$$u = [u_x, u_y], \nabla f = \left[\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right] f$$

$$\nabla u = \begin{bmatrix} \frac{\partial u_x}{\partial x} & \frac{\partial u_x}{\partial y} \\ \frac{\partial u_y}{\partial x} & \frac{\partial u_y}{\partial y} \end{bmatrix} = \begin{bmatrix} \nabla u_x \\ \nabla u_y \end{bmatrix}$$

$$\begin{aligned} \nabla.\left(H \nabla u\right) &= \begin{bmatrix} \nabla.\left(H \nabla u_x\right) \\ \nabla.\left(H \nabla u_y\right) \end{bmatrix} = \begin{bmatrix} \nabla H.\nabla u_x + H \nabla.\left(\nabla u_x\right) \\ \nabla H.\nabla u_y + H \nabla.\left(\nabla u_y\right) \end{bmatrix} \\ &= \nabla H. \begin{bmatrix} \nabla u_x \\ \nabla u_y \end{bmatrix} + H \nabla. \begin{bmatrix} \nabla u_x \\ \nabla u_y \end{bmatrix} = \nabla H.\nabla u + H \nabla.\left(\nabla u\right). \end{aligned}$$