Homework 2

UCLA MAE 263F, Fall 2024
Mechanics of Flexible Structures and Soft Robots

He Kai Lim[1]

***Abstract*— This report strives to provide all deliverables for Homework 2 in partial fulfilment of the requirements for UCLA MAE 263F, Fall 2024 (Mechanics of Flexible Structures and Soft Robots)**

I. INTRODUCTION

A complete simulation of three dimensional (3D) elastic rods is a fundamental building block to simulating complex structures in the real world. This builds on prior work with 2D simulations on beams, where the fundamental physics of bending and stretching are considered. The complexity with extension into the 3D domain stems from the inclusion of twist — nodes can rotate about the tangential axis between each other.

II. METHODS

The method of discrete simulation is heavily referenced from the course notes of UCLA MAE 263F, Fall 2024. Specific to this homework assignment, the fundamental physics incorporated in this simulation are described below.

A. Elastic Energies

We note that the total elastic energy of an elastic rod is

$$E_{elastic} = \underbrace{\sum_{k=1}^{N-1} E_k^s}_{stretching} + \underbrace{\sum_{k=2}^{N-1} E_k^b}_{bending} + \underbrace{\sum_{k=2}^{N-1} E_k^t}_{twisting} \qquad (1)$$

between all N nodes k.

In the generalization from 2D to 3D, we note that $stretching$ and $bending$ energy formulations in 3D are intuitive. The novelty in this work stems from the incorporation of *twisting* energy, represented as

$$\sum_{k=2}^{N-1} E_k^t = \frac{1}{2} GJ \tau_k^2 \frac{1}{\bar{l}_k} \qquad (2)$$

where GJ is the twisting stiffness (shear modulus times polar moment of inertia), τ_k is the integrated twist at node k.

[1]He Kai Lim is with the Department of Mechanical and Aerospace Engineering, University of California Los Angeles, Los Angeles, CA 90095 USA limhekai@ucla.edu

B. Simulation Scenario

An elastic rod with a total length $l = 20$cm is naturally curved with radius $R_n = 2$cm. The location of its N nodes at $t = 0$ are

$$x_k = [R_n \cos{(k-1)\Delta\theta}, R_n \sin{(k-1)\Delta\theta}, 0]$$

where $\Delta\theta = \frac{l}{R_n}\frac{1}{N-1}$. The twist angles θ^k $(k = 1, ..., N-1)$ at $t = 0$ are 0. The first two nodes and the first twist angle remain fixed throughout the simulation (i.e. one end is clamped). The physical parameters are: density $\rho = 1000\text{kg/m}^3$, cross-sectional radius $r_0 = 1$mm, Young's modulus $E = 10$MPa , shear modulus $G = \frac{E}{3}$ (corresponding to an incompressible material), and gravitational acceleration $g = [0, 0, -9.81]^T$.

The number of nodes N and the step size Δt are investigated in sensitivity analysis across multiple simulations thusly presented below.

III. RESULTS

A. General Results

We observe that the elastic rod begins the simulation as a coiled planar circle, but stretches under gravity to become a helical coil (spring shape). With the time available for this project, the most accurate simulation of this scenario comprised $N = 400$ nodes and $\Delta t = 0.005$ s. This simulation required 83 mins to complete computation on the author's personal computer.

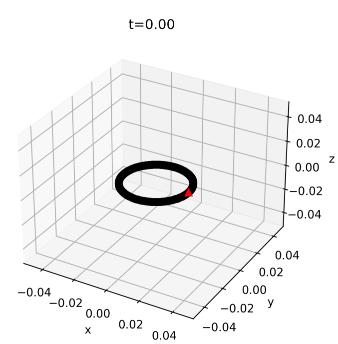

Fig. 1. The most accurate simulation, starting pose ($N = 400$, $\Delta t = 0.005$s)

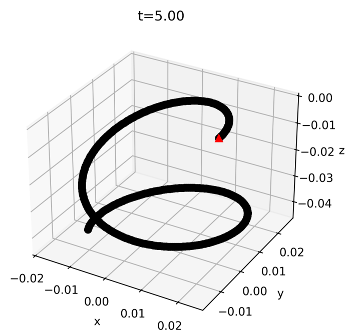

Fig. 2. The most accurate simulation, ending pose ($N = 400$, $\Delta t = 0.005$s)

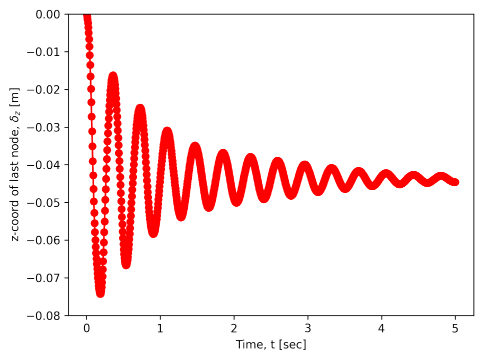

Fig. 3. The most accurate simulation, plot of z-coordinates for last node ($N = 400$, $\Delta t = 0.005$s)

B. Sensitivity Analysis

Sensitivity analysis was performed to determine the requirements on the refinement of mesh for accurate results. The mesh is constructed parametrically by defining the number of nodes N and the time-step in the simulation Δt.

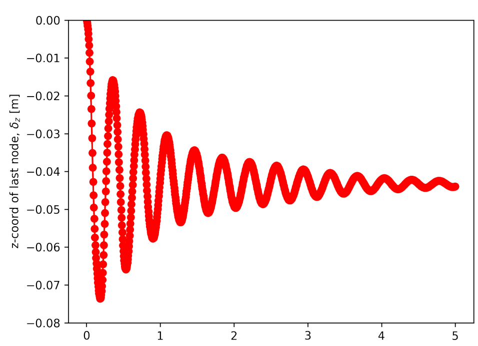

Fig. 4. Decreasing Number of Nodes (k), plot of z-coordinates for last node ($N = 100$, $\Delta t = 0.005$s)

It is found that decreasing the number of nodes does not have significant impact on the results, as shown in Fig. 4. However, when we further increase the refinement of $\Delta t : 0.005 \rightarrow 0.001$, there is noticeable difference. The damping effects on the system in converging to a steady state of $z \approx -0.04$ are less pronounced, and so steady state is not achieved in the $5s$ duration of this simulation.

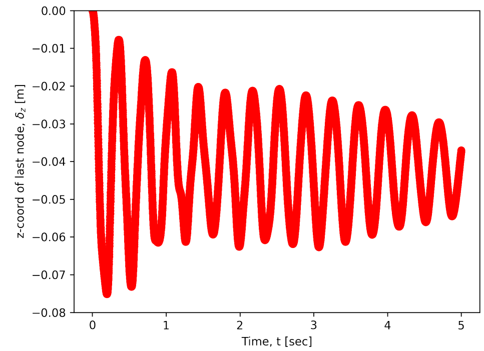

Fig. 5. Decreasing time steps for more refinement (Δt), plot of z-coordinates for last node ($N = 50$, $\Delta t = 0.001$s)

In either case of adjusting either K or Δt, the trade off is computational time. We note that refinement of Δt is linearly correlated to the number of steps to be computed. In this case, each subsequent step of the simulation should take less iterations of Newton-Raphson to converge as the system approaches a steady state. Still, as shown in Fig. 5, if there is no steady state achieved in the simulation, then the computational time for each step is approximately constant, and so the computational time for this simulation is linearly correlated with the refinement in Δt.

Alternatively, increasing the density of nodes K has less noticeable effects on computational time, because the physics of nodes that are linked close together mean that there is less variance, and so the computational increase with a higher node density is not a linear correlation; it is intuitively logarithmic in nature.

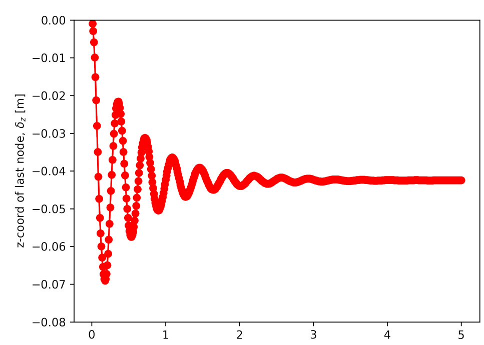

Fig. 6. Nominal plot of z-coordinates for last node ($N = 50$, $\Delta t = 0.01$s)

For completness, we also furnish the nominal plot of z-coordinates in the last node for $N = 50$, $\Delta t = 0.01$s, as is furnished in Chapter 7 of the text in this course.

ACKNOWLEDGMENT

Material used in this report are taken from the course UCLA MAE 263F, Fall 2024.

REFERENCES

[1] M. Khalid Jawed, Singmin Lim, Discrete Simulation of Slender Structures, UCLA Fall 2024 MAE 263F