

## Stat 6021: Homework Set 6

1. You will use the `birthwt` data set from the `MASS` package for this question. The data were collected at Baystate Medical Center, Springfield, Mass during 1986. The data contain information regarding weights of newborn babies as well as potential predictors. For this question, we will focus on using two predictors: *age*, the mother's age in years, and *race*, the mother's race which is coded as 1 for white, 2 for black, and 3 for other. The response variable is *bwt*, the weight of the baby at birth in grams.
  - (a) Produce a scatterplot of *bwt* against *age*. Be sure to have separate plots and overlay the regression lines for each of the three racial categories. Based on this plot, explain why there is an interaction effect between the age of the mother and the race of the mother.
  - (b) Fit a regression equation with interaction between the two predictors. How does this regression equation relate the age of the mother and the weight of the baby at birth for each of the three racial categories?
2. (No R required) This question is based on data about teacher salaries from the 50 states plus DC (so  $n = 51$ ) in the mid 1980s. The variables are:
  - *PAY*,  $y$ : average annual public school teacher salary, in dollars.
  - *SPEND*,  $x_1$ : Spending on public schools per student, in dollars.
  - *AREA*: Region (North, South, West).

Table 1 below provides some summary statistics of the data:

Region	$n$	Mean PAY	Mean SPEND
North	21	\$24424	\$3901
South	17	\$22894	\$3274
West	13	\$26159	\$3919

Table 1: Summary Statistics of Teacher Pay

- (a) Based only on Table 1, briefly comment on the relationship between geographic area and mean teacher pay.

- (b) Based only on Table 1, briefly comment on the relationship between mean public school expenditure (per student) and mean teacher pay.
- (c) Briefly explain why using a multiple linear regression model with teacher pay as the response variable with geographic area and public school expenditure (per student) can give further insight into the relationship(s) between these variables.
3. (No R required) This question is a continuation of question 2. A regression with interaction was fitted, i.e.,

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 I_2 + \beta_3 I_3 + \beta_4 x_1 \cdot I_2 + \beta_5 x_1 \cdot I_3,$$

where  $I_2$  and  $I_3$  are the dummy codes for *AREA*.  $I_2 = 1$  if *AREA* = South, 0 otherwise, and  $I_3 = 1$  if *AREA* = West, 0 otherwise.

The following output from R for the extra sums of squares is shown below.

Analysis of Variance Table						
Response: PAY						
	Df	Sum Sq	Mean Sq	F value	Pr(>F)	
SPEND	1	608555015	608555015	117.7856	3.764e-14	***
AREA	2	22606468	11303234	2.1877	0.1240	
SPEND:AREA	2	9720281	-----	-----	-----	
Residuals	45	232498501	5166633			

- (a) Carry out a hypothesis test to see if the interaction terms are significant.
- (b) Regardless of your answer from part 3a, suppose the interaction terms are dropped. The following is output from the model without interaction.

Coefficients:					
	Estimate	Std. Error	t value	Pr(> t )	
(Intercept)	1.160e+04	1.334e+03	8.690	2.43e-11	***
SPEND	3.289e+00	3.176e-01	10.354	1.03e-13	***
AREASouth	5.294e+02	7.669e+02	0.690	0.4934	
AREAWest	1.674e+03	8.012e+02	2.089	0.0422	*
#####					
##Variance-Covariance matrix for beta hats##					
#####					
	(Intercept)	SPEND	AREASouth	AREAWest	
(Intercept)	1780535.6980	-393.5597348	-491859.07243	-2.381145e+05	
SPEND	-393.5597	0.1008967	63.18227	-1.870101e+00	
AREASouth	-491859.0724	63.1822716	588126.71689	2.442380e+05	
AREAWest	-238114.5499	-1.8701007	244238.02959	6.418738e+05	

What is the reference class for this model?

- (c) What is the estimate of  $\beta_2$ ? Give an interpretation of this value.
- (d) Using the Bonferroni procedure, compute the 95% family confidence intervals for the difference in mean response for *PAY* between teachers in the
  - i. North region and the South region;
  - ii. North region and the West region;
  - iii. South region and the West region.
- (e) What do your intervals from part 3d indicate about the effect of geographic region on mean annual salary for teachers?