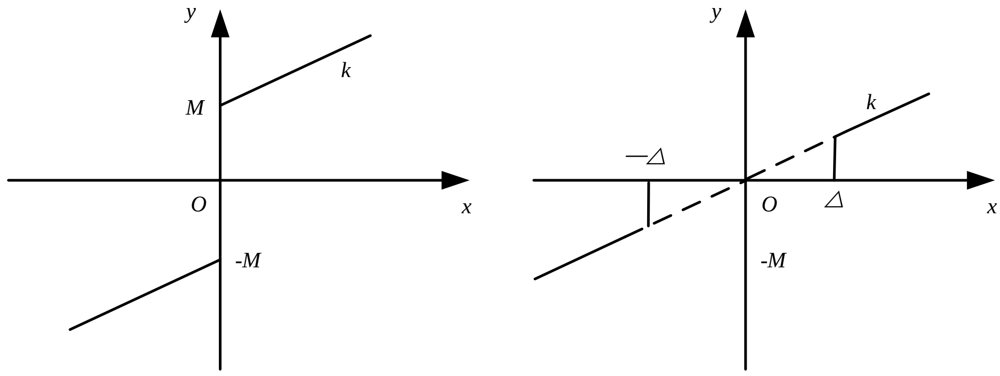
自动控制理论（1）作业十四答案

作业内容：在教材第七章内容和电子讲义的基础上，试解答以下题目。
学习目的：非线性系统分析
提交时间：12月12日上课交，或交电子版致网络学堂截至12月12日24时

书上 7.3

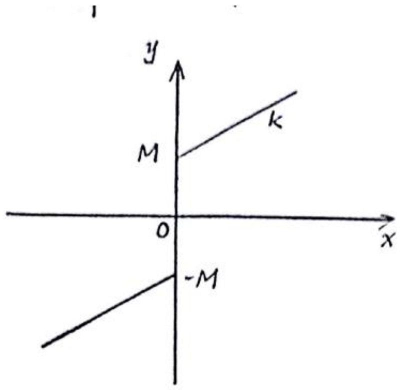
7.3 试求图7.E.2所示非线性特性的描述函数$N(X)$，并画出$N(X)$与$-1/N(X)$的图像。



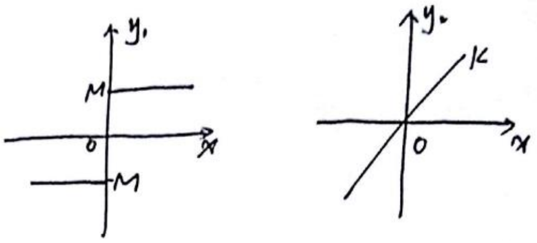
(a) (b)

图7.E.2

解：(a)

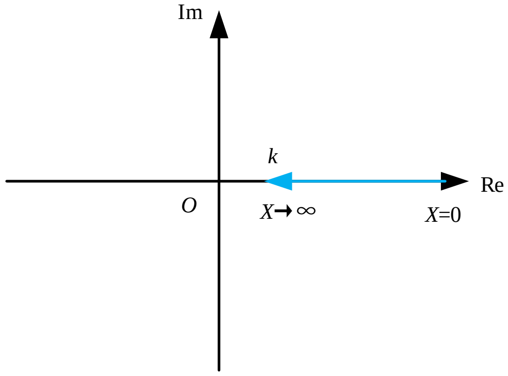


图示非线性性可分解为下图两个环节并联

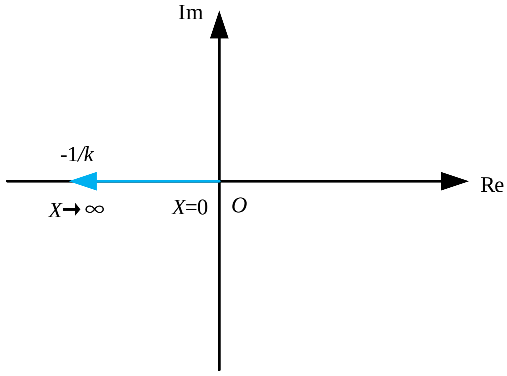


$$N(X)=N_1(X)+N_2(X)$$

$$=\frac{4M}{\pi X}+K$$

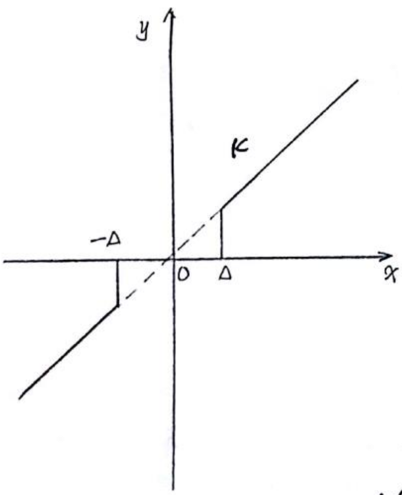


$N(X)$的图像

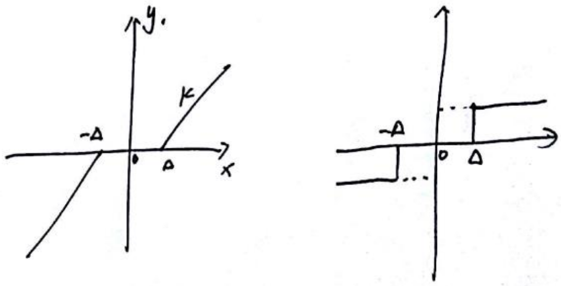


$-1/N(X)$的图像

(b)



图示非线性可分解为下图两个环节并联.

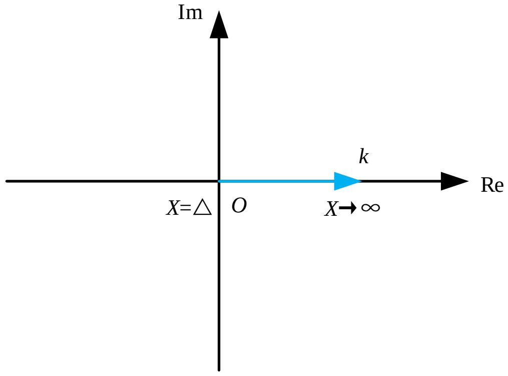


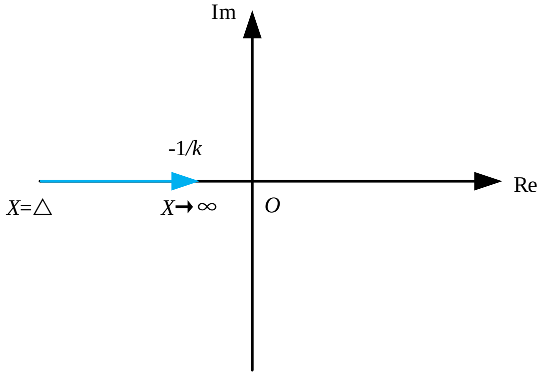
$$N(X)=N_1(X)+N_2(X)$$

$$=k-\frac{2k}{\pi}\left[\arcsin\frac{\Delta}{X}+\frac{\Delta}{X}\sqrt{1-\left(\frac{\Delta}{X}\right)^2}\,\right]+\frac{4M}{\pi X}\sqrt{1-\left(\frac{\Delta}{X}\right)^2}$$

$(\Delta\le X)$

$$=k+\frac{2k}{\pi}\left[\frac{\Delta}{X}\sqrt{1-\left(\frac{\Delta}{X}\right)^2}-\arcsin\frac{\Delta}{X}\right](X\ge\Delta)$$





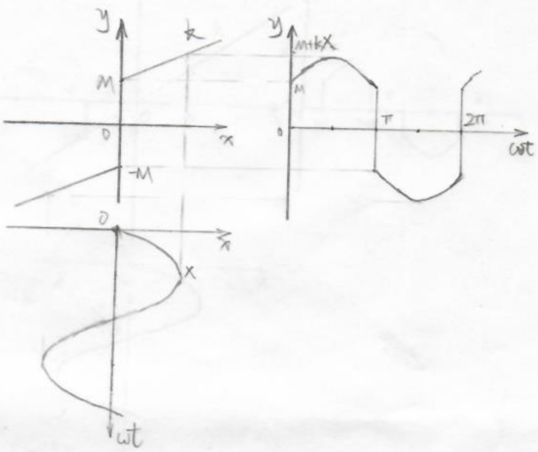
$N(X)$的图像　　　　　　　　　　$-1/N(X)$的图像

注：另一种解法是先计算输出函数$y(t)$，再计算A_1和B_1，从而计算$N(X)$。

7.3 (a) $y(t)$为奇函数, $A_1=0$

$$B_1=\frac{1}{\pi}\int_0^{2\pi} y(t)\sin\omega t\, d(\omega t)$$

$$y(t)=\begin{cases} kX\sin\omega t+M, & \omega t\in[0,\pi) \\ kX\sin\omega t-M, & \omega t\in[\pi,2\pi) \end{cases}$$



$$\therefore B_1=\frac{1}{\pi}\int_0^{2\pi} kX\sin^2\omega t\, d\omega t+\frac{1}{\pi}\int_0^{\pi} M\sin\omega t\, d\omega t-\frac{1}{\pi}\int_{\pi}^{2\pi} M\sin\omega t\, d\omega t$$

$$=\frac{kX}{\pi}\int_0^{2\pi}\frac{1-\cos 2\omega t}{2}\, d\omega t+\frac{2M}{\pi}\int_0^{\pi}\sin\omega t\, d\omega t$$

$$=\frac{kX}{\pi}\pi+\frac{4M}{\pi}=kX+\frac{4M}{\pi}$$

$$\therefore N(X)=\frac{B_1}{X}=k+\frac{4M}{\pi X}$$

以$\frac{M}{X}$为横轴，$\frac{N(X)}{k}$为纵轴 绘制描述函数图象

$$\frac{N(X)}{k}=1+\frac{4}{\pi k}\cdot\frac{M}{X}$$

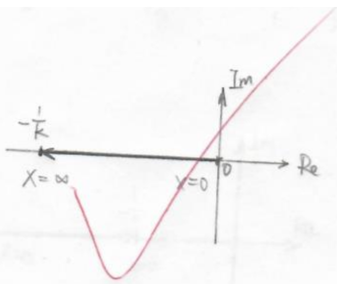
而，$-\frac{1}{N(X)}=-\frac{1}{k+\frac{4M}{\pi X}}$

$X:\ 0\to\infty$

$\frac{4M}{\pi X}:\ \infty\to 0$

$k+\frac{4M}{\pi X}:\ \infty\to k$

$-\frac{1}{N(X)}:\ 0\to -\frac{1}{k}$



(b) $y(x)=\begin{cases} kX\sin\omega t, & \omega t\in[\alpha_d,\pi-\alpha_d]\cup[\pi+\alpha_d,2\pi-\alpha_d] \\ 0, & \text{其他}\end{cases}$

$y(x)$为奇函数，$A_1=0$

$$B_1=\frac{1}{\pi}\int_0^{2\pi} y(t)\sin\omega t\,d\omega t$$
$$=\frac{4}{\pi}\int_{\alpha_d}^{\frac{\pi}{2}} kX\sin^2\omega t\,d\omega t$$
$$=\frac{2kX}{\pi}\int_{\alpha_d}^{\frac{\pi}{2}}(1-\cos 2\omega t)\,d\omega t$$
$$=\frac{2kX}{\pi}\left(\omega t-\frac{1}{2}\sin 2\omega t\right)\Big|_{\omega t=\alpha_d}^{\frac{\pi}{2}}$$
$$=\frac{kX}{\pi}\left(\pi-2\alpha_d+\begin{smallmatrix}\cancel{\sin 2\alpha_d}\\ \sin 2\alpha_d\end{smallmatrix}\right)$$

$$\therefore N(X)=\frac{B_1}{X}=\frac{k}{\pi}(\pi-2\alpha_d-\sin 2\alpha_d)$$

其中 $\alpha_d=\arcsin\frac{\Delta}{X}$

$$\sin 2\alpha_d=2\sin\alpha_d\cos\alpha_d=2\frac{\Delta}{X}\cdot\sqrt{1-\left(\frac{\Delta}{X}\right)^2}$$

$$\therefore N(X)=k-\frac{2k}{\pi}\arcsin\frac{\Delta}{X}+\frac{2k}{\pi}\cdot\frac{\Delta}{X}\sqrt{1-\left(\frac{\Delta}{X}\right)^2}$$

以$\frac{N(X)}{k}$为纵轴，$\frac{\Delta}{X}$为横轴绘制描述函数图象

记$\frac{\Delta}{X}=w$，$\frac{N(X)}{k}=1-\frac{2k}{\pi}\arcsin w+\frac{2k}{\pi}w\sqrt{1-w^2}$

而 $-\frac{1}{N(X)}=-\frac{k}{1-\frac{2}{\pi}\arcsin\frac{\Delta}{X}+\frac{2}{\pi}\frac{\Delta}{X}\sqrt{1-\left(\frac{\Delta}{X}\right)^2}}$

$X:\ \Delta\to\infty$

~~$\sin\alpha_d$~~ $\frac{\Delta}{X}: 1\to 0$

~~$\alpha_d: \frac{\pi}{2}\to 0$~~

$\frac{N(X)}{k}: 0\to 1$

$-\frac{1}{N(X)}: -\infty\to -\frac{1}{k}$