# Types of Models: WHAT's in the BOX

Conceptual.............Mathematical

Static………….Dynamic :*TIME*

Lumped.............Spatially Distributed: *SPACE*

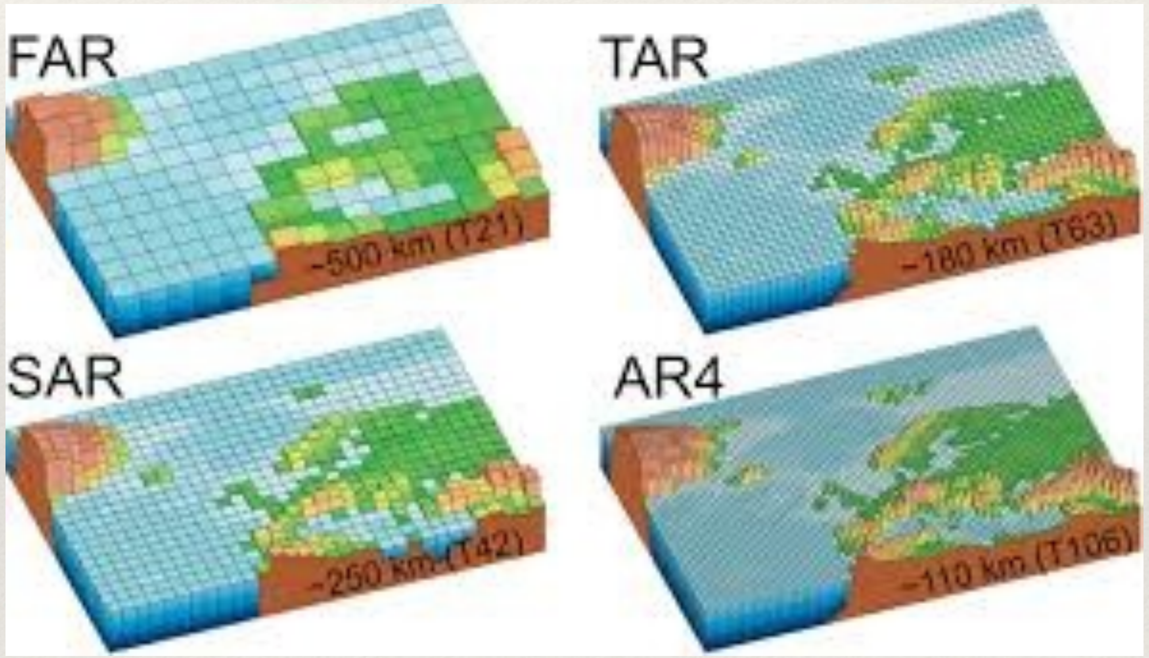
Stochastic.............Deterministic

Abstract………….Physically/Process Based

but biggest differences may often be the degree specific processes/parameters are accounted for

# Lumped ...Spatially distributed

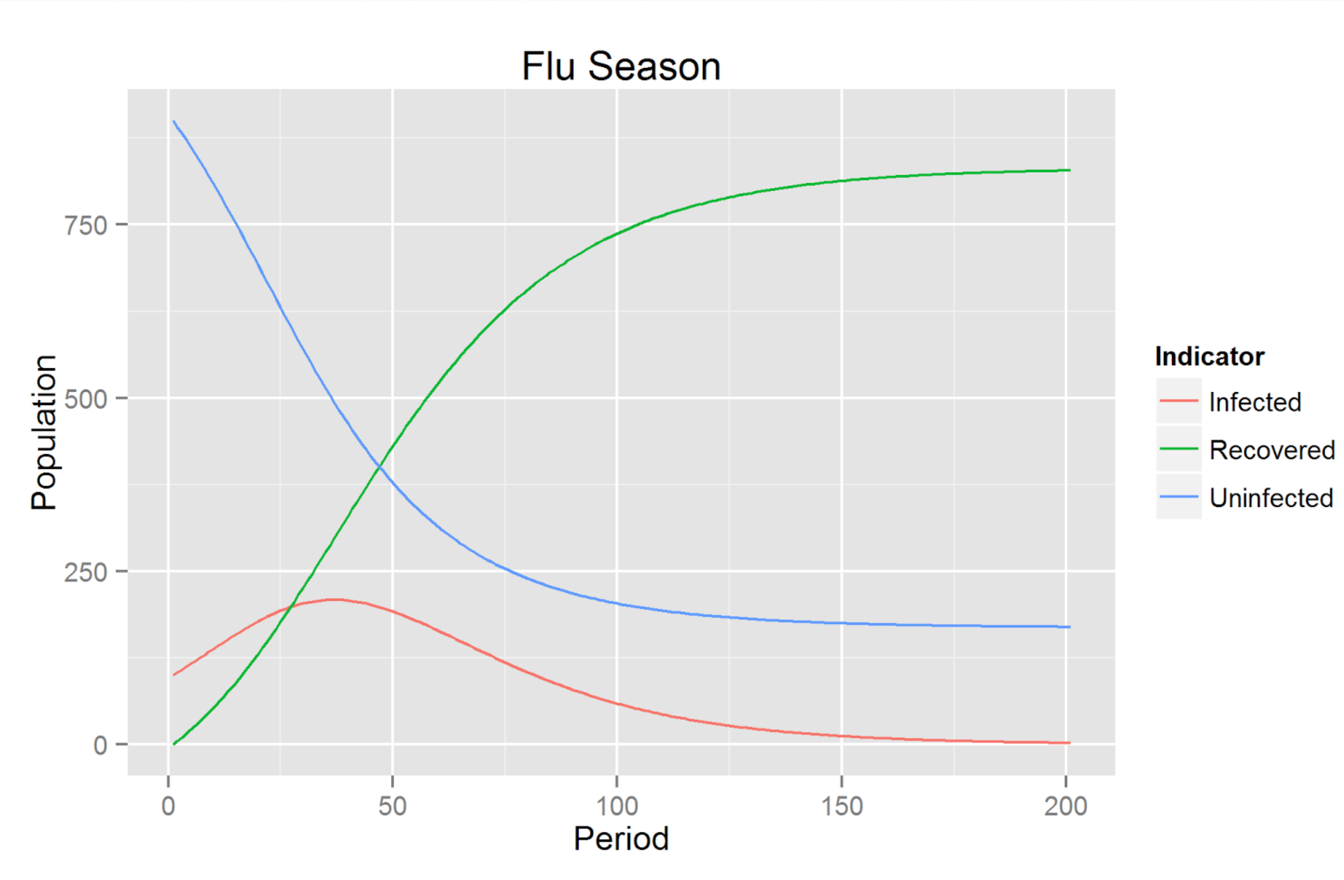
- Lumped - single point in space, or space doesn't matter
- Spatially distributed - model is applied to different "patches" in space
    - spatial units are independent
    - **spatial units interact with each other**




http://eo.ucar.edu/staff/rrussell/climate/modeling/climate_model_resolution.html

# Static- Dynamic Time Varying

- Static - Processes or Variables modeled do not evolve with time
- Dynamic - model elements evolve through time - and variables/results at one time step typically depends on previous time step




http://www.econometricsbysimulation.com/2013/05/sir-model-flue-season-dynamic.html

# Dynamics - connection in space and time

- Dynamic modeling is common in environmental problems solving
- Similar issues: what happens at one place, depends on neighbors; what happens at one time; depends on previous time
- Space - two way; Time is usually one-way
- Dynamic system modeling - quickly becomes complex (Engineering degrees spend a lot of time on this; there are books, entire journals etc on this topic)

# Dynamics models

- Many environmental problems and questions can be related to
    - Diffusion
    - Population
- Both often require dynamics models; and both often require thinking about dynamics in space and in time

# Dynamic Systems

Some useful terminology

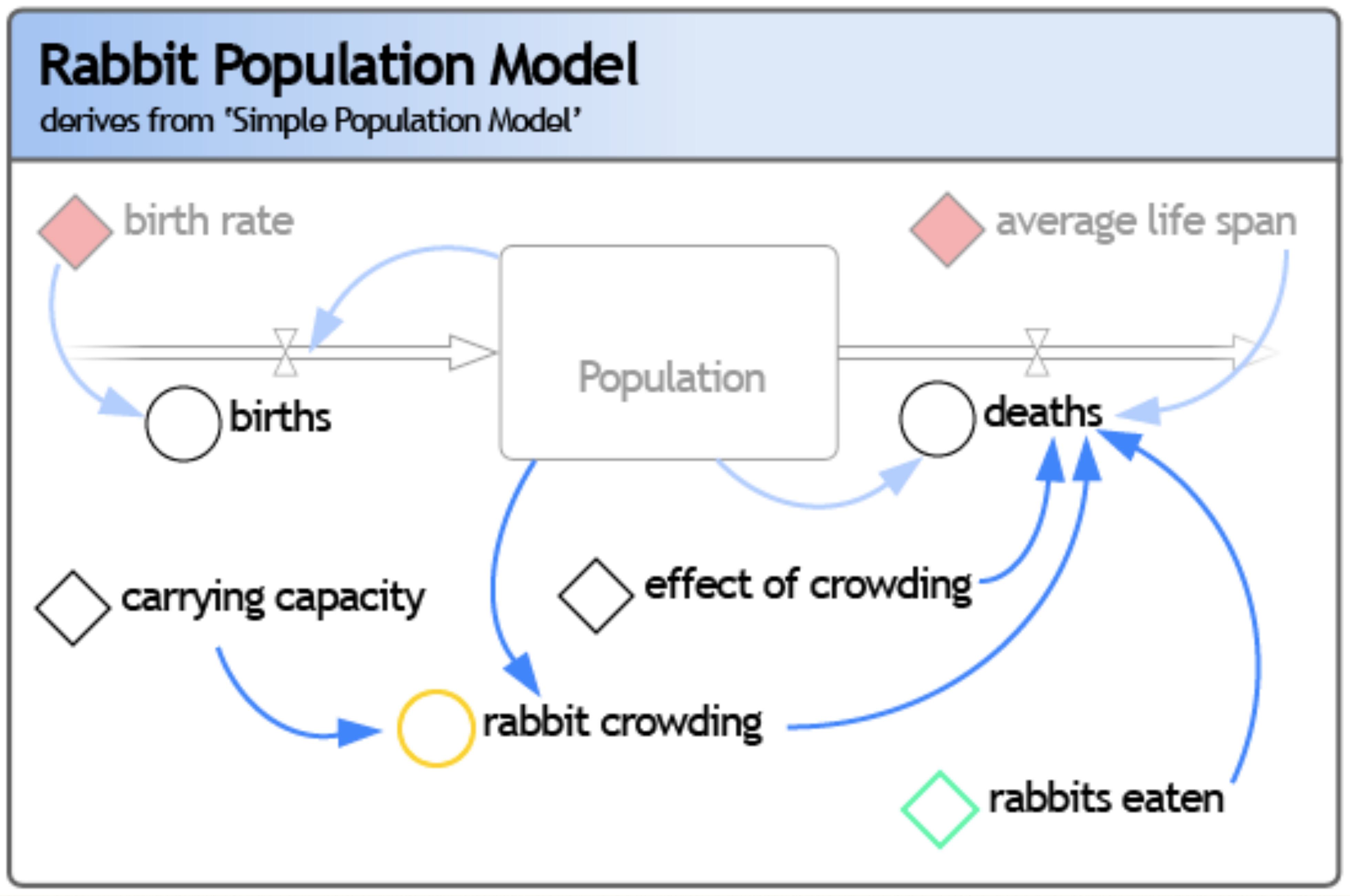
- *stocks* - variables that evolve over time
- *flows* - transfers between variables or from the system
- *parameters* - values that controls the relationship between stocks and flows
- *sink* - something that absorbs flows
- *source* - something that generates flows

# Dynamic Systems

- *System state*: value of all variables need to describe the "entity that evolves through time" at a particular point in time
  - usually think of these as stores (soil moisture, bank account balance, number of individuals in a particular age class)
- *State-space*: description of the entity may require multiple variables - for a watershed this could be soil moisture, water currently in dam and water stored in trees, and for each "grid" in a watershed)
- *State-space trajectories*: how the system state evolves through time, often involving transfers/flows between different stocks
- *Initial conditions*: values to describe the system state at the beginning
- *Dynamic systems modeling*: there at least one feedback loop

# Dynamic Systems

## Nature

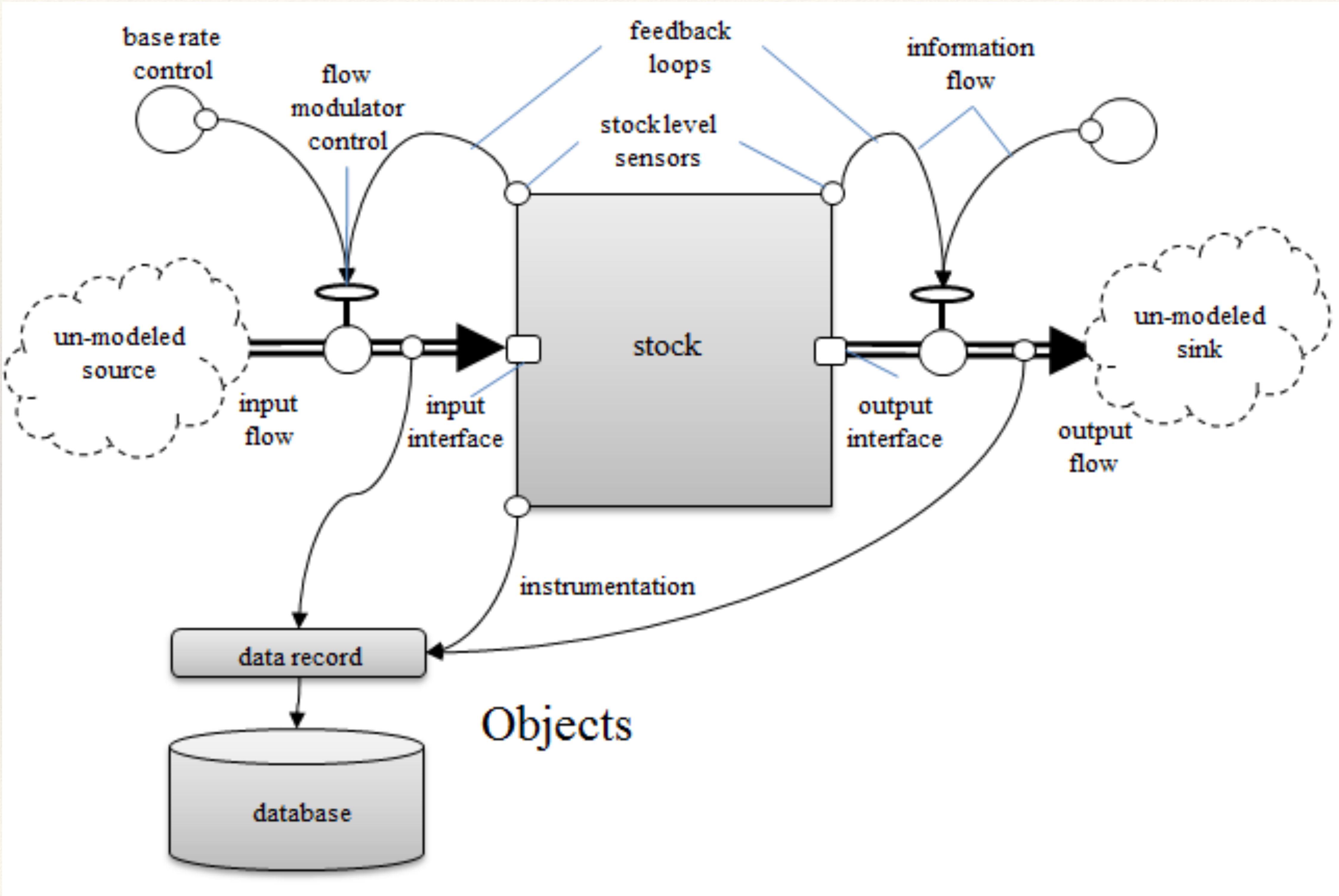



Notice how the change (flow in/out) of stock (population) depends on current value of stock

Feedback loop!

# Dynamic Systems

## Human Engineered



# Dynamic Systems

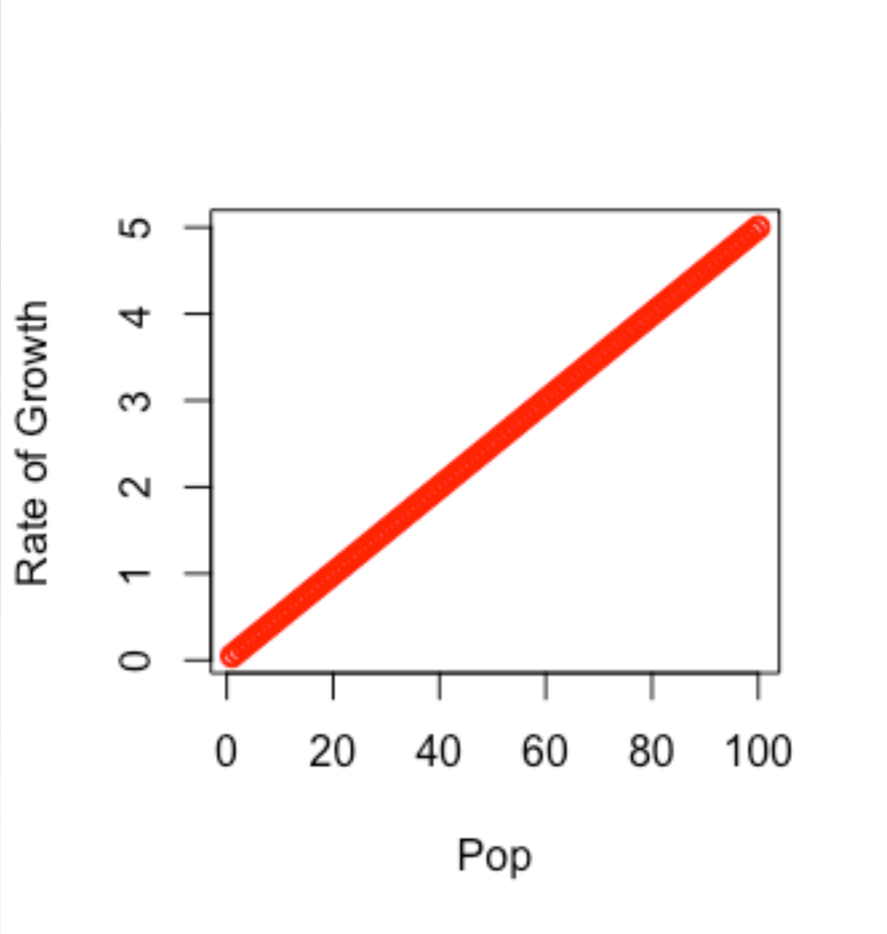
- dynamic systems have **feedback** loops
  - positive feedback
  - negative feedback
- feedback loops often lead to highly non-linear responses
- IF you have a feedback loop you need:
  - *difference* and *differential* equations: basically describe how the state evolves through time

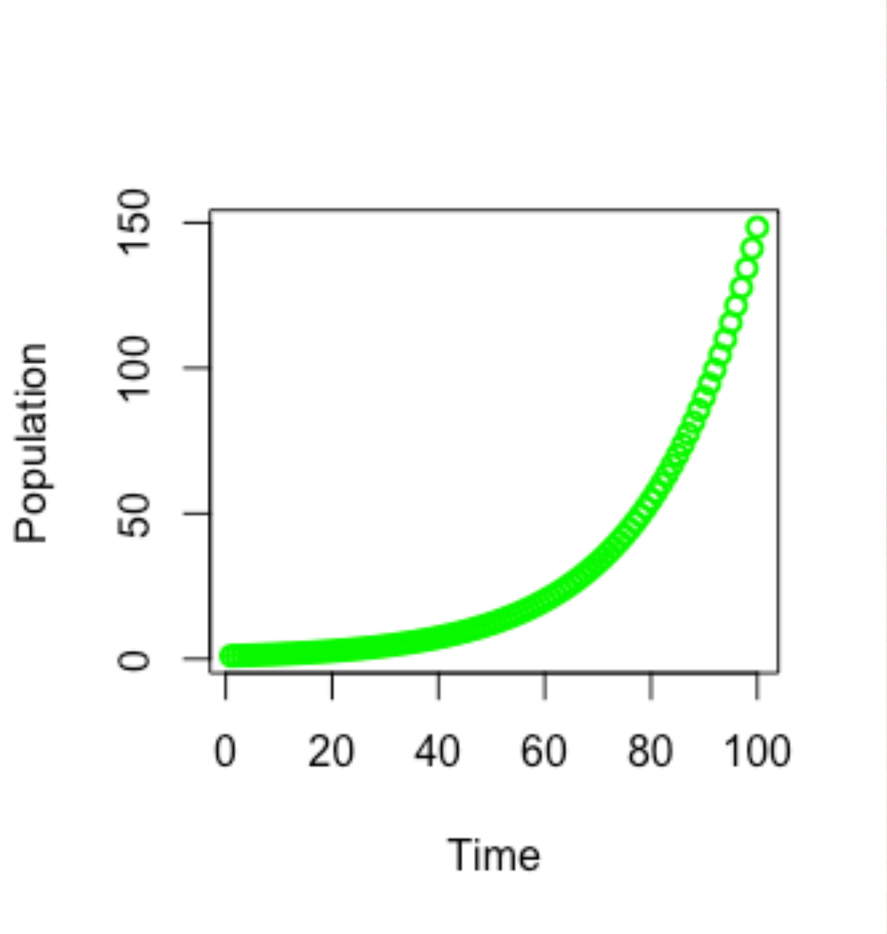
# Dynamic Systems

- Dynamic system may lead to stable or unstable states over time
  - stable …converge over time to a set of values or a repeated pattern
  - unstable…grow to infinity
  - chaotic - high sensitivity to initial conditions
  - for the same dynamic system (same set of equations), whether you are stable or unstable can depend on initial conditions and parameters

# Exponential Growth - Simple Dynamic System

- rate of growth(change) = r * population(density)
- differential equation
- $dP/dt = rP$



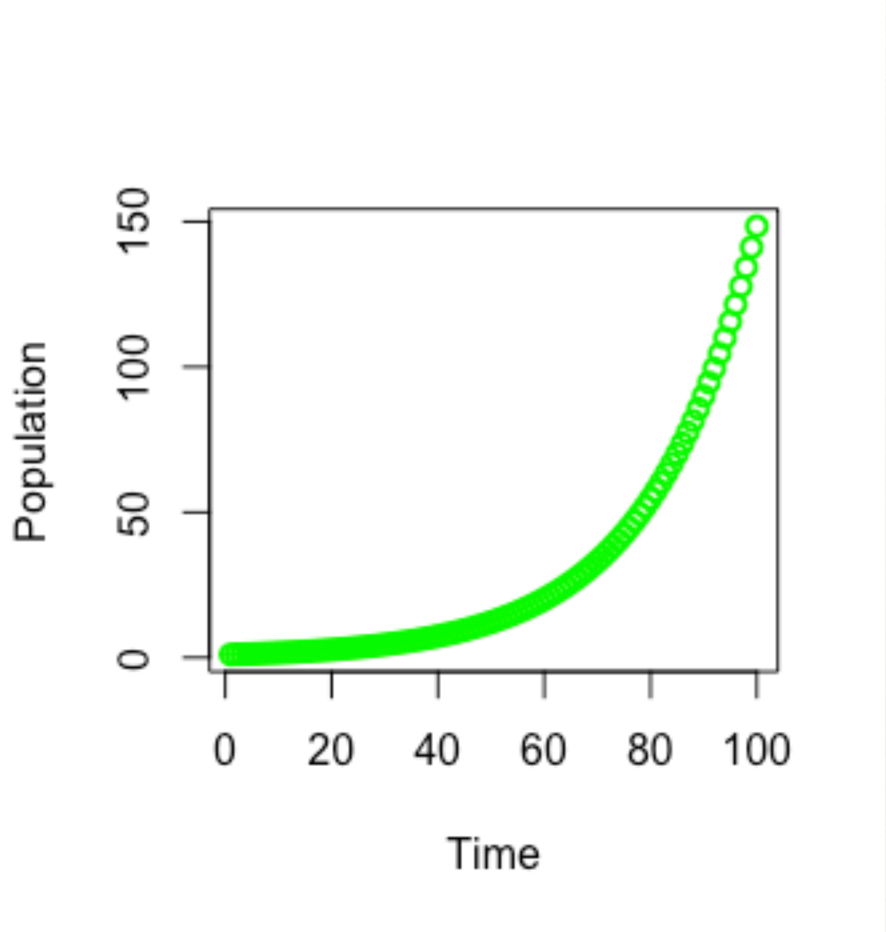



# Exponential Growth - Simple Dynamic System

- differential equation
- $dP/dt = rP$
- an analytic solution exists so we can write Population as a function of time (integrating both sides)
  - $P = P0 * \exp(rt)$

# Exponential Growth - Simple Dynamic System

```
#'  Simple population growth
#' @param T  period of growth
#' @param P initial population
#' @param r intrinsic growth rate
#' @return population at time T
#'
exppop = function(T,P0,r) {
   P = P0 * exp(r*T)
   return(P)
}
```




See Rmarkdown *SimpleDynamicModels*.Rmd

# Exponential Growth - Simple Dynamic System

**But what if we couldn't 'solve' it analytically ????**

Integrate the differential equation step by step

Also called numerical integration!

R has tools to help you do this!

**First** you need to code your differential equation as a function

# Integration, or Solving Differential Equations

- We want the value of the dependent variation (population) over a range of values for independent variable (time)
- We know how dependent variable is changing (that's the differential equation) $dP/dt = rP$
- For each P we can approximate the next P after a small time period
  - $Pt+1 = P + dP/dt$. Timestep
  - But as P changes $dP/dt$ changes so we have to keep time step small (really small if possible)

# Exponential Growth - Simple Dynamic System

Use R's ODE solver in the de**S**olve package - note the S

ODE works with ordinary differential equations…

Ordinary differential equations have derivatives of only one variable ((population) with respect to (time).,..it can have multiple order derivatives

dp/d**t** + d2p/d**t** + c = 0

Partial differential equations have derivatives with respect to more than one variable (think of spatial issues 3du/d**x** + 4du/d**y** = 0

**Numerical Integration**

Use R's ODE solver in the de**S**olve package

ODE requires initial conditions

Values of independent variable that you want results for

The function itself and parameters (as a single list)

# Example: Dynamic Systems

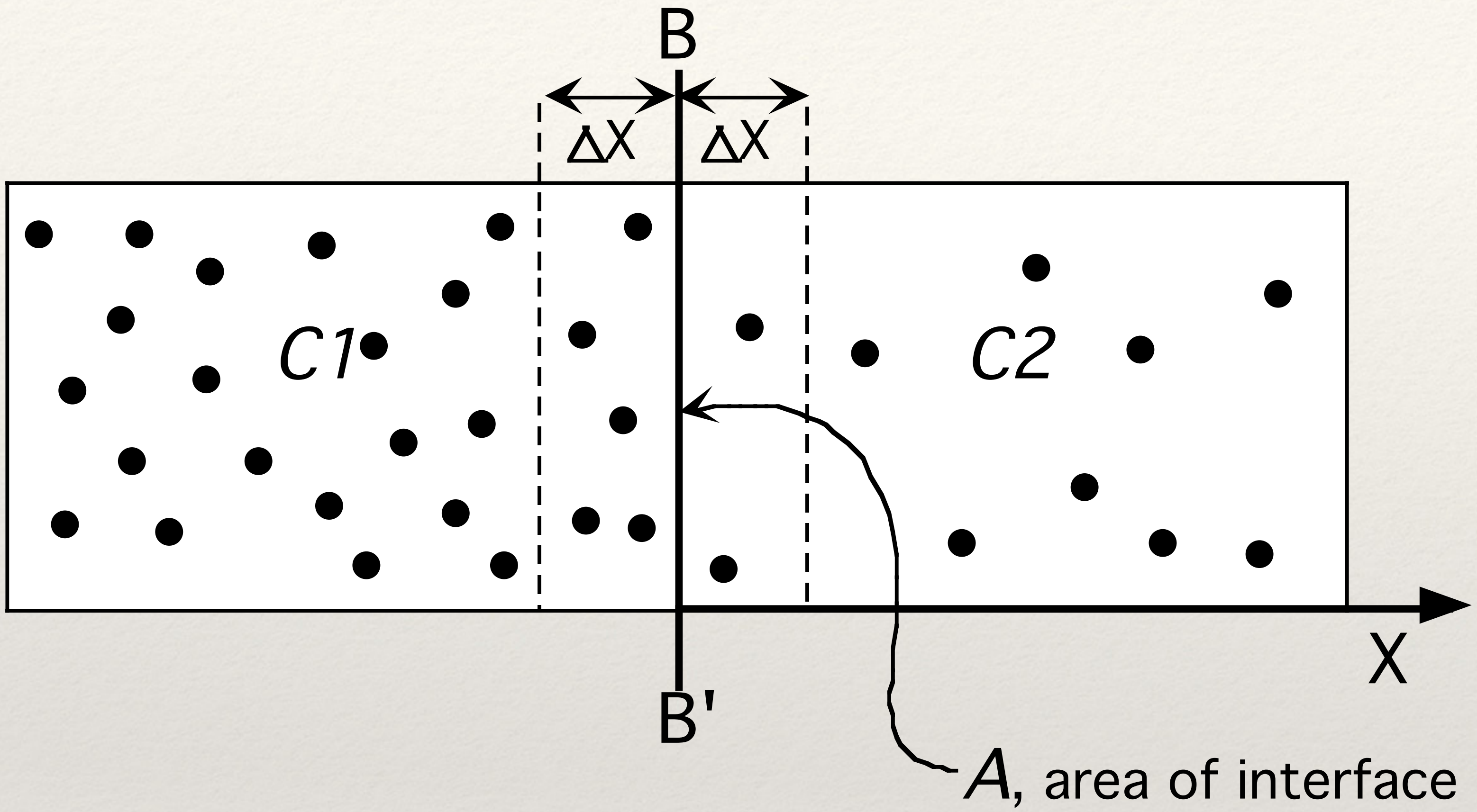
- Diffusion - Ficks Law
- flux (mass/time) =

  -Diffusivity * concentration gradient * area

  a concentration gradient is  C2-C1 / length

# Example: Dynamic Systems

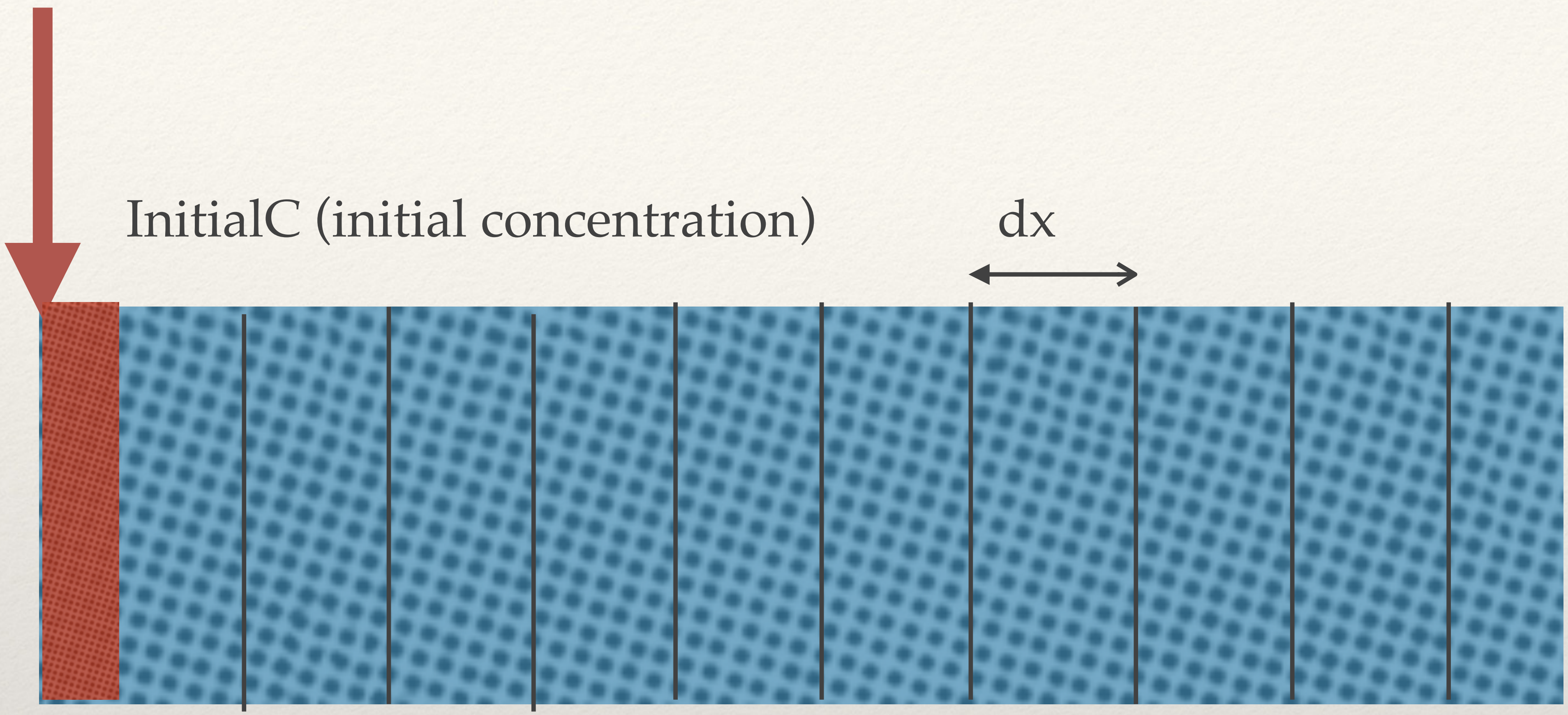
- Diffusion




How big that delta X is, depends on characteristics of the particles - often represented as a diffusivity (D) term

$$(5) \qquad q_x = \frac{0.5\Delta X A(C1 - C2)}{\Delta t}.$$

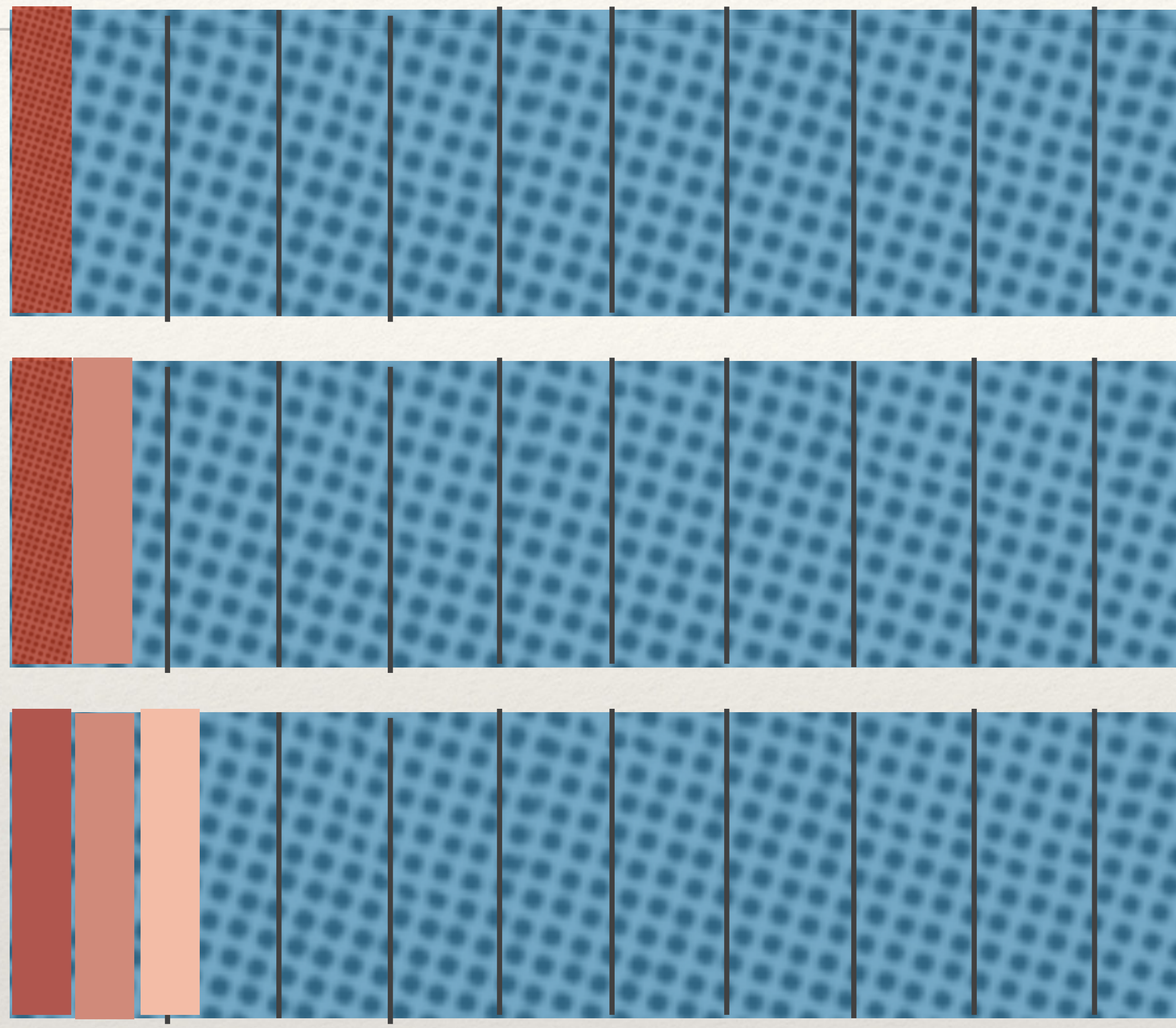
# Diffusion Example: Difference equation




Length = nx * dx

Nx: number of “boxes”
dx: discretization of space

# Diffusion Example



Timestep 1

Timestep 2

Timestep 3

**State evolves through time**
**(state space trajectory)**

…

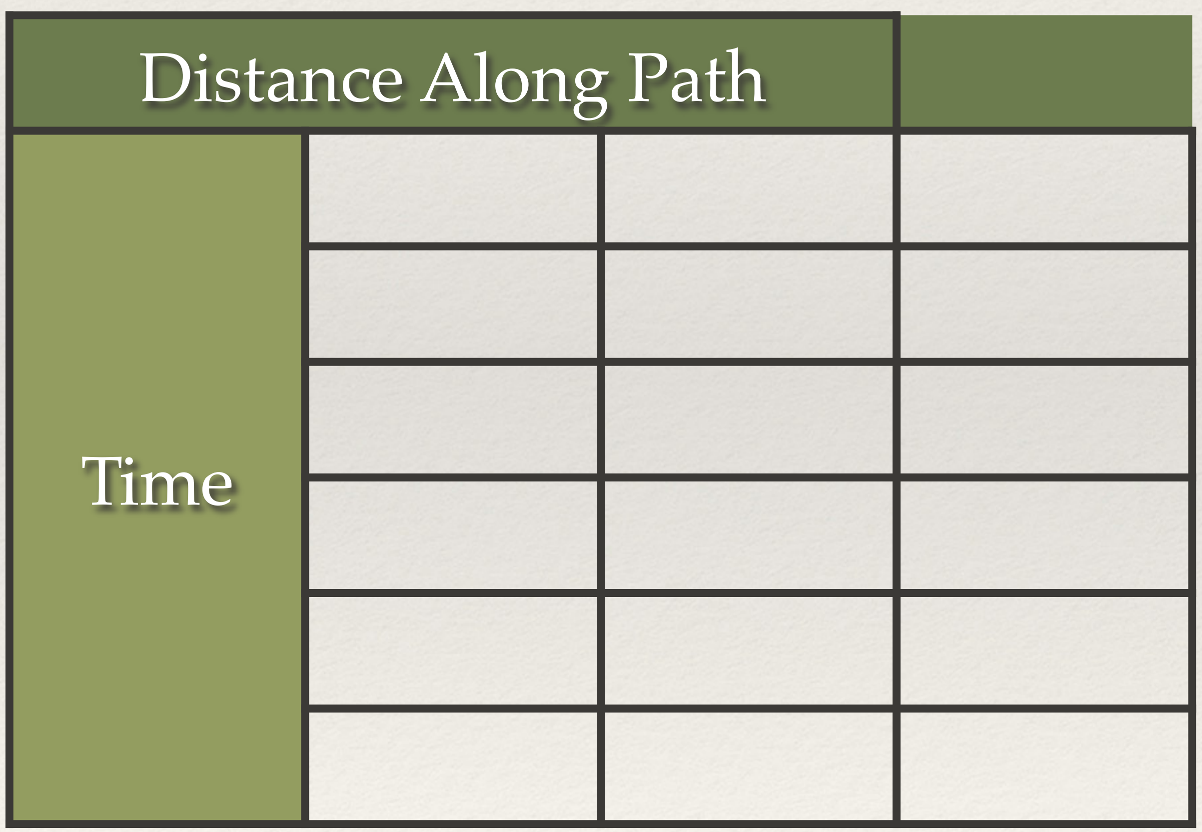


Total Simulation time = nt*dt

nt : number of time steps
dt - discretization of time

# Dynamic Modeling

- data structure to store the state (conc)
- 2-d array (rows are time, columns  are distance along path)
- use it to track concentration through time



# Dynamic - Diffusion modeling

- Choosing the appropriate time and space step is important - if either are too large then it is easy to overshoot and create unstable oscillations
- These are a due to using a discrete model (dividing things into units) to model what is actually a continuous process
- Trade-off - computational efficiency vs stability
- Differential equations help us to think through this problem - but often implemented in discrete ways