

2.

$$1) \neg \exists (x) (P(x) \wedge Q(x)) \equiv \neg \neg \forall x \neg (P(x) \wedge Q(x))$$

$$\equiv \forall x (\neg P(x) \vee \neg Q(x)) \equiv \forall x (P(x) \rightarrow \neg Q(x))$$

$$2) \neg \forall (x) (P(x) \rightarrow Q(x)) \equiv \neg \neg \exists (x) \neg (P(x) \rightarrow Q(x))$$

$$\equiv \exists x \neg (\neg P(x) \vee Q(x)) \equiv \exists x (P(x) \wedge \neg Q(x))$$

$$3) P(x): x \text{ 为正数} \quad Q(x): x \text{ 为负数} \quad R(x, y): x \text{ 比 } y \text{ 小}$$

$$\text{"不存在比负数小的正数"}: \neg \exists (x) (P(x) \wedge \exists y (Q(y) \wedge R(x, y)))$$

$$\text{"所有正数都比负数大"} \Leftrightarrow \text{"所有正数都不比负数小"}:$$

$$\forall x (P(x) \rightarrow \forall y (Q(y) \rightarrow \neg R(x, y)))$$

$$\equiv \forall x \forall y (P(x) \rightarrow (Q(y) \rightarrow \neg R(x, y)))$$

$$\equiv \neg \exists x \exists y \neg (P(x) \rightarrow (Q(y) \rightarrow \neg R(x, y)))$$

$$\equiv \neg \exists x \exists y (P(x) \wedge \neg (Q(y) \rightarrow \neg R(x, y)))$$

$$\equiv \neg \exists x \exists y (P(x) \wedge Q(y) \wedge R(x, y))$$

$$\equiv \neg \exists (x) (P(x) \wedge \exists y (Q(y) \wedge R(x, y)))$$

(4). $P(x, y)$: x, y 为相等角 $Q(x, y)$: x, y 为对顶角

"不是所有相等的角都是对顶角": $\neg \forall x \forall y (P(x, y) \rightarrow Q(x, y))$

"存在二者相等的角不是对顶角": $\exists x \exists y (P(x, y) \wedge \neg Q(x, y))$

$$\exists x \exists y (P(x, y) \wedge \neg Q(x, y)) \equiv \neg \forall x \forall y \neg (P(x, y) \wedge \neg Q(x, y))$$

$$\equiv \neg \forall x \forall y (\neg P(x, y) \vee Q(x, y)) \equiv \neg \forall x \forall y (P(x, y) \rightarrow Q(x, y))$$

3.

(1). $A(x, y)$: x 喜欢 y $B(x, y)$: x 通过 y $C(x)$: x 快乐 $D(x, y)$: x 上课认真听讲

$E(x)$: 努力学习 $F(x)$: x 聪明 $G(x)$: x 是人 $H(x)$: x 是课 $I(x)$: x 是考试

① "任何喜欢人智课并通过人智考试的人都是快乐的"

$$\forall x (G(x) \wedge A(x, \text{人智课}) \wedge B(x, \text{人智考试}) \rightarrow C(x))$$

② "任何人上课认真听讲的人都喜欢这门课"

$$\forall x \forall y (G(x) \wedge H(y) \wedge D(x, y) \rightarrow A(x, y))$$

③ "努力学习的人上课都能认真听讲"

$$\forall x \forall y (G(x) \wedge H(y) \wedge E(x) \rightarrow D(x, y))$$

④ "聪明或努力学习的人可以通过所有考试"

$$\forall x \forall y (G(x) \wedge (F(x) \vee E(x)) \wedge I(y) \rightarrow B(x, y))$$

⑤ "小明是人": $G(\text{小明})$

⑥ "人智考试是考试": $I(\text{人智考试})$

⑦ "小明努力学习": $E(\text{小明})$

⑧ "人智课是课": $H(\text{人智课})$

(2). 后向链接

小明是快乐的 $C(\text{小明})$

← ① $G(\text{小明}) \wedge A(\text{小明}, \text{人智课}) \wedge B(\text{小明}, \text{人智考试})$

$G(\text{小明}) \wedge H(\text{人智课}) \wedge D(\text{小明}, \text{人智课})$

$G(\text{小明}) \wedge (E(\text{小明}) \vee E(\text{小明})) \wedge I(\text{人智考试})$

$G(\text{小明}) \wedge H(\text{人智课}) \wedge E(\text{小明})$

∴ 小明是快乐的