UNIVERSITY OF VIRGINIA
DATA SCIENCE INSTITUTE

CS 5012: Foundations of Computer Science

Homework: Logic, Sets, Functions and Relations

QUESTIONS:

(Q1) [20 pts.]

Given the following predicates and their meanings

1. P(x,y) : x > y
2. Q(x,y) : x ≤ y
3. R(x) : x–7 = 2
4. S(x) : x > 9

If the universe of discourse is the real numbers, give the truth value (**true** or **false**) of each of the following propositions:

(i) $(\exists x)\ R(x)$
(ii) $(\forall y)[\neg S(y)]$
(iii) $(\forall x)(\exists y)\ P(x,y)$
(iv) $(\exists y)(\forall x)\ Q(x,y)$
(v) $(\forall x)(\forall y)[P(x,y) \vee Q(x,y)]$
(vi) $(\exists x)\ S(x) \wedge \neg(\forall x)\ R(x)$
(vii) $(\exists y)(\forall x)[S(y) \wedge Q(x,y)]$
(viii) $(\forall x)(\forall y)[\{R(x) \wedge S(y)\} \rightarrow Q(x,y)]$

(Q2) [10 pts.]

Which of the following sentences has the logical form $\mathbf{(p \wedge q) \rightarrow r}$?

1. If you don't attend the wedding, then Sam will be angry with you
2. Matt is happy and so are Sam and Fae
3. If it rains and it snows then flooding will result
4. Students will play football or students will play soccer; but they will not attend classes
5. Gene is smart and strong, additionally he is a good swimmer

(Q3) [10 pts.]

Which of the following formulas represents the sentence, "*If there are no fruit in the market then the farmers didn't plant fruit trees or the farmers didn't water the trees*"

p means *There are no fruit in the market*
q means *Farmers didn't plant fruit trees*
r means *Farmers didn't water the trees*

1. $\neg p \rightarrow q$
2. $p \rightarrow q \vee r$
3. $(p \rightarrow q) \vee \neg r$
4. $p \rightarrow q \vee \neg r$
5. $p \vee q \rightarrow \neg r$

(Q4) [15 pts.]
Show $\mathbf{[p \wedge (p \rightarrow q)] \rightarrow q}$ is a tautology.

(Q5) [15 pts.]
Argue that set A and set A' (the compliment of A) are disjoint.

(Q6) [10 pts.]
Which of the following is a one-to-one function?
1. { (1,2), (2,3), (3,4), (4,5), (3,7), (2,2) }
2. x = 5
3. x=5, 10 < y < 25
4. { (1,2), (2,3), (3,4), (2,5), (3,7) }
5. { (1,2), (2,4), (3,6), (4,8) }

(Q7) [20 pts.]
Let U = {x : x is an integer and $2 \leq x \leq 10$}.
In each of the following cases, determine whether $A \subseteq B$, $B \subseteq A$, both or neither:

(i) A = {x : x is odd } B = {x : x is a multiple of 3}
(ii) A = {x : x is even} B = {x : x^2 is even}
(iii) A = {x : x is even } B = {x : x is a power of 2}
(iv) A = {x : 2x + 1 > 7} B = {x : $x^2 > 20$}
(v) A = {x : $\sqrt{x} \in \mathbb{Z}$} B = {x : x is a power of 2 or 3} (see **note** below)
(vi) A = {x : $\sqrt{x} \leq 2$} B = {x : x is a perfect square}
(vii) A = {x : $x^2 - 3x + 2 = 0$} B = {x : x + 7 is a perfect square}

Note: $\mathbb{Z}$ denotes the set of all integers

GRADING:
- A maximum of **100 points** can be obtained on this homework assignment.

SUBMITTING:
- Submit on Collab
- Submit 1 **PDF** document as your homework
- You must work *individually* on this homework
- Your submitted homework must be typed
- At the top of your document be sure to include your name and computing ID