Types of Models: WHAT's in the BOX

Conceptual.............Mathematical

Static………….Dynamic :*TIME*

Lumped.............Spatially Distributed: *SPACE*

Stochastic.............Deterministic

Abstract………….Physically/Process Based

but biggest differences may often be the degree specific processes/parameters are accounted for

Dynamic Models

- Exact versus numerical interaction (ODE solver)
- Some dynamic models are clearly discrete (not continuous as in diffusion)
- Age structured population models are often represented as a "system of equations" that evolve the age structure over time
- Sometimes called a population "matrix" model

Matrix (Age) Population Models

- Suppose a population has individuals in different age groups (lets say 4)
- Populations of individuals in each group: n0(t), n1(t), n2(t), n3(t), where t is time
- Group lump all individuals in that age range (even though in reality there will be a range of ages
- Dynamically model the evolution of that population

Matrix (Age) Population Models

- To evolve the population we also need to think about births, deaths and aging
  - births depend on fertility rates of the different groups
  - aging simply evolves one group to the next
  - death remove individuals from a group

many ways of defining growth

- if b is birth rate and d is death rate
- $n(t+1) = n(t) + (b-d)\, n(t)$
- $n(t+1) = (1+r)\, n(t)$ where r is an intrinsic rate of increase (proportional/per capita rate of change)
- $n(t+1) = l\, n(t)$ = where l is really the finite (geometric growth rate)

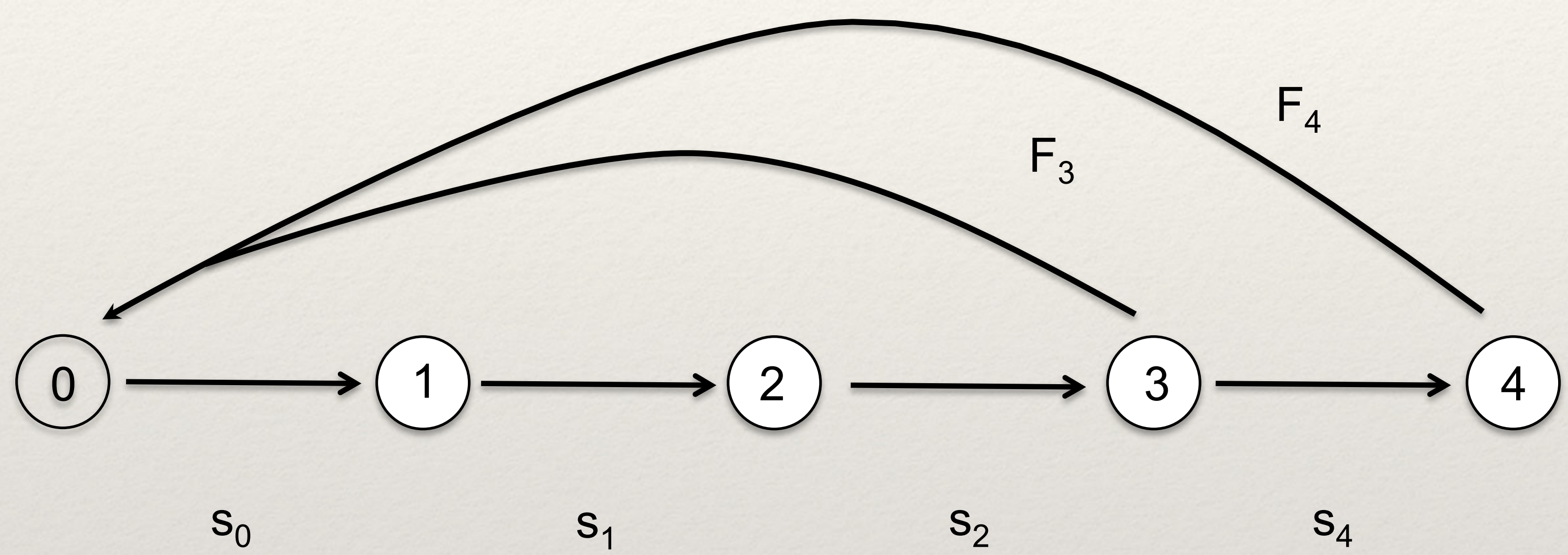
Matrix (Age) Population Models

- For multiple age classes we look survival probability
- define a survival parameter *Sk* that gives the fraction of individuals that survive from age class k to age class k+1,
  - for example $n2(t + 1) = S1 * n1(t)$.
  - in this case, the time step/increment must be the same as the increment between age classes!
  - we often work with 1 year but could be 1 month (but age classes would also be 1 month apart)

Matrix (Age) Population Models

- Births are little trickier because they may come from multiple age classes, so define the
- parameter *Fj* as the per capita fertility in age class j.
- The newly born all enter into age class 1, i.e.
- $n1(t + 1) = F_2{*}n2(t) + F_3{*}n3(t)$
- assume that population census is right after breeding
- note that fertility is not fecundity (birth per capita but included survivability - live almost year to be included)

- Use a matrix to keep track of populations in each age group



Leslie Matrix

- putting fertility and survivability together

$$L = \begin{bmatrix} F_0 & F_1 & F_2 & F_3 \\ S_0 & 0 & 0 & 0 \\ 0 & S_1 & 0 & 0 \\ 0 & 0 & S_2 & 0 \end{bmatrix}$$

Stability

- you can compute *l* (growth rate) of entire population by summing all the age classes
    - n = sum(n0+n1+n2…)
    - $l = n(t+1)/n(t)$
- a stable age distribution is one where even though total population may change the proportion in each age class stays the same
- at the point you will reach a asymptotic growth rate

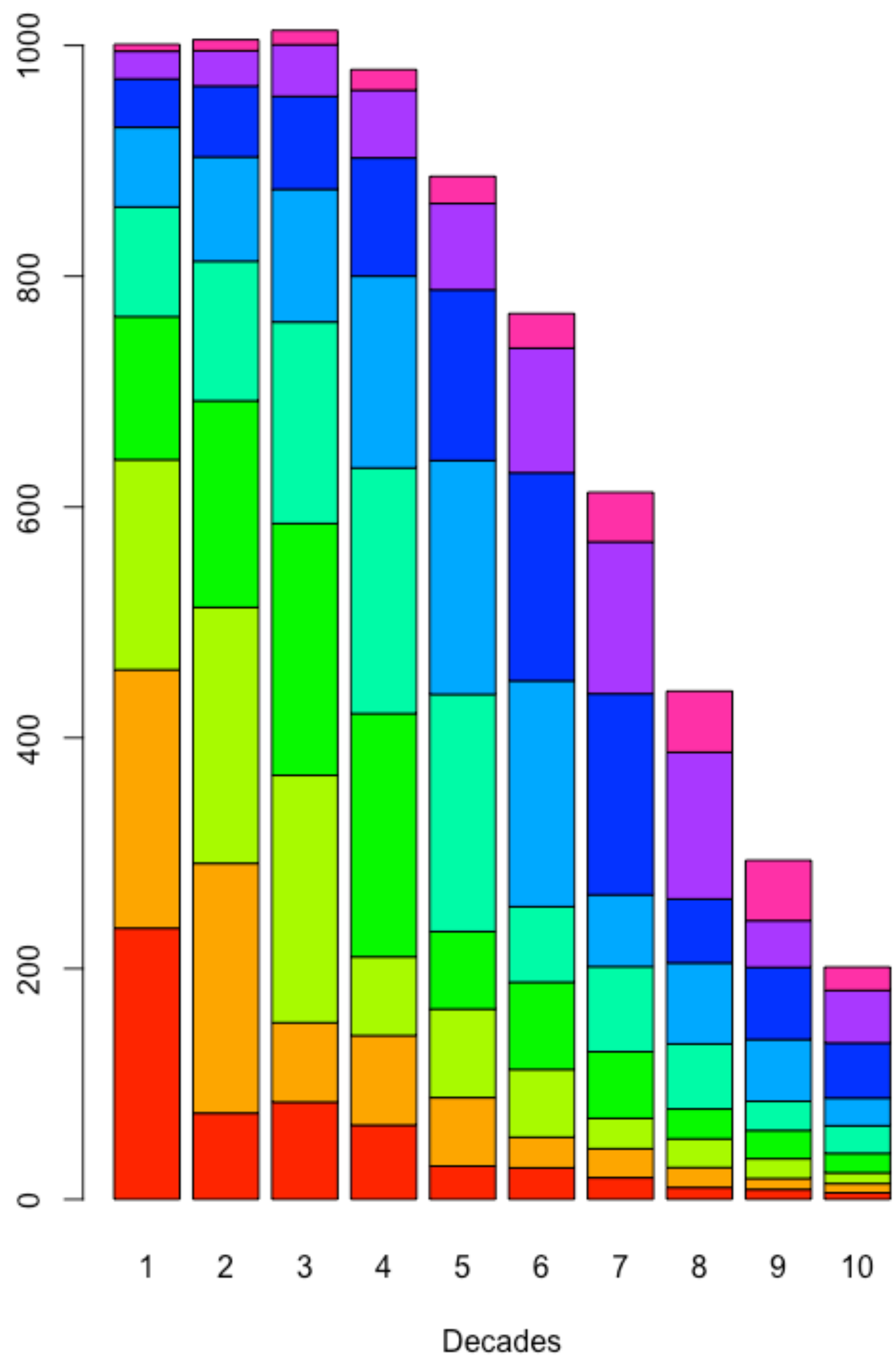
Putting all together

- we can write a function to evolve a population through time
- inputs = survivability, fertility, initial population, time steps
- output = final population matrix

Matrix Population Models

A typical problem:

Around 1980 China announced a goal of reducing its population from about 1 billion to about 700 million people. To do so, China was encouraging one-child households. Assume that about one quarter of the female population (about one eighth of the total population) between the ages of 10-19 had one child in any given decade, about half the female population between the ages of 20-29 had one child, and that about a quarter of the female population between the ages of 30-39 had one child. About how many decades will it take for China to achieve its goal?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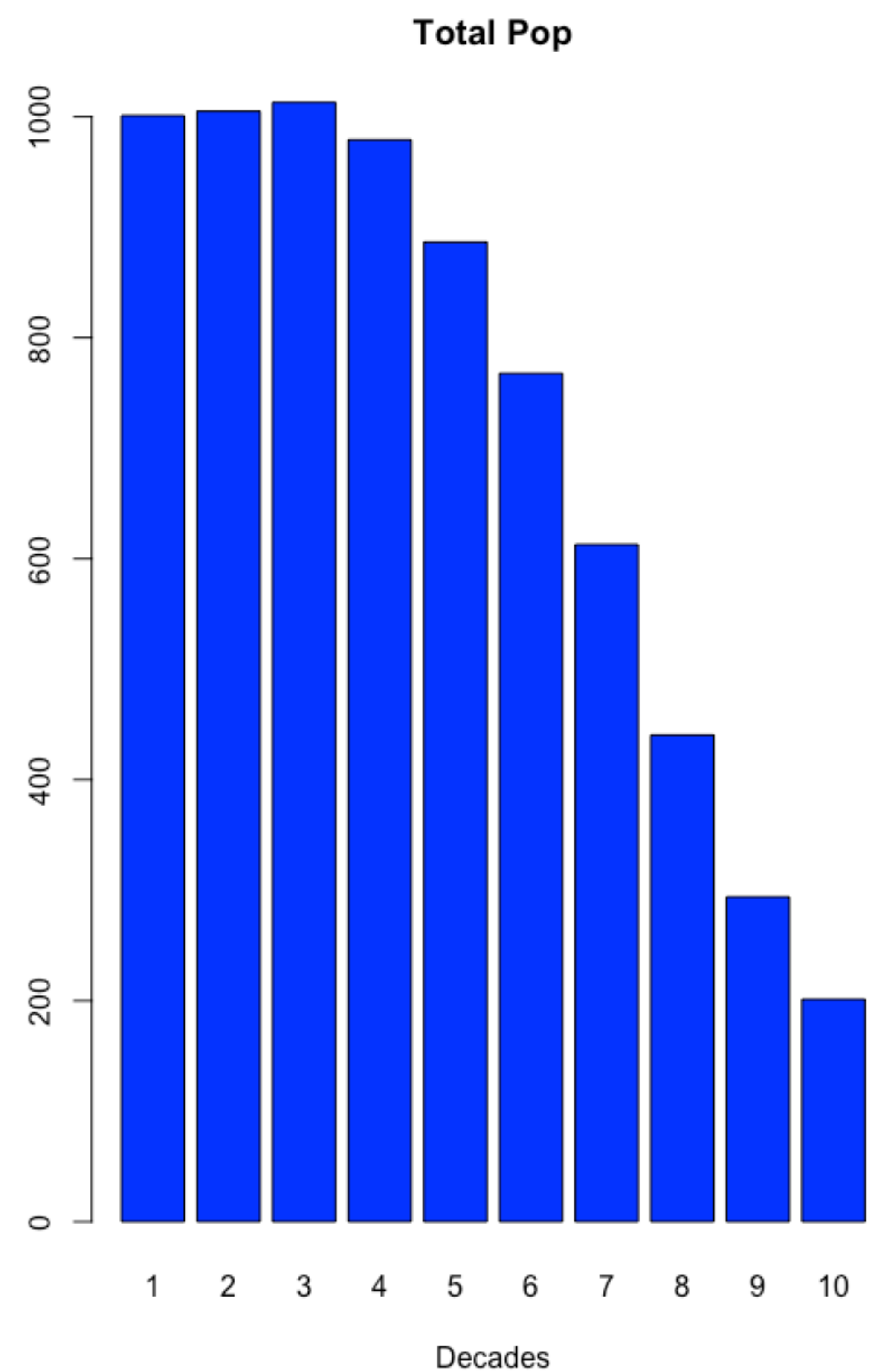
Decades


Total Pop
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Decades