EE683 Robot Control: Assignment2

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***Index Terms*—Passive Control, Robot, Manipulator**

I. Nonlinear Control

All of the control in the nature is non-linear control. Because linear control is also non-linear control and robotic manipulator is of the typical examples of nonlinear control. Non-linear control can be divided into two categories. It's autonomous and non-autonomous control respectively. If the autonomous and non-autonomous control is expressed in equation,

$$\begin{aligned} \dot{x} &= f(x) \\ \dot{x} &= f(x,t) \end{aligned} \tag{1}$$

The typical difference between autonomous and non-autonomous control is uniform stability. Before talking about uniform stability, we should to know about Lyapunov stability and uniform stability. Lyapunov stability .also known as stability, is defined for both autonomous and non-autonomous system. The biggest difference between Lyapunov stability and uniform stability is the independence of time.

$$\begin{aligned} &\forall \epsilon, \exists \delta(\epsilon) = \delta(\epsilon, t_0) s.t. \\ &\|x(0)\| < \delta \rightarrow \|x(t)\| < \epsilon, \forall t \geq 0 \end{aligned} \tag{2}$$

As shown as (2) Lyapunov stability is dependent of t_0, but in the definition of uniform stability:

$$\begin{aligned} &\forall \epsilon, \exists \delta(\epsilon) = \delta(\epsilon) s.t. \\ &\|x(0)\| < \delta \rightarrow \|x(t)\| < \epsilon, \forall t \geq t_0 \geq 0 \end{aligned} \tag{3}$$

In the (3), the definition of δ is independent of t_0. Autonomous control has not state t, so it can always has uniform stability. In order for non-autonomous control be uniform stable, it should following conditions shall be met:

$$W_1(x) \leq V(x,t) \leq W_2(x) \tag{4}$$

And in order for asymptotically stable, it should following conditions shall be met:

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} < -W_3(x) \tag{5}$$

A. Motion control

From now on, let's explain about motion control. Motion control can be explained in two ways: joint space and task space. Motion control in joint space is consist of rotation matrices. Also the state is angle of the joint and the robot dynamics equation is consist of torque, angular velocity and joint angle:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau \tag{6}$$

The meanings of each term are mass, Coriolis and centrifugal forces, D control, P control and gravity. On the other hand, task space is consist of translation and rotation. The rotation part can be represent other expression e.g, Euler angle, exponential coordinate. The robot dynamic equation in task space is following:

$$\Lambda(q)\ddot{V} + \Gamma(q,\dot{q})\dot{V} + \zeta(q) = F_b \tag{7}$$

1) Joint Space: In (6), τ is sourced torque from the robot. And the τ is following:

$$\tau = M\ddot{q}^{ref} + C\dot{q} + g \tag{8}$$

And in the PD control closed loop system, q^{ref} is following:

$$\ddot{q} = q^{ref} = \ddot{q}_d + K_d(\dot{q}_d - \dot{q}) + K_p(q_d - q) \tag{9}$$

If (6) is summarized using (8) and (9):

$$(\ddot{q}_d - \ddot{q}) + K_d(\dot{q}_d = \dot{q}) + K_p(q_d - q) = 0 \tag{10}$$

For tracking the desired state with PD controller, the state is changed from q to $e = q - q_d$ and the τ is following:

$$\tau = M\ddot{q}_d + C\dot{q}_d + g + K_d(\dot{d}_d - \dot{q}) + K_p(q_d - q) \tag{11}$$

the result of the equation in non-autonomous system is:

$$M(e+q_d(t))\ddot{e} + C(e+q_d(t), \dot{e}+\dot{q}_d(t))\dot{e} + K_d\dot{e} + K_p e = 0 \tag{12}$$

The time state is added in the (12) because the non-autonomous system is dependent of time. In the autonomous system, the time can be erased.

To analysis of PD controller Barbalat's lemma(Lyapunov-like analysis), the following conditions should be satisfied for asymptotically stability: $V(x,t)$ is lower bounded, $\dot{V}(x,t)$ is negative semi-definite and is uniformly continuous in time. Each conditions can be expressed like following equations:

$$\begin{aligned}
V(e,\dot{e},t) &= \frac{1}{2}\dot{e}^T M(e,t)\dot{e} + \frac{1}{2}e^T K_p e \leq V(e_0,\dot{e}_0,0) \\
\dot{V}(e,\dot{e},t) &= \frac{1}{2}\dot{e}^T \dot{M}(e,t)\dot{e} + \dot{e}^T M(e,t)\ddot{e} + e^T K_p \dot{e} \leq 0 \\
\ddot{V} &= -2\dot{e}^T K_d \ddot{e} \leq \infty
\end{aligned} \tag{13}$$

Finally the PID controller is the addition of I(integral) controller to the PD controller. The I controller is product of I gain and sum of the error. Torque sourced from robot τ is following:

$$\tau = -K\left(\underline{e} + K_{\mathrm{v}}e + K_I \int e\right) \tag{14}$$

2) Task Space: Before explain about the controller in task space, we should to know the Jacobian matrix. In the robotic manipulator, if know about the body twist of each joints and kinematics, can calculate the body twist of end effector easily using Jacobian matrix. In other words, the Jacobian matrix is a matrix that calculates the task variable when multiplied by the state.

$$\dot{p} = J(q)\dot{q} \tag{15}$$

Robot dynamic equation in task space is similar with in joint space. Replace the τ to body wrench and power $\tau^T\dot{q}$ using following equation:

$$\begin{aligned}
&\tau^T\dot{q} = F_b^T V_b = F_b^T J(q)\dot{q} \\
&\therefore \tau = J^T(q)F_b
\end{aligned} \tag{16}$$

Finally, we can get Robot dynamic equation in task space:

$$M(q)\dot{V}_b + \Gamma(q,\dot{q})V_b + \zeta(q) = F_b \tag{17}$$

The controller in the task space can be done in the same way with joint space. But there are some cautions in task space. First, the integration of body twist V_b has no physical meaning. Second, to change force F_b to torque τ, using $\tau = J^T(q)F_b$. Third, singularity. The Jacobian matrix has not full rank.

Jacobian matrix can be expressed:

$$\begin{aligned}
J &= U\Sigma V^T \\
J^{-1} &= V\Sigma^{-1}U^T
\end{aligned} \tag{18}$$

Σ in (18) is a orthogonal matrix consisting of several single values. If one element σ_i close to 0, $\frac{1}{\sigma_i}$ in Σ^{-1} go to very large number, and the singularity occurs.

One of the method to avoid singularity is damped least squared. To summarize the damped least squared method, minimize the $\|\dot{p} - Jq\dot{q}\| + \mu^2\|\dot{q}\|^2$ instead of $\|\dot{p} - Jq\dot{q}\|$. The solution of the method is following:

$$\begin{aligned}
&\dot{q} = (J^TJ + \mu^2 I)^{-1}J^T\dot{p} \\
&\Sigma^{\dagger} = \operatorname{diag}\left\{\frac{\sigma_1}{\sigma_1^2+\mu^2}, \frac{\sigma_2}{\sigma_2^2+\mu^2}, \cdots, \frac{\sigma_n}{\sigma_n^2+\mu^2}\right\}
\end{aligned} \tag{19}$$

Using this method, the increase of reciprocal $\frac{\sigma_i^2+\mu^2}{\sigma_i}$ is not very significant due to the denominator factor even when approaching zero.

II. Regulation Control

A. PD Regulation

In the regulation controller, the desired state q_d value is constant and also $\dot{d}_q$ and $\ddot{d}_q$ is zero. For PD regulation control in the joint space (6), the torque from robot joint is following:

$$\tau = -K_d\dot{q} + K_p(q_d - q) \tag{20}$$

and equation of closed-loop dynamics is following:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + K_d\dot{q} + K_p(q - q_d) + g(q) = 0 \tag{21}$$

The robot in closed-loop dynamics system (21), D control term and P control term make robot behaves like a damper and spring. The gravity term is external force that makes the robot deviate form the desired.

B. PD Regulation with External Interaction

If external interaction τ_{ext} is added in the PD regulation control, the robot dynamic system equation is changed.

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau + \tau_{ext} \tag{22}$$

The joint torque from the robot is same as (20) and the result of closed-loop dynamics is following:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + K_d\dot{q} + K_p(q - q_d) + g(q) = \tau_{ext} \tag{23}$$

A simple example of external interaction is the addition of torque τ_{ext} to the joint when external force f_{ext} is applied to the robotic manipulator. To convert the external force f_{ext} to external torque τ_{ext}, we should use the Jacobian matrix as like as:

$$\tau_{ext} = J(q)^T f_{ext} \tag{24}$$

C. PD Regulation with Gravity Compensation

If we don't compensate the gravity, the robot joint can't maintain the joint angle on the air. So we should add torque amount of gravity at the joint of robot.

$$\tau = -K_d\dot{q} + K_p(q_d - q) + g(q) \tag{25}$$

calculated by substituting (6) for (25),

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + K_d\dot{q} + K_p(q - q_d) = 0 \tag{26}$$

the gravity term is canceled in the closed-loop dynamics equation (26). Finally, when gravity is complemented and external force is added to PD regulation control, the robot dynamic equation (6) right side is added torque, and the gravity term is added to the torque sourced from robot, the result closed-loop dynamic equation is following:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + K_d\dot{q} + K_p(q - q_d) = \tau_{ext} \tag{27}$$

PD regulation in task space is similar with in joint space. In robot dynamic equation in task space

$$\Lambda(q)\ddot{V} + \Gamma(q,\dot{q})\dot{V} + \zeta(q) = f \tag{28}$$

The right side of the equation (17) means the force sourced from robot. To compensate the gravity, the required force from robot is following:

$$f = -K_d\dot{p} + K_p(p_d - p) + \zeta(q) \tag{29}$$

and the result of gravity compensation closed-loop control equation is following:

$$\Lambda(q)\ddot{p} + \Gamma(q,\dot{q})\dot{p} + K_d\dot{p} + K_p(p - p_d) = 0 \tag{30}$$

Robots are powered by motors, so need to converted force to torque.

$$\begin{aligned}\tau &= J(q)^T f \\ &= J(q)^T\left(-K_d\dot{p} + K_p\left(p_d - p\right) + \zeta(q)\right)\end{aligned} \tag{31}$$

III. Compliance Control

Compliance control can be categorized in two types. One is explicit force control and the other is implicit force control. Explicit force control as known as direct force control, use the force of the tool-tip. This control method need measured force from sensor, and control loop contain force.

Implicit force is also called indirect force control. This controller doesn't need to measure the force directly, and there is no force error in the control loop. The force is applied to the environment and use position for force control. In this paper, I'll talk about Impedance control and admittance control.

A. Impedance Control

Impedance control suppose there are mass-spring-damper between the robot and the environment. Because of mass-spring-damper system that controller assumed, the it can't control accurate contact force and doesn't need environment deformation of reaction force.

Robot torque of impedance control τ structure is following:

$$\tau = M\ddot{q}_d + C\dot{q}_d + g + K_d(\dot{q}_d - \dot{q}) + K_p(q_d - q) \tag{32}$$

and closed-loop dynamics equation is:

$$M(q)\ddot{e} + C(q,\dot{q})\dot{e} + K_d\dot{e} + K_p e = 0 \tag{33}$$

The state of the closed-loop dynamics (33) is error e not q. And if the external force τ_{ext} is implied to environment, the closed-loop dynamics is following:

$$M(q)\ddot{e} + C(q,\dot{q})\dot{e} + K_d\dot{e} + K_p e = \tau_{ext} \tag{34}$$

If we change the impedance in (34), can call it arbitrary impedance controller. The robot torque τ include inverse dynamics terms:

$$\begin{aligned}\tau =& C\dot{q}_d + g - \tau_{ext} \\ &+ M(\ddot{q}_d + M_n^{-1}(-C_n\dot{e} - K_d\dot{e} - K_p e + \tau_{ext}))\end{aligned} \tag{35}$$

To use this controller, it needs to measure τ_{ext} and high computation power for calculating of dynamic parameter M and C.

B. Admittance Control

The admittance controller is inverse of impedance controller. This controller is used when robot can't detect the force or can't access to low-level interface. And this case, we can use only velocity and position information for controller interface. Admittance controller input a state q_f through a admittance filter before position controller. The admittance filter is following:

$$q_f = \frac{1}{M_d s^2 + K_d s + K_p} \tag{36}$$

The K_p is lower, the higher q_f. So admittance controller easily become unstable when use low K_p or K_p as zero. To use admittance controller in task space, we should convert translation and rotation to joint position using inverse kinematics. Stiffness and damping matrices are (2,0) tensor, therefore,

$$K_{p/d,task} = J^{-T}(q)K_{p/d,joint}J^{-1}(q) \tag{37}$$

Finally, the compliance controller is divided into impedance controller and admittance controller. And the closed-loop dynamics of compliance controller with external force:

$$\begin{aligned} M(q)\ddot{e} + C(q,\dot{q})\dot{q} + K_d\dot{q} + K_p(q-q_d) &= \tau_{ext} \\ \Lambda(q)\ddot{V} + \Gamma(q,\dot{q})\dot{p} + K_d\dot{p} + K_p(p-p_d) &= f_{ext} \end{aligned} \tag{38}$$

The robot's behavior becomes harder with higher values and softer with smaller values.

IV. Passivity-based Control

A. Passivity

Passivity is a property of a physical system and it base on the concept of energy. It can be described flow of input energy and output energy through the system and energy is never generated in closed-loop dynamics. So input energy E_{in} is always greater than or equal to output energy E_{out}:

$$E_{out} \leq E_{in} \tag{39}$$

For example of energy of a simple mechanical system, we can consider a simple mass-spring-damper system without gravity. The dynamics equation is following:

$$m\ddot{x} + b\dot{x} + kx = f \tag{40}$$

and the energy is sum of kinetic energy and potential energy of spring:

$$E(x,\dot{x}) = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2 \geq 0, \forall x, \dot{x} \tag{41}$$

we can calculate energy flow by differential (41):

$$\frac{d}{dt}E(x,\dot{x}) = m\dot{x}\ddot{x} + kx\dot{x} = f\dot{x} - b\dot{x}^2 \tag{42}$$

The integration reveals the energy initially stored, the energy dissipated and the input and output power.

$$\begin{aligned} E(x(t),\dot{x}(t)) = E(x(t_0),\dot{x}(t_0)) \\ + \int_{t_0}^{t} f(\tau)\dot{x}(\tau) \\ - \int_{t_0}^{t} b\dot{x}^2 d\tau \end{aligned} \tag{43}$$

If there is not input/output mechanical power and damper, it will be oscillate forever and the system energy is equal to initial system energy. It can be said that there is no energy flow:

$$E(x(t),\dot{x}(t)) = E(x(t_0),\dot{x}(t_0)) = const \tag{44}$$

If dissipated energy and input/output power exist:

$$\begin{aligned} E(x(t),\dot{x}(t)) - E(x(t_0),\dot{x}(t_0)) \\ \leq \int_{t_0}^{t} f(\tau)\dot{x}(\tau) - \int_{t_0}^{t} b\dot{x}^2 d\tau \end{aligned} \tag{45}$$

The total extractable energy $-E(x(t_0),\dot{x}(t_0))$ is always less than or equal to initial stored energy $\int_{t_0}^{t} f(\tau)\dot{x}(\tau)$:

$$-E(x(t_0),\dot{x}(t_0)) \leq \int_{t_0}^{t} f(\tau)\dot{x}(\tau) \tag{46}$$

B. Passivity for Memoryless Function

Originally, passivity was defined for memoryless(static) functions $y = h(u)$. The memoryless function does not have dynamics.

The power flowing into the system is never negative and the system never produce energy internally. For example, mechanical damper and electrical resistance are kind of memoryless function. The passivity function $y = h(u)$ must lie in the first and third quadrant in scalar case.

But in MIMO dynamical systems, we can describe the system using by ordinary differential equation:

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{aligned} \tag{47}$$

C. Dissipativity

Passivity is special case of dissipativity. The input and output are same dimension on Dissipativity system. Dissipativity system has a continuous lower bounded function and supply rate. The supply rate is a pair of input and output $w(u,y)$. So if the system follow the following equations, we can say that system is dissipativity system.

$$\begin{aligned} S(x(t)) - S(x(t_0)) \leq \int_{t_0}^{t} w(u(s),y(s))ds \\ \dot{S}(X(t)) \leq w(u(t),y(t)) \end{aligned} \tag{48}$$

If the supply rate $w(x,y)$ is following:

$$w(u,y) = y^T u + \delta^T u + \epsilon^T y, \delta \geq 0, \epsilon \geq 0 \tag{49}$$

we can say input-output pair (u,y) is passive.

There are some special cases of dissipativity. If δ, ϵ are equal to zero and $\dot{S} = y^T u$, it means this system doesn't dissipate, and if δ or ϵ is greater than zero, each are input strictly passive and output strictly passive. If both δ and ϵ are greater than zero, this is very strictly passive system.

The passivity can expressed as following as:

$$S(x(t)) \leq S(x(t_0)) + \int_{t_0}^{t} y^T(s)u(s)ds \tag{50}$$

$S(x(t))$ means current energy and $S(x(t_0))$ is initial energy and $\int_{t_0}^{t} y^T(s)u(s)ds$ is supplied energy from outside. Namely, there is not internal generation of energy in the system. The extractable energy never can greater than initial stored energy $S(x(t_0))$.

Passivity and Lyapunov stability are similar. Passivity consider the input and output energy, but Lyapunov stability consider about system state. If the energy flow $y^T u$ is greater than or equal to zero, storage function automatically implies Lyapunov stability.

D. Interconnection of Passivity Systems

Passivity systems can interconnection parallel and/or feedback. In the case of parallel interconnection, one input value $u = u_1 = u_2$ through separate systemsΣ_1, Σ_2 and the output of each systems y_1, y_2 are combined as one output y. If express parallel interconnection equation, is following:

$$\begin{aligned}\dot{S}(x) &= \dot{S}_1(x_1) + \dot{S}_2(x_2)\\ &\leq y_1^T u_1 + y_2^T u_2 = (y_1 + y_2)^T u = y^T u\end{aligned} \tag{51}$$

The pair of parallel interconnection system is (u, y)

Feedback interconnection of passivity system also has two systems but the output of one system is used as another system's input. When the system is combined, the sign is different:

$$\begin{aligned}\dot{S}(x) &= \dot{S}_1(x_1) + \dot{S}_2(x_2)\\ &\leq y_1^T (v_1 + y_2) + y_2^T (v_2 - y_1)\\ &= \left[y_1^T y_2^T\right]^T \cdot \left[u_1^T u_2^T\right]\end{aligned} \tag{52}$$

The pair of Feedback interconnection system is $([v_1^T, v_2^T]^T, [y_1^T y_2^T]^T)$

V. Passivity of Manipulators

For modeling of the manipulator contact with environment in the task space, we assume that robot push spring element for passivity. To prove the passivity of environment interaction, calculate the input $-\dot{x}$ and output F_{ext} pair.

$$\begin{aligned}\int F_{ext}^T(-\dot{x}) &= \int x^T K_{env}\dot{x}\\ &= \frac{1}{2}x^T K_{env} x - E(0) \geq -E(0) = 0\end{aligned} \tag{53}$$

The I/O pair $(-\dot{x}, F_{ext})$ is lower bounded and environment is passive. This I/O pair can be extended to p term instead of x. Let's consider passivity of environment interaction in joint space. To calculate the speed of translation $\dot{p}$, we can use equation (15). Input the calculated $-\dot{p}$ to environment, it output the external force $J^T(q)$. We can calculate the joint torque using Transpose of Jacobian:

$$\tau_{ext} = F_{ext} J^T(q) \tag{54}$$

The robot with environmental interaction use feedback interconnection. The feedback interconnection set of three sub systems; controller, robot and environment. Each subsystem are passive, so the whole system is passive. If human apply the force to the system, it can make the system active. Because it is not internal energy but generated energy. But we assume that human is intelligent, so human act passively like can even damp out the energy.

Controller of passivity-based control is same as (25). We can calculate the storage function $S(q, \dot{q})$ sum of kinetic energy and potential energy of virtual spring:

$$\begin{aligned}S(q, \dot{q} &= \frac{1}{2}\dot{q}^T M(q)\dot{q} + \frac{1}{2}(q - q_d)^T K_p (q - q_d)\\ \dot{S} &= -\dot{q}^T K_d \dot{q} + \dot{q}^T \tau_{ext} \leq \dot{q}^T \tau_{ext}\end{aligned} \tag{55}$$

PD controller is acting like virtual spring and virtual damper and this is also passive system.

PID controller is not passive system. The integrator is not a physical element. I-action generates more and more force to achieve desired position and it increase spring potential energy. In the result, the system will break passivity at some point.

VI. Port-Hamiltoninan System

Mass-spring system can be thought of in two passive systems. State, input and output of mass dynamics are $\dot{p}, f_k, v_k$ and spring dynamics are $\dot{x}, v_p, f_p$. The equation of dynamics is following:

$$\mathcal{K} : \begin{cases} \dot{p} = f_k \\ v_k = \frac{\partial K}{\partial p} = \frac{p}{m} (= \dot{x}) \end{cases} \qquad \mathcal{U} : \begin{cases} \dot{x} = v_p \\ f_p = \frac{\partial U}{\partial x} = kx \end{cases} \tag{56}$$

If we arrange two dynamics to substituting values:

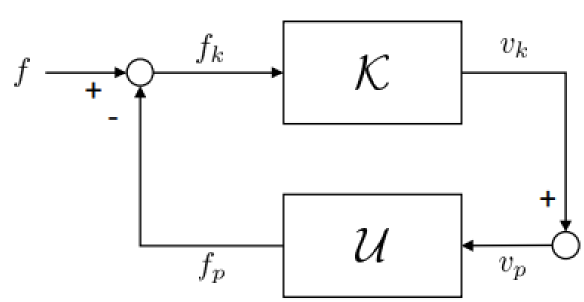

Fig. 1. Energy exchange between kinetic energy and potential energy.

$$\begin{bmatrix} \dot{x} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial x} \\ \frac{\partial H}{\partial p} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} f \tag{57}$$

We can call this $H(x, p) = K(p) + U(x)$ is Hamiltonian. To find passivity of input output pair, calculate the derivative of H. First term of (57) is skew-symmetric, so the result of $\dot{H}$ is $f^T v_k$. The two subsystems K and U exchange energy in a power-preserving way and the subsystem K exchanges energy with the external world through the (f, v_k). If we add dissipation term (damping) in the system, dissipation term is added to decrease the total energy:

$$\begin{bmatrix} \dot{x} \\ \dot{p} \end{bmatrix} = \left(\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & b \end{bmatrix} \right) \begin{bmatrix} \frac{\partial H}{\partial x} \\ \frac{\partial H}{\partial p} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{58}$$

Most of cases, stored energy(Hamiltonian function) $H(x)$ is less than or equal to total energy because of the internal dissipation $R(x)$ decrease the internal energy. To express energy flow in a equation:

$$\dot{H} = -\frac{\partial H^T}{\partial x} R(x) \frac{\partial H}{\partial x} + \frac{\partial H^T}{\partial x} g(x) u \leq y^T u \tag{59}$$

We can generalize the composition of two port-Hamiltonian system as (56) and (58).

$$\begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = & \begin{bmatrix} J_1(x_1) & g_1(x_1) g_2^T(x_2) \\ -g_2(x_2) g_1^T(x_1) & J_2(x_2) \end{bmatrix} \\ & - \begin{bmatrix} R_1(x_1) & 0 \\ 0 & R_2(x_2) \end{bmatrix} \begin{bmatrix} \frac{\partial H_1}{\partial x_1} \\ \frac{\partial H_2}{\partial x_2} \end{bmatrix} \end{aligned} \tag{60}$$

The J in the (60) is internal power-preserving interconnection between subcomponents like skew-symmetric matrix in (58). The composited Hamiltonian function $H(x_1, X_2)$ is sum of $H_1(x_1)$ and $H_2(x_2)$ it gives scalability and modulability.

To express the robot system and controller in port-Hamiltoninan form, each system can be interconnect using a Dirac structure. Dirac structure can exchange energy without any loss between input and output ports (u_c, y_c). Robot also can interact with passive environment or system through another input and output ports (τ_{ext}, y) in a power-preserving way.

A. *Interconnection and Damping Assignment Passivity-Based Control*

The goal of Interconnection and Damping Assignment Passivity-Based Control(IDA-PBC) is to find a control law $u = \beta(x)$. In IDA-PBC, $J_d(x)$ term means Interconnection and $R_d(x)$ means Damping.

The main idea of IDA-PBC is using sum of existing matrices and additional matrices. To find additional stored energy $H_a(x)$, we have to solve PDE using left annihilator $g^{\perp}$.

$$\dot{x} = [J_d(x) - R_d(x)] \frac{\partial H_d}{\partial x}(x) \tag{61}$$

B. *Energy-tank method*

Energy-tank is one of passivation technique. suppose we have a controller τ_c but it is not considered passivity. And this closed-loop dynamics may generate energy and make system active. The main idea of energy-tank method is the store the energy in the virtual tank and controller consume stored energy. The system dynamics of tank as follows:

$$\begin{aligned} \dot{x}_t &= \frac{1}{x_t} D(x) + u_t \\ y_t &= x_t \end{aligned} \tag{62}$$

$D(x)$ in (62) is a dissipation rate of the port-Hamiltonian system. Energy exchange between controller and energy tank through Dirac structure to lossless energy exchange. The result of closed-loop dynamics remains the same with normal port-Hamiltonian equation. But the interconnection matrix becomes singular when $x_t = 0$ and it implies depletion of energy tank. Smaller x_t controller generate more energy, and the system is becoming active. To solve this problem, add a switching parameter $\alpha(t)$ as follows:

$$\begin{cases} \alpha = 1 \text{ if } T(x_t) \geq \epsilon > 0 \\ \alpha = 0 \text{ if } T(x_t) < \epsilon \end{cases} \tag{63}$$

α can activate and deactivate the tank until the tank is refilled.

C. *Time domain Passivity approach: PO/PC method*

Passivity observers and passivity controller(PO/PC) is another straightforward method to passivate a controller or a system. Passivity observer(PO) check passivity condition of input and output port of controller and robot $PO1 = \Sigma u_1^T y_1$. Passivity controller(PC) if PO1 is negative, PC inject damper to recover passivity condition to dissipate the generated energy

PO/PC is robust against communication delay, energy leakage occurring when discretizing signals and active environment like active humans.

VII. Experiment

The purpose of this experiment is to compare PD control with PID control about passivity. The robot used in the experiment was a 2 DOF manipulator Fig. 2 and it using torque control with gravity compensation.

The simulator "Fig. 2" downloaded the open source from github (https://github.com/mws262/MATLABImpedanceControlExample). I removed all the previously applied controllers and used the ones I made myself except gravity compensation term. You can check my code on github (https://github.com/hdh7485/EE683).

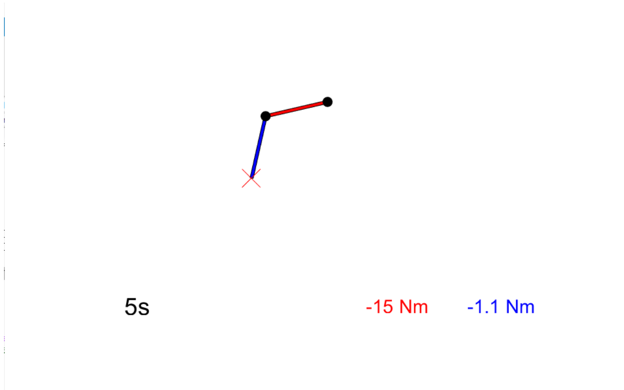

Fig. 2. Simulator used in experiment

2 DOF manipulator needs to know the angle of each joint in order to reach the desired point. The angle of each joint can be obtained using inverse kinematics. I used the second law of cosine to calculate the angle.

Torque for each joint was obtained using a (11) and (14) and controller energy was calculated by multiplying the torque of each joint by the angular velocity.

To prove the manipulator system is passive, I compared the integral of input and output energy and negative initial energy as follow:

$$\begin{aligned} \int -\dot{q}^T \tau_c = \int -\dot{q}^T (K_p(q_d - q) - K_d \dot{q}) \\ \geq -E(0) \end{aligned} \tag{64}$$

The calculated result of negative initial energy was -26.9090. The PD "Fig. 3" and PID Control "Fig. 4" experiments showed that the total energy of each was -8.9 and -29.5.

According to the (64), PD control is passive and PID control is active.

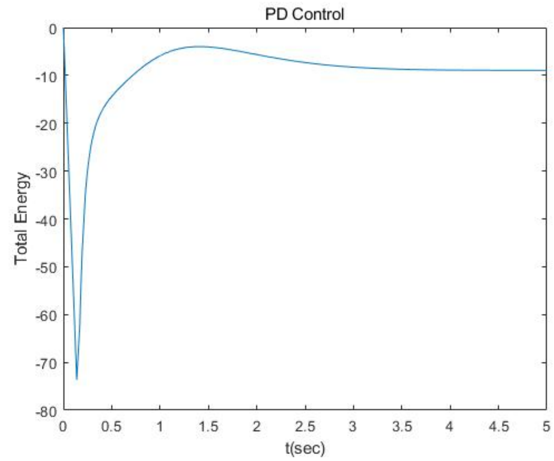

Fig. 3. Total energy of PD controller: -8.9, the total energy is bigger than negative initial energy. So this controller can be seen as passive.

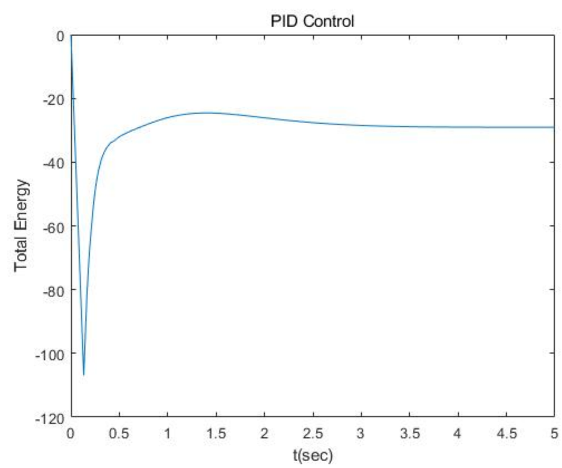

Fig. 4. Total energy of PID controller: -29.5, the total energy is smaller than negative initial energy. So this controller can be seen as active.