

restart;

$$x0 := \frac{(-b + \sqrt{b^2 - 4 \cdot a \cdot c})}{(2 \cdot a)}$$

$$\frac{1}{2} \frac{-b + \sqrt{b^2 - 4 a c}}{a} \quad (1)$$

$$x1 := \frac{(-b - \sqrt{b^2 - 4 \cdot a \cdot c})}{(2 \cdot a)}$$

$$\frac{1}{2} \frac{-b - \sqrt{b^2 - 4 a c}}{a} \quad (2)$$

rules := (phiX ≥ 0) and (phiY ≥ 0) and (costX > 0) and (costY > 0);

$$0 \leq \text{phiX} \text{ and } 0 \leq \text{phiY} \text{ and } 0 < \text{costX} \text{ and } 0 < \text{costY} \quad (3)$$

Comparing solution from quadratic formula for 2d solution above with 1d solution.

bigger := max(x0, x1);

$$\max\left(\frac{1}{2} \frac{-b - \sqrt{b^2 - 4 a c}}{a}, \frac{1}{2} \frac{-b + \sqrt{b^2 - 4 a c}}{a}\right) \quad (4)$$

solutionIDX := phiX + costX;

$$\text{phiX} + \text{costX} \quad (5)$$

solutionIDY := phiY + costY;

$$\text{phiY} + \text{costY} \quad (6)$$

a := costY² + costX²;

$$\text{costY}^2 + \text{costX}^2 \quad (7)$$

b := -2 · (phiX · costY² + phiY · costX²);

$$-2 \text{ phiX costY}^2 - 2 \text{ phiY costX}^2 \quad (8)$$

c := phiX² · costY² + phiY² · costX² - costX² · costY²;

$$\text{phiX}^2 \text{ costY}^2 + \text{phiY}^2 \text{ costX}^2 - \text{costX}^2 \text{ costY}^2 \quad (9)$$

bigger > solutionIDX;

$$\text{phiX} + \text{costX} < \max\left(\frac{1}{2} \frac{1}{\text{costY}^2 + \text{costX}^2} \left(2 \text{ phiX costY}^2 + 2 \text{ phiY costX}^2\right. \quad (10)$$

$$\left. - \left((-2 \text{ phiX costY}^2 - 2 \text{ phiY costX}^2)^2 - 4 (\text{costY}^2 + \text{costX}^2) (\text{phiX}^2 \text{ costY}^2 \right. \right.$$

$$\left. + \text{phiY}^2 \text{ costX}^2 - \text{costX}^2 \text{ costY}^2) \right)^{1/2} \Big), \frac{1}{2} \frac{1}{\text{costY}^2 + \text{costX}^2} \left(2 \text{ phiX costY}^2 \right.$$

$$\left. + 2 \text{ phiY costX}^2 \right.$$

$$\left. + \left((-2 \text{ phiX costY}^2 - 2 \text{ phiY costX}^2)^2 - 4 (\text{costY}^2 + \text{costX}^2) (\text{phiX}^2 \text{ costY}^2 \right. \right.$$

$$\left. + \text{phiY}^2 \text{ costX}^2 - \text{costX}^2 \text{ costY}^2) \right)^{1/2} \Big)$$

solve({x0, a, b, c, phiX ≥ 0, phiY ≥ 0, costX > 0, costY > 0}, {phiX, phiY, costX, costY});

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solve(bigger > solution1DX, {phiX, phiY, costX, costY});
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Warning, solutions may have been lost

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solve(x0 > x1 = true, {a, b, c});
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Warning, solving for expressions other than names or functions is not recommended.