

## \* Blackbox Optimization:

- General Aim: Given a certain domain and target function defined over the domain find the minimum/maximum.
- If we know the analytic form  $\rightarrow$  we can use gradient
- If we don't know?  $x \rightarrow \boxed{\text{problem}} \rightarrow f(x) \rightarrow$  Black box optimization!

### Algorithm 9 Random Search

```

1: Best  $\leftarrow$  some initial random candidate solution
2: repeat
3:    $S \leftarrow$  a random candidate solution
4:   if Quality( $S$ )  $>$  Quality(Best) then
5:     Best  $\leftarrow S$ 
6: until Best is the ideal solution or we have run out of time
7: return Best

```

### Algorithm 4 Hill-Climbing

```

1:  $S \leftarrow$  some initial candidate solution
2: repeat
3:    $R \leftarrow \text{Tweak}(\text{Copy}(S))$  Small modification/adjust on current solution in order to explore nearby solutions
4:   if Quality( $R$ )  $>$  Quality( $S$ ) then
5:      $S \leftarrow R$ 
6: until  $S$  is the ideal solution or we have run out of time
7: return  $S$ 

```

### Algorithm 5 Steepest Ascent Hill-Climbing

```

1:  $n \leftarrow$  number of tweaks desired to sample the gradient
2:  $S \leftarrow$  some initial candidate solution
3: repeat
4:    $R \leftarrow \text{Tweak}(\text{Copy}(S))$ 
5:   for  $n - 1$  times do
6:      $W \leftarrow \text{Tweak}(\text{Copy}(S))$ 
7:     if Quality( $W$ )  $>$  Quality( $R$ ) then
8:        $R \leftarrow W$ 
9:     if Quality( $R$ )  $>$  Quality( $S$ ) then
10:     $S \leftarrow R$ 
11: until  $S$  is the ideal solution or we have run out of time
12: return  $S$ 

```

### Algorithm 14 Tabu Search

```

1:  $I \leftarrow$  Desired maximum tabu list length
2:  $n \leftarrow$  number of tweaks desired to sample the gradient

3:  $S \leftarrow$  some initial candidate solution
4:  $Best \leftarrow S$ 
5:  $L \leftarrow \{\}$  a tabu list of maximum length  $I$  # first-in first-out
6: Enqueue  $S$  into  $L$  → kuyruğa ekle
7: repeat
8:   if Length( $L$ )  $> I$  then
9:     Remove oldest element from  $L$ 
10:     $R \leftarrow \text{Tweak}(\text{Copy}(S))$ 
11:    for  $n - 1$  times do
12:       $W \leftarrow \text{Tweak}(\text{Copy}(S))$ 
13:      if  $W \notin L$  and (Quality( $W$ )  $>$  Quality( $R$ ) or  $R \in L$ ) then
14:         $R \leftarrow W$ 
15:      if  $R \notin L$  then
16:         $S \leftarrow R$ 
17:        Enqueue  $R$  into  $L$ 
18:      if Quality( $S$ )  $>$  Quality(Best) then
19:        Best  $\leftarrow S$ 
20: until Best is the ideal solution or we have run out of time
21: return Best

```

*L'ün elementi düşer  
bir tweak bulmaya  
acilis yorum (R)*

*eni candidate olabileceğimiz eylem*

### Algorithm 7 Generate a Random Real-Valued Vector

```

1:  $min \leftarrow$  minimum desired vector element value
2:  $max \leftarrow$  maximum desired vector element value

3:  $\vec{v} \leftarrow$  a new vector  $\langle v_1, v_2, \dots, v_l \rangle$ 
4: for  $i$  from 1 to  $l$  do
5:    $v_i \leftarrow$  random number chosen uniformly between  $min$  and  $max$  inclusive
6: return  $\vec{v}$ 

```

### Algorithm 8 Bounded Uniform Convolution

```

1:  $\vec{v} \leftarrow$  vector  $\langle v_1, v_2, \dots, v_l \rangle$  to be convolved
2:  $p \leftarrow$  probability of adding noise to an element in the vector
3:  $r \leftarrow$  half-range of uniform noise
4:  $min \leftarrow$  minimum desired vector element value
5:  $max \leftarrow$  maximum desired vector element value

6: for  $i$  from 1 to  $l$  do
7:   if  $p \geq$  random number chosen uniformly from 0.0 to 1.0 then
8:     repeat
9:        $n \leftarrow$  random number chosen uniformly from  $-r$  to  $r$  inclusive
10:      until  $min \leq v_i + n \leq max$ 
11:       $v_i \leftarrow v_i + n$ 
12: return  $\vec{v}$ 

```

## Population Based Methods

- Idea: Evolve a population (multiset) of candidate solutions using the concepts of survival of the fitness, variation and inheritance

Initialization: Randomly generates the initial population of candidate solutions (String, real vectors)

Evaluation: Evaluates the population of candidate solutions using given fitness function

Selection: Selects promising solutions from current population by making more copies of better solutions

Variation: Processes selected solutions to generate new candidate solutions that share similarities with selected solutions but novel in some way

Replacement: Incorporates new candidate solutions into the original population

### Algorithm 17 An Abstract Generational Evolutionary Algorithm (EA)

```

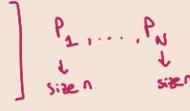
1:  $P \leftarrow$  Build Initial Population
2:  $Best \leftarrow \square$  ▷  $\square$  means "nobody yet"
3: repeat
4:   AssessFitness( $P$ )
5:   for each individual  $P_i \in P$  do
6:     if  $Best = \square$  or  $Fitness(P_i) > Fitness(Best)$  then ▷ Remember, Fitness is just Quality
7:        $Best \leftarrow P_i$ 
8:    $P \leftarrow Join(P, Breed(P))$  → Breed: mevcut P'den yeni bireyler dogumak →
9: until  $Best$  is the ideal solution or we have run out of time
10: return  $Best$ 

```

## A simple Genetic Algorithm:

### • Generate the initial population

```
generate(n,N)
  P = new population of size N;
  for i=1 to N
    P[i] = generate_random(n);
  return P;
```



### • Evaluation

```
evaluate(P,f,n,N)
  fv = new array of N real numbers;
  for i=1 to N
    fv[i]=f(P[i],n);
```

N adet fv  
her bir P: icin

```

P = generate(n,N)
fv = evaluate(Lf, f, n, N)
S = selection(P, fv, n, N)
O = variation(S, P, n, N)
P = replacement(O, P, n, N)

```

$n$ : # of bits  
 $N$ : population size  
 $f$ : objective function  
 $pm$ : probability of mutation  
 $pc$ : probability of crossover

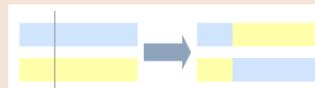
```
binary_tournament_selection (P, fv, n, N)
  S = new population of Size N;
  for i=1 to N
    a=rand(1,N); → N adet random number
    b=rand(1,N-1); → N-1 adet random number
    ayni_birde if (b>=a) b++;
    if (fv[a]>fv[b])
      S[i]=P[a];
    else
      S[i]=P[b];
  return S;
```

ex: a=1  
b=1 → b=2  
a=1  
b=2 → b=3  
a=1  
b=3 → b=4

```
variation(S,n,N,pm,pc)
  O = new population of size N;
  shuffle(S,n,N); randomly reorder strings in the population (P)
  for i=1 to N with step 2
    O[i] = mutation(S[i],n,pm);
    O[i+1] = mutation(S[i+1],n,pm);
    if (rand01()<pc) rand01: random uniform [0,1]
      crossover(O[i],O[i+1],n);
  return O;
```

```
replace_all(O,P,n,N)
  new_P = new population of N strings;
  for i=1 to N
    new_P[i]=O[i];
  return new_P;
```

\* crossover: combines bits and pieces of promising solutions



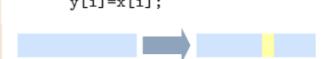
onepoint\_crossover(x,y,n)
 cp=random(2,n);
 for i=cp to n
 exchange(x[i],y[i]);



twopoint\_crossover(x,y,n)
 cp1=random(1,n);
 cp2=random(1,n);
 if (cp1>cp2)
 exchange(cp1,cp2);
 for i=cp1 to cp2
 exchange(x[i],y[i]);

\* Mutation: makes small perturbations to promising solutions

```
mutation(x,n)
  y=new binary string of n bits;
  for i=1 to n
    if (rand01())<pm)
      y[i]=1-x[i];
    else
      y[i]=x[i];
```



## Real-Coded Genetic Algorithms

- To extend the same principle to another coding, we need to implement new variation operators
- **Real-Valued Operators:** Candidate solutions are vectors of real values  
Random initialization generates a random number from an interval  
Operators must deal with real-valued parameters

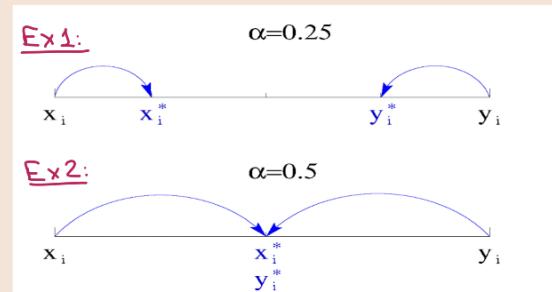
### Arithmetic Crossover:

Two real-valued parents:  $x = (x_1, \dots, x_n)$ ,  $y = (y_1, \dots, y_n)$

Choose random variable  $i \in \{1, 2, \dots, n\}$

First child is:  $O_1(x_1, \dots, x_{i-1}, \alpha y_i + (1-\alpha)x_i, y_{i+1}, \dots, y_n)$

Second child:  $O_2(y_1, \dots, y_{i-1}, \alpha x_i + (1-\alpha)y_i, x_{i+1}, \dots, x_n)$



### Simple Mutation:

Change each variable with a fixed probability

To change a variable, start by generating a random number from  $[-\delta, \delta]$  where  $\delta > 0$  is a small number (Add that number to the variable)

### Gaussian Mutation:

Change is generated according to Gaussian Dist.  $N(0, \sigma^2) \rightarrow \sigma^2$  variance of mutation steps (number)

### Permutations:

Ordering and sequencing problems form a special class

Ex: Travelling Salesman Problem (TSP)

Given  $n$  cities, find a complete tour with min. length

Label the cities  $1, 2, \dots, n$

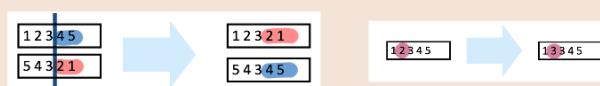
One complete tour is one permutation (ex:  $n=4$ ; [1 2 3 4] or [3 4 1 2] are OK.)

Search space is huge  $\rightarrow$  for 10 cities,  $30!$  possible tours

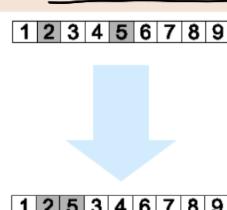
Can we apply genetic algorithm to TSP?

YES. But crossover and mutation needs to be adapted to the new representation

Single crossover and mutation operators lead to inadmissible solutions

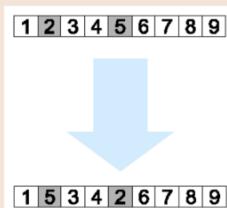


#### Insert Mutation for Permutations



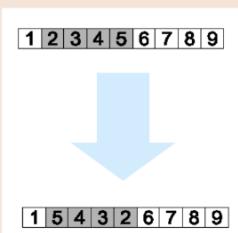
- Pick two allele (feature) values at random
- Move the second to follow the first, shift the rest
- This preserves most of the order and adjacency info

#### Swap Mutation for Permutations



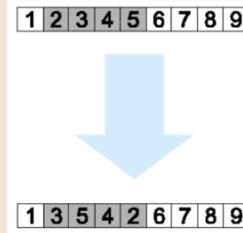
- Pick two alleles at random and swap their position
- Preserves most of adjacency info, disrupts order more

#### Inversion Mutation for Permutations



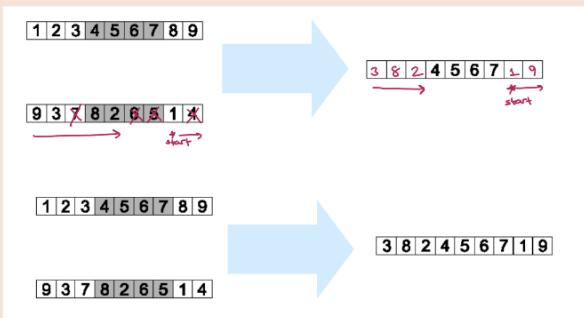
- Pick two alleles at random and invert the substring between them
- Preserves more adjacency info but disruptive of order info

#### Scramble Mutation for Permutations



- Pick a subset of genes at random
- Randomly rearrange the alleles in those positions

• Order-1 Crossover for Permutations



- Choose an arbitrary part from first parent
- Copy this part to the first child
- Copy numbers that are not in the first parent to the first child  
(Start right from the cut point of the copied part, using the order of second parent and wrapping around at the end)
- Same for the second child with parent roles reversed.