Multivariable control of a Winding machine

This study aims at simulating the behavior of a lab scale system represented by a winding machine in open-loop and in closed-loop. The objective is to study and understand the following topics:

- The analysis of the behavior of a multivariable system
- The modeling around an operating point
- The state-feedback control of a multivariable control

1. Process description

The system considered here is a lab scale winding machine (Figure 1) representing a subsystem of many industrial systems as sheet and film processes, steel industries, etc. The system is composed of three reels driven by DC motors (M_1, M_2, and M_3), gears reduction coupled with the reels, and a plastic strip (Figure 1). Motor M_1 corresponds to the unwinding reel, M_3 is the rewinding reel, and M_2 is the traction reel. The angular velocity of motor M_2 (Ω_2) and the strip tensions between the reels (T_1, T_3) are measured using a tachometer and tension-meters, respectively. Each motor is driven by a local controller. Torque control is achieved for motors M_1 and M_3, while speed control is achieved for motor M_2. For a multivariable control application, a dSPACE board associated with Matlab™/Simulink is used. Data are given between -1 and 1 and correspond to voltages between -10V and 10V.

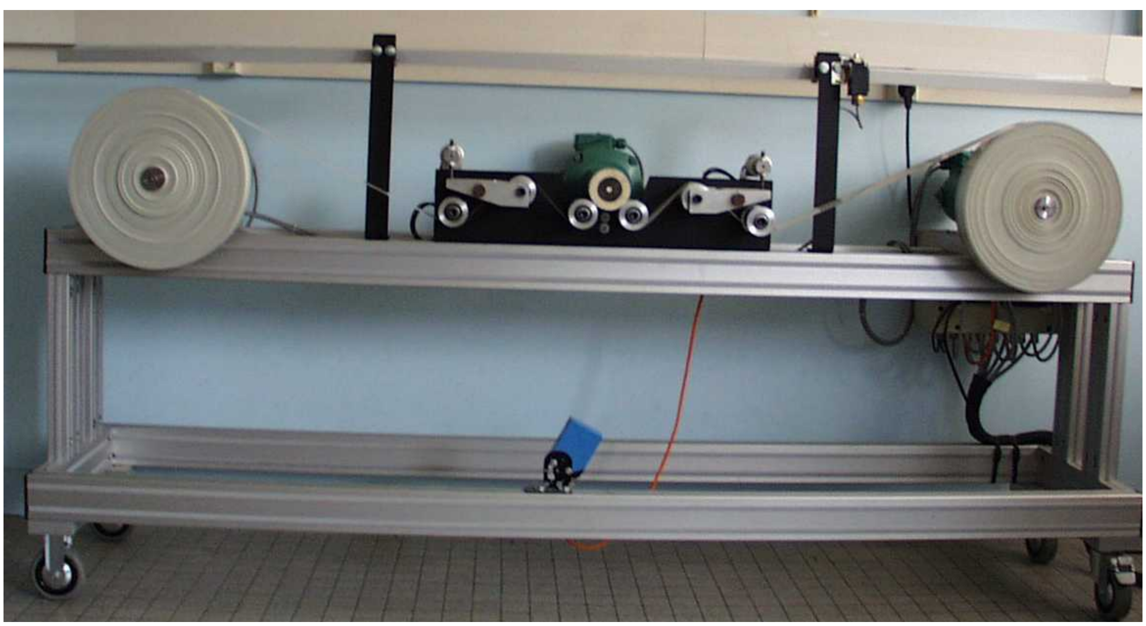

Figure 1. The Winding Machine

The control inputs of the three motors are U_1, U_2, and U_3. U_1, and U_3 correspond to the current set points I_1^* and I_3^* of the local controller. U_2 is the input voltage of motor M_2.

In winding processes, the main goal usually consists of controlling tensions T_1 and T_3 and the linear velocity of the strip. Here the linear velocity is not available for measurement, but since the traction reel radius is constant, the linear velocity can be controlled by the angular velocity Ω_2. The internal representation of this system can be illustrated by Figure 2.

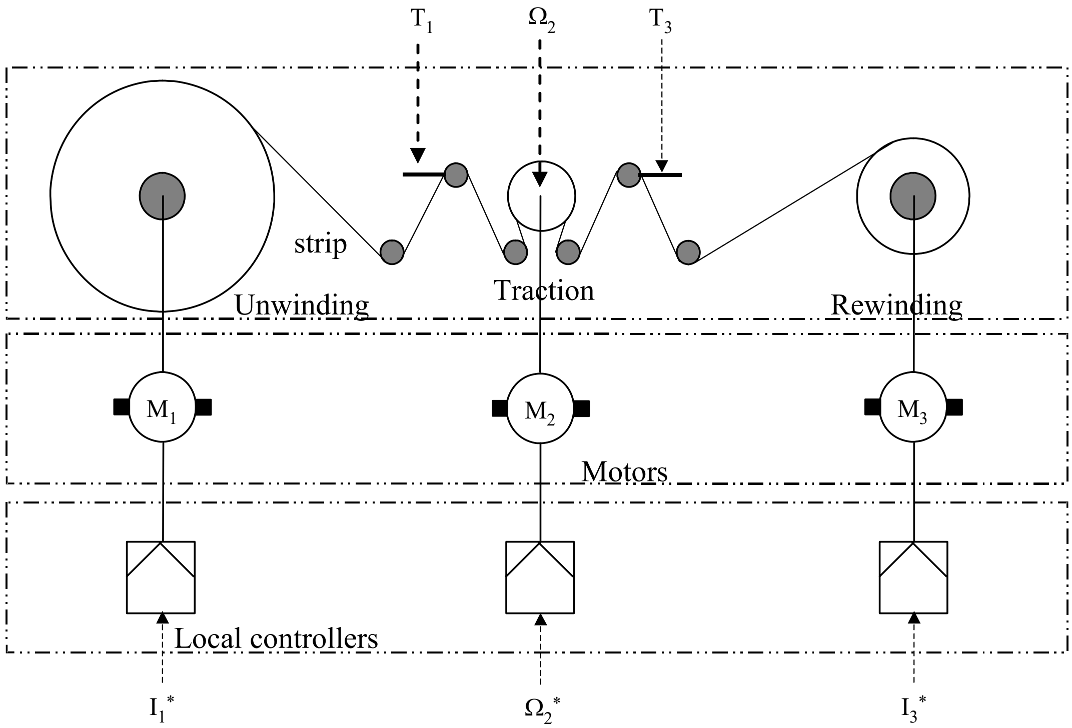

Figure 2. Internal representation of the winding machine.

Figure 3 illustrates a simplified multivariable block-diagram of the winding machine.

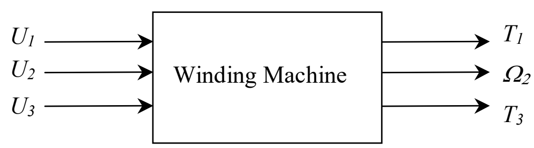

Figure 3. Block diagram of the winding machine.

2. Selection of an operating point

The system is nonlinear. Therefore, it is required to select an operating zone where the system behavior can be considered as linear.

The operating point is considered in open-loop to have the system operating properly. Here, selecting an operating point is achieved experimentally: It consists of applying inputs U_{10}, U_{20}, and U_{30} such that the strip is unwound from one side and rewound on the other side without having too high or too low strip tensions. This is a time consuming task. After many tests, the following operating point has been selected:

$U_{10} = 0.10$; $U_{20} = 0.45$; $U_{30} = 0.30$; $T_{10} = 0.40$; $\Omega_{20} = 0.48$; $T_{30} = 0.19$

3. ***Analysis of the multivariable property of the system:***

In order to check if the system can be considered as multivariable, we have to analyze the influence of each input on the system outputs. One input is varying around the operating point while the others are kept constant. The following figures show the results for a sampling rate Ts = 0.1s.

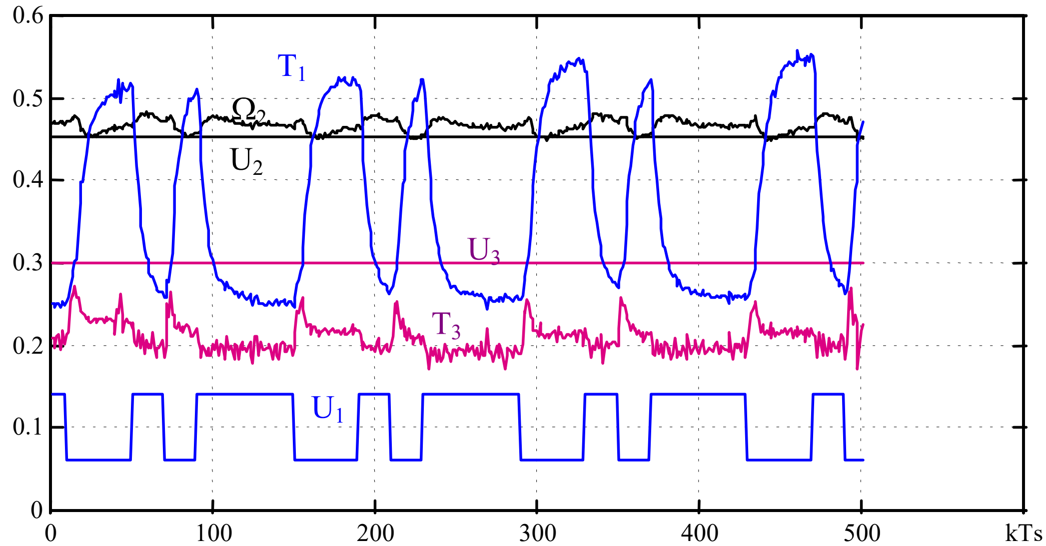

Figure 4.a. Influence of the control input U_1.

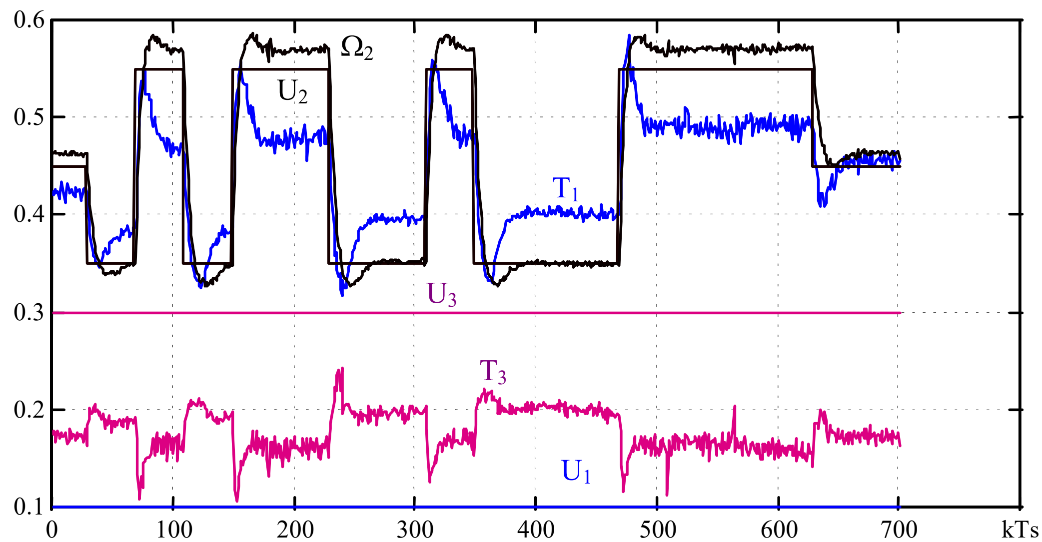

Figure 4.b. Influence of the control input U_2.

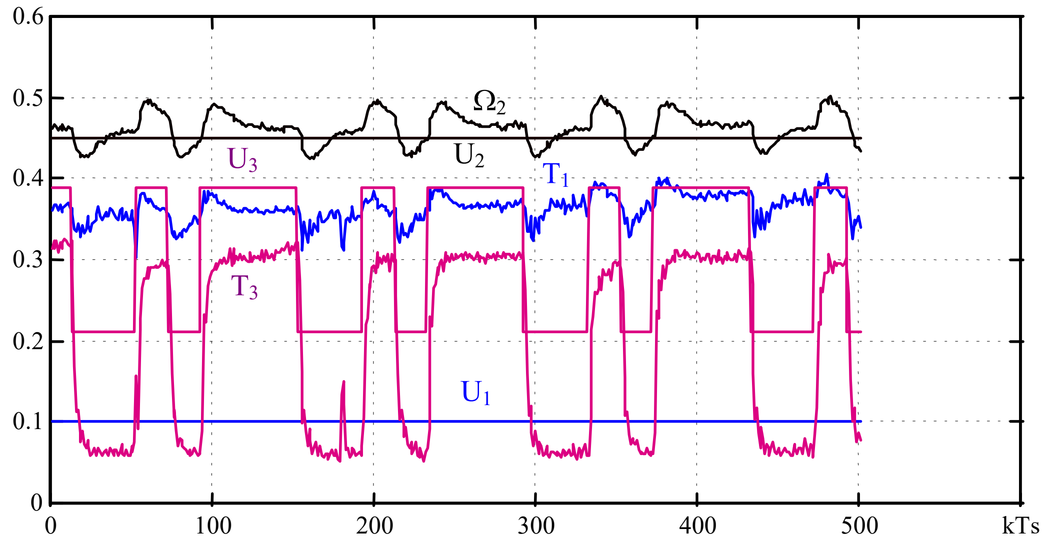

Figure 4.c. Influence of the control input U3.

3.1. What is the duration of these experiments?

3.2. Analyze these results and describe the influence of each control input on the system outputs.

4. System Modeling and Identification

The system model can be obtained by writing the physical equations of the system or experimentally. For this system, the physical equations are highly nonlinear and make use of partial differential equations which are not easy to write and handle. Therefore, experiments have been conducted to get a linearized model of the system around the following operating point.

$$U_0 = [-0.15 \quad 0.6 \quad 0.15]^T \qquad\qquad y_0 = [0.6 \quad 0.5 \quad 0.4]^T$$

Remark: *This operating point is different from the previous one, because maintenance operations have been done on the system which changes the operating point. This has no effect on the work you have to do or on the analysis of the system.*

The system is considered to be linear around a given operating point, and the corresponding analytical model is obtained using an ARX structure (An Identification method not detailed here). This model describes the dynamical behavior of the system in terms of input/output variations u and y around the operating point (u_0, y_0). The data set used for the parameter identification step is composed of Pseudo-Random Binary Sequence signals applied to the system and their corresponding outputs. This data set is displayed on Figures 5. The sampling rate is $T_s = 0.1$ s. The signals collected via the dSPACE board are given in the interval [-1, 1] corresponding to [-10 V, 10 V].

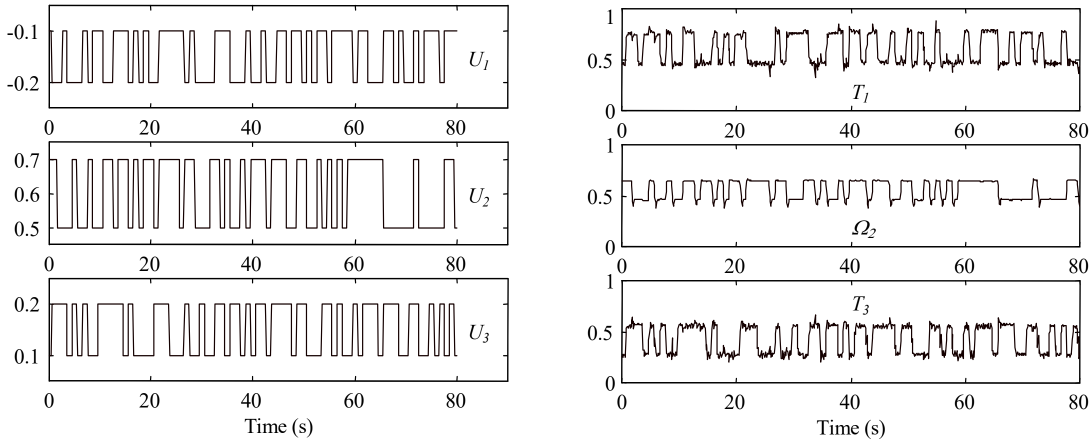

Figure 5. The Input/Output signals used for parameters identification.

Therefore, the linearized model of the winding machine around the operating point (U_0, Y_0) is given by the following discrete state-space representation:

$$x(k+1) = \mathrm{A}\,x(k) + \mathrm{B}\,u(k)$$
$$y(k) = \mathrm{C}\,x(k),$$

with

$$x = \begin{bmatrix} T_1 \\ \Omega_2 \\ T_3 \end{bmatrix}, u = \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}, \mathrm{A} = \begin{bmatrix} 0.4126 & 0 & -0.0196 \\ 0.0333 & 0.5207 & -0.0413 \\ -0.0101 & 0 & 0.2571 \end{bmatrix}, \mathrm{B} = \begin{bmatrix} -1.7734 & 0.0696 & 0.0734 \\ 0.0928 & 0.4658 & 0.1051 \\ -0.0424 & -0.093 & 2.0752 \end{bmatrix}.$$

C is the identity matrix I_3.

4.1. Check the controllability and the observability of this system.

5. Simulation in open-loop

5.1. Use Simulink to simulate the behavior of the variations of the system inputs/outputs in open-loop around the operating point. For the control inputs u_1, u_2 and u_3, choose square signals of appropriate magnitude around zero. These signals corresponding to the variations of the global control inputs applied to the real system lead to variations of the system outputs around the operating point. Analyze and understand the behavior of the system outputs.

5.2. Modify the previous Simulink model in order to display the actual input/output signals taking the operating point into account.

6. Design of a multivariable tracking control law (Closed-loop behavior)

Controlling a system consists in designing a control input 'u' to apply to the system in order to have a desired behavior. In tracking control, the number of outputs that have to follow a reference input vector, 'y_r', must be less than or equal to the number of control inputs. For this system the number of the control inputs (3 inputs) is equal to the number of the outputs (3 outputs: T_1, T_3, and Ω_2).

Refer to lecture notes (Chapter 5) to design a tracking control law:

6.1. Consider a comparator and integrator vector z, and write the matrices of the augmented system.

The nominal feedback control law of this system can be computed by:

$$u(k) = -\mathrm{K}\,\tilde{x}(k) = -\begin{bmatrix}\mathrm{K}_1 & \mathrm{K}_2\end{bmatrix}\begin{bmatrix}x(k)\\ z(k)\end{bmatrix}, \quad (12)$$

with $\tilde{x} = [x^T \quad z^T]^T$ and $\mathrm{K} = [\mathrm{K}_1 \quad \mathrm{K}_2]$ is the feedback gain matrix.

The objective now is to calculate this feedback gain matrix. This matrix can be obtained by pole placement technique.

6.2. Pole Placement: **In “Matlab Command window” calculate this feedback gain matrix K using the function « place » by selecting appropriate poles for the closed-loop system.**

The main performances required for the closed-loop system are:

- Stability
- Accuracy
- Rapidity
- Small overshoot
- Disturbance rejection
- Acceptable solicitation of the actuators.

The accuracy is guaranteed by the presence of the integrator in the control law. The stability, the rapidity, the overshoot, and the solicitation of the actuators depend on the placement of the poles of the closed-loop system.

6.3. Use Simulink to implement the control law designed above and analyze the system behavior in closed-loop. Use appropriate square signals for the reference inputs.

6.4. Modify the poles of the closed-loop system and study their influence on the feedback gain matrix and therefore on the performances of the closed-loop system.

These performances can be analyzed visually by reading the time responses of the outputs and the eventual overshoots. This can also be achieved by calculating the norm of the tracking error signal between each output and its reference. The Matlab function « **norm** » allows calculating the norm of a vector.

6.5. Fault Diagnostic

1. How to represent a loss in the effectiveness of Motor 3 by 50%?
2. What is the effect of this fault on the system outputs?
3. Design a **fault detection module** using a state observer allowing **to detect** such a fault when it occurs. Give all conditions and steps needed to design this module. Complete the previous closed-loop block diagram by adding the state observer block diagram.
4. This is not enough to know which actuator is faulty. Explain how to design a module allowing **to isolate** the fault that may occur on the actuators. Give the concept and the block diagram of the fault isolation module. Implement this module on the Matlab/Simulink program

5. Assuming that the loss of effectiveness on Motor3 has been detected, isolated, and estimated, would it be possible to compensate for the effect of this fault? If yes, explain how to implement a **fault-tolerant control module** for such an actuator fault.