NEUR503: Computational Neuroscience

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Abstract

1 Model Neurons: Neuroelectronics

1.1 Electrical Properties of Neurons

By convention, the potential of the extracellular fluid outside a neuron is defined to be zero. When a neuron is inactive, the excess internal negative charge causes the potential inside the cell membrane to be negative.

Under normal conditions, neuronal membrane potentials vary over a range from about -90mV to +50mV.

Reversal potentials (a.k.a Nernst potentials).
A channel's reversal potential is the membrane potential at which the net current for that ion is zero. They are calculated by the Nernst equation.

Voltage, ions, and currents.
Current is sum of the flow of ions through channels, while the direction of the flow of ions is determined by the voltage difference.

1.2 The Hodgkin-Huxley Model: Dynamics

The Hodgkin-Huxley model for the generation of the action potential, in its single-compartment form, is constructed by writing the membrane current in equation 5.6 as the sum of a leakage current, a delayed-rectified K^+ current, and a transient Na^+ current,

$$i_m = \bar{g}_L(V - E_L) + \bar{g}_K n^4(V - E_K) + \bar{g}_{Na} m^3 h(V - E_{Na}). \qquad (5.25)$$

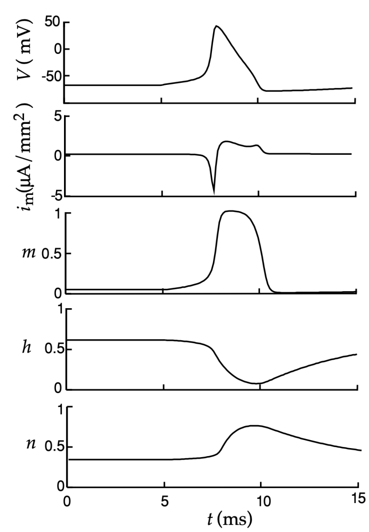
The maximal conductances and reversal potentials used in the model are

$$\bar{g}_L = 0.003\,\mathrm{mS/mm^2}, \quad \bar{g}_K = 0.36\,\mathrm{mS/mm^2}, \quad \bar{g}_{Na} = 1.2\,\mathrm{mS/mm^2},$$

$$E_L = -54.387\,\mathrm{mV}, \quad E_K = -77\,\mathrm{mV}, \quad \text{and} \quad E_{Na} = 50\,\mathrm{mV}.$$

The full model consists of equation 5.25 for the membrane current, and equations of the form 5.17 for the gating variables n, m, and h. These equations can be integrated numerically, using the methods described in appendices A and B.

Temporal evolution of dynamic variables.
The initial rise of the membrane potential, prior to the action potential, seen in the upper panel of figure below, is due to the injection of a positive electrode current into the model starting at $t = 5ms$.



The dynamics of V, m, h, and n in the Hodgkin-Huxley model during the firing of an action potential. The upper-most trace is the membrane potential, the second trace is the membrane current produced by the sum of the Hodgkin-Huxley K^+ and Na^+ conductances, and subsequent traces show the temporal evolution of m, h, and n. Current injection was initiated at

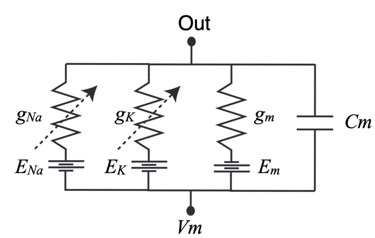
When the current drives the membrane potential up to about -55mV, the m variable that describes activation of the Na^+ conductance suddenly jumps from nearly 0 to a value near 1.

Initially, the h variable, expressing the degree of inactivation of the Na^+ conductance, is around 0.6. Thus, for a brief period both m and h are significantly different from 0. This causes a large influx of Na^+ ions, producing the sharp downward spike of inward current shown in the second trace from the top. The inward current pulse causes the membrane potential to rise rapidly to around 50 mV (near the Na^+ equilibrium potential). The rapid increase in both V and m is due to a positive feedback effect. Depolarization of the membrane potential causes m to increase, and the resulting activation of the Na^+ conductance makes V increase. The rise in the membrane potential causes the Na^+ conductance to inactivate by driving h toward 0. This shuts off the Na^+ current. In addition, the rise in V activates the K^+ conductance by driving n toward 1. This increases the K^+ current, which drives the membrane potential back down to negative values. The final recovery involves the readjustment of m, h, and n to their initial values.

1.3 *The Hodgkin-Huxley Model: Formalization*

$$I_c + I_m + I_{Na} + I_K = 0, \tag{1}$$

$$C\frac{dV_m}{dt} + g_m(V_m - E_m) + \bar{g_K} n^4 (V_m - E_K) + \bar{g_{Na}} m^3 h (V_m - E_{Na}) = 0, \tag{2}$$



where n, m, h are gating functions.

- I_c is the **capacitive current**. It arices from charging or discharging the membrane capacitor C.
- I_m is the **leakage (or membrane) current**. Given by Ohm's law, with conductance g_m and reversal potential E_m.
- $\bar{g_K}, \bar{g_{Na}}$ denotes the **maximum conductance** for that ion channel.
- The gating functions n, m, h give the fraction of open channels for that ion type. So the effective conductance would be $\bar{g_K} n^4$ and $\bar{g_{Na}} m^3 h$ etc.

Exponent on the gating function? It relects how many subunit must be open simutaneously to open the channel.

variability neq randomness
Autocorrelation: counts the number of coincidences spikes

References