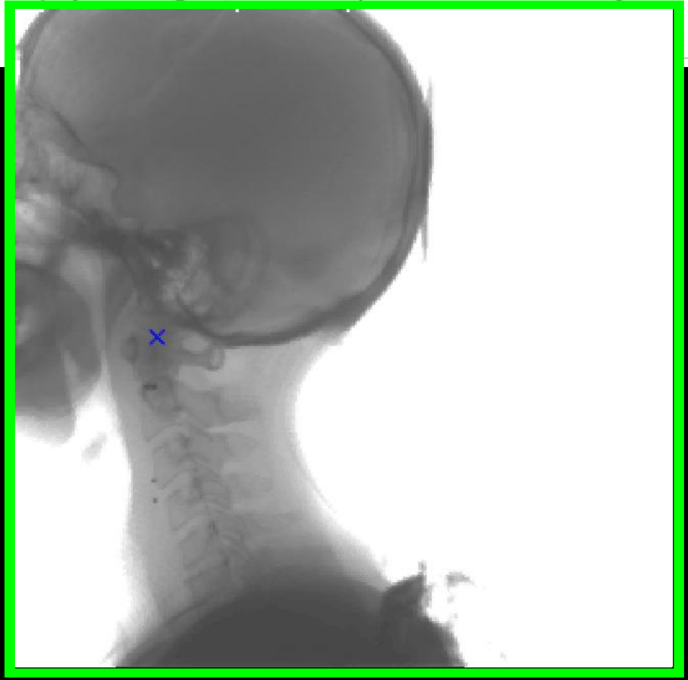


3D Filtered Backprojection

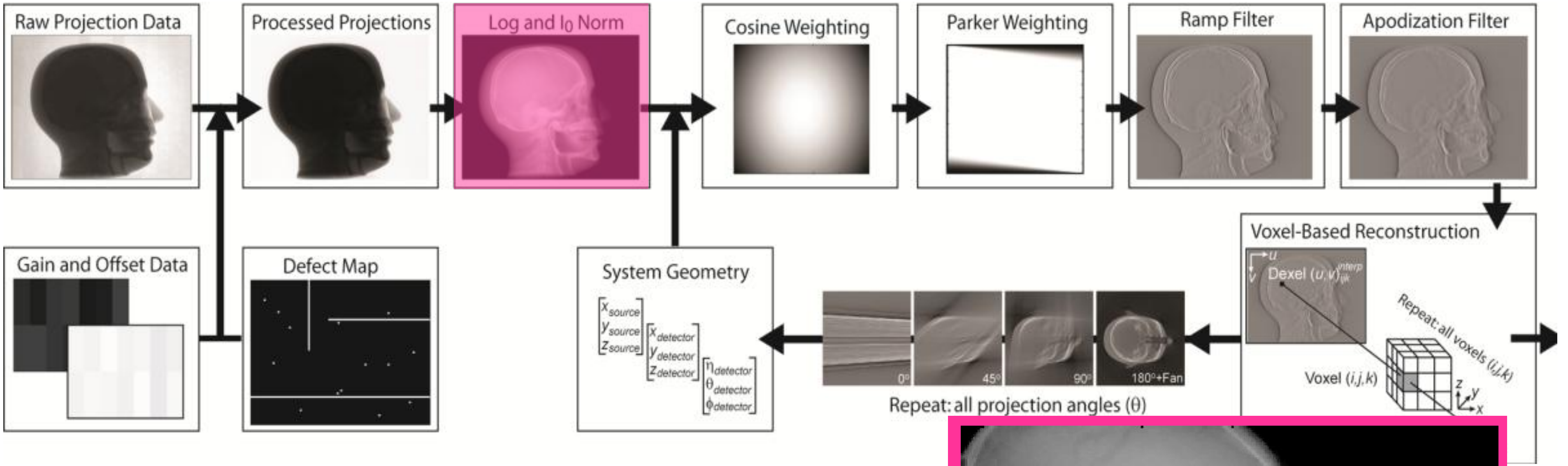


Offset-Gain (and Defect) Correction

$$I_{proc}(u,v) = K \frac{I_{raw}(u,v) - \overline{I_{offset}(u,v)}}{\overline{I_{gain}(u,v)} - \overline{I_{offset}(u,v)}}$$



3D Filtered Backprojection

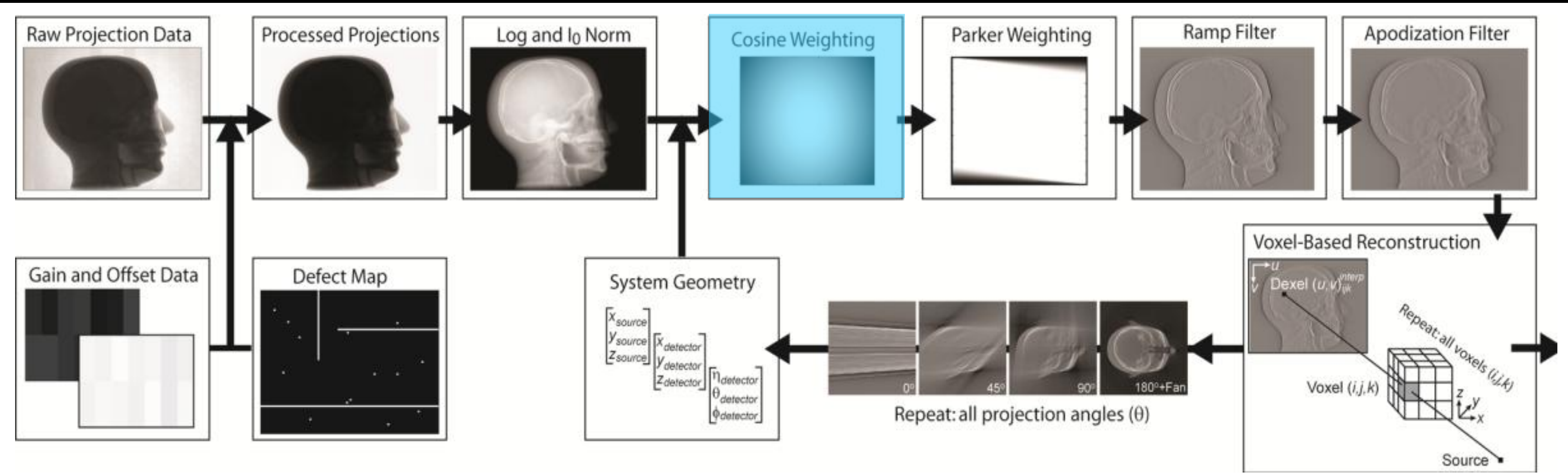


Log Normalization

$$p_1(u,v;\theta) = \ln\left(\frac{p_0}{p(u,v;\theta)}\right)$$

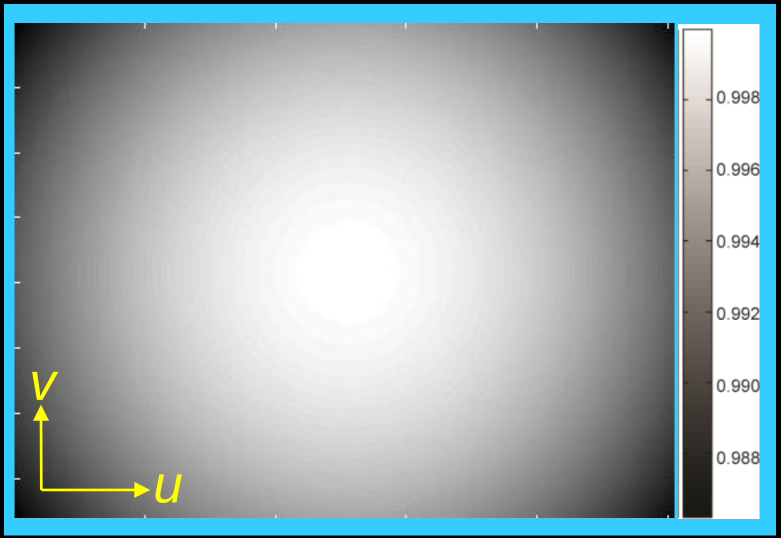
$$= \int_0^{SDD} \mu(x,y,z)\,dy$$

3D Filtered Backprojection

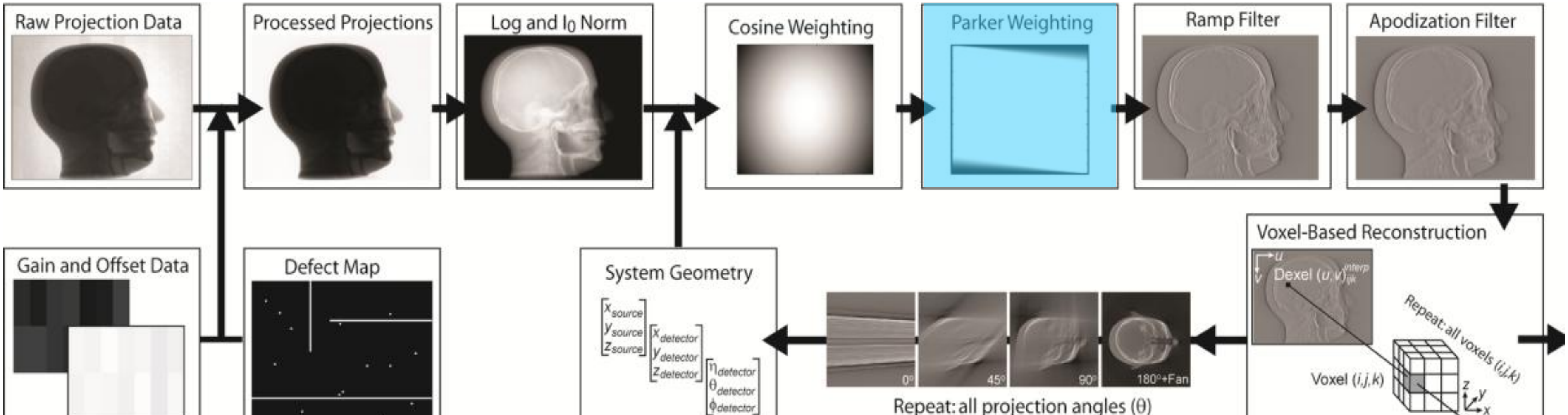


Cosine Weighting (Feldkamp Weights)

$$p_2(u,v;\theta) = p_1(u,v;\theta)\left[\frac{SDD}{\sqrt{SDD^2+u^2+v^2}}\right]$$



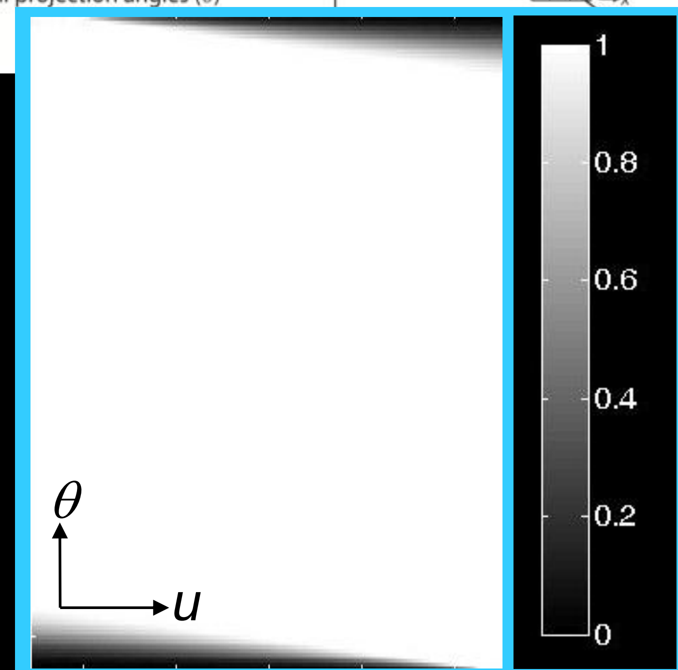
3D Filtered Backprojection



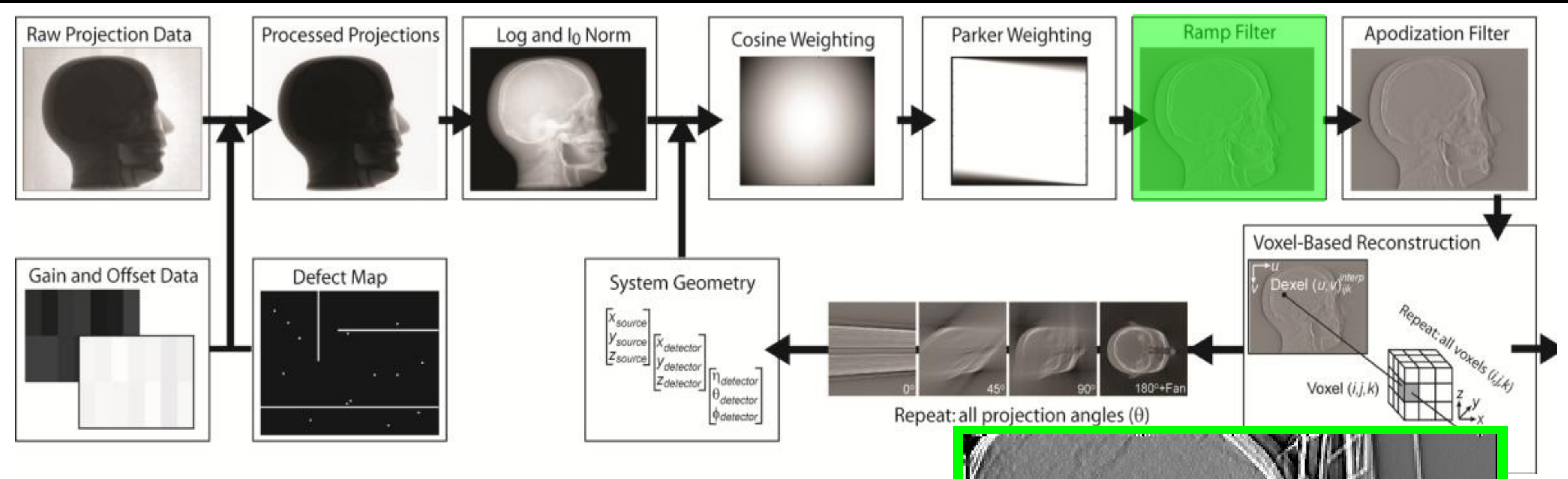
Data Redundancy (Parker) Weights

$$p_3(u,v;\theta) = p_2(u,v;\theta)\,w_3(u;\theta)$$

$$w_3(u;\theta) = \begin{cases} \sin^2\left(\dfrac{\pi\theta}{4\left(\frac{1}{2}\phi_{fan} - \phi(u-u_0)\right)}\right) & \text{for } 0 \le \theta \le \phi_{fan} - 2\phi(u-u_0) \\ 1 & \text{for } \phi_{fan} - 2\phi(u-u_0) \le \theta \le \pi - 2\phi(u-u_0) \\ \sin^2\left(\dfrac{\pi(\pi + \phi_{fan} - \theta)}{4\left(\frac{1}{2}\phi_{fan} + \phi(u-u_0)\right)}\right) & \text{for } \pi - 2\phi(u-u_0) \le \theta \le \pi + \phi_{fan} \end{cases}$$



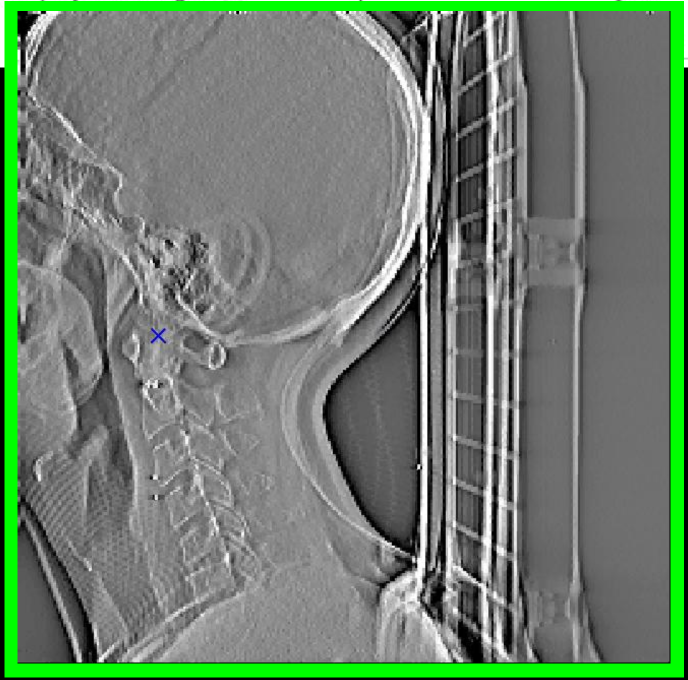
3D Filtered Backprojection



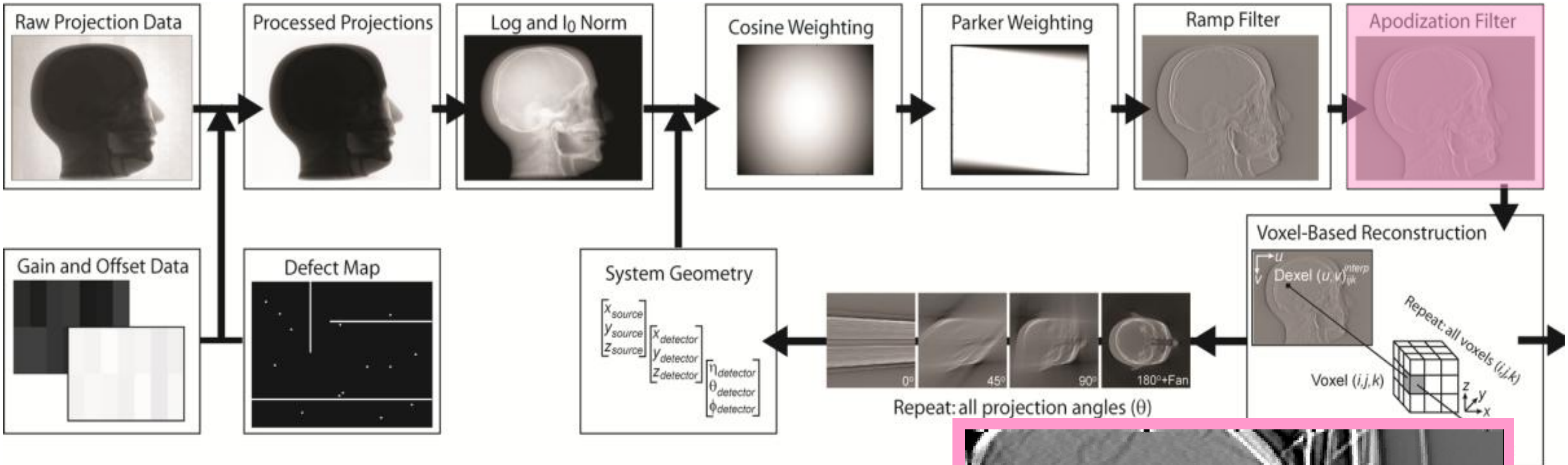
Ramp Filter

$$p_4(u,v;\theta) = FT^{-1}\left[FT\left[p_3(u,v;\theta)\right]\left|\rho\right|\right]$$

$$= p_3(u,v;\theta) * \left(-\frac{1}{2\pi^2 u^2}\right)$$



3D Filtered Backprojection

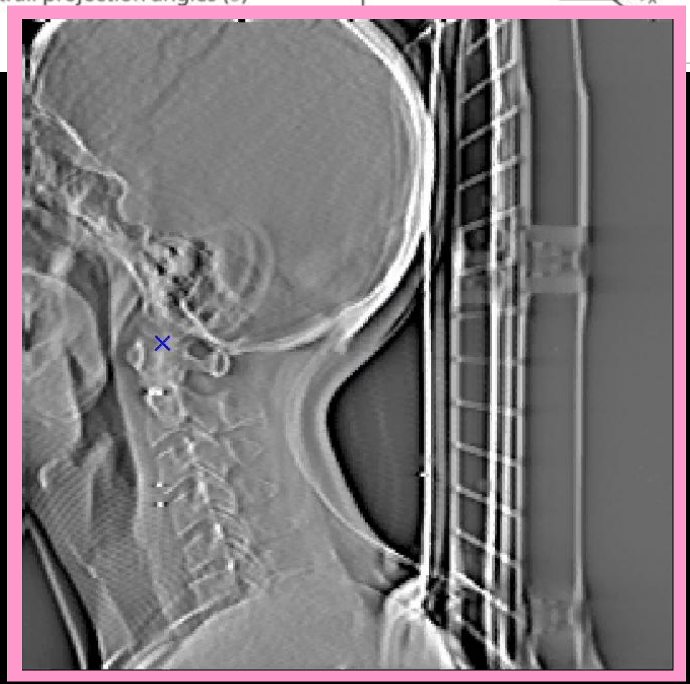


Smoothing / Apodization Filter

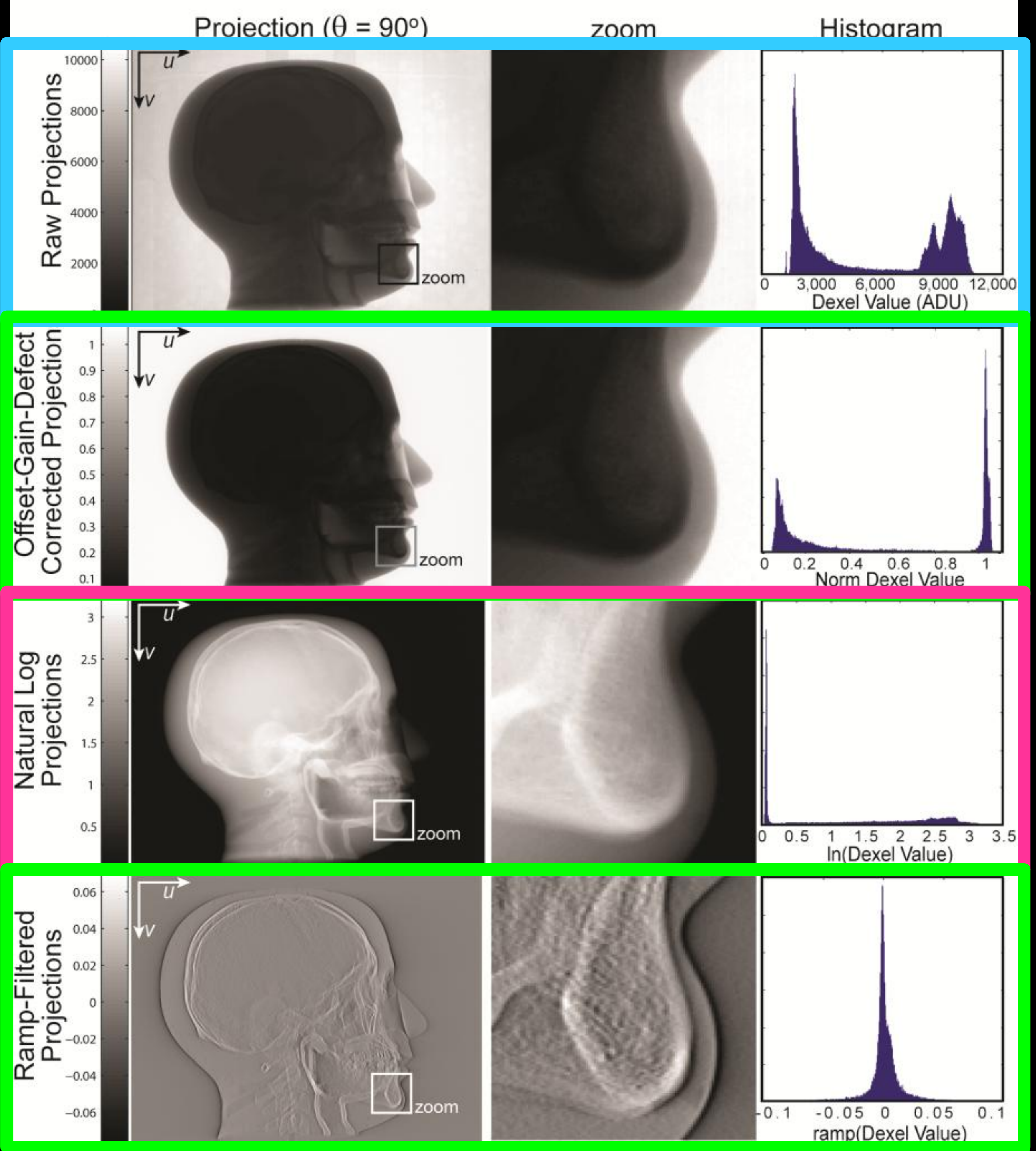
$$p_5(u,v;\theta) = FT^{-1}\left[FT\left[p_4(u,v;\theta)\right]T_{win}(f)\right]$$

$$= p_4(u,v;\theta) * t_{win}(u,v)$$

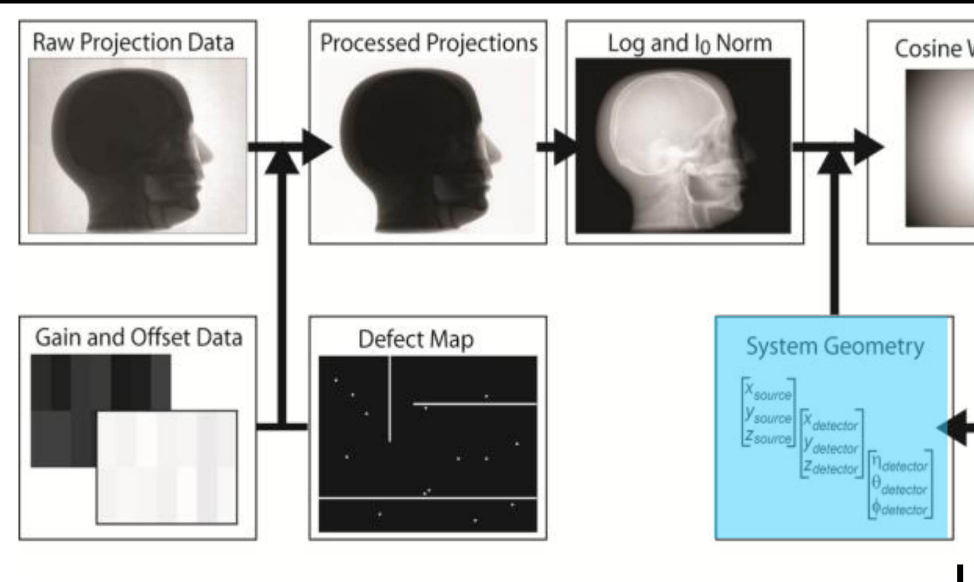
$$T_{win}(f) = h_{win} + (1 - h_{win})\cos(2\pi f\Delta)$$

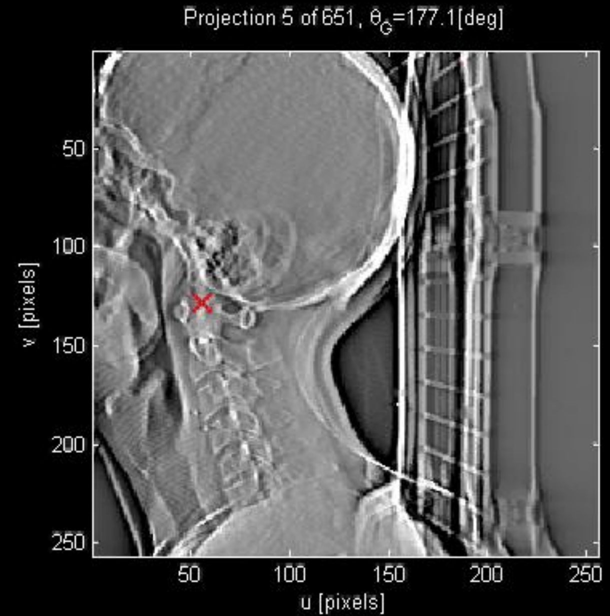


Evolution of the Projection Data

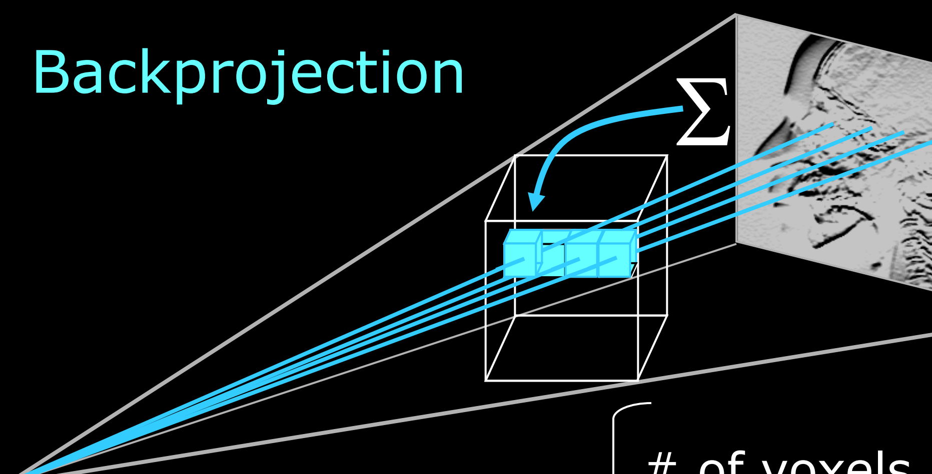


3D Filtered Ba

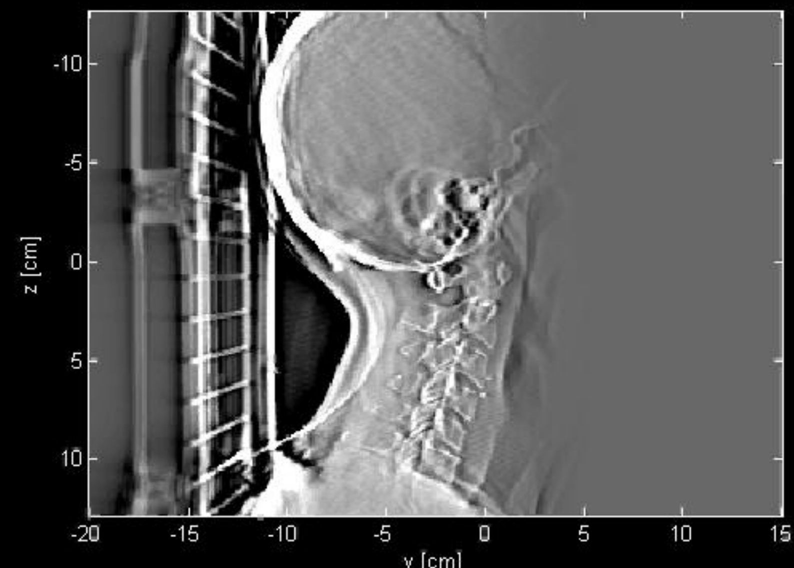




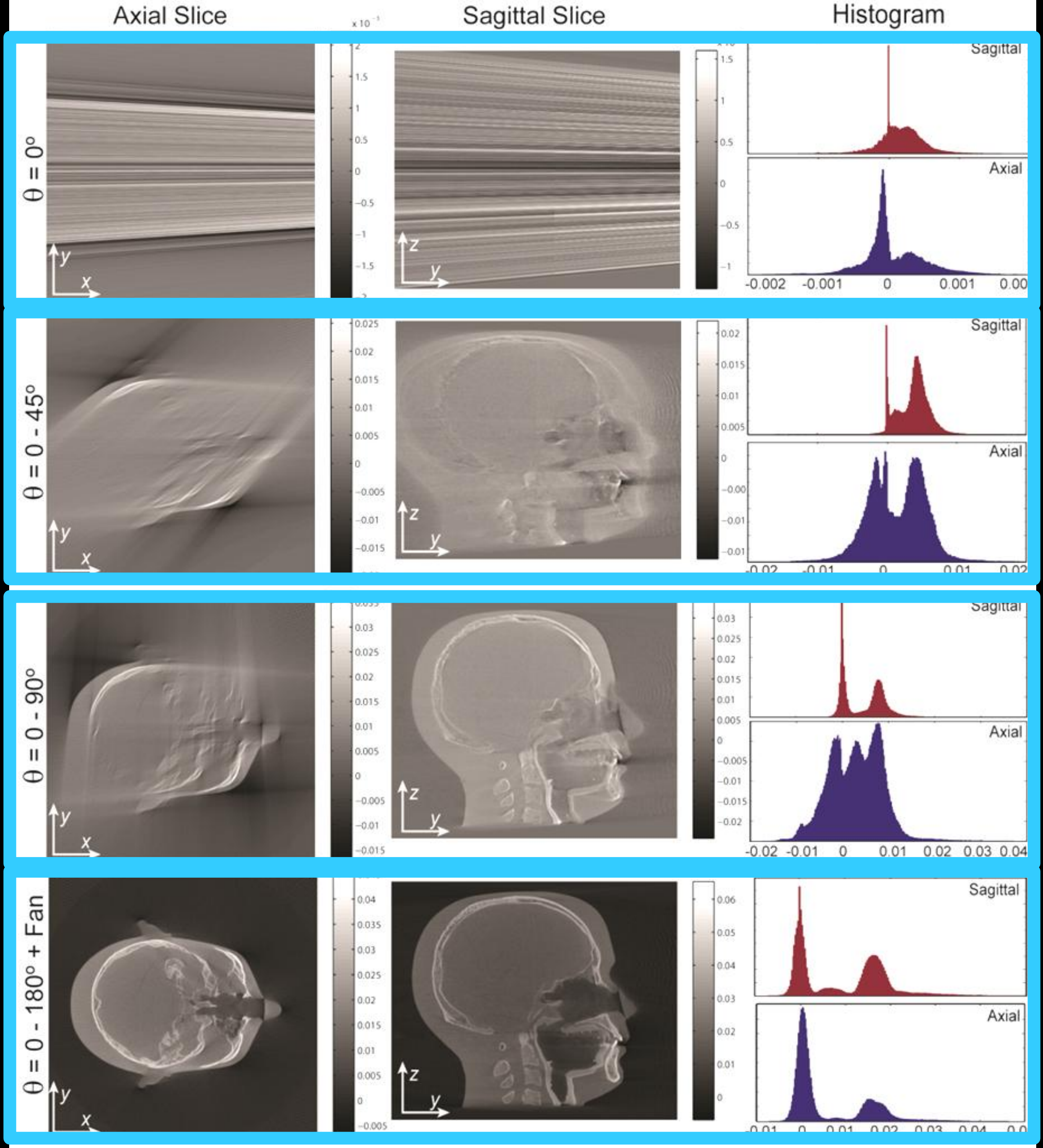
Backprojection



Repeat × { # of voxels
# of projecti



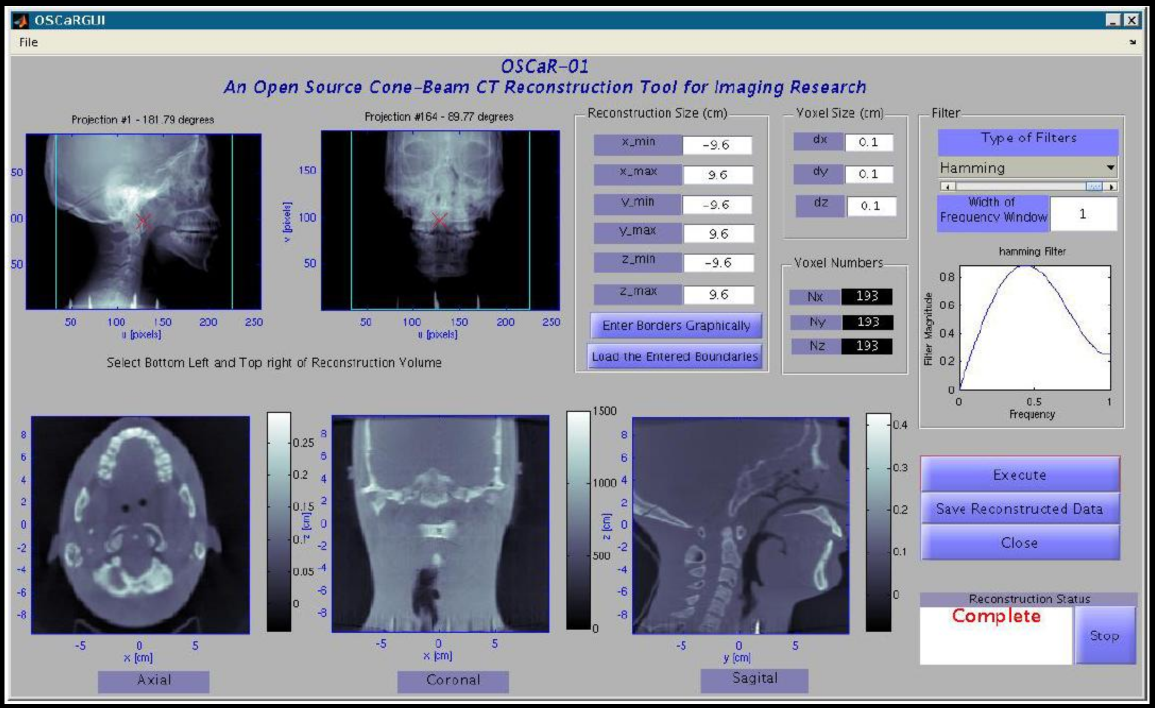
Evolution of the 3D Recon



Open-Source Resources

OSCaR

Open-Source Cone-Beam Reconstructor



MATLAB
- Source code (m)
- Function call (m)
- Executable UI (exe)

Intended user
- Education
- Research

Input data types
- jpg, raw, png, etc.

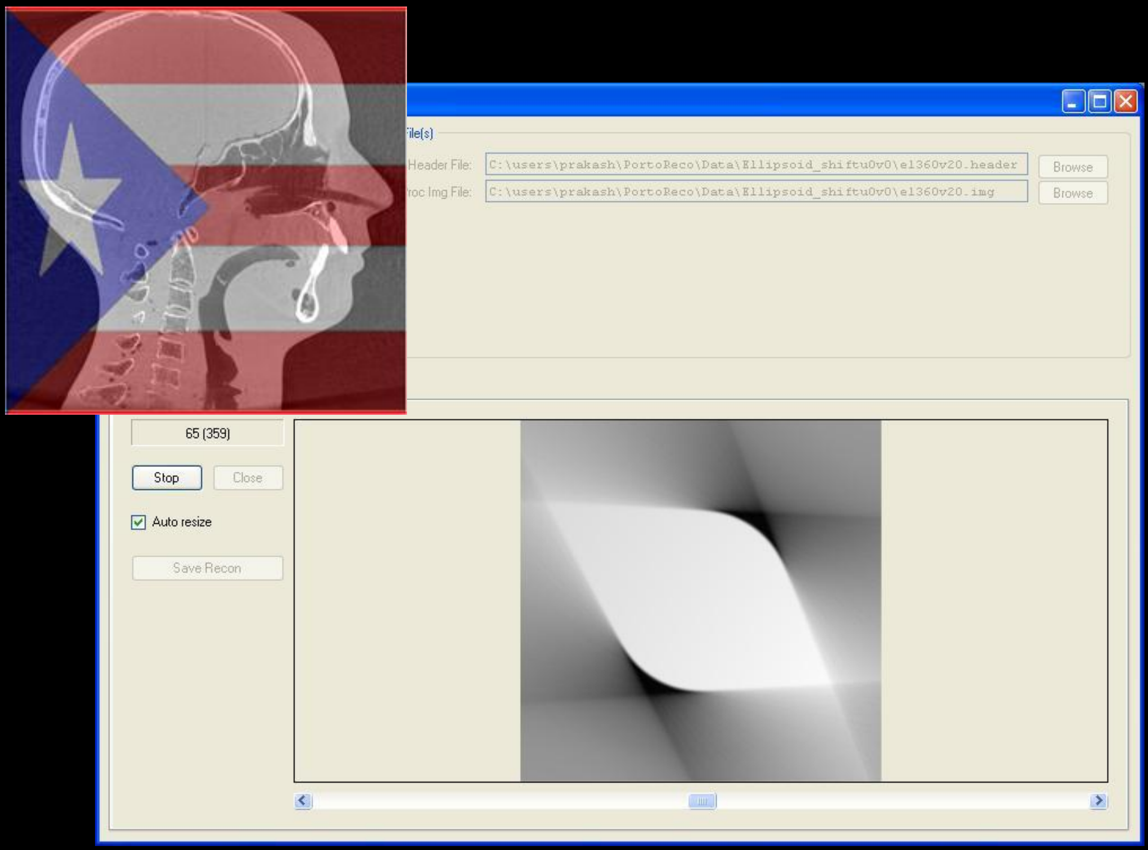
Flexible, transparent
- Filters, voxel size,
- Reconstruction filters

www.jhu.edu/istar/downloads

Open-Source Resources

PortoRECO

Portable Reconstruction



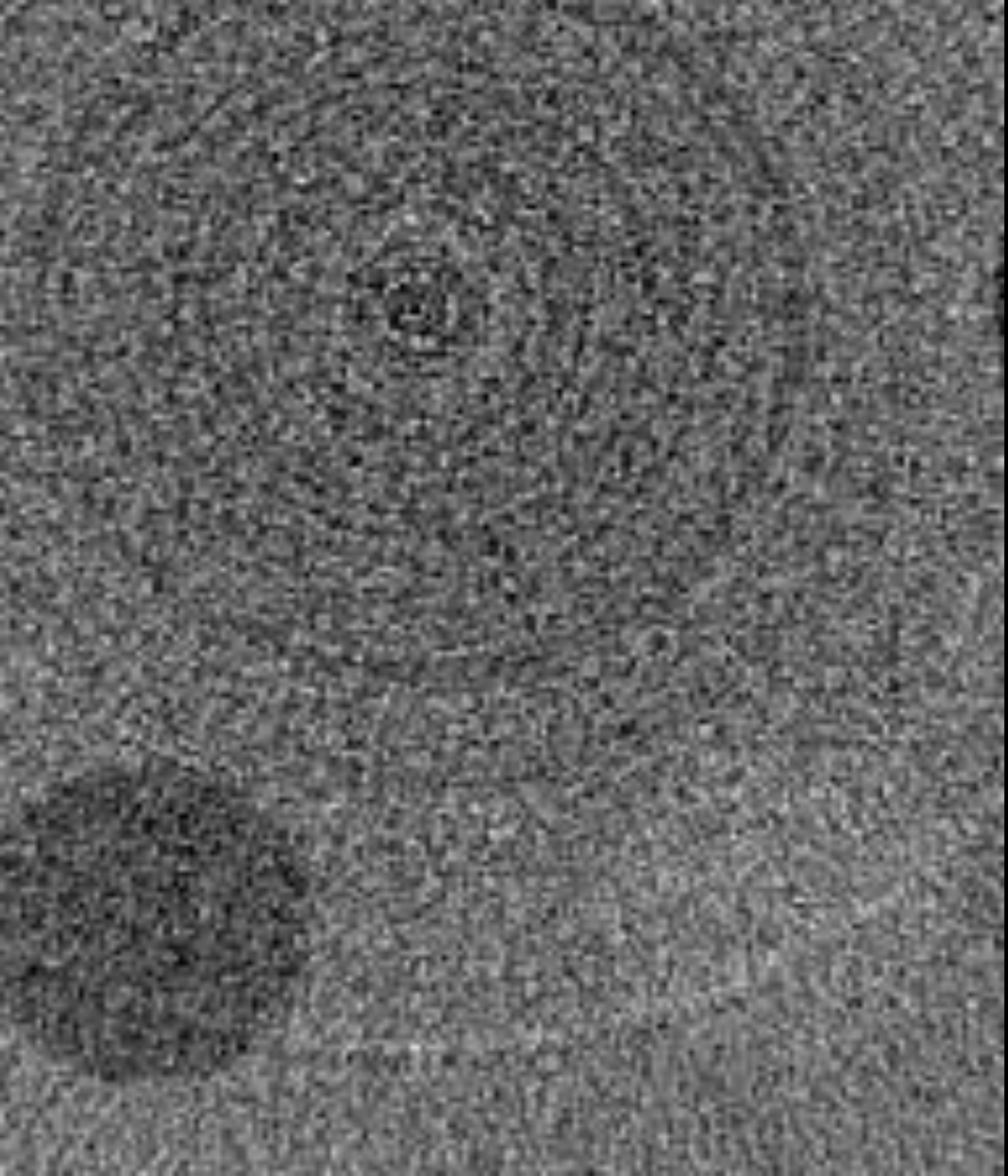
C# / C++
- Source code (C#)
- Executable UI (exe)

Intended user
- Education
- Research

Input data types
- jpg, raw, png, etc.

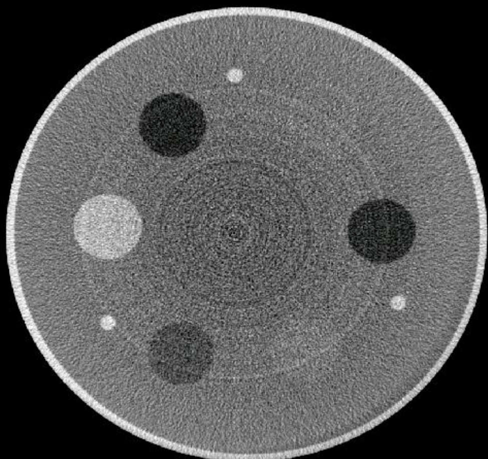
Flexible, transparent
- Filters, voxel size,
- Reconstruction filters

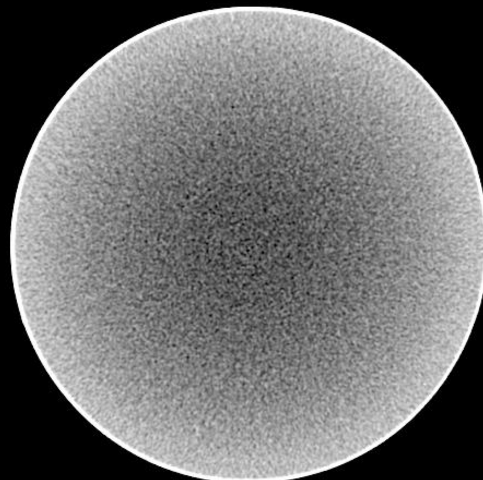
www.jhu.edu/istar/downloads

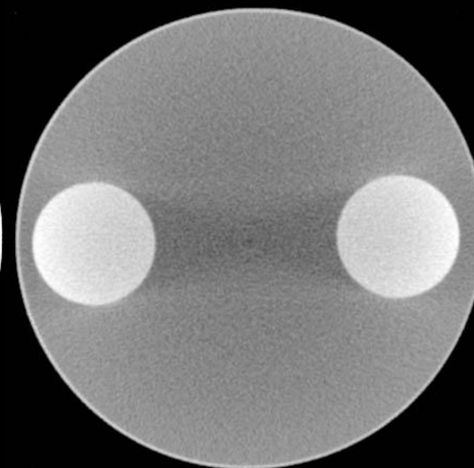


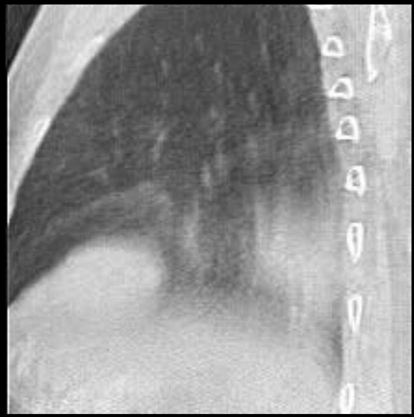
Artifacts

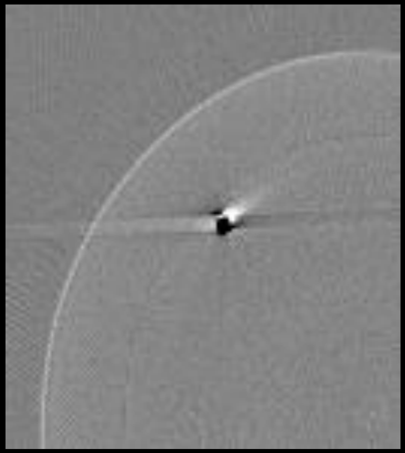
Artifacts

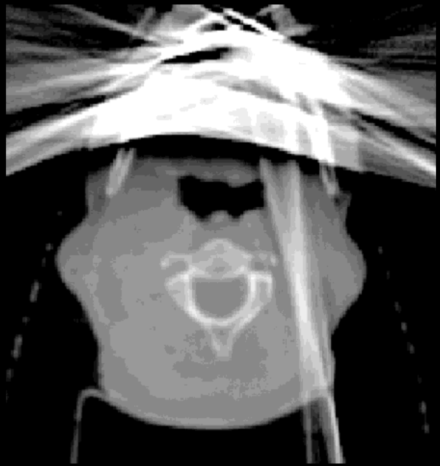

Rings


Shading

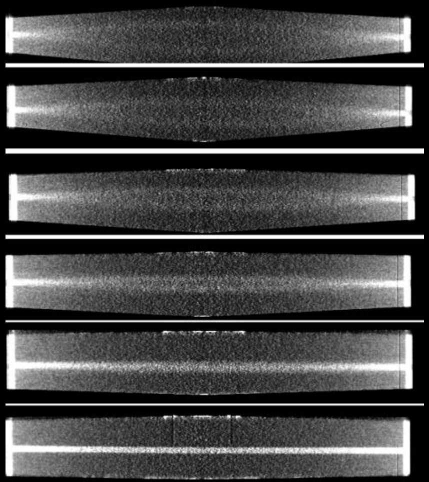

Streaks


Motion


Lag


Metal


Truncation


"Cone-Beam"

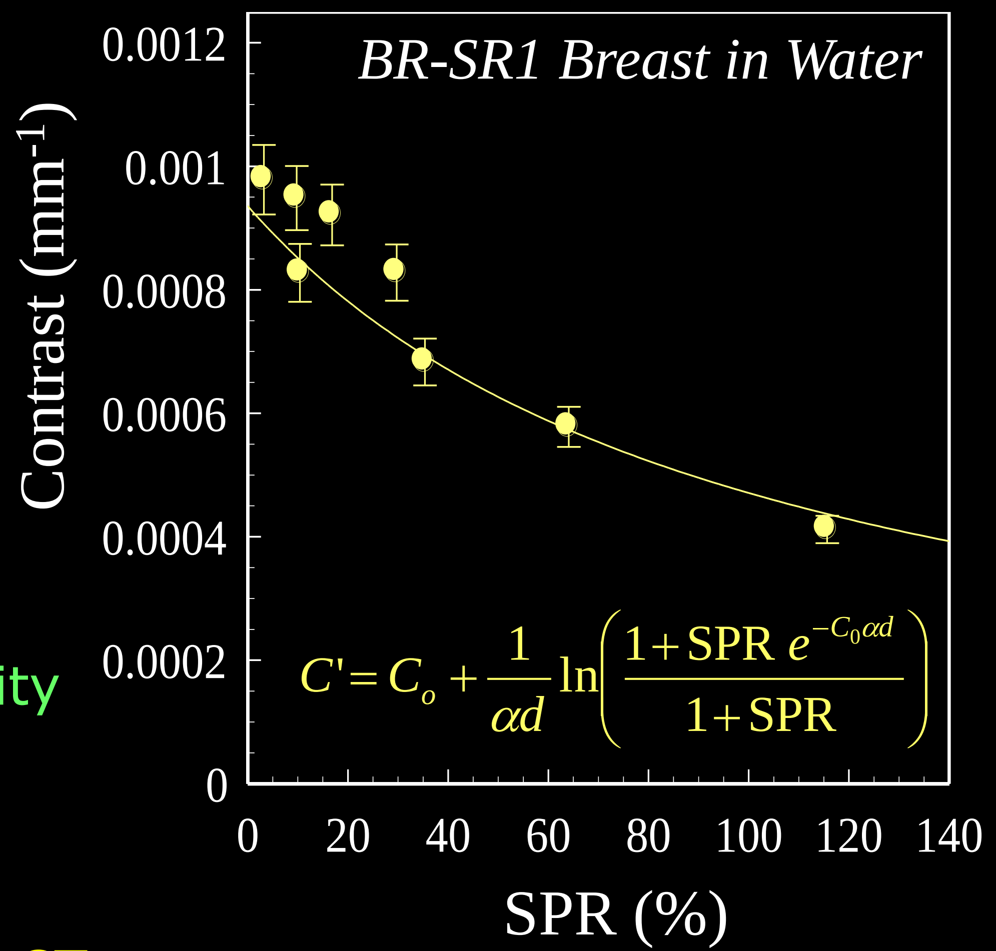
Artifacts: X-ray Scatter

1) Reduced contrast
   Reduction of ΔCT#

2) Image artifacts
   Cupping and streaks

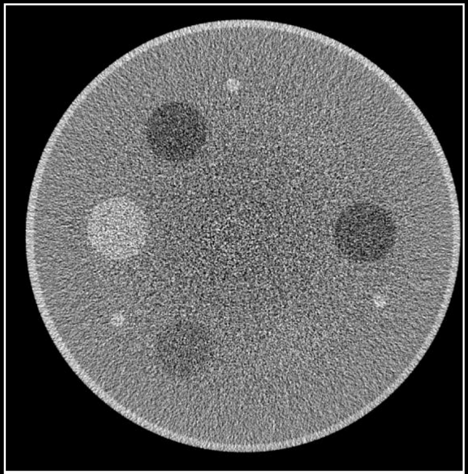
3) Increased image noise
   Reduced DQE

Reduced soft-tissue detectability

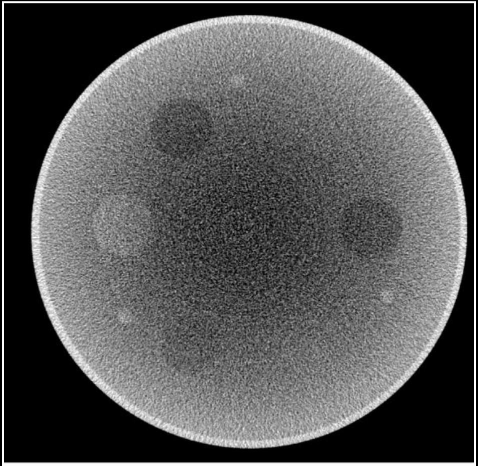


A big problem for cone-beam CT:
SPR is very large for large cone angles (i.e., large FOV)

Artifacts: X-ray Scatter



SPR = 7%



SPR = 120%

1) Reduced contrast
   Reduction of ΔCT#

2) Image artifacts
   Cupping and streaks

3) Increased image noise
   Reduced DQE

Reduced soft-tissue detectability

A big problem for cone-beam CT:
SPR is very large for large cone angles (i.e., large FOV)

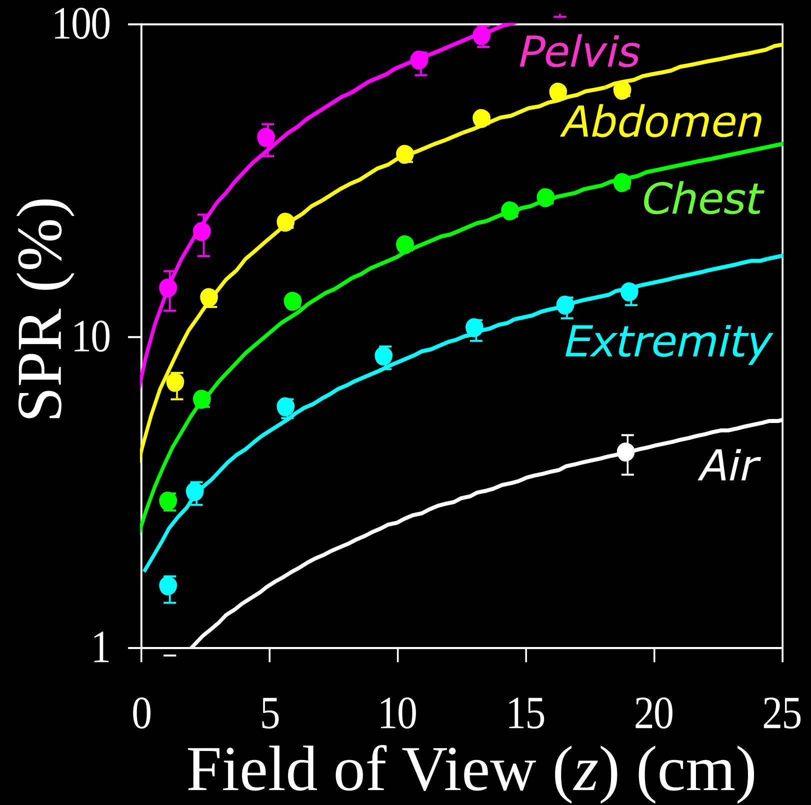
Artifacts: X-ray Scatter

1) Reduced contrast
   Reduction of ΔCT#

2) Image artifacts
   Cupping and streaks

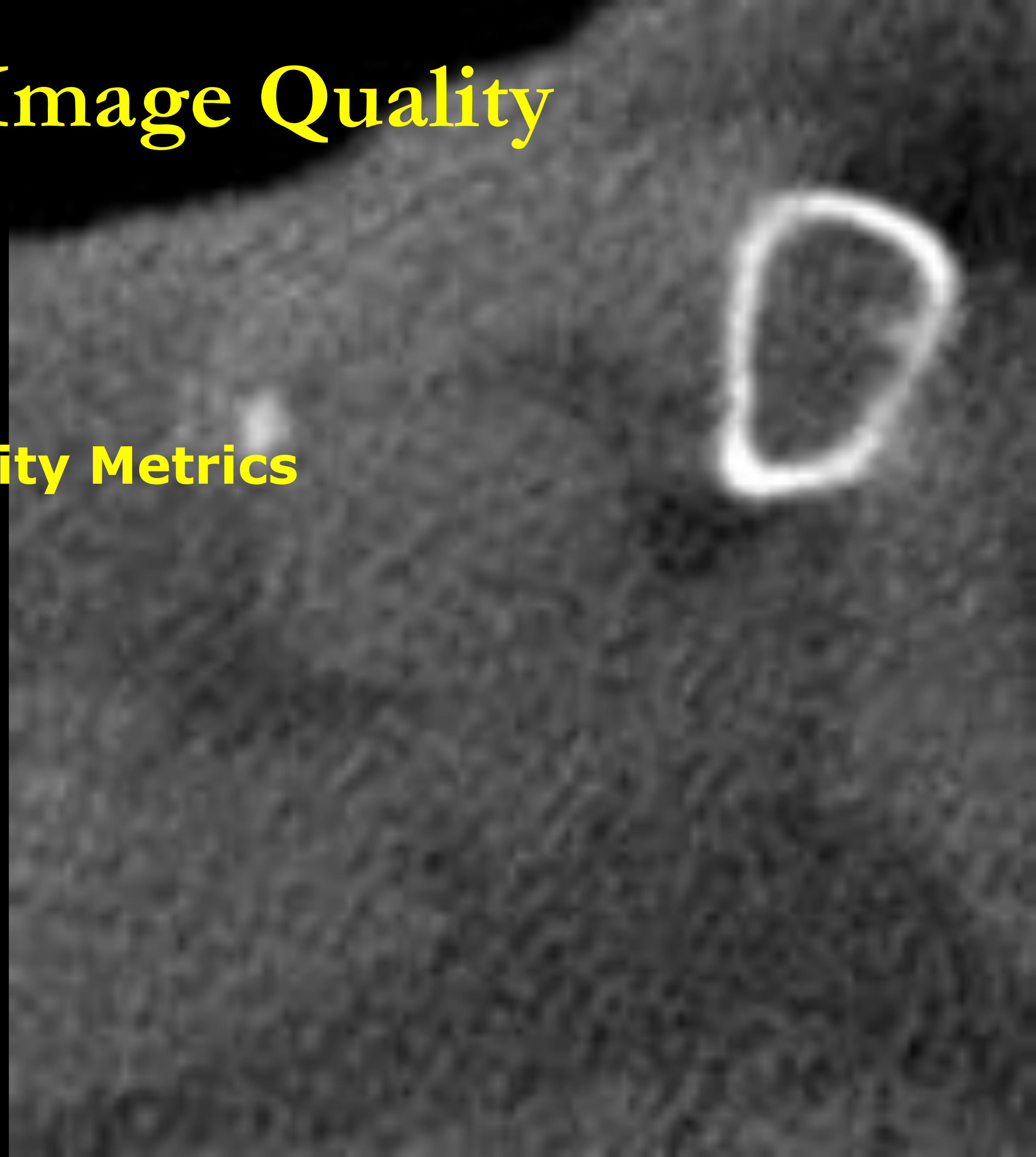
3) Increased image noise
   Reduced DQE

Reduced soft-tissue detectability



A big problem for cone-beam CT:
SPR is very large for large cone angles (i.e., large FOV)

Image Quality

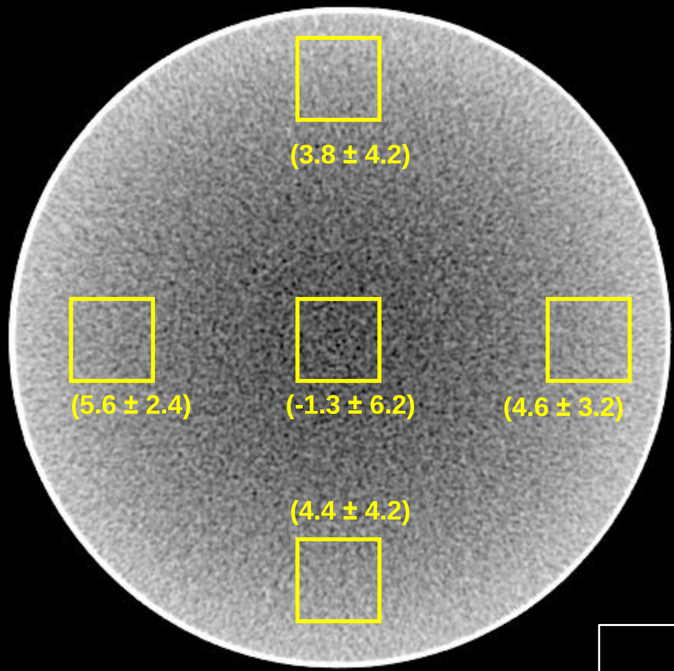


Key Image Quality Metrics

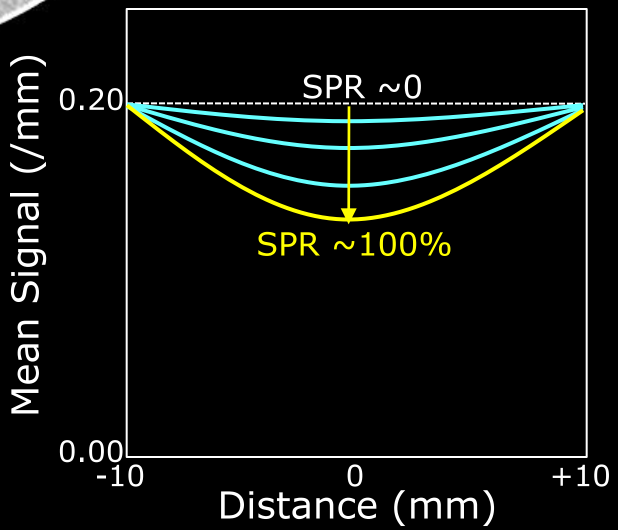
- Image Uniformity
- CT # Accuracy
- Spatial resolution
- Contrast resolution
- Noise (and NPS)
- CNR
- NEQ

Uniformity / Stationarity

- Signal Uniformity
  - Stationarity of the mean
    - Shading artifacts
    - Beam-hardening
    - Truncation
- Noise Uniformity
  - Stationarity of the noise
  - WSS of second-order statistic
    - Physical effects:
      - Quantum noise
      - Bowtie filter
    - Sampling effects:
      - Intrinsic to FBP
      - Number of projections
      - View aliasing



$$\Delta_{HU} = (4.6\text{-}1.3)\ \text{HU} = 3.3\ \text{HU}$$



Uniformity / Stationarity

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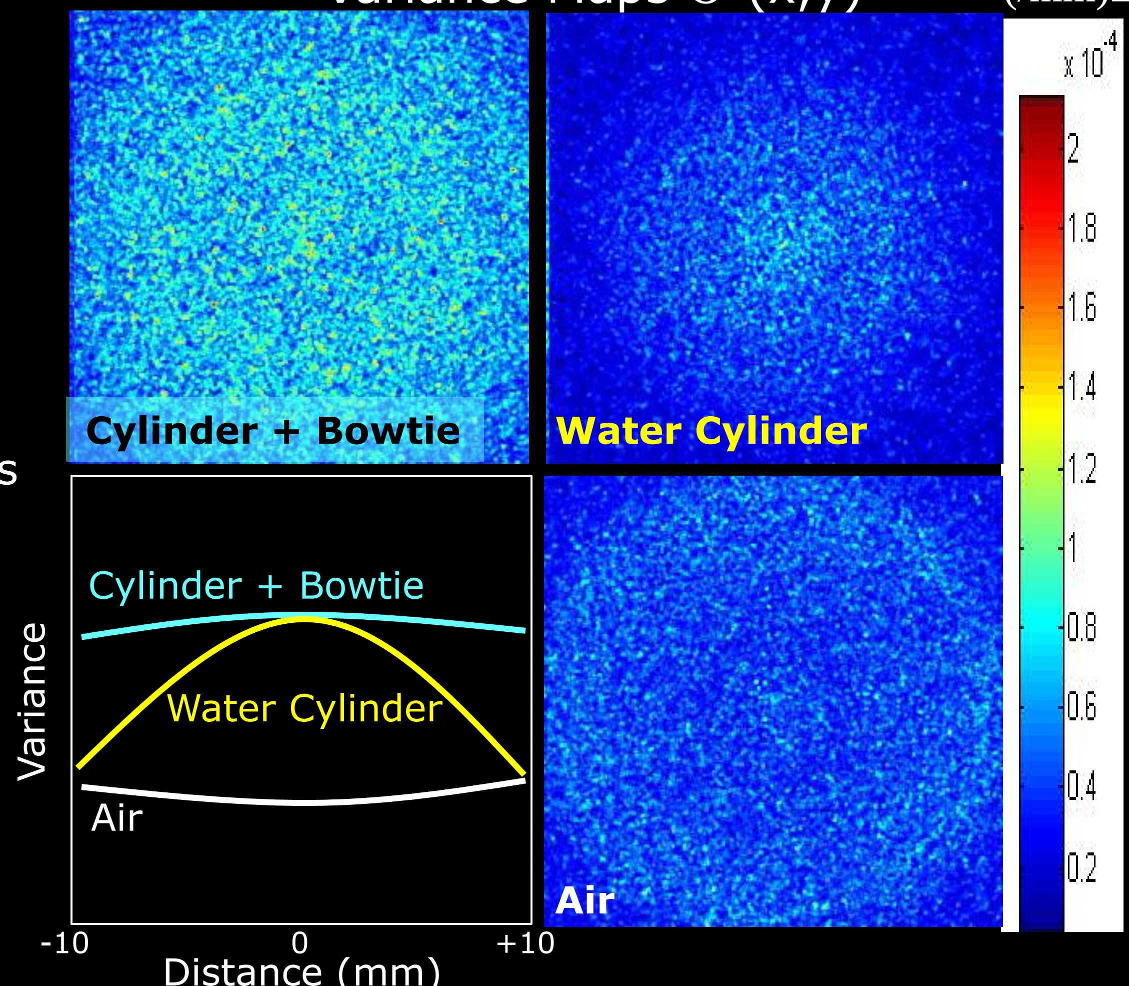
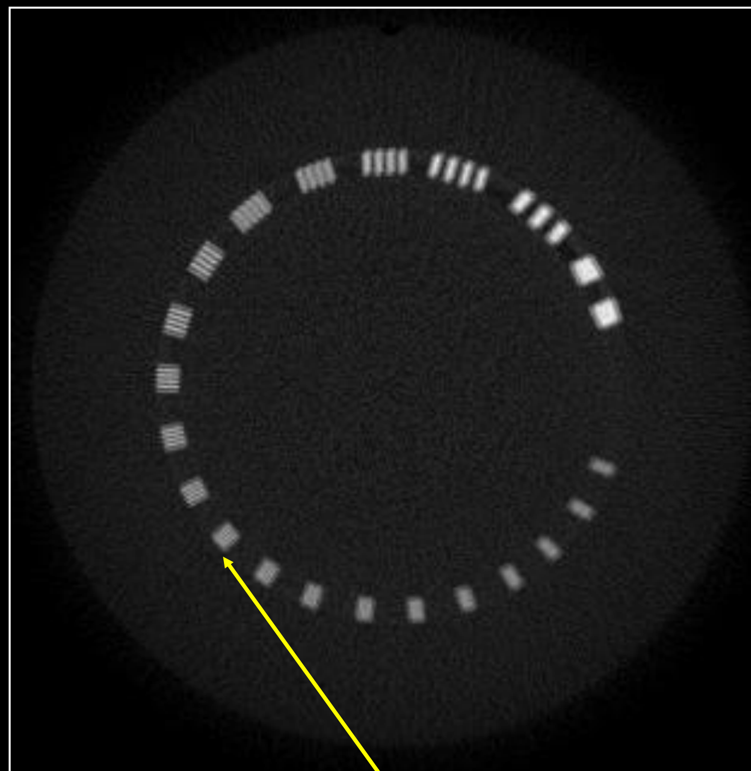
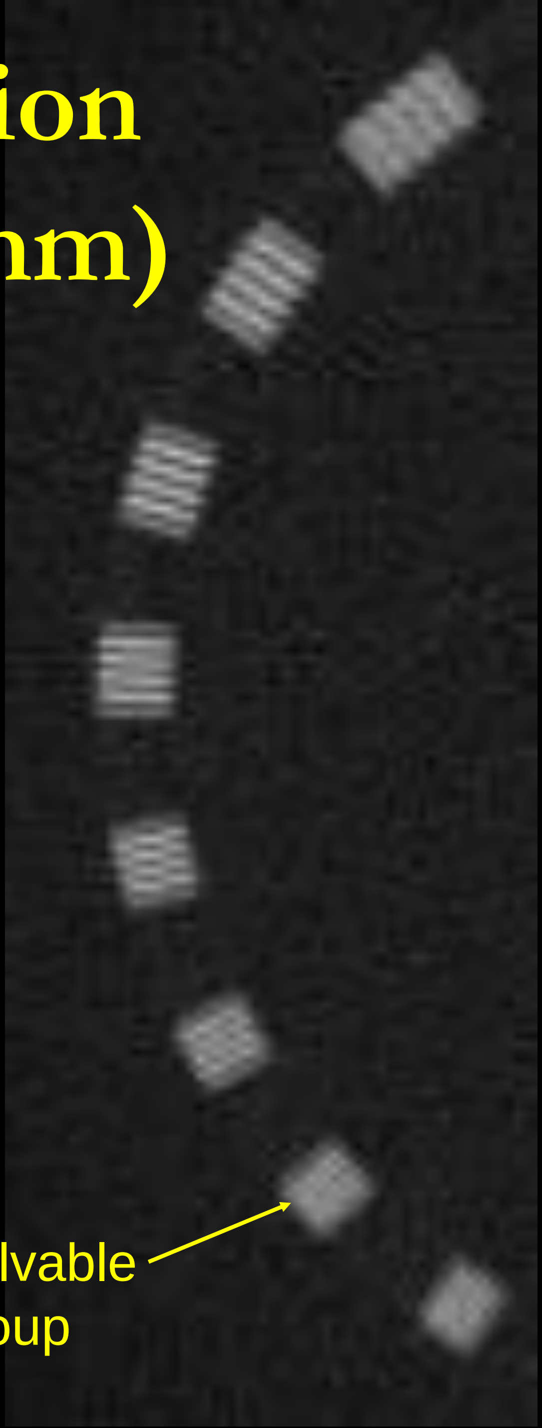
Variance Maps $\sigma^2(x,y)$



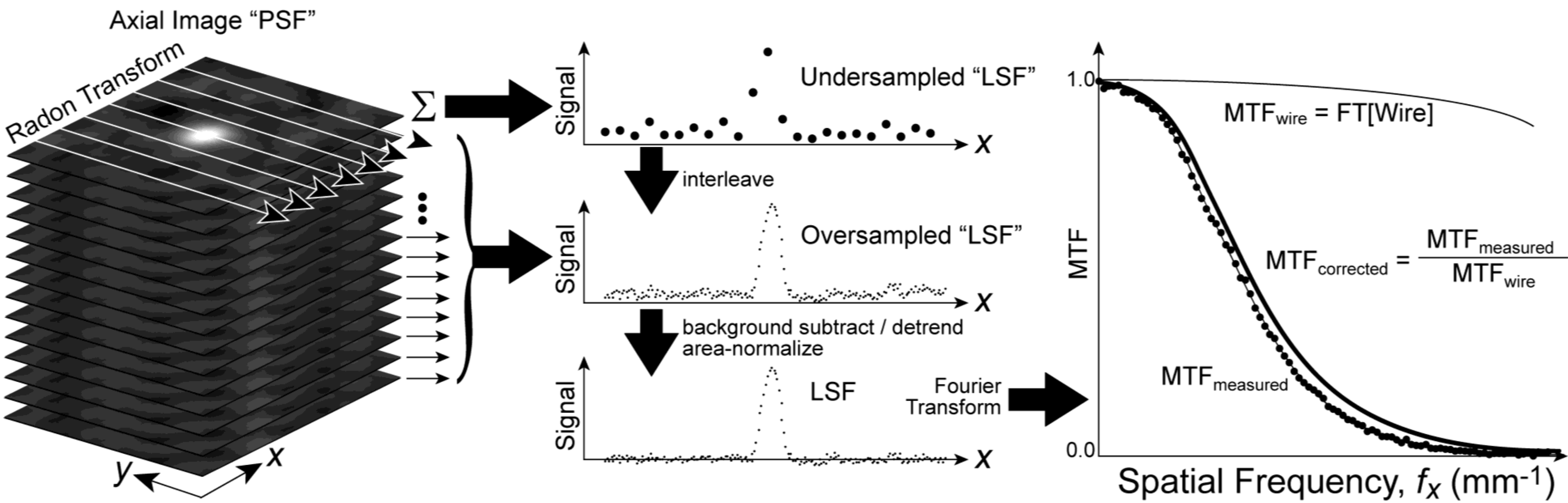
Spatial Resolution (line-pairs per mm)



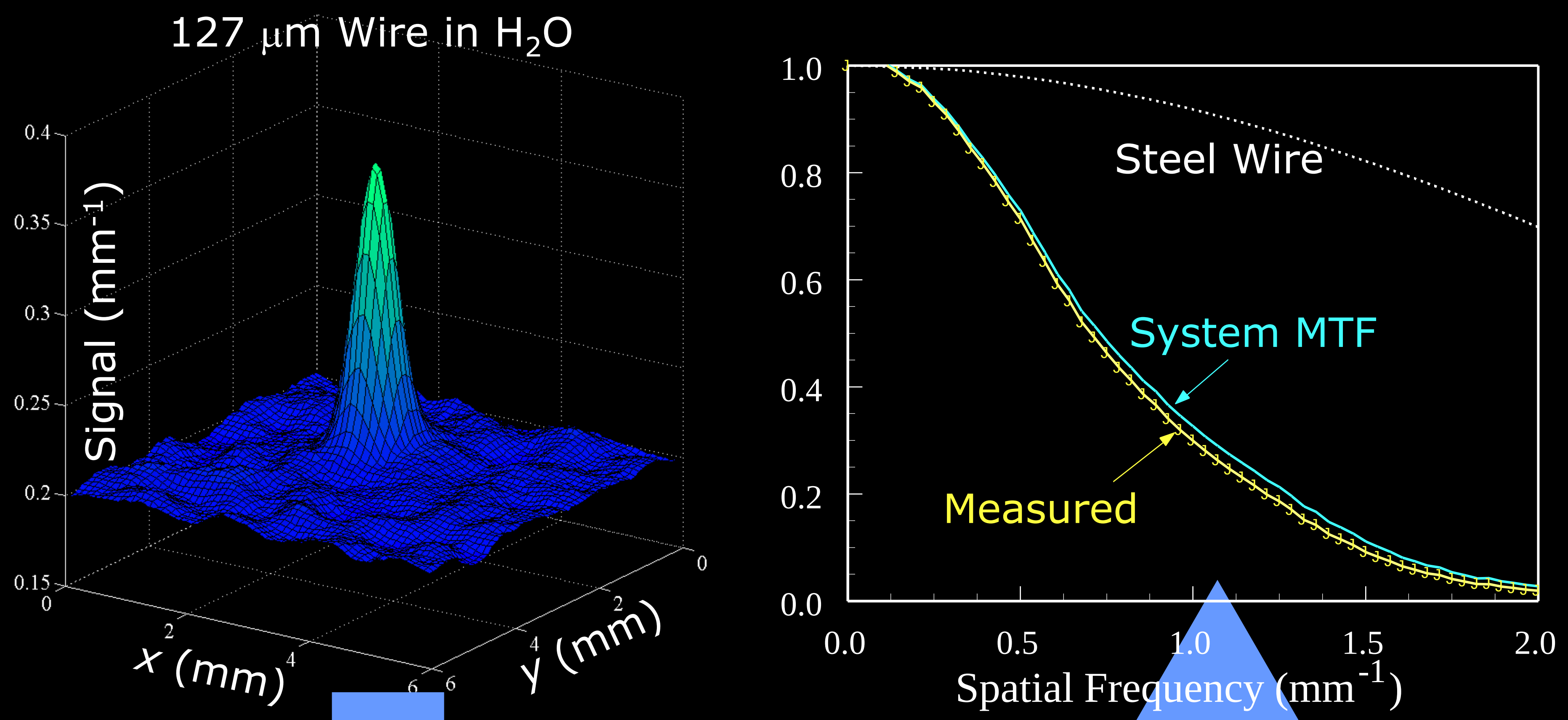


Minimum resolvable line-pair group

Spatial Resolution (Modulation Transfer Function)



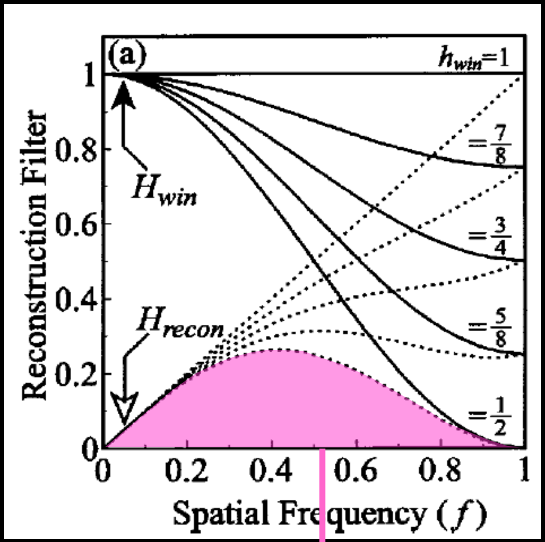
Spatial Resolution (Modulation Transfer Function)



$$MTF\left(f_x, f_y\right) = \left|FT\left[LSF\left(x, y\right)\right]\right|$$

Image Noise

- <u>CT image noise depends on</u>
  - Dose
  - Detector efficiency
  - Voxel size
    - Axial, a_{xy}
    - Slice thickness, a_z
  - Reconstruction filter



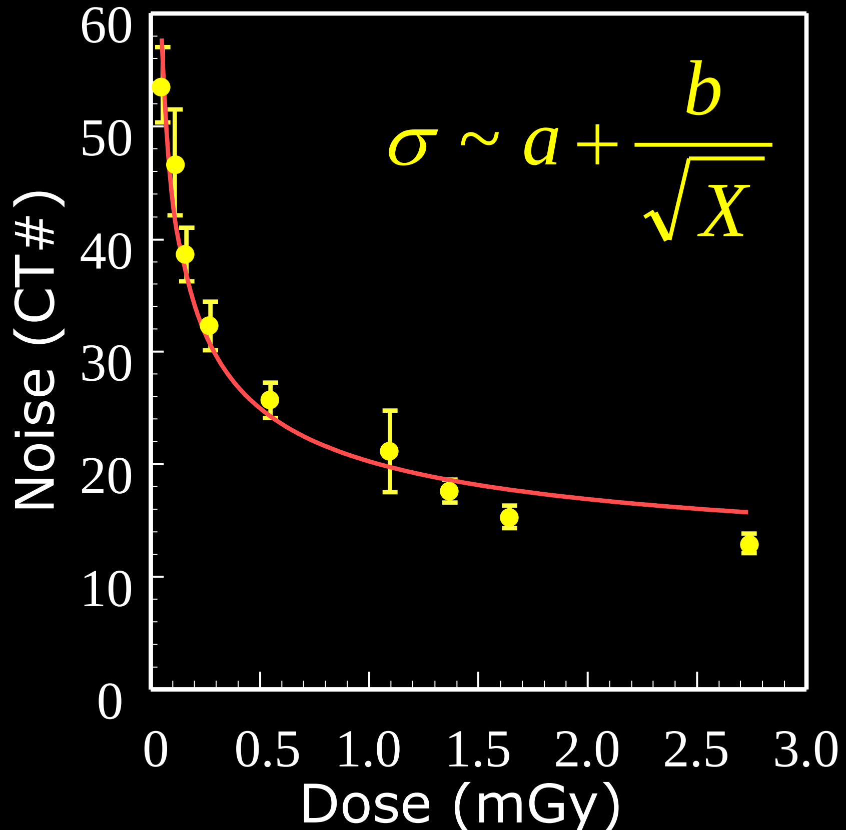
$$\sigma^2 = \frac{k_E}{D_o}\frac{1}{\eta}\frac{K_{xy}}{a_{xy}^3 a_z}$$

$$\sigma \propto \sqrt{\frac{1}{D_o}} \propto \sqrt{\frac{1}{a_{xy}^3}} \propto \sqrt{\frac{1}{a_z}}$$

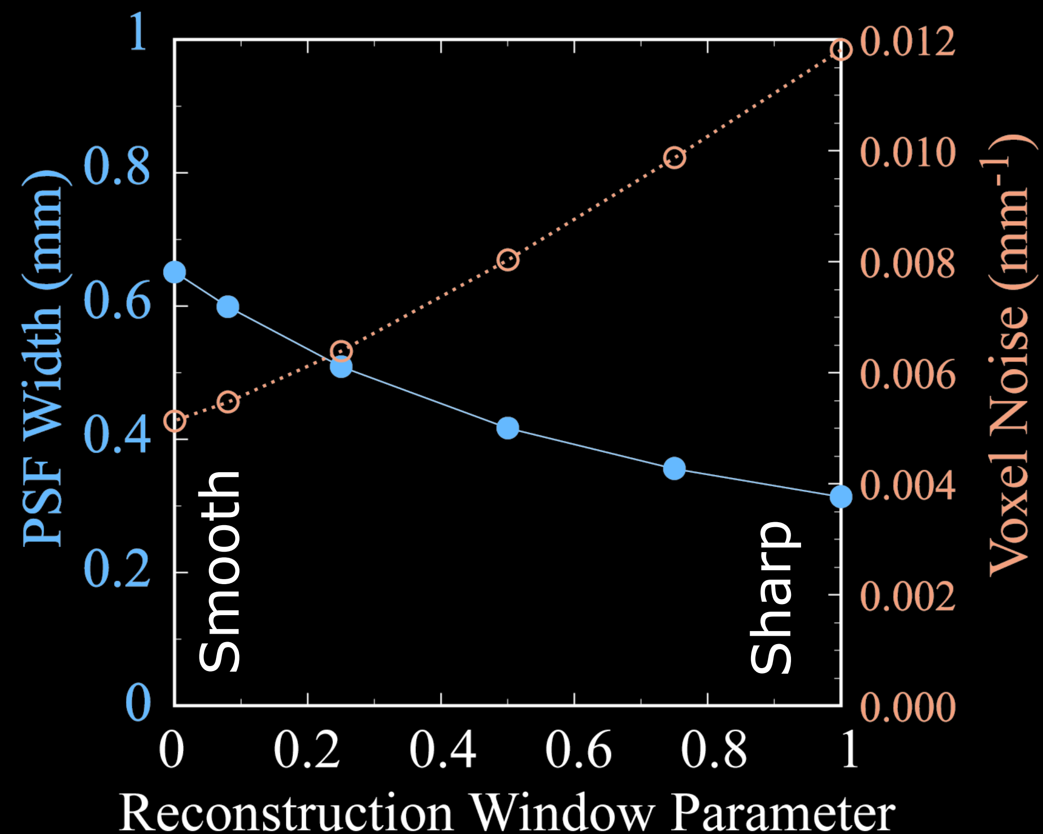
Barrett, Gordon, and Hershel (1976)

Image Noise

Dose



Reconstruction Filter



Noise-Power Spectrum

- The NPS describes

  Frequency content (correlation)

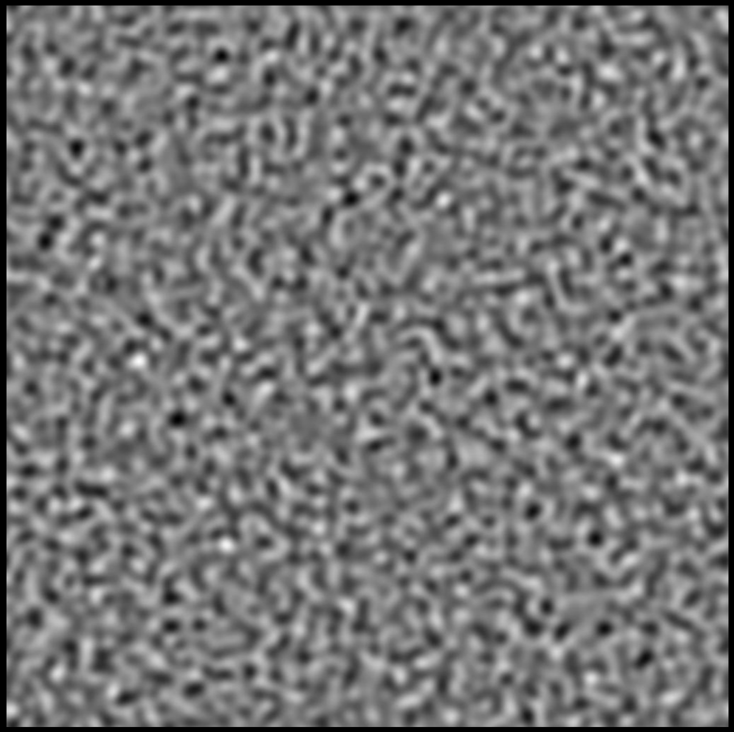
$$NPS\left(f_x, f_y, f_z\right) = \frac{a_x}{L_x}\frac{a_y}{L_y}\frac{a_z}{L_z}\left\langle\left|DFT\left\{\Delta I\left(x, y, z\right)\right\}\right|^2\right\rangle$$

  Magnitude of fluctuations

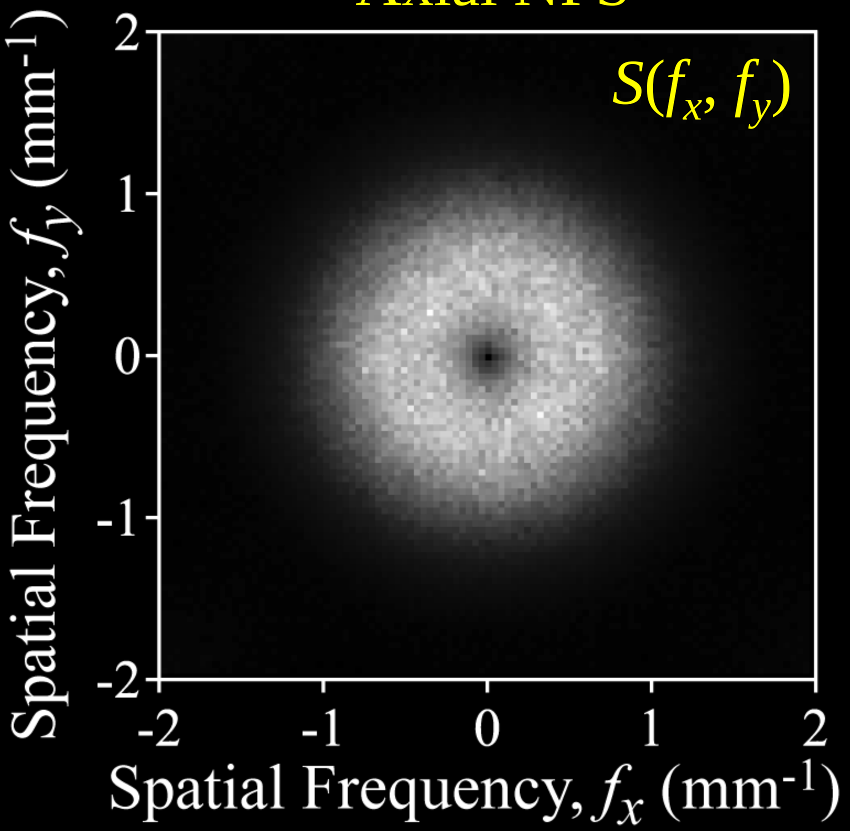
$$\sigma^2 = \iiint NPS\left(f_x, f_y, f_z\right) df_x\, df_y\, df_z$$

Noise-Power Spectrum

Axial Plane (x,y)

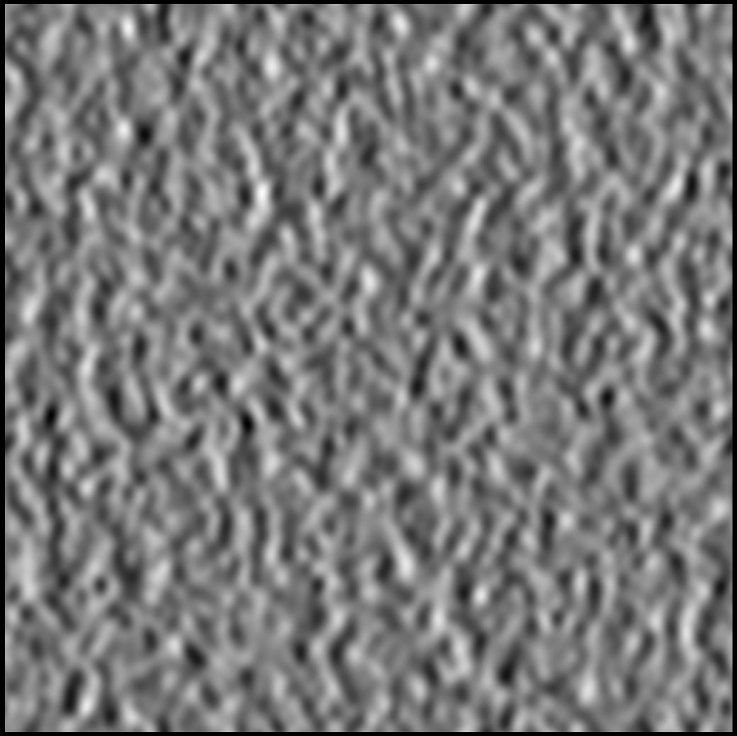


Axial NPS

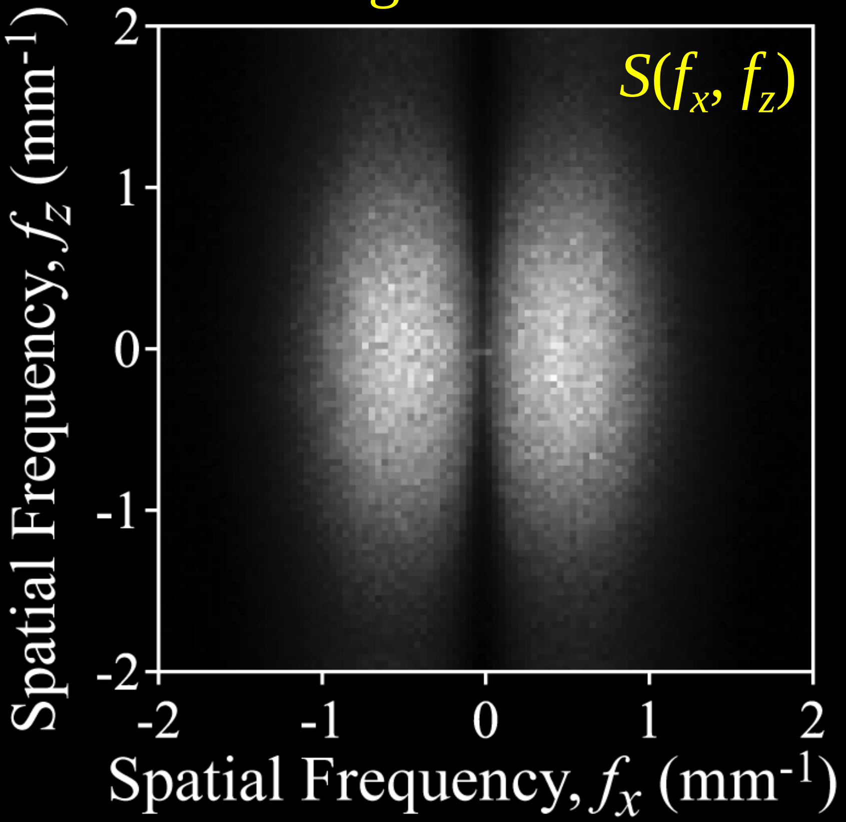


Noise-Power Spectrum

Sagittal Plane (x,z)

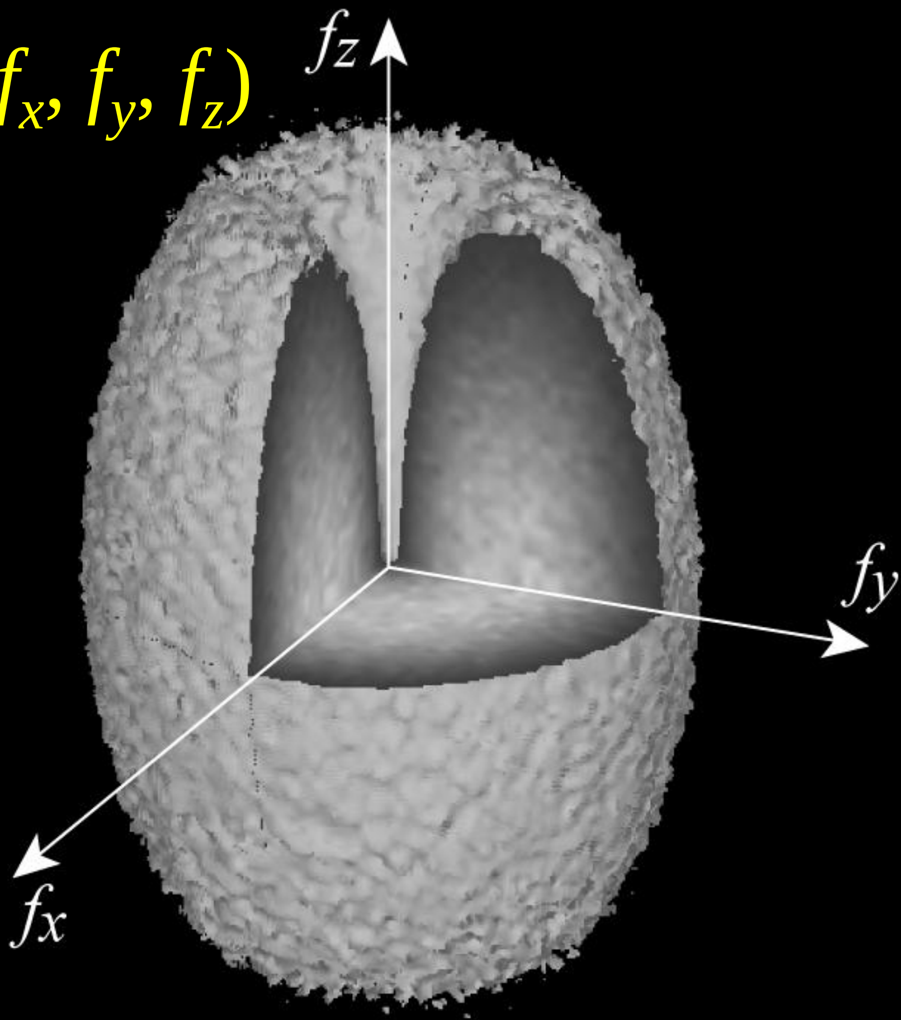


Sagittal NPS



Noise-Power Spectrum

$NPS(f_x, f_y, f_z)$



- <u>Axial domain:</u>
  "Filtered-ramp"
  Mid-Pass
- <u>Longitudinal domain:</u>
  "Band-limited"
  Low-Pass

Contrast

A "large-area transfer characteristic"

Defined:

- As an absolute difference in mean pixel values:

$$C = \left| \overline{\mu}_1 - \overline{\mu}_2 \right|$$

For example:
C = |0.18 cm^{-1} – 0.20 cm^{-1}|
= 0.02 cm^{-2}
or
C = |-100 HU – 0 HU|
= 100 HU

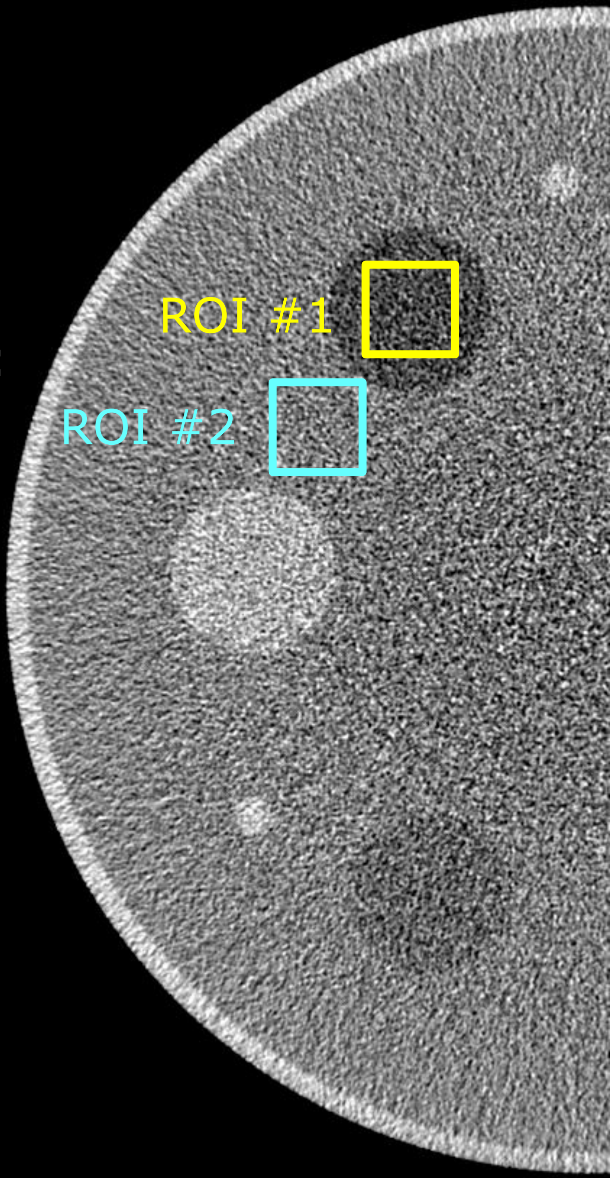
- As a relative difference in mean pixel values:

$$C = \frac{\left| \overline{\mu}_1 - \overline{\mu}_2 \right|}{\left(\overline{\mu}_1 + \overline{\mu}_2 \right) / 2}$$

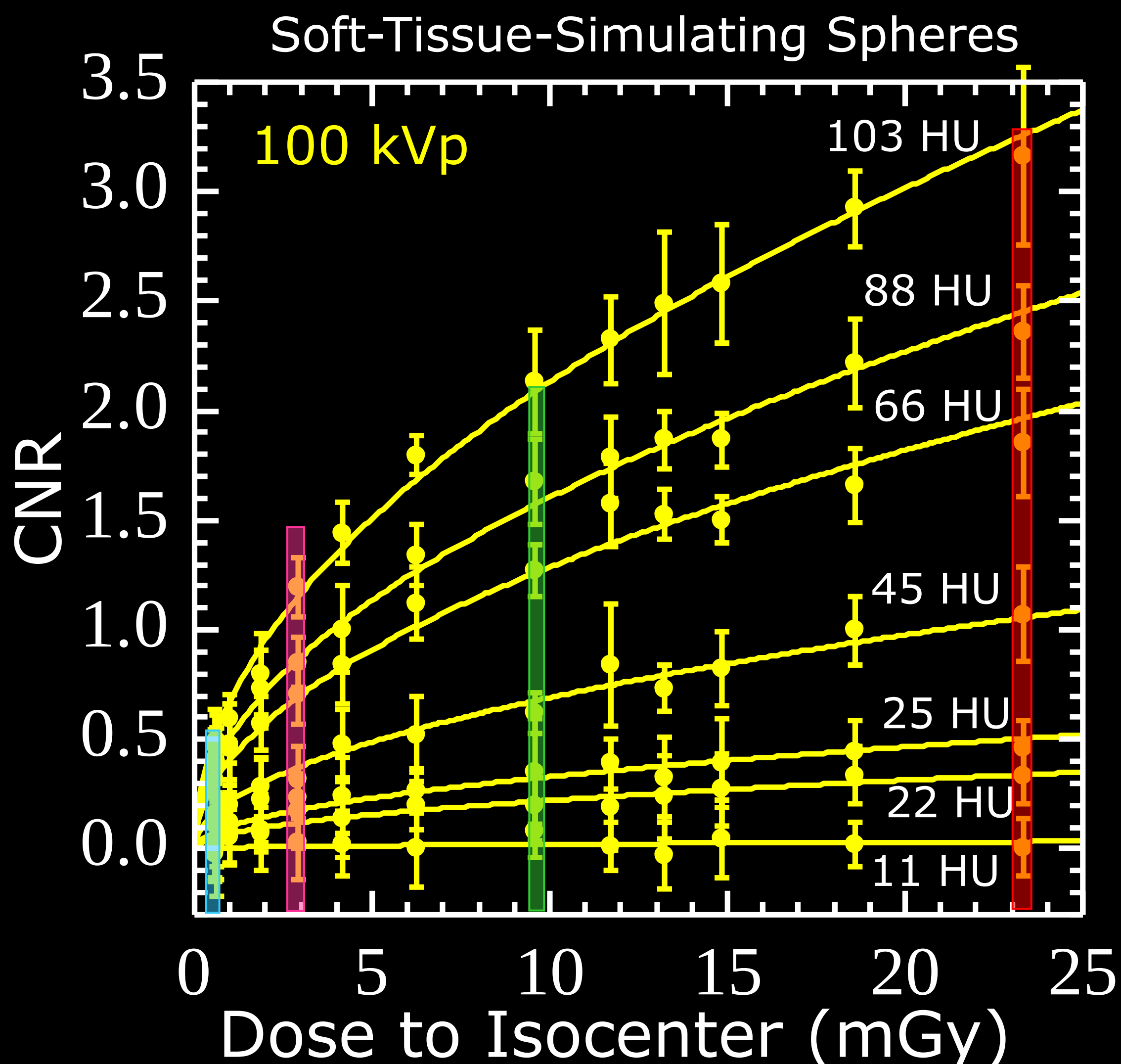
For example:

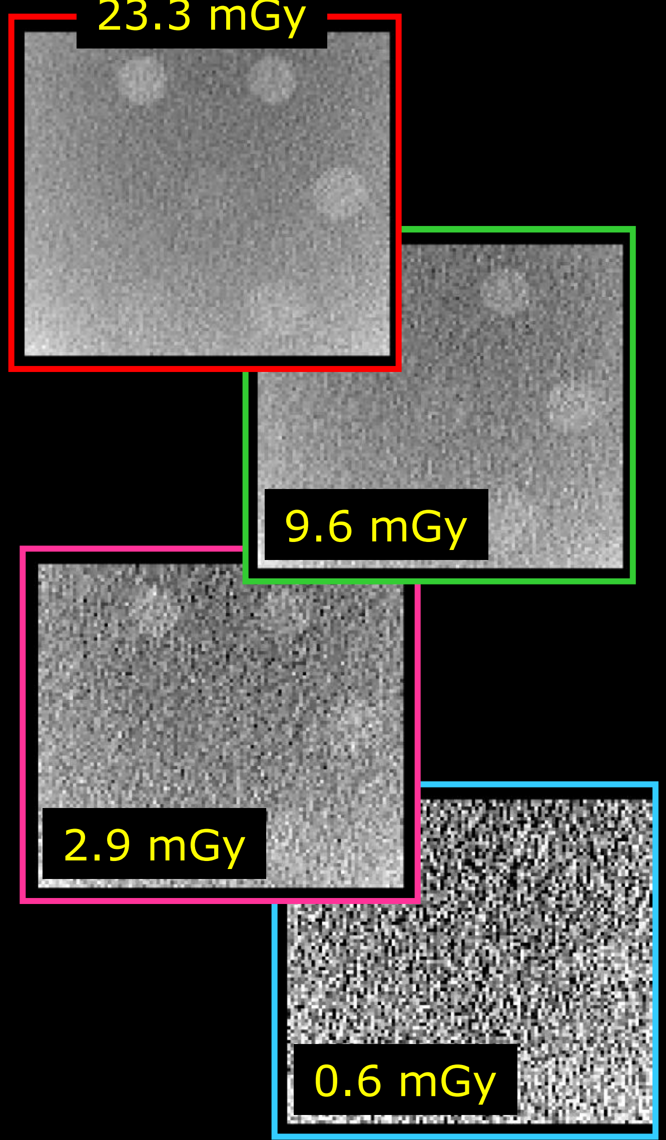
$$C = \frac{|0.18\ \text{cm}^{-1} - 0.20\ \text{cm}^{-1}|}{0.19\ \text{cm}^{-1}}$$

~ 10%



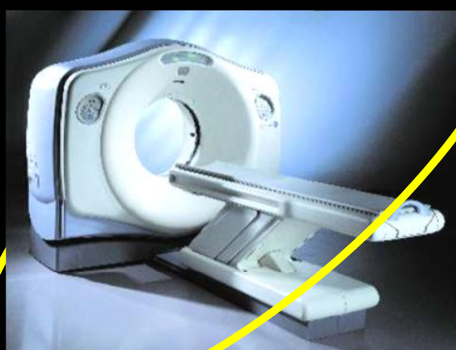
Contrast-to-Noise Ratio





Proliferation of CT Technologies and Applications

Diagnostic Imaging

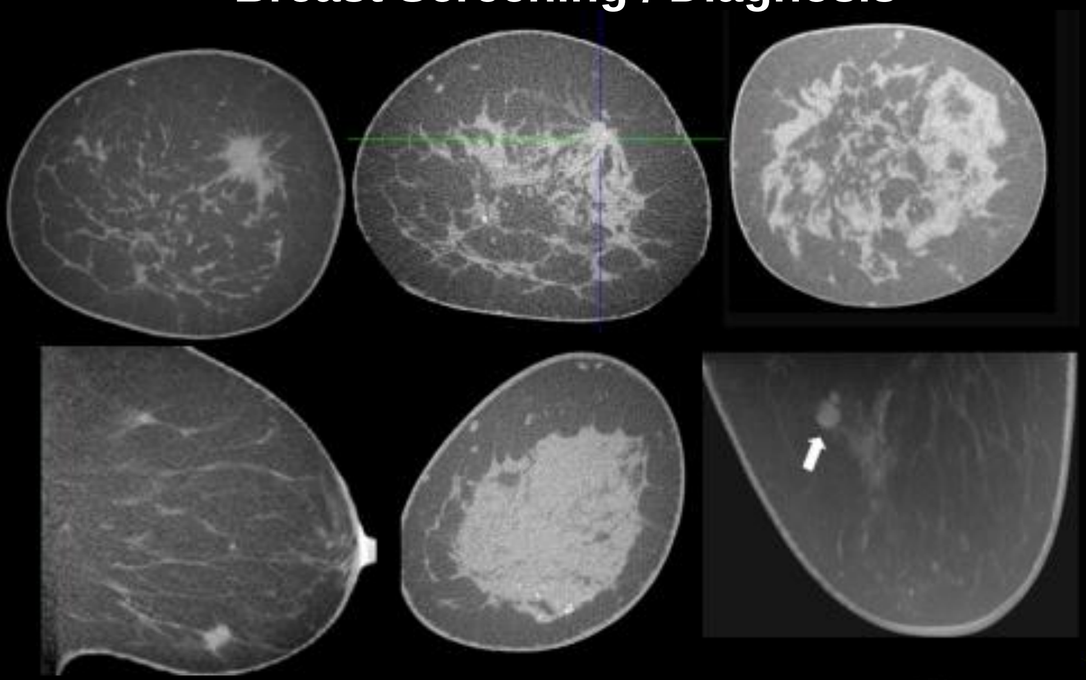


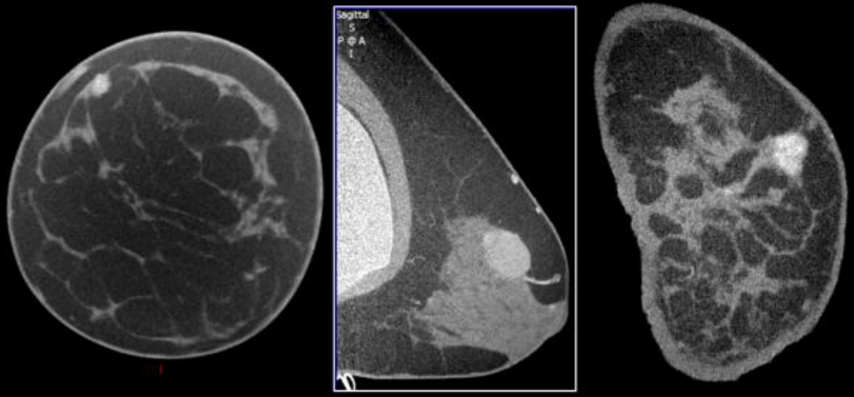
Specialty Applications and Image-Guided Procedures

Trade names removed.

Proliferation of CT Technologies and Applications

Breast Screening / Diagnosis





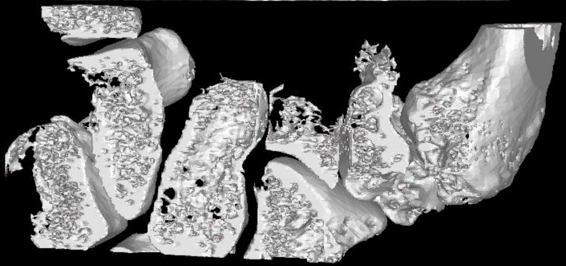
Iodine-Enhanced Tumor Imaging

J. M. Boone (UC Davis)

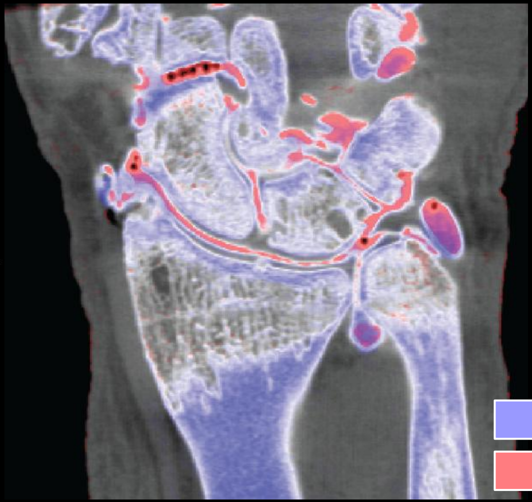
Proliferation of CT Technologies and Applications

Upper Extremities

Morphology, Function, and Quantitation



Dual-Energy CBCT



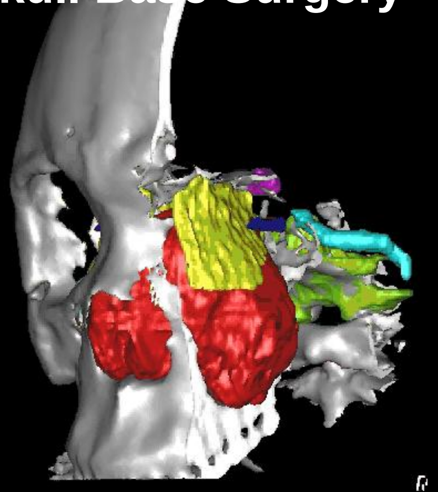
Iodine
Calcium

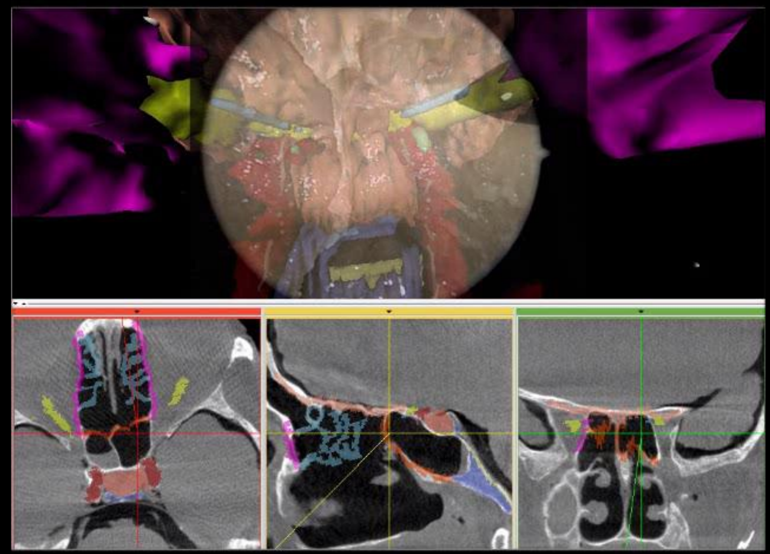
Lower Extremities

Weight-Bearing

Proliferation of CT Technologies and Applications

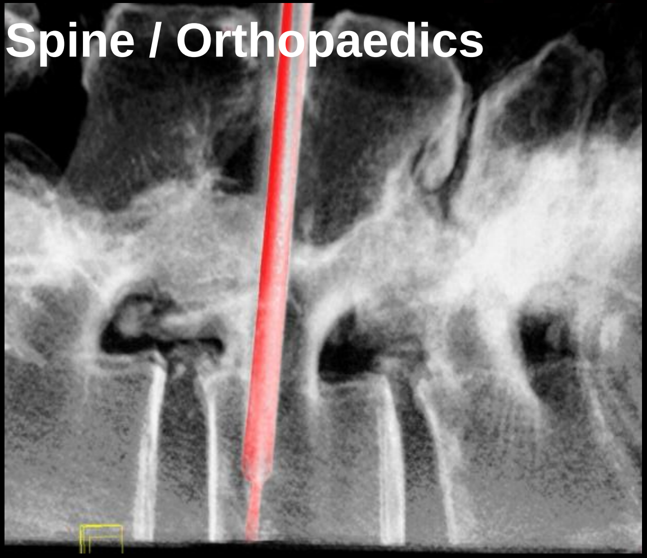
Skull Base Surgery



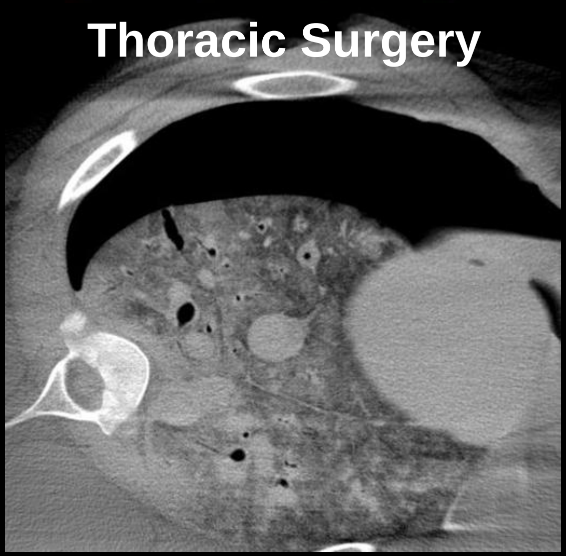


CBCT C-Arm

Spine / Orthopaedics

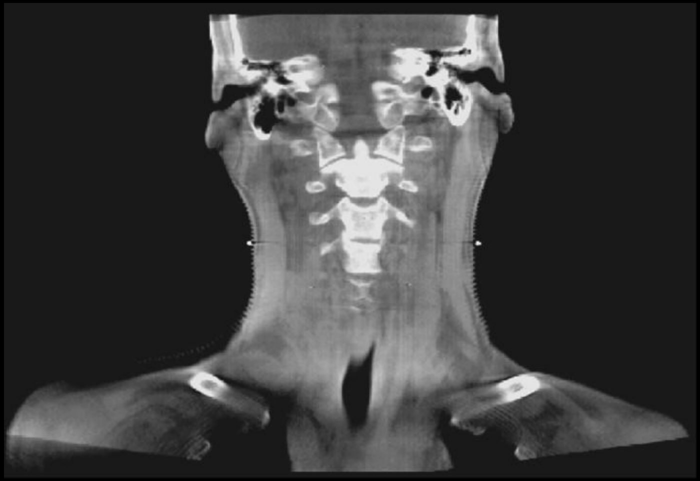


Thoracic Surgery

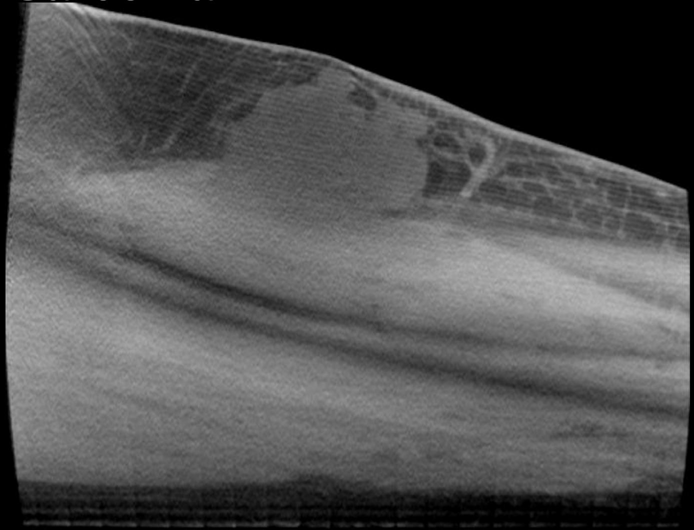


Proliferation of CT Technologies and Applications

Head and Neck

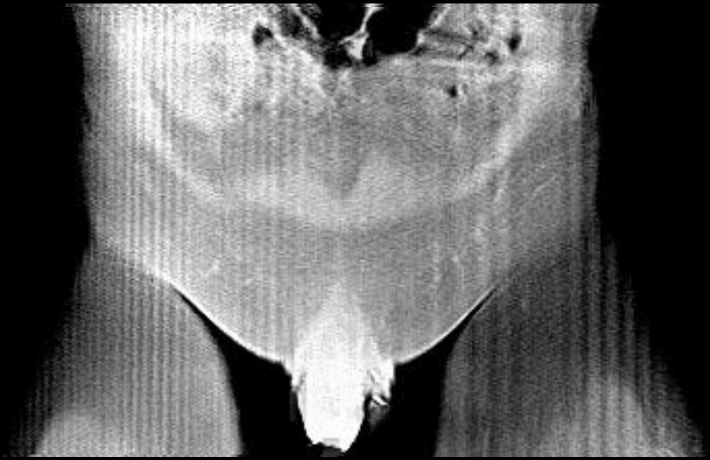


Sarcoma

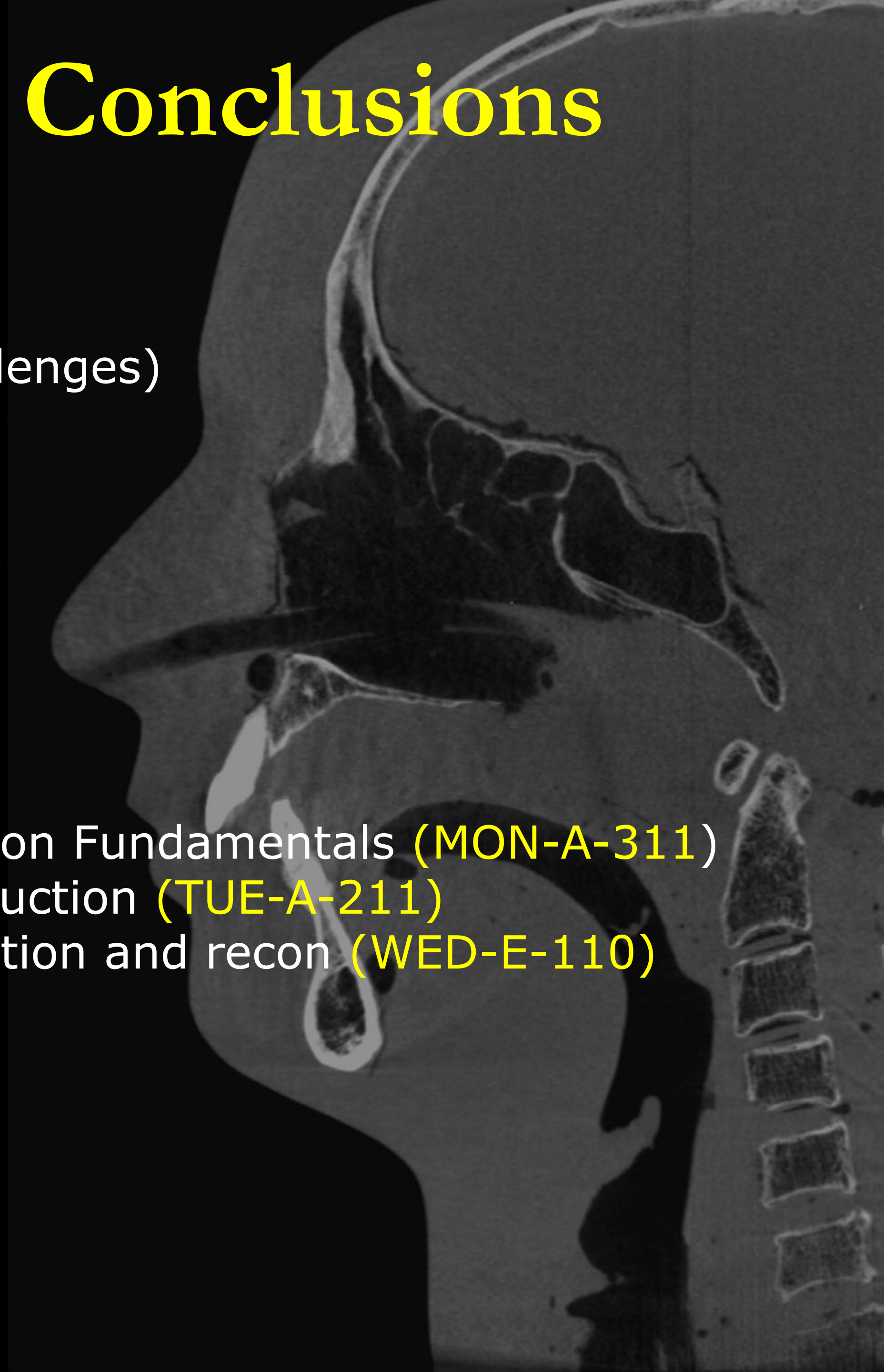


Lung

Prostate



Summary and Conclusions



Proliferation of CT

Increased utilization (and related challenges)
New technology (e.g., cone-beam CT)
Specialty applications

Open Source

OSCaR, PortoRECO
Others? (Please contact me.)

More to Come – This Week:

1.) Siewerdsen – Filtered Backprojection Fundamentals (MON-A-311)
2.) Fessler – Iterative Image Reconstruction (TUE-A-211)
3.) Yu – Optimization of image acquisition and recon (WED-E-110)

Information and Handouts:
www.jhu.edu/istar