In [1]:
```
%matplotlib inline
import matplotlib.pyplot as plt
import numpy as np
```

Lab 5 -- Nebular HII regions

In this lab we will explore the physics of photoionized regions, which are seen surrounding newly formed stars or white dwarfs.

1. Strömgren sphere of ionized gas surrounding a massive star

Consider a newly formed massive star radiating a total ionizing ($h\nu > 13.6 \, \rm eV$) photon rate of Q_*, which is surrounded by a medium with hydrogen number density n_H.

Assume that within some spherical volume (the "Strömgren sphere"), the hydrogen atoms absorb (and are ionized by) every single one of these photons, resulting in an entirely ionized gas, while outside of this volume the gas is entirely neutral. By setting the ionization and recombination rates equal within that volume, calculate its radius $R_{\rm st}$ (the "Strömgren radius").

$[\textit{Hint: } \textrm{What are the units of Q? What about the units of recombination rate?}]$ $[\textrm{If every photon in the inner region results in an ionization what is the ionization rate?}]$

Like in the pre-lab video, you can write the result in terms of the hydrogen recombination coefficient, α_H. Do your scalings of the Strömgren radius with Q_* and n_H make sense?

$$Rst = \sqrt[3]{\frac{3 Q_*}{4 \pi n_H^2 \alpha_H (T)}}$$

My scalings of the Stromgren radius with Q *and nH make sense because it goes ~ Q* / nH^2

2. Typical size of a photoionized nebula

Assuming a star of effective (surface) temperature $T = 3\times10^4$ K and radius $R = 2\times10^{11}$ cm, use the approximate expression from the pre-lab video to calculate the integrated ionizing photon rate Q_* for hydrogen, with ionization energy of $h \nu_i = 13.6$ eV.

Then assuming $n_H = 10^2 \, \rm cm^{-3}$ and $\alpha_H = 3.3\times10^{-13} \, \rm cm^3 \, \rm s^{-1}$, calculate the Strömgren radius. How large is the ionized nebula compared to the size of the massive star? How about compared to the width of a Galactic spiral arm of $\simeq 1$ kpc?

In [9]:
```
Rstar = 2e11 # cm
T = 3e4 # K
nH = 1e2 # cm^-3
aH = 3.3e-13 #cm^3/s
k = 1.3807e-16
hnu = 13.6 * 1.6022e-12 # erg
c = 3e10 # cm/s
h = 6.626e-27

kpc = 3.086e+21 # centimeters in a parsec

ui = hnu/(k*T)

qStar = 8*np.pi**2*(Rstar**2/c**2)*(k*T/h)**3*np.exp(-ui)*(2+2*ui+u
i**2)

print(qStar)

stromRadStar = (3*qStar/(4*np.pi*nH**2*aH))**(1/3)

print(stromRadStar)

print(stromRadStar/3.086e+21)
```

```
1.7891187480721121e+47
2.347896089546129e+18
0.0007608218047783956
```

The Stromgren radius of the ionized nebula is about 1e7 times larger than the radius of the star, which is about 0.07% the size of a galactic spiral arm

In [3]:
```
Rarm = 3.086e+21 # cm

qArm = 8*np.pi**2*(Rarm**2/c**2)*(k*T/h)**3*np.exp(-ui)*(2+2*ui+ui*
*2)

print(qArm)

stromRadArm = (3*qArm/(4*np.pi*nH**2*aH))**(1/3)

print(stromRadArm)
```

```
4.2598481551731957e+67
1.455226260230711e+25
```

3. Thickness of the transition from ionized to neutral

The thickness of the boundary where the gas goes from nearly fully ionized to neutral can be estimated as the mean free path of an ionizing photon in the neutral region. Using $\sigma_0 = 6.3 \times 10^{-18} \, \rm cm^2$, calculate this width and compare with your value of $R_{\rm st}$ from part 2.

How does the result compare to the assumption above of a very sharp transition from fully ionized to neutral gas?

In [8]:
```
sigma = 6.3e-18 # cm^2
l = 1/(nH * sigma)
print(l, 'cm')
```

```
1587301587301587.0 cm
```

This result is pretty large, so it goes against our assumption above of a very sharp transition from fully ionized to neutral gas.

4. Relative size of hydrogen and helium ionization zones

Using the Strömgren sphere model, calculate the ratio of the radius of a helium to hydrogen ionization zone in terms of their relative number of ionizing photons $Q_{\rm H}$ and $Q_{\rm He}$ and composition parameters X and Y.

Calculate this ratio for the first ionization state of helium for the assumed massive star properties above, with $n_H = 10^2$ cm^{-3}, $X = 3/4$, $Y = 1/4$, and noting that the ionization energy is $h \nu_i = 24.6$ eV.

In [14]:
```
x = 3/4 # hydrogen abundance
y = 1/4 # helium abundance
hnu2 = 24.6 * 1.6022e-12 # erg

ui2 = hnu2/(k*T)

# twice as many electrons in helium as in hydrogen, but 3 times as much hydrogen as helium, so
photonRatio = x / (2*y) # number of 13.6 eV photons to the number of 24.6 eV photons = 3/2

qH = 3*qStar
qHe = 2*8*np.pi**2*(Rstar**2/c**2)*(k*T/h)**3*np.exp(-ui2)*(2+2*ui2+ui2**2)

stromRatio = (3*qHe/(4*np.pi*nH**2*aH))**(1/3) / (3*qH/(4*np.pi*nH**2*aH))**(1/3)

stromRatio
```

Out[14]:
```
0.29726415612728285
```

So the ratio of Stromgren radii for this scenario is about 1/3, where the hydrogen Stromgren radius is roughly 3x larger than the helium stromgren radius.

5. Comparison to output from a numerical calculation with the Cloudy code

Compare your results with the same problem as run using the Cloudy photoionization code, which tracks in detail the ionization state and temperature of the nebula.

1) Fill in your values for the Strömgren radius below to plot your expected locations for a drop from full to zero ionization on the plot below.

2) How well does the Strömgren sphere assumption work?

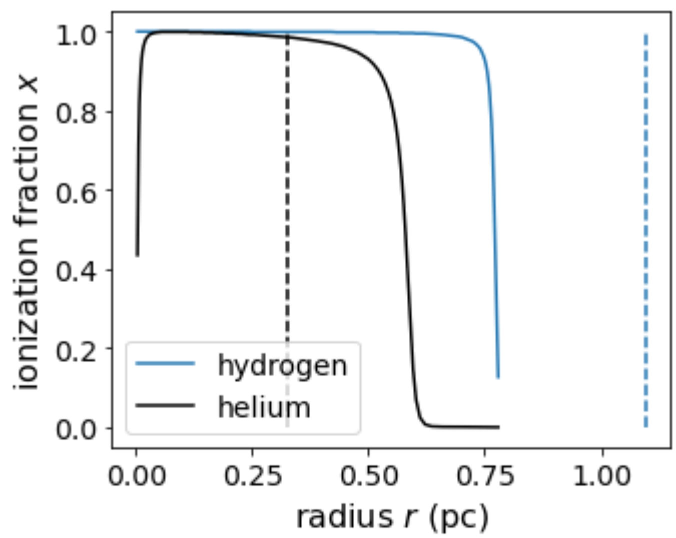
Surprisingly well!

In [15]:
```
# FILL IN YOUR VALUES FOR RSTROM HERE
rstrom_H = (3*qH/(4*np.pi*nH**2*aH))**(1/3) # change this to your c
alculated value for Hydrogen above
rstrom_He = (3*qHe/(4*np.pi*nH**2*aH))**(1/3) # change this to your
calculated value for Helium above

# load Cloudy output data
r,xH,xHe,xHe2 = np.load('cloudy_hii_example_Rx.npy')

# make a plot of ionization fractions
plt.figure(figsize=(5,4))
plt.plot(r/3.09e18,xH,label='hydrogen',color='C0')
plt.plot(r/3.09e18,xHe,label='helium',color='k')
plt.plot(rstrom_H/3.09e18+np.zeros(50),np.arange(50)/49.,color='C0
',linestyle='--')
plt.plot(rstrom_He/3.09e18+np.zeros(50),np.arange(50)/49.,color='k
',linestyle='--')
ax = plt.gca()
ax.xaxis.set_tick_params(labelsize=14)
ax.yaxis.set_tick_params(labelsize=14)
plt.xlabel('radius $r$ (pc)',fontsize=16); plt.ylabel(r'ionization
fraction $x$',fontsize=16)
plt.legend(frameon=True,fontsize=14)
```

Out[15]: <matplotlib.legend.Legend at 0x7fee1c3c6700>



The Stromgren sphere assumption seems reasonable!

6. Nebular spectrum

Cloudy also calculates the observed spectrum from the nebula, including both the initial stellar spectrum (here a Planck spectrum $B_\nu (T)$ with $T = 3\times10^4$ K) and the additional radiation from the ionized gas in the nebula.

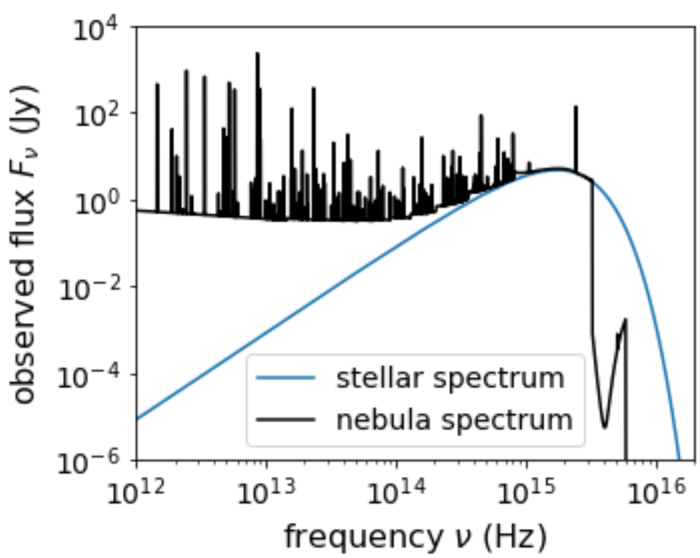
What are the major differences between the incident (stellar) spectrum, and the observed one from the entire nebula? Why is the high energy flux of the star greatly reduced in the observed spectrum? Where did that energy go?

Comment on any radiation processes (emission or absorption) that you think might be contributing to the shape of the observed nebula spectrum.

In [16]:
```
# load Cloudy output data
nu,Fnu_trans,Fnu_diff,Fnu_inc = np.load('cloudy_hii_example_spec.np
y')

# make a plot of the stellar (blue) and observed (black) spectra
plt.figure(figsize=(5,4))
plt.loglog(nu,Fnu_inc,label='stellar spectrum',color='C0')
plt.loglog(nu,Fnu_trans,label='nebula spectrum',color='k')
ax = plt.gca()
ax.xaxis.set_tick_params(labelsize=14)
ax.yaxis.set_tick_params(labelsize=14)
plt.xlabel(r'frequency $\nu$ (Hz)',fontsize=16); plt.ylabel(r'obser
ved flux $F_\nu$ (Jy)',fontsize=16)
plt.legend(frameon=True,fontsize=14)
plt.ylim(1e-6,1e4); plt.xlim(1e12,2e16)
```

Out[16]: (1000000000000.0, 2e+16)



The major differences between the stellar spectrum and the nebula spectrum are the fluxes at low and high frequencies. At low frequencies, the nebula spectrum has much a larger flux (specifically emission lines) compared to the stellar spectrum. At high frequencies, you can see the nebula spectrum dip around 3e15 hz, which corresponds to the ionizing energy of H-alpha, so we see absorption lines at that point.