```
restart;
phiX2 := phiX * phiX;
```

$$phiX^2 \tag{1}$$

```
phiY2 := phiY * phiY;
```

$$phiY^2 \tag{2}$$

```
phiX2
```

$$phiX^2 \tag{3}$$

```
costX2 := costX * costX;
```

$$costX^2 \tag{4}$$

```
costY2 := costY * costY;
```

$$costY^2 \tag{5}$$

```
a := costX2 + costY2;
```

$$costX^2 + costY^2 \tag{6}$$

```
b := -2 * (phiX * costY2 + phiY * costX2);
```

$$-2\, phiX\, costY^2 - 2\, phiY\, costX^2 \tag{7}$$

```
c := phiX2 * costY2 + phiY2 * costX2 - costX2 * costY2;
```

$$phiX^2\, costY^2 + phiY^2\, costX^2 - costX^2\, costY^2 \tag{8}$$

```
f := b · b < 4 · a · c;
```

$$\left(-2\, phiX\, costY^2 - 2\, phiY\, costX^2\right)^2 < 4\left(costX^2 + costY^2\right)\left(phiX^2\, costY^2 + phiY^2\, costX^2 - costX^2\, costY^2\right) \tag{9}$$

```
simplify(f);
```

$$4\left(phiX\, costY^2 + phiY\, costX^2\right)^2 < -4\left(costX^2 + costY^2\right)\left(-phiX^2\, costY^2 - phiY^2\, costX^2 + costX^2\, costY^2\right) \tag{10}$$

```
phiX ≥ 0;
```

$$0 \le phiX \tag{11}$$

```
phiY ≥ 0;
```

$$0 \le phiY \tag{12}$$

```
costX > 0;
```

$$0 < costX \tag{13}$$

```
costY > 0;
```

$$0 < costY \tag{14}$$

```
f
```

$$\left(-2\, phiX\, costY^2 - 2\, phiY\, costX^2\right)^2 < 4\left(costX^2 + costY^2\right)\left(phiX^2\, costY^2 + phiY^2\, costX^2 - costX^2\, costY^2\right) \tag{15}$$

```
simplify(f);
```

$$4\left(phiX\, costY^2 + phiY\, costX^2\right)^2 < -4\left(costX^2 + costY^2\right)\left(-phiX^2\, costY^2 - phiY^2\, costX^2 + costX^2\, costY^2\right) \tag{16}$$

```
solve(f);
Warning, solutions may have been lost
solve(f, {phiX, phiY, costX, costY});
```

Warning, solutions may have been lost

$solve(f, \{phiX\});$

f

$$\left(-2\, phiX\, costY^2 - 2\, phiY\, costX^2\right)^2 < 4\left(costX^2 + costY^2\right)\left(phiX^2\, costY^2 + phiY^2\, costX^2 - costX^2\, costY^2\right) \tag{17}$$

$solve(\{f, phiX \geq 0, phiY \geq 0, costX > 0, costY > 0\});$

Warning, solutions may have been lost

$solve(\{f, phiX \geq 0, phiY \geq 0, costX > 0, costY > 0\}, phiX);$

Warning, solutions may have been lost

$solve(\{f, phiX \geq 0, phiY \geq 0, costX > 0, costY > 0\}, \{phiX, phiY, costX, costY\});$

Warning, solutions may have been lost

$g := b \cdot b < 4 \cdot a \cdot c;$

$$\left(-2\, phiX\, costY^2 - 2\, phiY\, costX^2\right)^2 < 4\left(costX^2 + costY^2\right)\left(phiX^2\, costY^2 + phiY^2\, costX^2 - costX^2\, costY^2\right) \tag{18}$$

$g := bb \cdot bb < 4 \cdot aa \cdot cc;$

$$bb^2 < 4\, aa\, cc \tag{19}$$

$solve\ (g);$

$$`*`\left(bb^2 < 4\, aa\, cc, solve\right) \tag{20}$$

$solve(g);$

$$\left\{-\mathrm{signum}(cc)\, aa < -\frac{1}{4}\,\frac{\mathrm{signum}(cc)\, bb^2}{cc}\right\} \tag{21}$$

g

$$bb^2 < 4\, aa\, cc \tag{22}$$

$expand(f);$

$$4\, costY^4\, phiX^2 + 8\, phiX\, costY^2\, phiY\, costX^2 + 4\, costX^4\, phiY^2 < 4\, costX^2\, phiX^2\, costY^2 + 4\, costX^4\, phiY^2 - 4\, costX^4\, costY^2 + 4\, costY^4\, phiX^2 + 4\, costY^2\, phiY^2\, costX^2 - 4\, costY^4\, costX^2 \tag{23}$$

f

$$\left(-2\, phiX\, costY^2 - 2\, phiY\, costX^2\right)^2 < 4\left(costX^2 + costY^2\right)\left(phiX^2\, costY^2 + phiY^2\, costX^2 - costX^2\, costY^2\right) \tag{24}$$

$simplify(expand(f));$

$$4\, costY^4\, phiX^2 + 8\, phiX\, costY^2\, phiY\, costX^2 + 4\, costX^4\, phiY^2 < 4\, costX^2\, phiX^2\, costY^2 + 4\, costX^4\, phiY^2 - 4\, costX^4\, costY^2 + 4\, costY^4\, phiX^2 + 4\, costY^2\, phiY^2\, costX^2 - 4\, costY^4\, costX^2 \tag{25}$$

$\frac{f}{4};$

$$\frac{1}{4}\left(-2\, phiX\, costY^2 - 2\, phiY\, costX^2\right)^2 < \left(costX^2 + costY^2\right)\left(phiX^2\, costY^2 + phiY^2\, costX^2 - costX^2\, costY^2\right) \tag{26}$$

$$expand\left(\frac{f}{4}\right);$$

$$costY^4\,phiX^2 + 2\,phiX\,costY^2\,phiY\,costX^2 + costX^4\,phiY^2 < costX^2\,phiX^2\,costY^2 + costX^4\,phiY^2 - costX^4\,costY^2 + costY^4\,phiX^2 + costY^2\,phiY^2\,costX^2 - costY^4\,costX^2 \quad (27)$$

$$f4 := expand\left(\frac{f}{4}\right);$$

$$costY^4\,phiX^2 + 2\,phiX\,costY^2\,phiY\,costX^2 + costX^4\,phiY^2 < costX^2\,phiX^2\,costY^2 + costX^4\,phiY^2 - costX^4\,costY^2 + costY^4\,phiX^2 + costY^2\,phiY^2\,costX^2 - costY^4\,costX^2 \quad (28)$$

$$f4$$

$$costY^4\,phiX^2 + 2\,phiX\,costY^2\,phiY\,costX^2 + costX^4\,phiY^2 < costX^2\,phiX^2\,costY^2 + costX^4\,phiY^2 - costX^4\,costY^2 + costY^4\,phiX^2 + costY^2\,phiY^2\,costX^2 - costY^4\,costX^2 \quad (29)$$

$$solve(f4);$$

Warning, solutions may have been lost

$$solve(\{f4, costX > 0, cosY > 0, phiX \geq 0, phiY \geq 0\}, phiX);$$

Warning, solutions may have been lost