1 Algoritmi

1.1 Metoda Bisecției

Algorithm 1: Metoda Bisecției

input : $a; b; f; \epsilon$ (precizia dorită)

do

  $x = \frac{a+b}{2}$;

  if $f(a) \cdot f(x) \geq 0$ **then**

    $b = x$;

  else

    $a = x$;

while $b - a \geq \epsilon$;

output: $x = \frac{a+b}{2}$

1.2 Metoda Tangentei

Algorithm 2: Metoda Tangentei

input : $a; b; f; \epsilon$

$y = a$;

if $f(y) \cdot f''(y) > 0$ **then**

  $y = b$;

do

  $x = y$;

  $y = \phi(x)$;

while $|y - x| \geq \epsilon$;

output: y

Ecuații pentru Metoda Tangentei:

- $\phi(x) = x - \frac{f(x)}{f'(x)}$
- $f'(x) = \frac{f(x_0+h)-f(x_0)}{h}$
- $h = 0.1$

1.3 Serie Alternantă cu ϵ

Algorithm 3: Serie Alternantă cu ϵ

input : $S' = 0; i = 0; \text{semn} = -1; \epsilon = 10^{-2}$
do
  $S = S'$;
  $i = i + 1$;
  $\text{semn} = \text{semn} \cdot (-1)$;
  $S' = S + \frac{\text{semn}}{i}$
while $|S' - S| \geq \epsilon$;
output: S

1.4 Serie cu termeni pozitivi

Algorithm 4: Serie cu termeni pozitivi

input : $S = \frac{2}{3}; x = \frac{2}{3}; n = 0; \epsilon = 10^{-2}$
do
  $n = n + 1$;
  $x = x \cdot \frac{n}{n+1} \cdot \frac{2}{3}$;
  $S = S + x$
while $3 \cdot x > \epsilon$;
output: S

2 Examen Calcul Numeric

$\boxed{1}$

Să se studieze teoretic seria $S = \sum_{n\geq 1} \frac{4^n}{n^3 \cdot 5^n}$ și să se facă algoritmul de calcul pentru calculul sumei seriei cu o precizie de $\epsilon = 10^{-3}$ (nu se cere calculul efectiv al sumei seriei)

$$S = \sum_{n\geq 1} \frac{4^n}{n^3 \cdot 5^n}, \epsilon = 10^{-3}$$

$$\frac{a_{n+1}}{a_n} = \frac{4^{n+1}}{(n+1)^3 \cdot 5^{n+1}} \cdot \frac{n^3 \cdot 5^n}{4^n} = \frac{\cancel{4^n} \cdot 4}{(n+1)^3 \cdot \cancel{5^n} \cdot 5} \cdot \frac{n^3 \cdot \cancel{5^n}}{\cancel{4^n}} = \frac{4}{5} \cdot \frac{n^3}{(n+1)^3} \Rightarrow$$

$$\Rightarrow \boxed{q = \frac{4}{5}}$$

$$R_{M-1} = a_M + a_{M+1} + ... \geq a_M + a_M \cdot q + a_M \cdot q^2 + ... \Rightarrow$$

$$\Rightarrow R_{M-1} \geq a_M(1 + q + q^2 + ...) \Rightarrow$$

$$\Rightarrow \boxed{R_{M-1} \geq a_M \cdot \frac{1}{1-q} \geq \epsilon}$$

Algoritm:

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input : $S = \frac{4}{5}; x = \frac{4}{5}; n = 0; \epsilon = 10^{-3}$
do
  $n = n + 1;$
  $x = x \cdot \frac{n^3}{(n+1)^3} \cdot \frac{4}{5};$
  $S = S + x;$
while $x \cdot 5 > \epsilon;$
output: S

$\boxed{2}$

Să se calculeze $\sqrt{8}$ cu o precizie de $\epsilon = 10^{-1}$ folosind metoda bisecției (metoda înjumătățiri intervalului).

$$f(x) = x^2 - 8$$

$$x^2 - 8 = 0 \Rightarrow x^2 = 8 \Rightarrow \boxed{x = \pm 2\sqrt{2}}$$

Calculăm $f(x)$ până găsim un interval $(-, +)$

$$f(0) = -8$$

$$f(1) = -7$$

$$\left.\begin{array}{l} f(2) = -4 \\ f(3) = 1 \end{array}\right\} \Rightarrow \text{luăm intervalul } \boxed{(a, b) = (2, 3)}$$

Calculăm $x = \frac{a+b}{2}$

$$\Rightarrow \boxed{x = \frac{5}{2}}$$

Verificăm $f(a) \cdot f(x) \geq 0$

$$f(2) \cdot f(\frac{5}{2}) \overset{?}{\geq} 0$$

$$-4 \cdot (\frac{25}{4} - 8) \overset{?}{\geq} 0$$

$$-4 \cdot (\frac{25-32}{4}) \overset{?}{\geq} 0$$

$$\underbrace{-4 \cdot -\frac{7}{4}}_{-\cdot-=+} \overset{?}{\geq} 0 \Rightarrow \text{F} \Rightarrow a = x$$

$$\Rightarrow \text{noul interval este: } \boxed{(a, b) = (\frac{5}{2}, 3)}$$

Verificăm $b - a \geq \epsilon$

$$3 - 2.5 \geq 0.1 \Rightarrow \text{A} \rightarrow \text{Continuăm}$$

$$x = \frac{\frac{5}{2} + 3}{2} = \frac{\frac{11}{2}}{2} \Rightarrow \boxed{x = \frac{11}{4}}$$

$$f(\frac{5}{2}) \cdot f(\frac{11}{4}) \overset{?}{\geq} 0$$

$$f(\frac{11}{4}) = \frac{121}{16} - 8 = \frac{121-128}{16} = -\frac{7}{10}$$

$$\Rightarrow f(\frac{5}{2}) \cdot f(\frac{11}{4}) \overset{?}{\geq} 0 \Rightarrow \text{F} \Rightarrow a = x$$

$$\Rightarrow \text{noul interval este: } \boxed{(a, b) = (\frac{11}{4}, 3)}$$

$$3 - \frac{11}{4} \overset{?}{\geq} 0.1 \Rightarrow \text{A} \Rightarrow \text{Continuăm}$$

$$x = \frac{\frac{11}{4} + 3}{2} = \frac{\frac{23}{4}}{2} = \frac{23}{8}$$

$$f(\frac{11}{4}) \cdot f(\frac{23}{8}) \overset{?}{\geq} 0$$

$$f(\frac{23}{8}) = \frac{529}{64} - 88 = \frac{529-512}{64} = \frac{17}{64}$$

$$f(\frac{11}{4}) \cdot f(\frac{23}{8}) \overset{?}{\geq} 0 \Rightarrow \text{A} \Rightarrow b = x$$

$$\Rightarrow \text{noul interval este: } \boxed{(a, b) = (\frac{11}{4}, \frac{23}{8})}$$

$$\frac{23}{8} - \frac{11}{4} \overset{?}{\geq} 0.1 \Rightarrow \text{A} \Rightarrow \text{Continuăm}$$

$$x = \frac{\frac{11}{4} + \frac{23}{8}}{2} = \frac{\frac{45}{8}}{2} = \frac{45}{16}$$

$$f(\frac{11}{4}) \cdot f(\frac{45}{16}) \overset{?}{\geq} 0$$

$$f(\frac{45}{16}) = \frac{2025}{256} - 8 = \frac{2025 - 2048}{256} = -\frac{23}{256}$$

$$f(\frac{11}{4}) \cdot f(\frac{45}{16}) \overset{?}{\geq} 0 \Rightarrow \text{F} \Rightarrow a = x$$

$$\Rightarrow \text{noul interval este: } \boxed{(a, b) = (\frac{45}{16}, \frac{23}{8})}$$

$$\frac{23}{8} - \frac{45}{16} \overset{?}{\geq} 0.1 \Rightarrow \text{F} \Rightarrow \text{Ne Oprim}$$

__**Rezultat:**__

$$x = \frac{\frac{45}{16} + \frac{23}{8}}{2} = \frac{\frac{91}{16}}{2} \Rightarrow \boxed{x = \frac{91}{32} \approx 2.84}$$

$\boxed{3}$

Să se facă separarea rădăcinilor reale în cazul ecuației $x^3 + 2x - 2 = 0$. Să se calculeze rădăcina reală folosind metoda tangentei cu o precizie de $\epsilon = 10^{-1}$

$$f'(x) = 3x + 2$$

$$f''(x) = 6x$$

Calculăm $f(x)$ până găsim un interval $(-, +)$

$$\left. \begin{array}{l} f(0) = -2 \\ f(1) = 1 \end{array} \right\} \Rightarrow \text{luăm intervalul: } (a, b) = (0, 1)$$

$$y = a \Rightarrow \boxed{y = 0}$$

Verificăm $f(y) \cdot f''(y) \overset{?}{\geq} 0$

$$f(0) \cdot f''(0) \overset{?}{\geq} 0$$

$$-2 \cdot 0 \overset{?}{\geq} 0 \Rightarrow \text{A} \Rightarrow y = b$$

$$\Rightarrow \boxed{y = 1}$$

$$x = y \Rightarrow \boxed{x = 1}$$

$$y=\phi(x)=x-\frac{f(x)}{f'(x)}$$

$$f'(x)=\frac{f(x+h)-f(x)}{h}, h=0.1$$

$$f'(1)=\frac{f(\frac{11}{10})-f(1)}{\frac{1}{10}}=\frac{\frac{1331}{1000}+\frac{22}{10}-2-1}{\frac{1}{10}}=\frac{\frac{1331+2200-3000}{100\cancel{0}}}{\frac{1}{\cancel{10}}}=\frac{531}{100}=5.31$$

$$\left.\begin{array}{l} f(1)=1 \\ f'(1)=5.31 \end{array}\right\} \Rightarrow \phi(1)=1-\frac{1}{5.31}=1-0.1889\approx 0.8 \Rightarrow \boxed{y=0.8}$$

Verificăm $|y-x| \overset{?}{\geq} 0.1$

$$|0.8-1| \overset{?}{\geq} 0.1$$

$$0.2 \overset{?}{\geq} 0.1 \Rightarrow \text{A} \Rightarrow \text{Continuăm}$$

$$x=y \Rightarrow \boxed{x=0.8}$$

$$\phi(0.8)=0.8-\frac{f(0.8)}{f'(0.8)}$$

$$f(0.8)=\frac{512}{1000}+\frac{16}{10}-2=\frac{512+1600-2000}{1000} \Rightarrow \boxed{f(0.8)=0.11}$$

$$f'(0.8)=\frac{f(0.9)-f(0.8)}{0.1}$$

$$f(0.9)=\frac{729}{1000}+\frac{18}{10}-2=\frac{729+1800-2000}{1000}=\underline{0.52}$$

$$f'(0.8)=\frac{\frac{52}{100}-\frac{11}{100}}{\frac{1}{10}}=\frac{\frac{41}{10\cancel{0}}}{\frac{1}{\cancel{10}}} \Rightarrow \boxed{f'(0.8)=4.1}$$

$$\phi(0.8)=0.8-\frac{0.11}{4.1}=0.8-0.026\approx 0.77 \Rightarrow \boxed{y=0.77}$$

$$|0.77-0.8 \overset{?}{\geq} 0.1 \Rightarrow \text{F} \Rightarrow \text{Ne Oprim}$$

<u>**Rezultat:**</u>

$$\boxed{y=0.77}$$

$\boxed{4}$

Să se rezolve sistemul liniar cu metoda lui Gauss:

$$\begin{cases} 3x+y-z=2 \\ x-y+2z=-3 \\ -2x+46-7z=11 \end{cases}$$

(se cere transformarea sistemului linear pas cu pas și rezultatul final)

$$\begin{cases} 3x + y - z = 2 \\ x - y + 2z = -3 \\ -2x + 46 - 7z = 11 \end{cases} \xrightarrow[:3]{\text{normalizăm i=1}} \begin{cases} x + \frac{1}{3}y - \frac{1}{3}z = \frac{2}{3} \\ x - y + 2z = -3 \\ -2x + 4y - 7z = 11 \end{cases} \xrightarrow[(i{=}1)\cdot(-a_{31})+(i{=}3)]{(i{=}1)\cdot(-a_{21})+(i{=}2)}$$

$$\begin{cases} x + \frac{1}{3}y - \frac{1}{3}z = \frac{2}{3} \\ -\frac{4}{3}y + \frac{7}{3}z = -\frac{11}{3} \\ \frac{14}{3}y - \frac{23}{3}z = \frac{37}{3} \end{cases} \xrightarrow[:-\frac{4}{3}]{\text{normalizăm i=2}} \begin{cases} x + \frac{1}{3}y - \frac{1}{3}z = \frac{2}{3} \\ y - \frac{7}{4}z = \frac{11}{4} \\ \frac{14}{3}y - \frac{23}{3}z = \frac{37}{3} \end{cases} \xrightarrow{(i{=}2)\cdot(-a_{31})+(i{=}3)}$$

$$\begin{cases} x + \frac{1}{3}y - \frac{1}{3}z = \frac{2}{3} \\ y - \frac{7}{4}z = \frac{11}{4} \\ \frac{1}{2}z = -\frac{1}{2} \end{cases} \xrightarrow{\textbf{Rezultat}} \boxed{\begin{array}{l} x = 0 \\ y = 1 \\ z = -1 \end{array}}$$