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Linear Motor Driven Inverted Pendulum and LQR Controller Design

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***Abstract* - Inverted pendulum can verify the effectiveness of controllers. Traditional inverted pendulum is driven by a rotational motor, and its control performance is influenced by the friction and gap of transmission mechanism. We proposed an innovative Direct Driven Inverted Pendulum (dDIP) which consists of an inverted pendulum (IP) mounted on a stage driven by an ironless permanent magnet linear synchronous motor (IPMLSM). In this paper, the structure of dDIP is first analyzed and its mathematic model is built. Then, a LQR controller is designed and numerical simulations in MATLAB are carried out. Finally, the dDIP is successfully controlled by a dSPACE controller and also a TI28DSP based control card. The results show that the dDIP can be stabilized to its upright position and at the same time the cart's displacement can be also regulated to zero after exerting a step disturbance to the dDIP. Experiments show that the transition time is within 4s, which indicates that dDIP has excellent dynamic characteristics that can not be reached by a traditional IP.**

Index Terms - inverted pendulum; linear synchronous motor; LQR; dSPACE.

List Of Abbreviation

IP	inverted pendulum
LSM	linear synchronous motor
IPMLIM	ironless permanent magnet linear synchronous motor
dDIP	direct driven inverted pendulum

I. Introduction

Inverted pendulum system, which has the characteristics of high order, instability, multivariable, nonlinear and strong coupling [1], is very similar with the control of biped robot [2], rocket launching and Pan & Tilt. What's more, it can effectively reflect many key points in the process of control, such as the problem of none-linear, robustness, stabilization and tracing of the system [3], [4], [5]. The control performance of inverted pendulum can be measured directly by the angle of pendulum, the displacement of cart and the transition time. We can use inverted pendulum to verify and compare the effectiveness of controller when an innovative theory or method of control comes out [6], [7]. Therefore, the research for control techniques for inverted pendulum has important theoretic and practical meaning. It is widely concerned by scholars.

Schaefer and Cannon used Bang-Bang theory [8] to control a crankshaft at the inverted position in 1966. That was a single and simple IP. In 2002, after 36 years of its first appearing, Prof. Hongxing Li of Beijing Normal University developed his original variable universe stable adaptive fuzzy controller [9] to control a complicated quadruple inverted pendulum, still using traditional structure. Nowadays inverted pendulum becomes an important experimental device for studying, researching and validation of different control theory [10]. Traditionally, typical inverted pendulum is driven by a rotating servo motor which drives the cart via transfer mechanism to keep the balance of inverted pendulum. With this kind of configuration, the problem is that the transmission friction and gap are included in the system. Transmission by flexible belt will also produce vibration, extension, and delay, and make the control system unpredictable [11]. The system can not reach the state of absolute stable, but moving around the balance point. To overcome the defect of the inverted pendulum driven by a rotating machine and make it reflect the reality of the control theory or method, this paper proposed a novel inverted pendulum which is driven by an ironless permanent magnet linear synchronous motor and we call it dDIP. Its mathematical model is analyzed and a corresponding control system is built. Experimental results from both digital simulation in MATLAB and real-time control in dSPACE are provided in the paper.

II. MODEL Analysis Of A Ddip

Linear motor is a new type driving device which can directly transform electric energy to mechanical linear motion and is called "direct transmission" or "zero clearance transmission" [12]. It has the advantages of high velocity, high acceleration, high accuracy, and no maximal travel length restriction. Linear motor can be used in industry, commercial, military and any other field where linear motion [13][14] is needed. Linear motor can be classified into linear induction motor, linear synchronous motor etc. The motor used in our system is an ironless permanent magnet linear synchronous motor which is developed by us. Its maximal velocity is 5 m/s; maximal acceleration is 100 m/s^2; rated thrust force is 98 N; the peak thrust force is 280 N and the stage's resolution is 5μm.

The dDIP consists of a linear motor, a pendulum, a pedestal and a rotary encoder, as shown in Fig. 1. The cart for inverted pendulum is attached to the mover of the linear motor by rigid connection. In this way, the mover can directly drive the cart to achieve linear motion without transfer mechanism.

A. Modeling of Inverted Pendulum (IP)

The physical model of a single IP is shown in Fig. 2; m is the mass of the pendulum while M is the mass of the pedestal; l is the length between the center of rotational shaft and the centroid of the pendulum. The inertia of the pendulum is I. F is the force exerted on the pedestal. x is the displacement of the pedestal. θ is the pendulum angle from vertical.

The assumptions are as follows: (1) the pendulum and the pedestal are both rigid body; (2) air resistance and friction force between pendulum and the bearing are ignored; (3) the direction of the arrowhead is positive direction of the vector.

Analyzing the physical model of the single IP, we can obtain the mathematical expression of IP as follows:

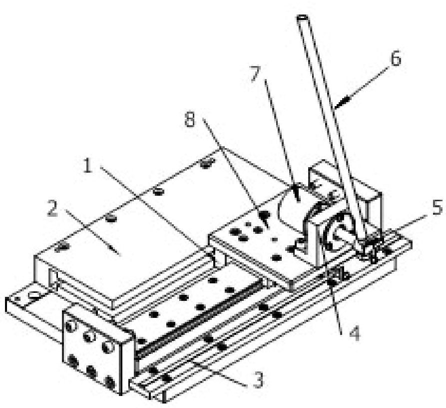
$$\begin{cases}(M+m)\ddot{x}+ml\ddot{\theta}\cos\theta+ml\dot{\theta}^2\sin\theta=F\\ (I+ml^2)\ddot{\theta}+ml\ddot{x}\cos\theta=mgl\sin\theta\end{cases} \tag{1}$$

While the IP is running, normally θ (radian) hardly changes at the equilibrium point and nears zero. Therefore, approximation processing can be made: $\cos\theta\approx 1$, $\sin\theta\approx\theta$, $(d\theta/dt)^2\approx 0$. With u representing the input force F, the expressions (1) can be simplified as follows:

$$\begin{cases}(M+m)\ddot{x}+ml\ddot{\theta}=u\\ (I+ml^2)\ddot{\theta}+ml\ddot{x}=mgl\theta\end{cases} \tag{2}$$

B. Modeling of linear motor

The frequency response of the linear motor is measured by using a dynamic signal analyzer Agilent 35670A. Agilent 35670A is a FFT type frequency spectrum/network analyzer with 4 channels.



1. mover 2. stator 3. linear encoder 4. pedestal
5. shaft 6. pendulum 7. rotary encoder 8. cart

Fig. 1 Inverted pendulum and linear synchronous motor

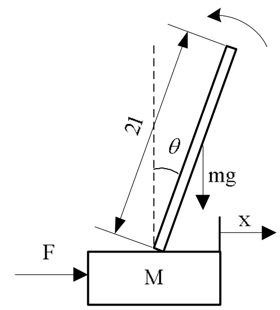


Fig. 2 Physical model of the single inverted pendulum

This standard apparatus can measure frequency spectrum, network, time domain and amplitude domain in the range of 0-100 KHz and can analyze frequency response, octave, harmonic distortion and order spectrum [15]. Agilent 35670A requires that the input is an analog signal, but the displacement of the linear motor's mover given by a linear encoder which has a resolution of 5μm is digital. So, a TMS320F2812 DSP is used to decode and count the digital signals. Then DSP convert the digital count value into analog voltage through DAC7731. In this way, Agilent 35670A can sweep sine to the linear motor.

We mounted the pedestal of the IP on the linear motor's mover without the pendulum while the sweeping process was in progress thus the mass M (including the mass of the angle encoder) of the pedestal is taken into account. The result of the measurement is the motor's frequency response within 1-100 Hz. Using fitting function of supplied by MATLAB, we obtained the linear motor's transfer function; where the input is voltage *u*, and the output was displacement *x*.

$$G(s)=\frac{X(s)}{U(s)}=\frac{1.869}{s^2+12.32s+0.4582} \tag{3}$$

To apply inverse Laplace transformation of (3), and the result can be expressed as:

$$1.869u=\ddot{x}+12.32\dot{x}+0.4582x \tag{4}$$

C. Modeling of dDIP

According to equation (4) we can get:

$$\ddot{x}=-12.32\dot{x}-0.4582x+1.869u \tag{5}$$

Combining (5) and (2) we can obtain the following equation:

$$\ddot{\theta}=\frac{0.4582ml}{I+ml^2}x+\frac{12.32ml}{I+ml^2}\dot{x}+\frac{mgl}{I+ml^2}\theta-\frac{1.869ml}{I+ml^2}u \tag{6}$$

Four state variables are chosen as follows:

$x_1=x, x_2=\dot{x}, x_3=\theta, x_4=\dot{\theta}$,

So the state vector is $X=\begin{bmatrix}x\\ \dot{x}\\ \theta\\ \dot{\theta}\end{bmatrix}$,

and the state space description of the dDIP is

$$\dot{X} = Ax + Bu = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -0.4582 & -12.32 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ a & b & c & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 1.869 \\ 0 \\ d \end{bmatrix} u \quad (7)$$

Where

$$a = \frac{0.4582ml}{I+ml^2}, b = \frac{12.32ml}{I+ml^2}, c = \frac{mgl}{I+ml^2}, d = -\frac{1.869ml}{I+ml^2}$$

Choose the outputs as follows:

$$y_1 = x, y_2 = \dot{x}, y_3 = \theta, y_4 = \dot{\theta}$$

So the output vector Y is:

$$Y = CX = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix} \quad (8)$$

By measurement, we know that $m = 0.1$kg, $l = 0.2415$ m, and we choose $g = 9.8$ m/s^2.

Substitute the value of the parameters mentioned in above into *a, b, c, d* and we get the matrices as follows:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -0.4582 & -12.32 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1.6918 & 45.4892 & 36.1846 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1.8690 \\ 0 \\ -6.9009 \end{bmatrix}$$

III. CONTROLLER DESIGN FOR THE PENDULSM AND SIMULATION

A. *Introduction of LQR*

Suppose that the state equation of the linear time-invariant system is:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \end{cases} \quad (9)$$

Introducing the optimal control performance index, that is, designing an input $u(t)$ to make the parameter J in the following equation is minimal.

$$J = \frac{1}{2}\int_0^\infty \left[x^T(t)Q(t)x(t) + u^T(t)R(t)u(t) \right] d(t) \quad (10)$$

Q and R denote the weighting matrix of state variable and input variable. If this system is disturbed and offset the zero state, the control u can make the system come back to zero state and J is minimal at the same time [16]. Here, the control value u is called optimal control. The control signal should be:

$$u(t) = -R^{-1}B^T P(t)x(t) = -Kx(t) \quad (11)$$

Where $P(t)$ is the solution of Riccati equation, K is the linear optimal feedback matrix. Now we only need to solve the Riccati equation (12):

$$PA + A^T P - PBR^{-1}B^T P + Q = 0 \quad (12)$$

Then we can get the value of P and K.

$$K = R^{-1}B^T P = [k_1, k_2, k_3, k_4]^T \quad (13)$$

B *Controller design for the dDIP and simulation*

According to the optimal control law, its optimality is totally depended on the selection of Q and R. However, there is no resolving method to choose these two matrices. The widespread method used to choose Q and R is simulation and trial. Generally, Q and R should be both diagonal matrix. If we hope smaller input, larger R is needed; if we hope the input of a state is smaller, the element in the corresponding column of Q needs to be larger.

When Q is fixed but R reduces, the transition time and the overshoot will be reduced; rise time and steady state error will increase. When R is fixed but Q increases, the transition time and the overshoot will be reduced, so is the change of the angle; but rise time and steady state error will increase at the same time [17]. We choose the weighting matrices Q = diag [5000, 0, 100, 0] and R=-70.7111 after simulation. We use MATLAB function K=*lqr* (A, B, Q, R) to solve the optimal problem. Run K=*lqr* (A, B, Q, R) in MATLAB, we can get:

$$K = [-70.9563 \quad -41.2621 \quad -85.5466 \quad -14.1238]$$

When applied state feedback, the state equation of the closed loop system is $(A - BK, B, C - DK, D)$.

The step response curve is as follows:

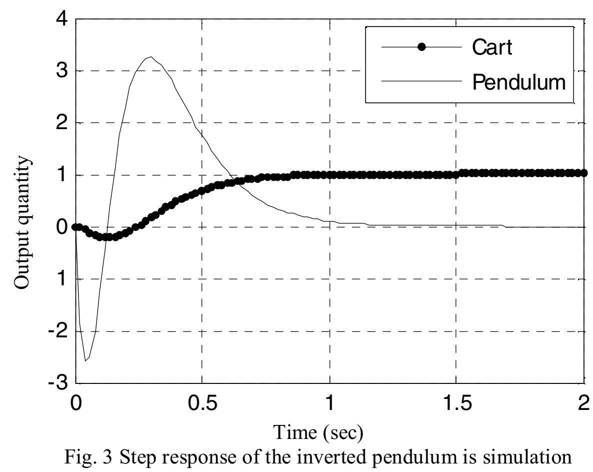


Fig. 3 Step response of the inverted pendulum is simulation

As shown in Fig. 3, the dDIP has good dynamic performance. The transition time of the angle and the displacement are both no more than 1.2s and the overshoot is not very large.

IV. EXPERIMENT RESULTS AND ANALYSIS

A Real-time control of dDIP

Fig. 4 Picture of the and linear synchronous motor driven inverted pendulum

Fig.4 is the picture of a dDIP, and Fig.5 shows the hardware architecture of the dDIP, including dSPACE real-time control card, BAS701515 servo drive from Elmo Corporation of Israel, rotary encoder, linear motor and inverted pendulum. dSPACE DS1104 is specifically designed for the development of high-speed multivariable digital controllers and real-time simulations in various fields. dSPACE receives angle signal from encoder and cart's position encoder, then dSPACE processes these signals to get $\theta, x, \dot{\theta}, \dot{x}$ four state variables.

Control system is built in simulink, as shown in Fig. 6, and the system's sample time is set as 0.01s. In Fig. 6, 'DS1104ENC_Angle' is the angle signal input block, and 'DS1104ENC_Position' is the linear displacement signal input block. The resolution of the linear encoder is 0.5 μm, and 'DS1104ENC_Position' will produce large output once motor moves a little. In order to match the output of 'DS1104ENC_Angle', we use 'div' block to reduce the value of DS1104ENC_Position's output.

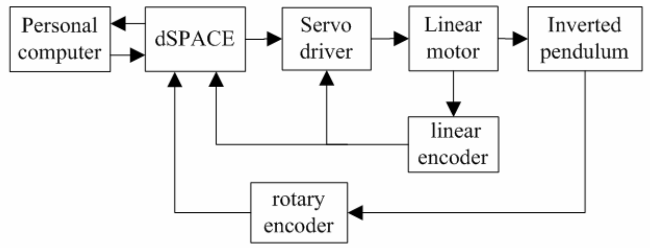


Fig. 5 Hardware architecture of the inverted pendulum and linear synchronous motor

During control process, the angle and angular velocity of the pendulum, the displacement and velocity of the cart are firstly multiplied by the feedback matrix and then summed up together; then the result is reduced to a certain ratio that permitted by the DAC; finally, the control value is given out via block 'DS1104DAC_C2' and passed to the motor driver and the controlling operation is achieved. To get better dynamic performance of IP, the feedback matrix can be regulated according to the value from MATLAB simulation in section 5.2. After several times of experiment, we select feedback matrix as follows:

$$K = \begin{bmatrix} -180.9563 & -10.2621 & -97.5466 & -17.1238 \end{bmatrix}$$

B Experiment result and analysis

After exerting step disturbance to the IP, the variation of pendulum angle, cart displacement and output voltage via DAC are displayed in the interface provided by ControlDesk, an operation tool offered by DS1104, as shown in Fig.7, which indicates that the pendulum reaches stabilization state immediately and keeps static. The transition time is less than 0.6s, and the cart would trace the step signal, and its transition time is within 4s. And this demonstrated that dDIP possesses an excellent dynamic characteristic. Once the elements of weighting matrix Q are reduced, the transition time is apparently increased.

V. Conclusion

This paper presents an innovative inverted pendulum driven by an ironless permanent magnet linear synchronous motor. LQR is used as a controller for the dDIP. It succeeds in using MATLAB for digital simulation and dSPACE to perform the real-time control over the dDIP.

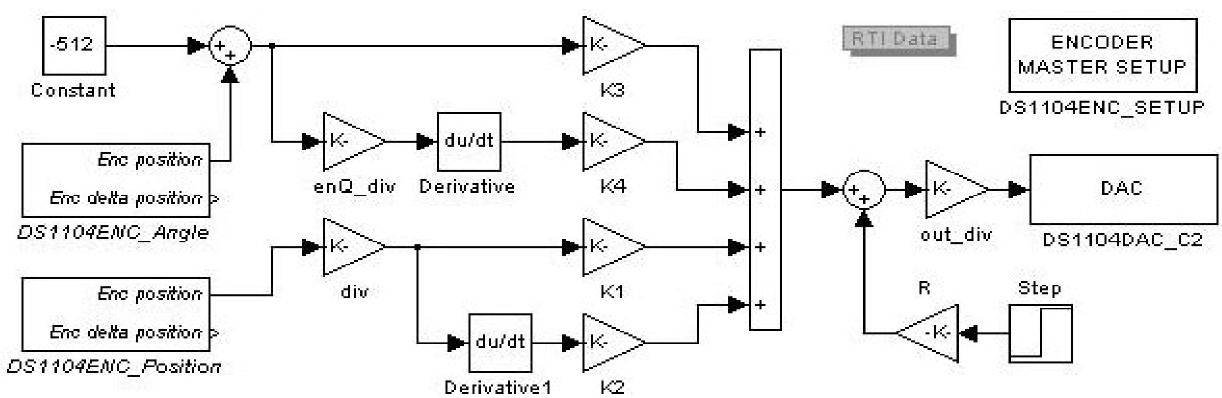


Fig. 6 LQR controller for the inverted pendulum

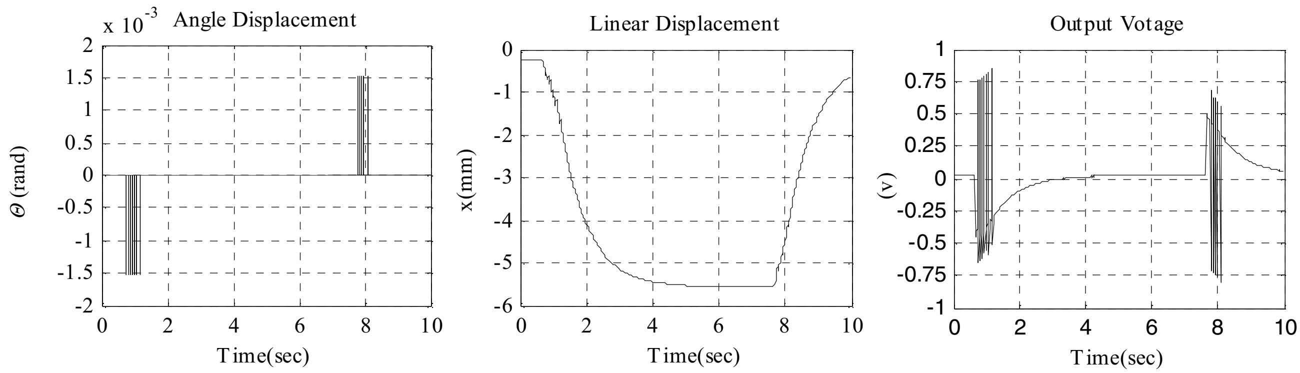


Fig. 7 Step responds of the inverted pendulum in real-time control

The dDIP has an excellent dynamic characteristic because of the direct driving of the linear motor. Its steady state, the balance point, can be reached, and the displacement of the cart and the angle of pendulum can be regulated to zero. If no disturbance exerted on the inverted pendulum, the pendulum can keep static.

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