

Let a_i, b_i be coprime positive integers for $i = 1, 2, \dots, k$, and m the least common multiple of b_1, \dots, b_k . Prove that the greatest common divisor of $a_1 \frac{m}{b_1}, \dots, a_k \frac{m}{b_k}$ equals the greatest common divisor of a_1, \dots, a_k .

设 a_i, b_i 是两两互质的正整数, 其中 $i = 1, 2, \dots, k$, 设 m 为 b_1, \dots, b_k 的最小公倍数。证明: $a_1 \frac{m}{b_1}, \dots, a_k \frac{m}{b_k}$ 的最大公约数等于 a_1, \dots, a_k 的最大公约数。

Let $x_0 = 5$ and $x_{n+1} = x_n + \frac{1}{x_n}$ ($n = 0, 1, 2, \dots$). Prove that $45 < x_{1000} < 45.1$.

设 $x_0 = 5$, 且 $x_{n+1} = x_n + \frac{1}{x_n}$ ($n = 0, 1, 2, \dots$)。证明: $45 < x_{1000} < 45.1$ 。