EE683 Robot Control: Assignment3

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***Index Terms*—Control, Robot, Disturbance Observer, Robust Internal-loop Compensator, DOB, RIC, Flexible Joint Robot, FJR, SEA**

I. Disturbance Observer

In the control system, the result of controller u input into the plant $P(s)$. In this process, the disturbance d is added and it is difficult to control the desired value. It is the disturbance observer that observes to remove the disturbance that occur at the control system and removes disturbance so controller can control more accurately.

For design the disturbance observer, suppose that we have a controller $K(s)$ designed for a nominal plant $P_n(s)$. In practice, real plant $P(s)$ is differ from nominal plant $P_n(s)$ and unknown disturbance is exist. But Disturbance Observer(DOB) makes real plant $P(s)$ behaves like nominal plant $P_n(s)$ and reject the disturbance d. Also robot controlled in low-frequency.

A. Basic idea of Disturbance Observer

Before we first explain the concept, suppose we know plant $P(s)$.

$$P_n(s) = P(s) \tag{1}$$

The actual plant does not have the same result as the controller, but has a value added to the disturbance $d(s)$. Then the plant $P(s)$ will result y, which will return to the controller as a feedback. At this time, we have to calculate the d of the disturbance, but to calculate the disturbance, multiply the plant's result y by the plant's inverse function $P{-}1(s)$.

$$u = yP^{-1}(s) \tag{2}$$

Then we can know the value added by the result of the disturbance and the controller, and we can subtract the value of the controller result to get the disturbance.

However, this method cannot be used as it is. Because of the vibration of causality in the process of finding the inverse function of the plant. Assuming $P(s)$ as $\frac{1}{ms}$, P^{-1} will be ms. Then $P^{-1}(s)$ will need future information, which will result in a violation of causality가.

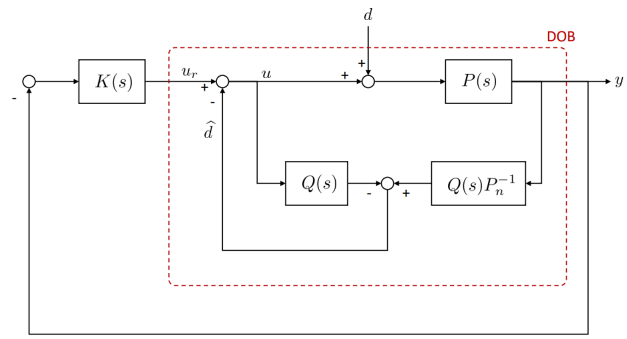

Fig. 1. Structure of DOB.

B. Violation of Causality

We use low pass filter $Q(s)$ to solve the violation of causality. To validate DOB, the estimated cost of $\hat{d}$ is as follows.

$$\widehat{d} = \frac{QP_n^{-1}P}{1 - Q + QP_n^{-1}P}d + \frac{QP_n^{-1}P - Q}{1 - Q + QP_n^{-1}P}u_r \tag{3}$$

In the low frequency, $Q(s) \approx 1$, the result of $\hat{d}$ is as follows.

$$\widehat{d} = d + \left(1 - P^{-1}P_n\right) u_r \tag{4}$$

If $\hat{d}$ is substituted for u, you can get the result of removing the disturbance $d(s)$.

$$u = u_r - \widehat{d} = -d + P^{-1}P_n u_r \tag{5}$$

C. Associated Transfer Function of DOB

The above explained that DOB operates in low frequency. This section, I will explain the reason in modifier. The equation for $y(s)$ is as follows.

$$\begin{aligned} X_Q(s) &= P_n(s) + \left(P(s) - P_n(s)\right) Q(s) \\ y(s) &= [\frac{P(s)P_n(s)}{X_Q(s)}]u_r(s) + [\frac{P_n(s)(1 - Q(s))}{X_Q(s)}]d(s) \end{aligned} \tag{6}$$

If the system is low frequency, $Q(s) \approx 1$ and substitute in(6)

$$y = P_n u_r \tag{7}$$

And $Q(s) \approx 0$ for high frequency. Substitute $Q(s)$ in (6) is as follows:

$$y = P_n u_r + d \tag{8}$$

If you look at the result (7) when low frequency, d does not exist, but if look at the result (8) when high frequency, we can see that d exists. In other words, the disturbance is removed only when it is low frequency.

D. *Design of Q-Filter*

Although we have described the $Q(s)$ as low pass filter(LPF) above, the Q-filter does not necessarily have to be LPF. Another commonly used Q-filter is 3-1 order Q-filter.

$$Q(s) = \frac{3(\tau s) + 1}{(\tau s)^3 + 3(\tau s)^2 + 3(\tau s) + 1} \tag{9}$$

In addition, more diverse Q-filters can be used to achieve better results depending on the situation. Usually, 1st order LPF is also working well.

II. Robust Internal-loop Compensator

Robust Internal-loop Compensator (RIC) has a capacitor $C(s)$ and copy of plant $P(s)$ in the system loop. Again, assume we already know $P_n(s)$. The Compensator C inputs the difference between the result y of $P(s)$ and the predicted $\hat{y}$ obtained through $P(s)$ inside the loop. The resulting u_{ric} subtracts u_r from the controller to calculate u.

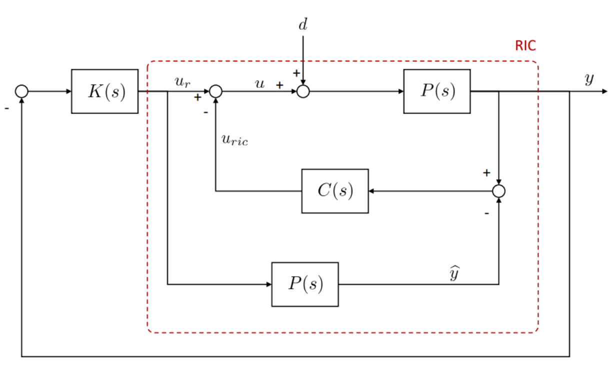

Fig. 2. Structure of RIC.

A. *Transfer Function of RIC*

The $u_{ric}(s)$ described above is calculated as follows.

$$u_{ric}(s) = \frac{P(s)C(s)}{1 + P(s)C(s)} d(s) \tag{10}$$

And calculate $y(s)$, it's as follows.

$$\begin{aligned} y(s) &= P(s)u_r + \frac{P(s)}{1 + P(s)C(s)} d(s) \\ &= P(s)u_r + \left(1 - \frac{P(s)C(s)}{1 + P(s)C(s)}\right) P(s)d(s) \end{aligned} \tag{11}$$

$y(s)$ includes two input values u_r and $d(s)$, and a common factor $\frac{s)C(s)}{1+P(s)C(s)}$ can be replaced with a forward function $Q(s)$. In other words, RIC is the same as DOB. If exchange $P(s)$ and $C(s)$ and calculate u_{ric}, we can will get the same result as low pass filter.

$$\begin{aligned} P(s) &= \frac{1}{s} \\ C(s) &= L \\ u_{ric}(s) &= \frac{L}{s + L} d(s) \end{aligned} \tag{12}$$

B. *Alternative Interpretation of RIC*

There are two plants inside the RIC, which can be thought of as having two systems in one loop at the same time. The results of each system and the expected results are as follows.

$$\begin{aligned} y &= P(s)\left(u_r + d - u_{ric}\right) \\ \hat{y} &= P(s)u_r \end{aligned} \tag{13}$$

Interpreting the RIC from a different perspective puts the difference between the two systems at the Compensator $C(s)$ and can be seen as a transfer function.

$$\begin{aligned} y - \hat{y} &= P(s)d - P(s)u_{ric} \\ u_{ric} &= C(s)(y - \hat{y}) \\ y - \hat{y} &= \frac{P(s)}{1 + P(s)C(s)} d(s) \end{aligned} \tag{14}$$

From this point of view, $C(s)$ serves to eliminate disturbance $d(t)$.

The above concept can be incorporated into the mechanical system. Let's say that the mechanical system we considered and the cloned system are the following formula.

$$\begin{aligned} m\ddot{x} &= f + f_d - f_{ric} \\ m\ddot{\hat{x}} &= f \end{aligned} \tag{15}$$

Subtract the two expressions.

$$m(\ddot{x} - \ddot{\hat{x}}) = f_d - f_{ric} \tag{16}$$

Then f_{ric} becomes a new control input.

$$f_{ric} = C(s)(x - \hat{x}) \tag{17}$$

(17)의 $C(s) = Ls$는 PD, PID 또는 D제어기로 사용될 수 있다.

Here you can change to momentum p to remove the mass m.

$$\begin{aligned} \dot{p} &= f + f_d - f_{ric} \\ \dot{\hat{p}} &= f \\ (\dot{p} - \dot{\hat{p}}) &= f_d - f_{ric} \\ f_{ric} &= L(p - \widehat{p}) \end{aligned} \tag{18}$$

C. Estimation

Let's put the resulting $C(s)$ to u_{ric} in a copied plant instead of a real plant. When calculate the difference between the two systems:

$$\begin{aligned} y &= P\left(d + u_r\right) \\ \widehat{y} &= P\left(u_r + u_{ric}\right) \end{aligned} \tag{19}$$

Calculation produces the same result angle as (14). This structure only makes predictions without compensation for the actual plant because u_{ric} is entered into the copied nominal plant.

III. Flexible Joint Robot

Flexible Joint Robot (FJR) models refer to structures spring-connected between actuators and links.

A. Series Elastic Actuator

The FJR is called a combination of the Series Elastic Actuators (SEAs). SEA is mainly used for soft joints and FJR is the word used for stiff joint. But recently, many people use two words mixed together. Another criterion for distinguishing the two is the research perspective. Use SEA to focus on describing the actuator's own technology (Linear Control), or the word FJR to describe non-linear control with multi-body links.

B. Basic Dynamics of FJR

The dynamics between the operator and link, which are not connected by spring and have infinity, or rigid connection, can be expressed as follows.

$$M\ddot{q} + C\dot{q} + g = \tau \tag{20}$$

An FJR with a spring connection and a finite stiffness is defined by two dynamics: motor, link

$$\begin{aligned} &B\ddot{\theta} + K_j(\theta - q) = \tau_m \\ &M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = K_j(\theta - q) + \tau_{ext} \end{aligned} \tag{21}$$

In (21), $K_j(\theta - q)$ is called a point torque and means a spring torque between the motor and the link. And τ_m is the torque generated by the motor and τ_{ext} means the external torque applied to the link.

To implement FJR, attach a Joint Torque Sensor(JTS) between the motor and the link. The sensor can measure the point torque $K_j(\theta - q)$ that occurs between the motor and the link.

C. Motor Position Feedback PD Control of FJR

The FJR dynamic model (21) is a 4th order dynamic with four input factors: $-K\theta$, $-K\dot{\eta}$, $-Kq$, $-K\dot{q}$ To PD control the motor's angle of θ_d, use the motor's angle of θ and the angular speed of $\dot{theta}$ [3]. (At this point an encoder is used to measure the motor angle θ and angular velocity $\dot{theta}$.)

$$\begin{aligned} \tau_m &= K_p\left(\theta_d - \theta\right) - K_d\dot{\theta} + g\left(q_d\right) \\ \theta_d &= q_d + K_j^{-1} g\left(q_d\right) \end{aligned} \tag{22}$$

IV. Friction Observer of FJR model

In FJR, the various friction forces of τ_f can cause the control to be as poor as desired. This friction force is generated by various factors, such as bearings, mainly by gear decelerators. And by this friction force, FJR dynamics can be expressed with the following changes:

$$\begin{aligned} &B\ddot{\theta} + K_j(\theta - q) = \tau_m + \tau_f \\ &M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = K_j(\theta - q) + \tau_{ext} \end{aligned} \tag{23}$$

A. Friction

Let's summarize the friction force before offsetting it. Friction forces are mainly described by Coulomb friction and Viscous friction. The Coulomb-Viscous fraction formula is as follows.

$$\tau_f = f_c \operatorname{sign}(\dot{\theta}) + f_v\dot{\theta} \tag{24}$$

Friction forces can be seen to increase proportionally to the speed when they begin to move. In analysis of the friction force in more detail, there is a phenomenon in which the friction force is reduced not directly in the low-speed section from the static friction force to the kinetic friction force, but by drawing a curve. This phenomenon is called stribeck effect at low velocity region. The Coulomb-Viscous Friction model (24) has a disadvantage that it is difficult to implement because both coulomb friction and viscous friction must be calculated. One of the most powerful models to replace this is the LuGre fiction model [1].

B. Friction Compensation with 1st order System

The friction force generated by the FJR can be measured and compensate using DOB or RIC. The Plant $P(s)$ described earlier is the motor's dynamics (23) and the extra d is replaced by friction τ_f. The difference between q from link dynamics and θ from motor dynamics multiplied by the K_j spring factor results in joint torque τ_j. That is, $K_j(\theta - q) = \tau_j$ is established.

Two system dynamics must be created to describe the above system using a 1st order low pass filter as (18) of RIC. First, motor dynamics (real dynamics) are as follows.

$$B\ddot{\theta} + K_j(\theta - q) = \tau_m + \tau_f \tag{25}$$

At (25) τ_j can be measured using JTS, and the torque τ_m of the motor equals the result of the controller calculated torque of τ_c minus the estimated friction torque of τ_f.

$$\tau_m = \tau_c - \hat{\tau}_f \tag{26}$$

The resulting motor dynamics are as follows:

$$\dot{p} = \tau_c - \tau_j + \tau_f - \widehat{\tau}_f \tag{27}$$

And nominal dynamics is as follows.

$$\dot{\hat{p}} = \tau_c - \tau_j \tag{28}$$

The results of (27) - (28) for estimation and compensation of friction force are as follows.

$$\begin{aligned} \dot{p} - \dot{\hat{p}} &= \tau_f - \widehat{\tau}_f \\ \hat{\tau}_f &= L(p - \hat{p}) \\ \frac{1}{L}\dot{\hat{\tau}}_f &= \tau_f - \hat{\tau}_f \\ \hat{\tau}_f &= \frac{L}{S+L}\tau_f \end{aligned} \tag{29}$$

In the (29), we should implement $\hat{\tau}_f = L(p - \hat{p})$. p is $B\dot{\beta}$, and $\hat{p}$ is available by integrate of $\dot{\hat{p}}$.

As a result of simulation of (29) in the coulomb friction model, it was estimized with curves because it went through LPF when there was no static friction, and over-estimation occurs when there is static friction.

C. Friction Compensation with 2nd order System

Small oscillation can occur when manipulator is controlled by friction compensation with 1st order system. This is due to static friction. Static friction forces prevent movement at the desired angle $\hat{\theta}$ and increase. And the fiction compensation torque $\hat{\tau}_f$ increases gradually, the manipulator moves at some point. And at some point the robot will stop, but the error is not zero. In terms of passivity, the robot's movement means that the kinetic energy is not zero, which is the energy generated. And it eventually breaks the passivity. As this process is repeated, the manipulator has an occlusion.

Using 2nd order motor dynamics, it can solve the problem of oscillation. The copied Nominal Dynamics with 2nd order motor dynamics are as follows.

$$B\ddot{\theta} = -\tau_j + \tau_c + \tau_f - \widehat{\tau}_f \tag{30}$$

$$B\ddot{\hat{\theta}} = -\tau_j + \tau_c \tag{31}$$

If subtract the two equations, (30) - (31)

$$B(\ddot{\theta} - \ddot{\hat{\theta}}) = \tau_f - \widehat{\tau}_f \tag{32}$$

(32) is a new equation that can replace (30) or (31). And 4th order system is created using two 2nd order system (32) and (31): θ, $\dot{\theta}$, e_{rn}, $\dot{e}_{rn}$

If analyze it using a (32),

$$\hat{\tau}_f = -K_p e_{en} - K_d \dot{e}_{rn} \tag{33}$$

Since (33)and (31) all use 2nd order PD control, passivity can be maintained.

In fact, the control gain of the 1st order friction observer can be well controlled to make it work well. In general, increasing the value of K_p works well. If you have to use a small K_p, there is a way to use an ad-hoc solution. An example of an ad-hoc solution is the method of creating deadzone in the construction capacity.

V. Residual Based Observer

The Residual Based Observer is the same as the RIC structure, which does not compensation, but only estimates. The target is estimate the τ_{ext} of link-side multi-body dynamics in (21)에서 link-side multi-body dynamics In (21), write down link-side multi-body dynamics using momentum term:

$$\dot{p} = C^T\dot{q} - g + \tau_j + \tau_{ext} \tag{34}$$

And nominal dynamics is as follows.

$$\dot{\hat{p}} = C^T\dot{q} - g + \tau_j + \widehat{\tau}_{ext} \tag{35}$$

To estimate the RIC, calculate (34) - (35):

$$\dot{p} - \dot{\hat{p}} = -\widehat{\tau}_{ext} + \tau_{ext} \tag{36}$$

$\hat{\tau}_{ext}$ can be calculated as follows:

$$\hat{\tau}_{ext} = L(p - \hat{p}) \tag{37}$$

In the (37), p can be calculated using $M\dot{q}$, $\hat{q}$ integrate (35) into the final target $\hat{\tau}_{ext}$.

$$\hat{\tau}_{ext} = L[p - \int (C^T\dot{q} - g + \tau_j + \widehat{\tau}_j)\mathrm{d}t] \tag{38}$$

In general, in (38) $\hat{\tau}_{ext}$ is sometimes used as a $r(t)$.

Residual based Observer can also perform a fault detection using the (38). On link dynamics with the addition of the Residual based server $r(t)$:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau_j + r(t) \tag{39}$$

When calculate $r(t)$ using the residual based observer,

$$r = L\left[p - \int \left(C^T\dot{q} - g + \tau_j + r\right) \mathrm{d}t\right] \tag{40}$$

If the $r(t)$ value is abnormally greater than the torque of τ_j measured by JTS, this is a fault.

Residual based observer (38) also can detect the collision.

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau_m + \tau_f + \tau_{ext} \tag{41}$$

Residual based observer can detect the collision using external torque τ_{ext}. But residual based observer can not distinguish friction torque τ_f and external torque τ_{ext}

$$r(t) = \frac{L}{s+L}\left(\tau_f + \tau_{ext}\right) \tag{42}$$

To calculate the τ_{ext}, it should be compensate the friction torque τ_f.

VI. Experiment and Result

The purpose of this experiment is implement LuGre friction model and compensate the friction using DOB. The robot used in the experiment was a 2 DOF manipulator and it using torque control with gravity compensation. You can check my code on github (https://github.com/hdh7485/EE683).

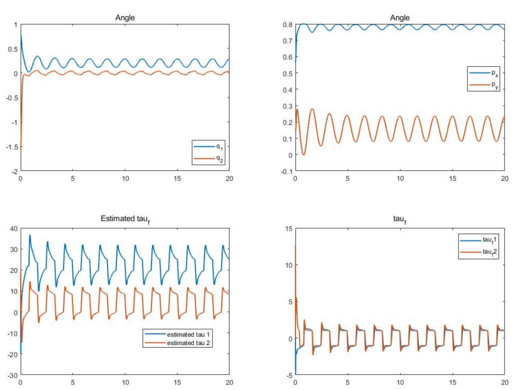

Fig. 3. Implement the LuGre friction model. It does not include Disturbance Observer.

In the "Fig. 3", the Manipulator is oscillated by LuGre friction before DOB is inserted.

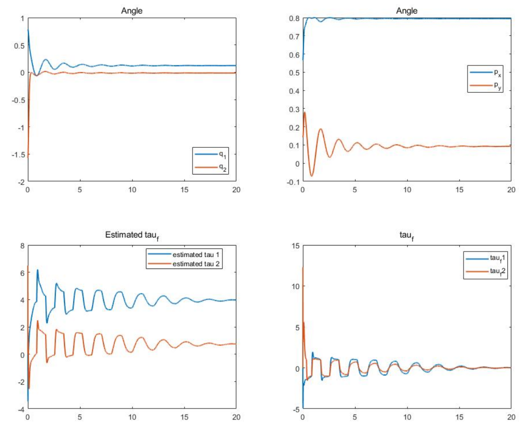

Fig. 4. Thanks to Disturbance Observer, the noise is decrease and position is converge to desired position.

However, after DOB is inserted, it converges in the desired position (0.2, 0) as shown in the "Fig. 4".

To implement the LuGre friction model, the following Matlab function was created [2].

```
function [F, z] = lugre
    (z, v, Fc, Fs, vs, sigma_0,
    sigma_1, sigma_2, ts)

    r = -(v/vs)^2;
    g = (Fc + (Fs - Fc)
        * exp(r)) / sigma_0;
    z_dot = v - abs(v) * z / g;
    z = z + z_dot * ts;
    F = sigma_0 * z
        + sigma_1 * z_dot
        + sigma_2 * v;
end
```

The parameters of the function are $z, v, Fc, FS, vs.sigma_0, sigma_1, sigma_2, ts$. z is previous friction, v put the rotational speed of the joint. And Fc, Fs put in friction constants, and $vsandsigma_0, sigma_1, sigma_2$ also put constant. ts put 0.001 second as sampling time of the controller.

Value of $Q(s)P_m^{-1}$ was same as calculated tau in the DOB. Originally a reverse of the plant matrix $P(s)^{-1}$, but actually it is the torque measured value using JTS. In this experiment, we used simulations, and didn't use JTS, so use the previously calculated tau.

Because low pass filter (LPF) is used, delay occurs and a slight smoothing phenomenon is seen. The lower the tau of the LPF, the more accurate it was to be seen to have an estimations. The LPF code has been implemented directly and the LPF that have been implemented are as follows.

```
function y =
    LPF(x, pre_y, ts, tau)

    y = (pre_y.*tau + x.*ts)
        /(tau + ts);
end
```

The smaller the tau of LPF, the more accurate the optimization was possible. However, due to causality violation, it cannot be zeroed in the actual model. The experiment used a value of 0.0001 for tau.

REFERENCES

[1] Karl Johanastrom and Carlos Canudas-De-Wit. Revisiting the lugre friction model. *IEEE Control systems magazine*, 28(6):101–114, 2008.

[2] Luc Le Tien, Alin Albu-Schaffer, Alessandro De Luca, and Gerd Hirzinger. Friction observer and compensation for control of robots with joint torque measurement. In *2008 IEEE/RSJ International Conference on Intelligent Robots and Systems*, pages 3789–3795. IEEE, 2008.

[3] Patrizio Tomei. A simple pd controller for robots with elastic joints. *IEEE Transactions on automatic control*, 36(10):1208–1213, 1991.