Types of Models: WHAT's in the BOX

Conceptual.............Mathematical

Static………….Dynamic :*TIME*

Lumped.............Spatially Distributed: *SPACE*

Stochastic.............Deterministic

Abstract………….Physically/Process Based

but biggest differences may often be the degree specific processes/parameters are accounted for

Dynamic Models

- solvers in R: for different types of differential equations;
- library(desolve)
- Two steps
  - model specification (implement your differential equation as a function)
  - model application (apply the differential equation to obtain an estimate)

Differential more than one variable

- For many dynamic systems, you may have more than one dependent variable that you are tracking through time (or space)
- These variables may interact
- If the differentials are all with respect to a single independent variable (e.g time) it is an ODE
    - use ODE solver in R

Predator / Prey Models - Competition
Vito Volterra and Alfred Lotka in 1925-6.

Alfred J. Lotka *(1880–1949) Founder of Theoretical Ecology*

Chemical Reactions

Vito Volterra *(1860–1940) Professor of Mathematical Physics, University of Rome*

Fish in the Adriatic Sea

Predator-Prey Models

- Predator-Prey models
- A simple approach that assumes
  - prey grow exponentially, with a fixed intrinsic growth rate
  - a fixed mortality rate of predators
  - a fixed rate of consumption/predation rate of prey by predators
  - a fixed conversion rate (ingestion rate) that determines how many "new" predators you get with predation
  - no environmental effects (e.g no carrying capacity)
- Note this has analog

Predator-Prey Model

- Prey
  - $d\text{prey}/dt = r_{prey} * \text{prey} - \alpha*\text{prey}*\text{pred}$
- Predator
  - $d\text{predator}/dt = \mathit{eff} * \alpha * \text{prey}*\text{pred} - \mathit{mort}*\text{pred}$

Predator-Prey Model

- As with diffusion, the basic form/ideas in this model can be applied elsewhere
  - economics (firm competition)
  - infectious disease

Differential equation - 1 independent variable "*time*" -ODE solver

As before, we need to generate our derivative function

inputs to function are (time, y, parameters)

outputs are derivatives

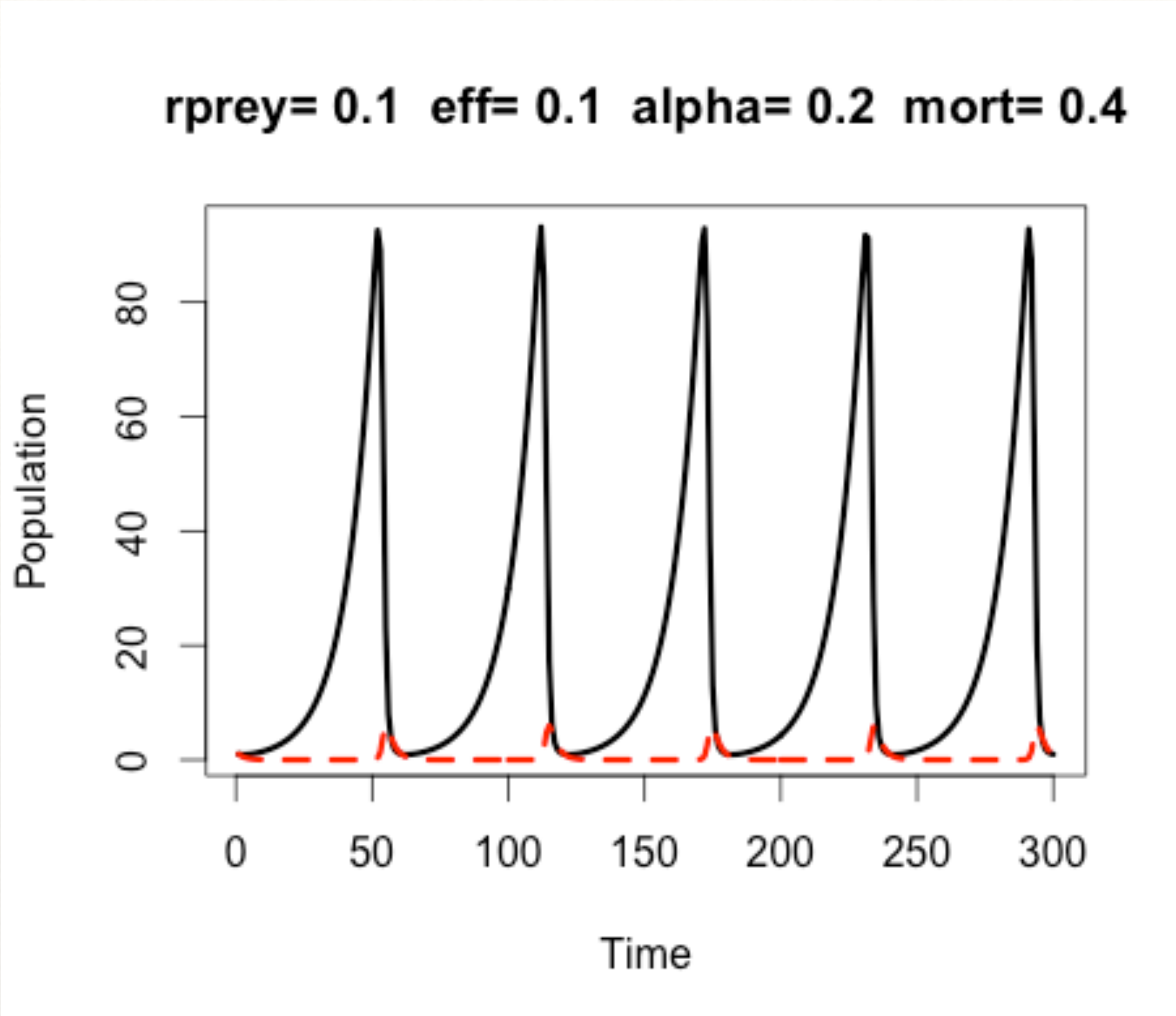
Our "y" or dependent variable now has two dimensions : pred, and prey; list with a pred and prey component to capture this

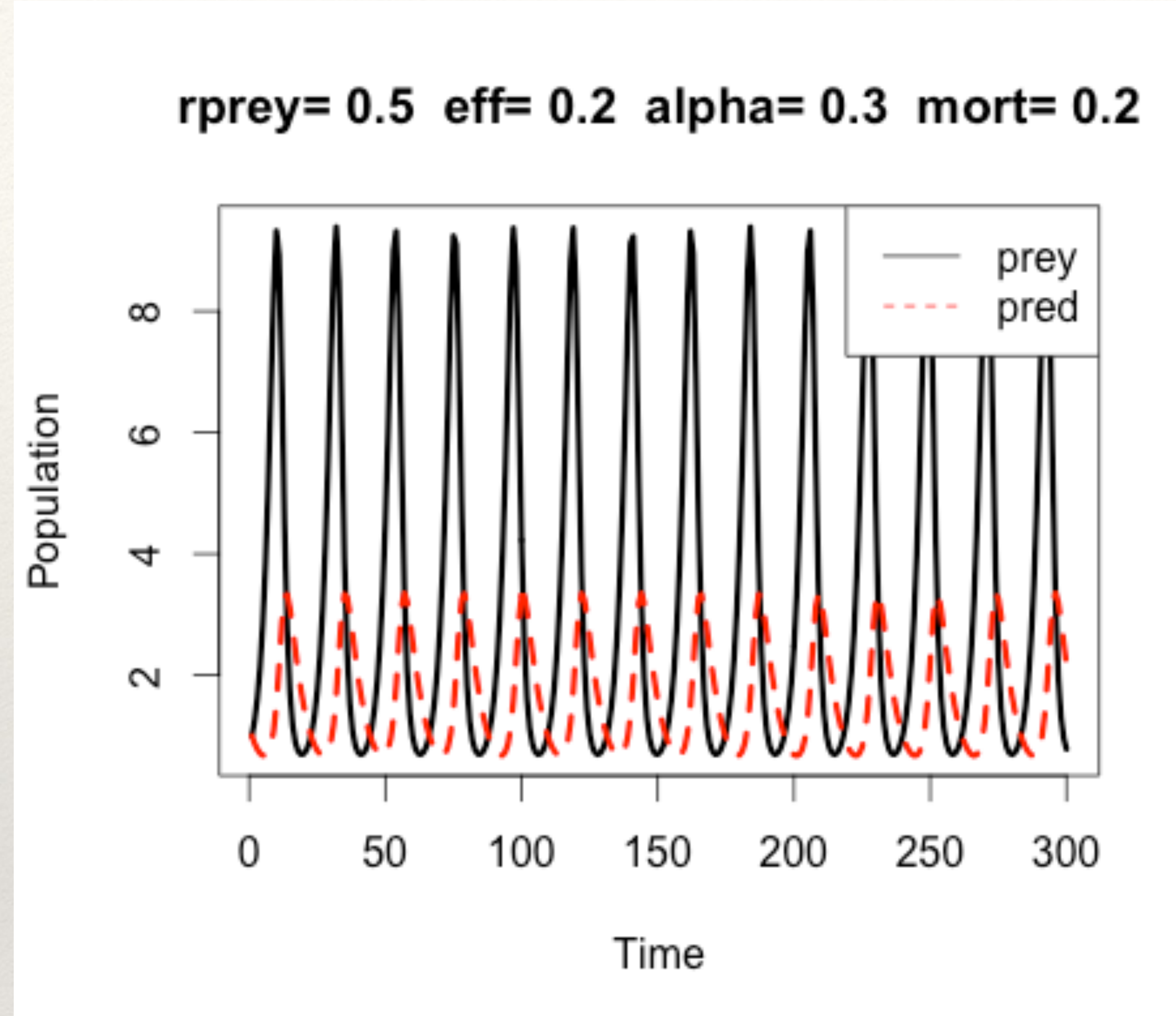
We will also have two derivatives - again we will use a list (2-d)

- Prey. $d\text{prey}/dt = r_{prey} * \text{prey} - \alpha * \text{prey} * \text{pred}$
- Predator. $d\text{predator}/dt = eff * \alpha * \text{prey} * \text{pred} - pmort * \text{pred}$

Predator-Prey Model

- Visualizing results: two interesting things
  - Populations over time
  - Population interactions with each other
- Common in dynamic systems of more than one variable
- Dynamic model of a firm (labor, production, profit)


rprey= 0.1 eff= 0.1 alpha= 0.2 mort= 0.4
Population
Time
0
20
40
60
80
0
50
100
150
200
250
300


rprey= 0.5 eff= 0.2 alpha= 0.3 mort= 0.2
prey
pred
Population
Time
2
4
6
8
0
50
100
150
200
250
300

When does population stop changing?

- Prey
  - $d\text{prey}/dt = r_{prey} * \text{prey} - \alpha*\text{prey}*\text{pred}$
- Predator
  - $d\text{predator}/dt = eff * \alpha* \text{prey}*\text{pred} - mort*\text{pred}$

When does population stop changing?

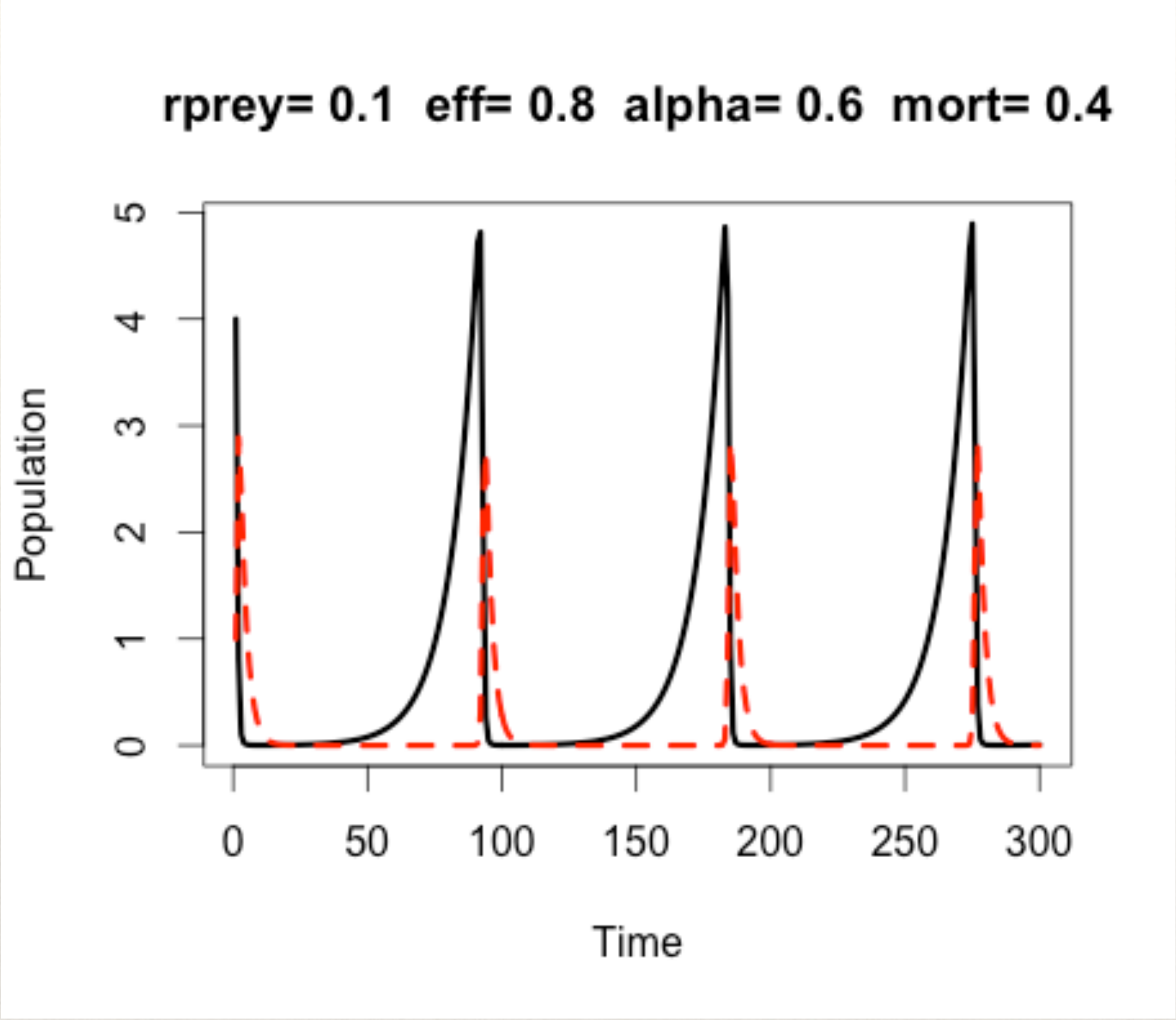
- derivative are zero
  - $0 = r_{prey}$ * prey - α*prey*pred
  - $0 = eff * \alpha$* prey*pred - $pmort$*pred
- occurs when pred = $r_{prey/\alpha}$
- and when prey = $pmort/(eff * \alpha)$

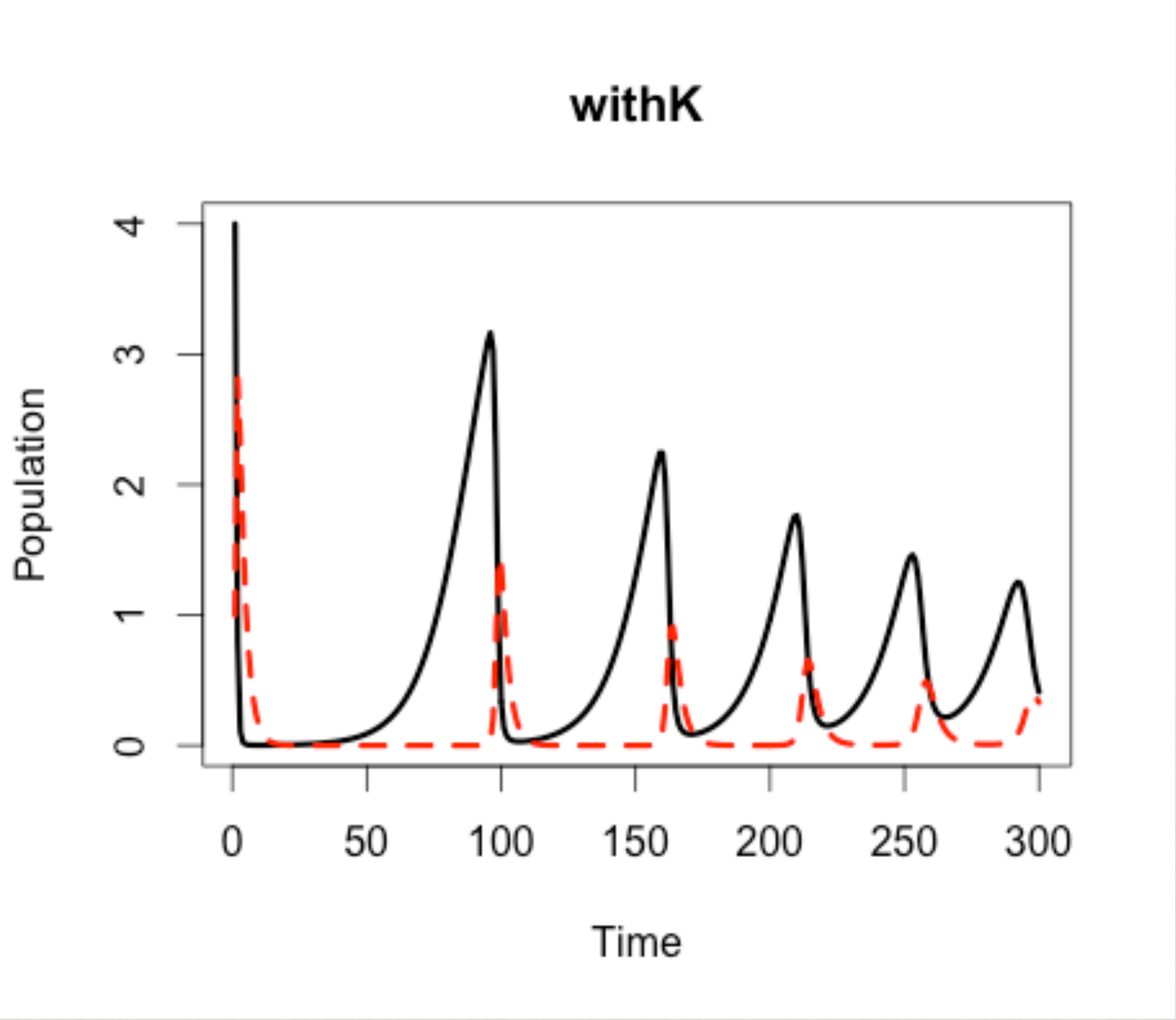
Predator-Prey Model

- Prey
  - $d\text{prey}/dt = (r_{prey}$ *(1-prey/K)*) prey - α*prey*pred
- Predator
  - $d\text{predator}/dt = \mathit{eff}$ * α* prey*pred - mort*pred

Expand to add in carrying capacity to the prey species

Predator-Prey Model





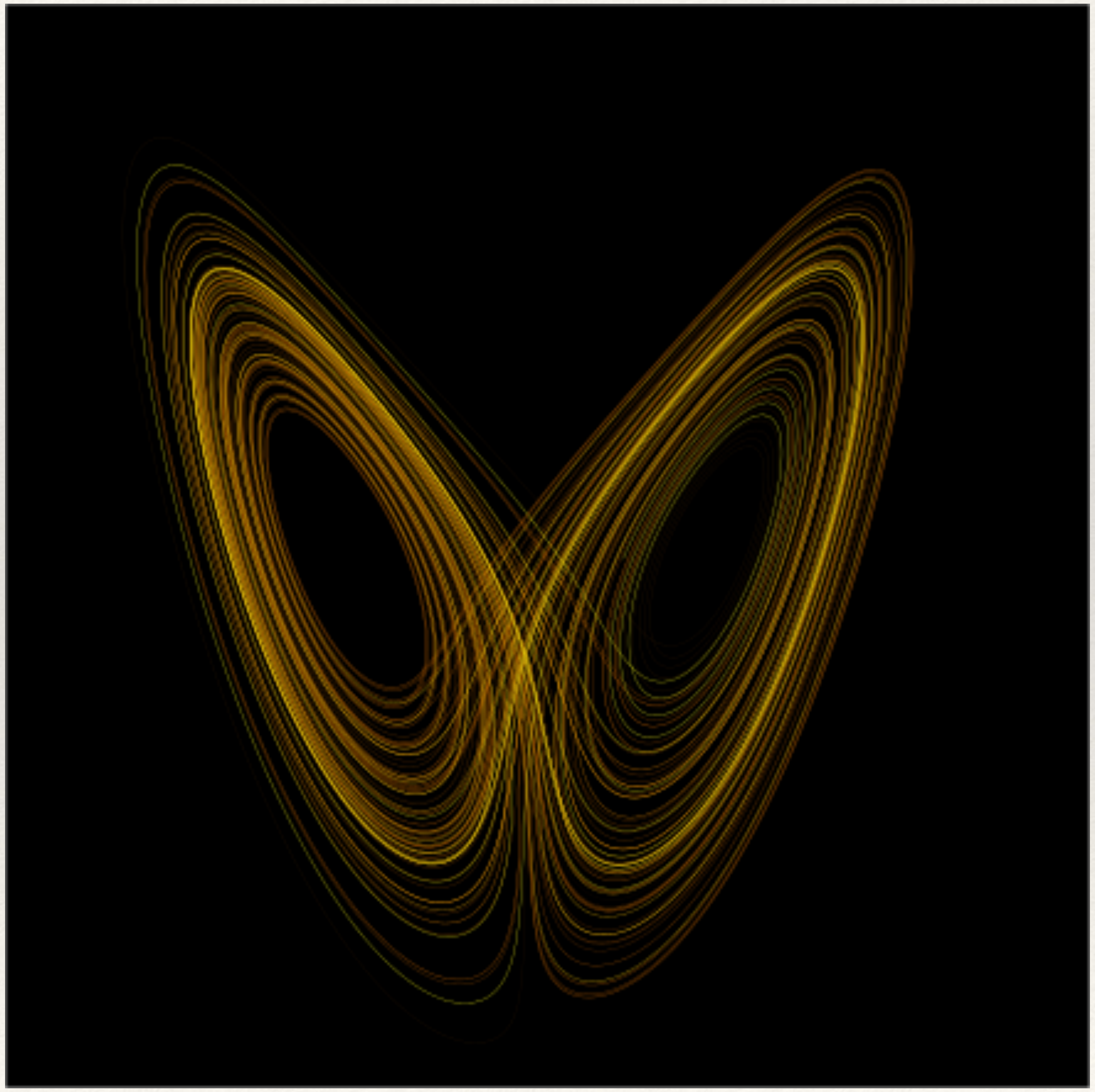
Competition Model

- Species A
  - $d\text{species}_{A} / dt = (r_A * (1\text{-species}_{A} / K_A)*)\ \text{species}_{A} - \alpha_{AB}*\text{speciesA}*\text{speciesB}$
- Species B
  - $d\text{species}_{B} / dt = (r_B * (1\text{-species}_{B} / K_B)*)\ \text{species}_{B} - \alpha_{BA}*\text{speciesA}*\text{speciesB}$

Similar to predictor - prey models - building on logistic growth

Dynamic Equations

- Lorenz equations are example of dynamic systems that can exhibit stable and chaotic states depending on parameters and initial conditions



A sample solution in the Lorenz attractor when $\rho = 28$, $\sigma = 10$, and $\beta = 8/3$

Dynamics

- Lorenz Equations (for fluid dynamics), P, R, B are parameters (fixed values), x,y,z variables that change with time (think of them as coordinates)

$$dx/dt = P(y - x)$$

$$dy/dt = Rx - y - xz$$

$$dz/dt = xy - By$$