MATH324: Statistics

Matthew He

January 10, 2025

Textbook: *Mathematical Statistics with Applications* by Dennis Wackerly, William Mendenhall, Richard L. Scheaffer.

1 Introduction

The objective of statistics is to make an inference about a population based on information contained in a sample from that population and to provide an associated measure of goodness for the inference.

1.1 Estimation

Populations are usually characterized by numerical descriptive measures called **parameters**. The objective of many statistical investigations is to estimate the value of one or more relevant parameters. We called the parameter of interest in the experiment the *target parameter*.

The process of estimating the value of a parameter is called **estimate**. One type of estimate is called a **point estimate**, if a single value, or point, is given as the estimate of some parameter like mean μ. Another type of estimation procedure tells us a parameter will fall between two numbers. In this type of estimation, two values are given and may be used to construct an interval that is intended to enclose the parameter of interest; thus, it is called an **interval estimate**.

Definition 1 (Estimator). *An **estimator** is a rule or formula that tells us how to calculate an estimate based on the measurements contained in a sample.*

1.2 Inequalities

Definition 2 (Markov's Inequality). *Let X be a random variable and h be a **non-negative** function; i.e. $h : \mathbb{R} \mapsto \mathbb{R} \cup \{0\}$.*
Suppose $\mathbb{E}[h(X)] < \infty$, then for some $\lambda > 0$, we have:

$$\mathbb{P}(h(x) \geq \lambda) \leq \frac{\mathbb{E}[h(x)]}{\lambda}.$$

Then we have Tchebysheff's Inequality, which is a special case of Markov's Inequality. Consider $h(x) = (x - \mu)^2$, we have:

Definition 3 (Tchebysheff's Inequality). *Let X be a random variable with finite mean μ and finite variance σ^2. Then for any $K > 0$, we have:*

$$\mathbb{P}(|X - \mu| \geq K\sigma) \leq \frac{1}{K^2}.$$

References