1. 建模：$\log P(y^{(i)}=k) = \beta_k x_i - \log z$.

归一化：$P(y^{(i)}=k) = e^{\beta_k x_i}/z$

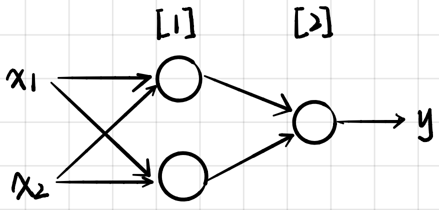
$$1=\sum_{k=1}^{K} P(y^{(i)}=k) = \sum_{k=1}^{K} e^{\beta_k x_i}/z.$$

$$\Rightarrow z = \sum_{k=1}^{K} e^{\beta_k x_i}$$

$$\Rightarrow P(y^{(i)}=k) = \frac{e^{\beta_k x_i}}{\sum_{k=1}^{K} e^{\beta_k x_i}}$$

2. (1)

model:



input: $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

parameter:

$W^{[1]} = \begin{bmatrix} w_{11}^{[1]} & w_{21}^{[1]} \\ w_{12}^{[1]} & w_{22}^{[1]} \end{bmatrix}$ $\quad W^{[2]} = \begin{bmatrix} w_{11}^{[2]} \\ w_{12}^{[2]} \end{bmatrix}$

$b^{[1]} = \begin{bmatrix} b_1^{[1]} \\ b_2^{[1]} \end{bmatrix}$ $\quad b^{[2]}$

$z^{[1]} = W^{[1]T}x + b^{[1]}$. $a^{[1]} = \mathrm{ReLU}(z^{[1]})$.

output: $y = z^{[2]} = W^{[2]T}a^{[1]} + b^{[2]}$.

取 $W^{[1]} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ $\quad b^{[1]} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

$W^{[2]} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ $\quad b^{[2]} = 0$.

分别令 $x = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

得到 $y = 0, 1, 1, 0$. 满足要求.

(2) 若激活函数为线性函数，则由矩阵运算为线性可知：

(1)中的model等价于

$$y = W^T x + b. \quad \text{其中 } W = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

所以要解决异或问题，就要使：

$$\begin{cases} w_1 + w_2 + b = 0 \\ b = 0 \\ w_1 + b = 1 \\ w_2 + b = 1 \end{cases}$$

该方程无解，故当激活函数为线性函数时，无法满足要求.

4. a) $z_1 \triangleq \begin{bmatrix} z_{11} \\ z_{12} \end{bmatrix} = \begin{bmatrix} w_1 & w_3 \\ w_2 & w_4 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_1 \end{bmatrix}$

$= \begin{bmatrix} 0.3775 \\ 0.3925 \end{bmatrix}$

$h = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = 6(z_1) = \begin{bmatrix} 0.5933 \\ 0.5969 \end{bmatrix}$

$z_2 \triangleq \begin{bmatrix} z_{21} \\ z_{22} \end{bmatrix} = \begin{bmatrix} w_5 & w_7 \\ w_6 & w_8 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} b_2 \\ b_2 \end{bmatrix}$

$= \begin{bmatrix} 1.0465 \\ 1.2249 \end{bmatrix}$

$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = 6(z_2) = \begin{bmatrix} 0.7401 \\ 0.7729 \end{bmatrix}$

b) $e = \frac{1}{2}(y_1 - O_1)^2 + \frac{1}{2}(y_2 - O_2)^2.$

$\frac{\partial e}{\partial w_5} = \frac{\partial e}{\partial y_1} \cdot \frac{\partial y_1}{\partial z_{21}} \cdot \frac{\partial z_{21}}{\partial w_5}$

$= (y_1 - O_1) \cdot y_1 (1 - y_1) \cdot h_1$

$= 0.07$

$\frac{\partial e}{\partial w_6} = \frac{\partial e}{\partial y_2} \cdot \frac{\partial y_2}{\partial z_{22}} \cdot \frac{\partial z_{22}}{\partial w_6}$

$= (y_2 - O_2) \cdot y_2 (1 - y_2) \cdot h_1$

$= -0.02$

c) $w_5' = w_5 - 0.1 \times \frac{\partial e}{\partial w_5} = 0.293$

$w_6' = w_6 - 0.1 \times \frac{\partial e}{\partial w_6} = 0.502$