Probability Review 1 - Definitions

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Bayesian Machine Learning

- Machine Learning - Uses data to provide automatic responses
- Bayesian methods - Demonstrated ability to manage uncertainty
- Bayesian Machine Learning can explicitly account for the uncertainty present in most machine learning problems
  - Extend applicability of ML
  - Solve more complex and important problems

Probability

- Randomness - Key component of the world
- Approaches to teaching
  - Measure theoretic
  - Heuristic & nonrigorous
- Definitions of probability
  - Frequentist
  - Bayesian
  - Axiomatic

Sample Space & Events

- Sample space - set of outcomes
  - Admission to UVA
  - Time until a car has a flat tire
- Event - subset of the sample space

Axioms of Probability

Given a sample space S and an event $E \subset S$, define a number $P(E)$ that satisfies the following conditions:

1. $0 \leq P(E) \leq 1$
2. $P(S) = 1$
3. For any $E_1, E_2, \ldots$ for which $E_i \cap E_j = \emptyset$ when $E_i \neq E_j$ then

$$P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i)$$

P(E) is the *probability* of E

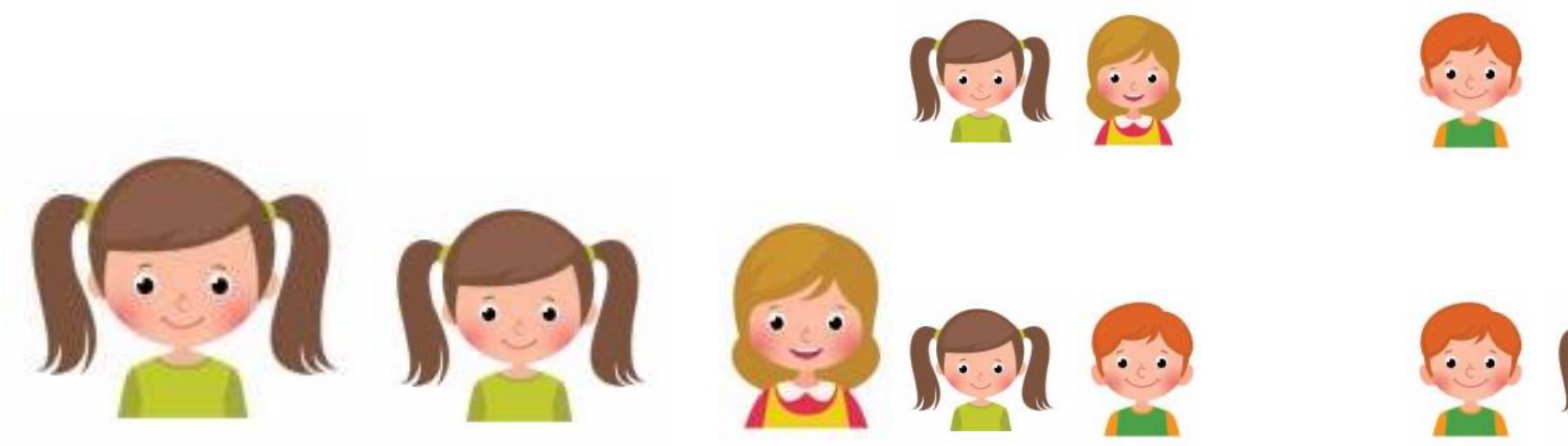
Conditional Probabilities

- Conditional probability of A given B:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

- What is the probability that a family of two children, one of which is a girl, has two girls?

Example - Family Composition



Example Solution

What is the probability that a family of two children, one of which is a girl, has two girls?

$$P(\text{two girls}|\text{one girl}) = \frac{P(\text{one girl} \cap \text{two girls})}{P(\text{one girl})}$$
$$= \frac{1/4}{3/4} = \frac{1}{3}$$

Statistical Independence & Sum Rule

- Statistical independence:

$$P(A \cap B) = P(A)P(B)$$
$$P(A, B) = P(A)P(B)$$
$$P(A|B) = P(A)$$

- Sum Rule

$$P(A) = \sum_{B:P(A,B)>0} P(A, B)$$