Predicate Logic (Pt.2)

CS 5012

UNIVERSITY OF VIRGINIA
DATA SCIENCE
INSTITUTE

Predicate Sentences and Formulae (1)

- **User supplies symbols for constants, functions and predicates**
- **Constant symbols**, which generally represent names and fixed numerical values:
  - Mary, 3, Green
- **Function symbols**, which map individuals to individuals
  - father-of(Mary) = John, color-of(Sky) = Blue
- **Predicate symbols**, which map individuals to truth values
  - greater(5,3), green(Grass) , color(Grass, Green)

Predicate Sentences and Formulae (2)

- The input provided by the user is combined with the symbols of the predicate logic to complete a formulae
- Variable symbols
  - E.g., x, y, foo
- **Connectives**
  - Same as in PL: not (¬), and (∧), or (∨), implies (→), if and only if (*biconditional* ↔)
- **Quantifiers**
  - Universal **∀x** or (Ax)
  - Existential **∃x** or (Ex)

Building Sentences (1)

- Predicate Logic sentences are built from Terms and Atoms
- A **term** (*denoting a real-world individual*) is a constant symbol, a variable symbol, or an n-place function of n terms.
  - x and $f(x_1, ..., x_n)$ are terms, where each x_i is a term
  - A term with no variables is a **ground term**
- An **atomic sentence** (*which has value true or false*) is an n-place predicate of n terms

Building Sentences (2)

- A **complex sentence** is formed from atomic sentences connected by the logical connectives:
  - ¬P, PVQ, P Λ Q, P→Q, P↔Q where P and Q are sentences
- A **quantified sentence** adds quantifiers ∀ and ∃
- A **well-formed formula (wff)** is a sentence containing no "free" variables. That is, all variables are "bound" by universal or existential quantifiers
  - (∀x)P(x,y) has x bound as a universally quantified variable, but y is free

Syntax of Predicate Logic (1)

Primitive vocabulary

(a) A set of terms:

- A set of individual constants: a, b, c, d, ...
  - John, Mary, Steve, Loren
- A set of individual variables: x, y, z, x_1, x_2, ...
  - he, she, it

Syntax of Predicate Logic (2)

Primitive vocabulary (cont'd)

(b) A set of predicates: P, Q, R, ...

- One-place predicates: *is happy, is boring*
- Two-place predicates: *like, hate, love, meet*
- Three-place predicates: *introduce, give*

(c) A binary identity predicate: =

(d) The connectives of propositional logic:

¬, Λ, V →, ↔

(e) Quantifiers: ∀, ∃

(f) brackets: ( ) [ ]

Syntax of Predicate Logic (3)

Syntactic rules

(a) If P is an n-place predicate and t_1, ..., t_n are all terms, then P(t_1, ..., t_n) is an atomic formula

- John is happy: H(j)
- John loves Mary: L(j,m)
- John introduced Mary to Sue: I(j,m,s)

(b) If t_1 and t_2 are individual constants or variables, then $t_1 = t_2$ is a formula.

- John is Bill: j=b

Syntax of Predicate Logic (4)

NOTE: Φ is Greek letter Phi and Ψ is Greek letter Psi, being used as symbols for 2 different formulae below

(c) If Φ is a formula, then ¬ Φ is a formula.

(d) If Φ and Ψ are formulas, then (Φ ∧ Ψ), (Φ ∨ Ψ),
(Φ → Ψ), and (Φ ↔ Ψ) are formulas too

(e) If Φ is a formula and x is a variable, then ∀xΦ, and ∃xΦ are formulas too

- Everyone is happy: ∀x H(x)
- Someone loves John: ∃x L(x, j)
- Everyone loves someone: ∀x ∃y L(x, y), ∃y ∀x L(x, y)

(f) Nothing else is a formula in predicate logic

Translate from English Sentences to Predicate Logic Formulae

NOTE: ∀ = *For all, for each, every, …*

∃ = *There exists, there is at least one, …*

(1) Every student is happy:

Right: ∀x [student(x) → happy(x)]

Wrong: ∀x [student(x) ∧ happy(x)]

(2) Some students are happy:

Right: ∃ x [student(x) ∧ happy(x)]

Wrong: ∃ x [student(x) → happy(x)]

1. *For all x, if x is a student, then x is happy*
2. *There exists an x, where x is a student and x is happy*

Translate from English Sentences to Predicate Logic Formulae

(3) No student complained:

- $\forall x$ [student(x) $\rightarrow$ $\neg$complained(x)]
- $\neg\exists$ x [student(x) $\wedge$ complained(x)]

(4) Not every student complained:

- $\neg\forall x$ [student(x) $\rightarrow$ complained(x)]
- $\exists$ x [student(x) $\wedge$ $\neg$complained(x)]

3. *- For all x, if member of set 1 then not member of set 2*
   - There doesn't exist an x, member of set 1 and member of set 2
4. *- For all x, it's false, if member of set 1 then member of set 2*
   - There exists an x, member of set 1 and not member of set 2

Are CS 5012 Class students good?

NOTE: S = *Class of CS 5012* (set of students)

x = *is a student* – an **Element** of the **Set S**

(5) Every student is very good:

Right: $\forall x\ [S(x) \rightarrow \text{very good}(x)]$

Wrong: $\forall x\ [S(x) \wedge \text{very good}(x)]$

(6) Some students are excellent:

Right: $\exists x\ [S(x) \wedge \text{excellent}(x)]$

Wrong: $\exists x\ [S(x) \rightarrow \text{excellent}(x)]$

5. *For every x, if x belongs to S, then x is very good*
6. *There exists an x, where x belongs to S and x is excellent*

Acknowledgments

Some of the contents of this set of slides have been adapted from many print media literary sources and websites. However, the following references deserve particular credit and acknowledgement

- Robert Wilensky, Professor Emeritus, Computer Science Department, UCLA, CA, USA
- CSEE, University of Maryland, Baltimore County , Baltimore, MD, USA
- Predicate Logic parts 1, 2, 3, 4 (2014), by Sandhya Sundaresan
- Institute for Software Systems, Hamburg University of Technology, Germany (originally based on slides of slides of J. Bowen)
- LE Thanh Sach, Ph.D., Department of Computer Science, HoChiMinh City University of Technology
- Richard Mayr, University of Edinburgh, UK
- Uta Priss, School of Computing, Edinburgh Napier University
- Farn Wang, Dept. of Electrical Engineering, National Taiwan University
- Department of Linguistics, Simon Fraser University, Canada