Propositional Logic

CS 5012

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Let’s Test your Knowledge

- The following slides give you a chance to refresh your knowledge and test your understanding of some of the basic concepts of **Propositional Logic**

Exercise 1

- Which of the following propositional formula(s) represent(s) the sentence, **"Tim will go to Charlottesville or Tim will go to Fairfax; if the former, he will visit our classroom"**

  Let the three atomic propositions above be p, q, & r

1. $p \rightarrow q \vee r$
2. $p \vee q \rightarrow r$
3. $(p \rightarrow q) \wedge (p \vee r)$
4. $p \vee \neg q \rightarrow r$
5. $(p \vee q) \wedge (p \rightarrow r)$

Exercise 2

- Which of the following sentences has the logical form $(p \wedge q) \rightarrow r$

1. If the baby is crying then you can't sleep
2. My students are smart and hardworking
3. Jane will go by bus but Tom will walk
4. If the sky is clear or the moon is full then night driving will be safe
5. If our team wins and all the students get A+ grades then everyone will be happy

Exercise 3

- A = **The field is flooded**
- B = **It rained 22 inches**
- C = **The plants will die**
- Make propositions from A, B and C to math the following:

1. $A \vee B$ e.g.: “The field is flooded or it rained 22 inches”
2. $A \wedge B$
3. $B \rightarrow A$
4. $(A \wedge B) \rightarrow C$
5. $(A \vee B) \rightarrow C$
6. $(\neg A \wedge \neg B) \rightarrow \neg C$

Exercise 4

- **Q1**: Why an atomic proposition can't be a tautology?
- **Q2**: Given:
  - A= **Kat met Clayton**
  - B= **Kat and Clayton had coffee together**
  - C= **Kat and Clayton went swimming**

  Present the following as symbolic formulas:
  - **Kat and Clayton had a cup of coffee then went swimming**
  - **Kat met Clayton then they had a cup of coffee and went swimming**
- **Q3**: Is this proposition a tautology: $B \rightarrow \neg(\neg B)$

Predicate Logic

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Some Classical Examples

Convert the following English sentences into Predicate Logic format:

1. Marcus was a man

1. Marcus was a Pompeian

Some Classical Examples (cont'd)

3. All Pompeiians were Romans

3. Caesar was a ruler

Some Classical Examples (cont'd)

5. All Romans were either loyal to Caesar or hated him

5. Everyone is loyal to someone

Some Classical Examples (cont'd)

7. People only try to assassinate rulers they are not loyal to

7. Marcus tried to assassinate Caesar

Expressing Exceptions (Predicate Logic)

- “Tiger was a cat. Tiger neither loved nor hated mice”
  - Cat(Tiger) ∧
    ¬[loved(Tiger, mice) ∨ hated(Tiger, mice)]

- “Jon is a banker. Jon likes money but does not like work”
  - Banker(Jon) ∧ like(Jon, money) ∧ ¬like(Jon, work)

Set Theory

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Practice Exercise 1

- U = {natural numbers};
- (U is a reference set which represents all the possible elements for the questions under study
  - A = {1, 2, 4, 6, 8, 10, 11, 12, 13, 14, 15}
  - B = {1, 3, 6, 7, 8}
- State whether each of the following is true or false:
  - (a) $2 \in A$
  - (b) $11 \in B$
  - (c) $4 \notin B$
  - (d) $A \in U$
  - (e) $6 \in (A \cap B)$
  - (f) $7 \in (A \cap B)$
  - (g) $4 \in (A \cup B)$
  - (h) A = {odd numbers}

Practice Exercise 2

- The following sets have been defined by using the set builder notation
- Set = { x | Include every x that meets the condition that follows }

- Write down one or two elements of each set:
  - (a) {x | (x> 10) ∩ (x < 100)}
  - (b) {x | (x is a capital city in Asia}
  - (c) {x | (x is capital of a state in the U.S.A.}
  - (d) {x | x is a capital city in Europe}
  - (e) {x | x = 2n - 5, x and n are natural numbers}
  - (f) {x | $4x^2 = 36$, x is an integer}
  - (g) {x | $x = n^2$, x and n are natural numbers}

Practice Exercise 3

State whether each of the following is true or false

a) $\emptyset = \{\ \}$
b) $\emptyset = \{0\}$
c) $(A \cup B)' = A' \cap B'$
d) $(A \cap B)' = A' \cup B'$
e) $(A \cup B)' = A \cap B$
f) $(A \cap B)' = A \cup B$
g) $\{1, 2, 3\} = \{3, 2, 1\} = \{2, 1, 3\}$
h) Let there be a set $S = \{1, 2, 4, 1, 5\}$
i) x is an *element* of A is denoted as: $x \subset A$
j) The # of elements in a set is called *cardinality*