1、$\sum_{i=1}^{n}(y_i-\bar{y})^2 = \sum_{i=1}^{n}(y_i-\hat{y}_i+\hat{y}_i-\bar{y})^2$

$= \sum_{i=1}^{n}(\hat{y}_i-\bar{y})^2+\sum_{i=1}^{n}(y_i-\hat{y}_i)^2 + 2\sum_{i=1}^{n}(y_i-\hat{y}_i)(\hat{y}_i-\bar{y})$

其中 $\hat{y}_i-\bar{y} = \hat{w}x_i+b-(\hat{w}\bar{x}+b) = \hat{w}(x_i-\bar{x})$， $\hat{w}=\dfrac{\sum_{i=1}^{n}(y_i-\bar{y})(x_i-\bar{x})}{\sum_{i=1}^{n}(x_i-\bar{x})^2}$

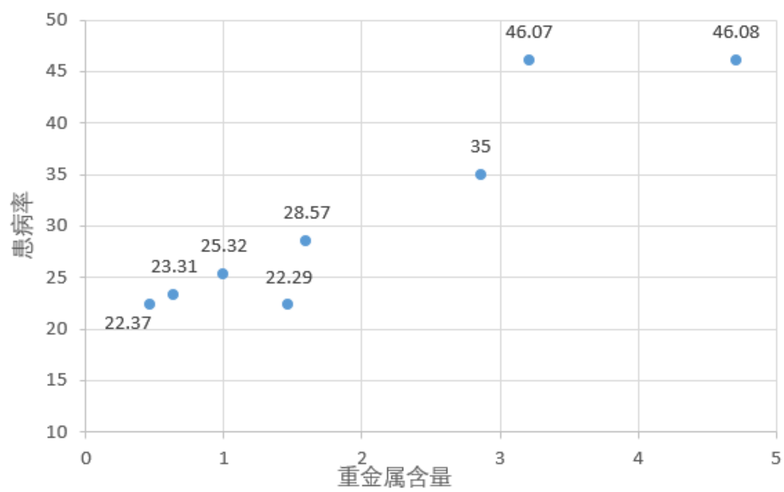
$y_i-\hat{y}_i = y_i-\bar{y}-\hat{w}(x_i-\bar{x})$

$\sum_{i=1}^{n}(y_i-\hat{y}_i)(\hat{y}_i-\bar{y}) = \sum_{i=1}^{n}\left[\hat{w}(y_i-\bar{y})(x_i-\bar{x})-\hat{w}^2(x_i-\bar{x})^2\right]$

$= \hat{w}\sum_{i=1}^{n}(y_i-\bar{y})(x_i-\bar{x}) - \hat{w}\cdot\dfrac{\sum_{i=1}^{n}(y_i-\bar{y})(x_i-\bar{x})}{\sum_{i=1}^{n}(x_i-\bar{x})^2}\cdot\sum_{i=1}^{n}(x_i-\bar{x})^2 = 0.$

故 $\sum_{i=1}^{n}(y_i-\bar{y})^2=\sum_{i=1}^{n}(\hat{y}_i-\bar{y})^2+\sum_{i=1}^{n}(y_i-\hat{y}_i)^2$.

2、(a)



(b) $\bar{x} = \dfrac{0.47+0.64+1.00+1.47+1.60+2.86+3.21+4.71}{8}$

$= 1.9950$.

$\bar{y} = \dfrac{22.37+23.31+25.32+22.29+28.57+35+46.07+46.08}{8}$

$= 31.1263$

$$\hat{w}=\frac{\sum_{i=1}^{8}(y_i-31.1263)(x_i-1.995)}{\sum_{i=1}^{8}(x_i-1.995)^2}=6.4218$$

$$\hat{b}=\bar{y}-\hat{w}\bar{x}=31.1263-6.4218\times1.995=18.3147$$

一元线性回归方程 $\hat{y}=6.4218x+18.3147$

$$r^2=\frac{\sum_{i=1}^{8}(\hat{y}_i-31.1263)^2}{\sum_{i=1}^{8}(y_i-31.1263)^2}=0.8718$$

3、(a) 不适用。因为最小二乘法是在自变量x的观测值准确时移动回归直线使得对应残差的平方和最小来实现的.而此题中自变量x的观测值存在误差，预测点与实际点的平方误差损失不能只用变量y的残差平方和表示。

(b) 回归直线 $(\vec{n},\vec{x}_0)$.

则 $\vec{x}_i=(x_i,y_i)^T$ 到回归直线的距离平方为 $[(\vec{x}_i-\vec{x}_0)\cdot\vec{n}]^2$

平方误差损失为 $\sum_{i=1}^{n}[(\vec{x}_i-\vec{x}_0)\cdot\vec{n}]^2$

(c) 设 $\vec{x}_0=(x_0,y_0)^T$ $\vec{n}=(\sin\theta,-\cos\theta)^T$

则 $\sum_{i=1}^{n}[(\vec{x}_i-\vec{x}_0)\cdot\vec{n}]^2=\sum_{i=1}^{n}[(x_i-x_0)\sin\theta-(y_i-y_0)\cos\theta]^2=D$

$$\left\{\frac{\partial D}{\partial x_0}=-2\sin\theta\sum_{i=1}^{n}[(x_i-x_0)\sin\theta-(y_i-y_0)\cos\theta]\right.$$

$$
\begin{cases}
\quad = -2n(\bar{x}-x_0)\sin^2\theta + n(\bar{y}-y_0)\sin 2\theta = 0. \\
\dfrac{\partial D}{\partial y_0} = 2\cos\theta \sum_{i=1}^{n}\left[(x_i-x_0)\sin\theta - (y_i-y_0)\cos\theta\right] \\
\quad = n(\bar{x}-x_0)\sin\theta - 2n(\bar{y}-y_0)\cos^2\theta = 0. \\
\dfrac{\partial D}{\partial \theta} = 2\sum_{i=1}^{n}\left[(x_i-x_0)\sin\theta - (y_i-y_0)\cos\theta\right]\left[(x_i-x_0)\cos\theta + (y_i-y_0)\sin\theta\right] \\
\quad = \left[\sum_{i=1}^{n}(x_i-x_0)^2 - \sum_{i=1}^{n}(y_i-y_0)^2\right]\sin 2\theta - 2\sum_{i=1}^{n}(x_i-x_0)(y_i-y_0)\cos 2\theta = 0.
\end{cases}
$$

则 $x_0=\bar{x}$ $\quad y_0=\bar{y}$. $\quad \tan 2\theta = \dfrac{2S_{xy}}{S_{xx}-S_{yy}}.$

$$\theta = \frac{1}{2}\arctan\frac{2S_{xy}}{S_{xx}-S_{yy}}.$$

当 $\vec{x_0}=(x_0,y_0)^T$ 和 $\vec{n}=(\sin\theta,-\cos\theta)^T$ 取如上值时平方损失表达式值最小.