

$$\begin{aligned}
2.1). \text{原式} &\equiv (\neg \exists x [P(x) \wedge Q(x)] \Rightarrow \forall x [P(x) \Rightarrow \neg Q(x)]) \wedge \\
&\quad (\forall x [P(x) \Rightarrow \neg Q(x)] \Rightarrow \neg \exists x [P(x) \wedge Q(x)]) \\
&\equiv (\exists x [P(x) \wedge Q(x)] \vee \forall x [\neg P(x) \vee \neg Q(x)]) \wedge \\
&\quad (\forall x [\neg P(x) \vee \neg Q(x)] \vee \neg \exists x [P(x) \wedge Q(x)]) \\
&\equiv (\exists x [P(x) \wedge Q(x)] \vee \forall x [\neg P(x) \vee \neg Q(x)]) \wedge \\
&\quad (\exists x [P(x) \wedge Q(x)] \vee \forall x [\neg P(x) \vee \neg Q(x)]) \\
&\equiv \exists x [P(x) \wedge Q(x)] \vee \neg \exists x [P(x) \wedge Q(x)] \\
&\equiv \text{TRUE} \quad (\text{原式得证})
\end{aligned}$$

$$\begin{aligned}
(2). \text{原式} &\equiv (\neg \forall x [P(x) \Rightarrow Q(x)] \Rightarrow \exists x [P(x) \wedge \neg Q(x)]) \wedge \\
&\quad (\exists x [P(x) \wedge \neg Q(x)] \Rightarrow \neg \forall x [P(x) \Rightarrow Q(x)]) \\
&\equiv (\forall x [\neg P(x) \vee Q(x)] \vee \exists x [P(x) \wedge \neg Q(x)]) \wedge \\
&\quad (\neg \exists x [P(x) \wedge \neg Q(x)] \vee \neg \forall x [\neg P(x) \vee Q(x)]) \\
&\equiv (\forall x [\neg P(x) \vee Q(x)] \vee \exists x [P(x) \wedge \neg Q(x)]) \wedge \\
&\quad (\forall x [\neg P(x) \vee Q(x)] \vee \exists x [P(x) \wedge \neg Q(x)]) \\
&\equiv (\forall x [\neg P(x) \vee Q(x)] \vee \neg \forall x [\neg P(x) \vee Q(x)]) \\
&\equiv \text{TRUE} \quad (\text{原式得证})
\end{aligned}$$

(3). $P(x)$: x 为比负数大的数.

$Q(x)$: x 是正数.

前-句话: $\neg \exists x Q(x) \wedge \neg P(x)$.

后-句话: $\forall x Q(x) \Rightarrow P(x)$.

$$\begin{aligned}
&\neg \exists x Q(x) \wedge \neg P(x) \Leftrightarrow \forall x Q(x) \Rightarrow P(x) \\
&\equiv (\neg \exists x Q(x) \wedge \neg P(x) \Rightarrow \forall x Q(x) \Rightarrow P(x)) \wedge (\forall x Q(x) \Rightarrow P(x) \Rightarrow \neg \exists x Q(x) \wedge \neg P(x)) \\
&\equiv (\exists x [Q(x) \wedge \neg P(x)] \vee \forall x [\neg Q(x) \vee P(x)]) \wedge (\neg \forall x [\neg Q(x) \vee P(x)] \vee \neg \exists x [Q(x) \wedge \neg P(x)]) \\
&\equiv (\exists x [Q(x) \wedge \neg P(x)] \vee \forall x [\neg Q(x) \vee P(x)]) \wedge (\exists x [Q(x) \wedge \neg P(x)] \vee \forall x [\neg Q(x) \vee P(x)]) \\
&\equiv (\exists x [Q(x) \wedge \neg P(x)]) \vee (\neg \exists x [Q(x) \wedge \neg P(x)]) \\
&\equiv \text{TRUE} \quad (\text{原式得证})
\end{aligned}$$

(4). $P(x, y)$: x, y 两角相等

$Q(x, y)$: x, y 是对顶角

前-句话: $\neg (\forall x \forall y P(x, y) \Rightarrow Q(x, y))$

后-句话: $\exists x \exists y [P(x, y) \wedge \neg Q(x, y)]$

$$\begin{aligned}
& \neg[\forall x\forall y P(x,y) \Rightarrow Q(x,y)] \Leftrightarrow [\exists x\exists y P(x,y) \wedge \neg Q(x,y)] \\
& \equiv \neg[\forall x\forall y P(x,y) \Rightarrow Q(x,y)] \Rightarrow [\exists x\exists y P(x,y) \wedge \neg Q(x,y)] \wedge \\
& \quad [\exists x\exists y P(x,y) \wedge \neg Q(x,y)] \Rightarrow \neg[\forall x\forall y P(x,y) \Rightarrow Q(x,y)] \\
& \equiv ([\forall x\forall y P(x,y) \wedge \neg Q(x,y)] \vee [\exists x\exists y P(x,y) \wedge \neg Q(x,y)]) \wedge \\
& \quad (\forall x\forall y [\neg P(x,y) \vee Q(x,y)] \vee \exists x\exists y [\neg P(x,y) \vee Q(x,y)]) \\
& \equiv (\forall x\forall y [\neg P(x,y) \vee Q(x,y)] \vee \exists x\exists y [P(x,y) \wedge \neg Q(x,y)]) \\
& \equiv \forall x\forall y [\neg P(x,y) \vee Q(x,y)] \vee \neg \forall x\forall y [P(x,y) \vee Q(x,y)] \\
& \equiv \text{TRUE.} \quad (\text{原式得证})
\end{aligned}$$

4. ①: $\forall x [N(x) \Rightarrow (GZ(x) \wedge I(x))]$

②: $\forall x [I(x) \Rightarrow (E(x) \vee O(x))]$

③: $\forall x [E(x) \Rightarrow I(S(x))]$

归结(待证) ~~证~~ $\forall x. N(x) \Rightarrow [O(x) \vee I(S(x))]$

化简取范式.

①: $\neg N(x) \vee (GZ(x) \wedge I(x))$

$\equiv (\neg N(x) \vee GZ(x)) \wedge (\neg N(x) \vee I(x))$

②: $\neg I(x) \vee E(x) \vee O(x)$

③: $\neg E(x) \vee I(S(x))$

④: (取反) $N(x) \wedge [\neg O(x) \wedge \neg I(S(x))]$
 $\equiv N(x) \wedge \neg O(x) \wedge \neg I(S(x))$

原子谓词.

$\neg N(x) \vee GZ(x)$ (1)

$\neg N(x) \vee I(x)$ (2)

$\neg I(x) \vee E(x) \vee O(x)$ (3)

$\neg E(x) \vee I(S(x))$ (4)

$\neg I(S(x))$ (5)

$\neg O(x)$ (6)

$N(x)$ (7)

(2)(7) 归结得到 $I(x)$.

再与(3)归结得到 $E(x) \vee O(x)$.

再与(4)归结得到 $E(x)$.

再与(5)归结得到 $I(S(x))$.

最后与(6)归结得到空谓词.

空谓词与(1)归结也必得到空谓词.

因此目标得证.