Homework 3 - Option 1

UCLA MAE 263F, Fall 2024
Mechanics of Flexible Structures and Soft Robots

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***Abstract*— This report strives to provide all deliverables for Homework 3 in partial fulfilment of the requirements for UCLA MAE 263F, Fall 2024 (Mechanics of Flexible Structures and Soft Robots)**

I. INTRODUCTION

Fits are important to perform good science, and machine-learned iterative methods are a useful method to create these linear fits when the data is unintuitive. There are two main types of fits: linear and nonlinear. Both need to be carefully examined to understand the methods of implementing machine learning methods in creating these fits.

II. PROBLEM 1 - LINEAR FITS

Here, we want to fit data to a linear equation:

$$y = mx + b \tag{1}$$

A. Theory

We use the Mean Squared Error (MSE) loss function:

$$\text{Loss}(m, b) = \frac{1}{N} \sum_{i=1}^{N} (y_i - (m \cdot x_i + b))^2 \tag{2}$$

The gradients of the loss function with respect to m and b are:

$$\frac{\partial \text{Loss}}{\partial m} = -\frac{2}{N} \sum_{i=1}^{N} x_i \cdot (y_i - (m \cdot x_i + b)) \tag{3}$$

$$\frac{\partial \text{Loss}}{\partial b} = -\frac{2}{N} \sum_{i=1}^{N} (y_i - (m \cdot x_i + b)) \tag{4}$$

Using gradient descent, we update the parameters m and b as follows:

$$m \leftarrow m - \eta \frac{\partial \text{Loss}}{\partial m}, b \leftarrow b - \eta \frac{\partial \text{Loss}}{\partial b} \tag{5}$$

where η is the learning rate.

B. Problem 1a: Compare the predicted values with the actual data

These are the results of fitting the model for 10000 epochs at a learning rate of 0.001.
m$_{f}itis - 0.011781642379859762$
b$_{f}itis0.06320409591699154$

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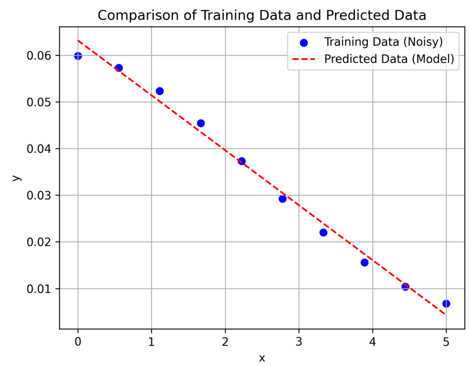

Fig. 1. TPredicted values y_red versus actual data y

C. Problem 1b: Experiment with the learning rate and number of epochs

Here I tried different values of the learning rate (η)

III. RESULTS

A. General Results

We observe that the elastic rod begins the simulation as a coiled planar circle, but stretches under gravity to become a helical coil (spring shape). With the time available for this project, the most accurate simulation of this scenario comprised $N = 400$ nodes and $\Delta t = 0.005$ s. This simulation required 83 mins to complete computation on the author's personal computer.

B. Sensitivity Analysis

Sensitivity analysis was performed to determine the requirements on the refinement of mesh for accurate results. The mesh is constructed parametrically by defining the number of nodes N and the time-step in the simulation Δt.

It is found that decreasing the number of nodes does not have significant impact on the results, as shown in Fig. **??**. However, when we further increase the refinement of $\Delta t : 0.005 \rightarrow 0.001$, there is noticeable difference. The damping effects on the system in converging to a steady state of $z \approx -0.04$ are less pronounced, and so steady state is not achieved in the $5s$ duration of this simulation.

In either case of adjusting either K or Δt, the trade off is computational time. We note that refinement of Δt is linearly correlated to the number of steps to be computed. In this case, each subsequent step of the simulation should take less iterations of Newton-Raphson to converge as the system approaches a steady state. Still, as shown in Fig. **??**, if there is no steady state achieved in the simulation, then the computational time for each step is approximately constant, and so the computational time for this simulation is linearly correlated with the refinement in Δt.

Alternatively, increasing the density of nodes K has less noticeable effects on computational time, because the physics of nodes that are linked close together mean that there is less variance, and so the computational increase with a higher node density is not a linear correlation; it is intuitively logarithmic in nature.

For completness, we also furnish the nominal plot of z-coordinates in the last node for $N = 50$, $\Delta t = 0.01$s, as is furnished in Chapter 7 of the text in this course.

ACKNOWLEDGMENT

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REFERENCES

[1] M. Khalid Jawed, Singmin Lim, Discrete Simulation of Slender Structures, UCLA Fall 2024 MAE 263F