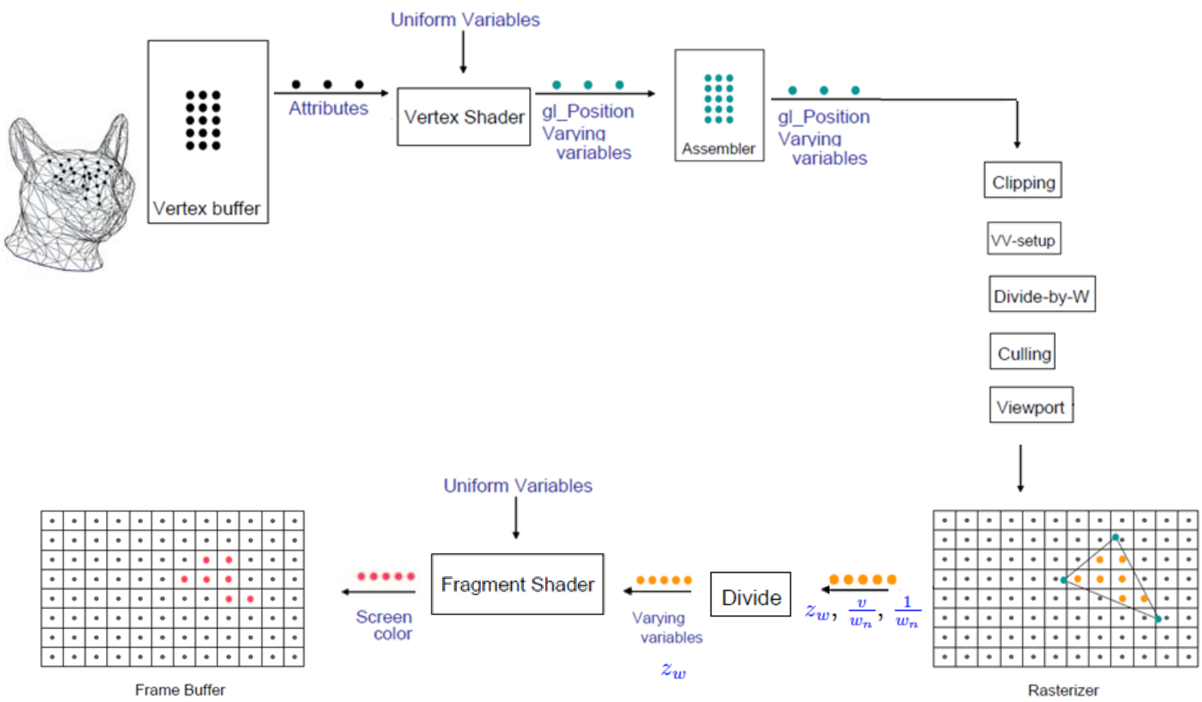
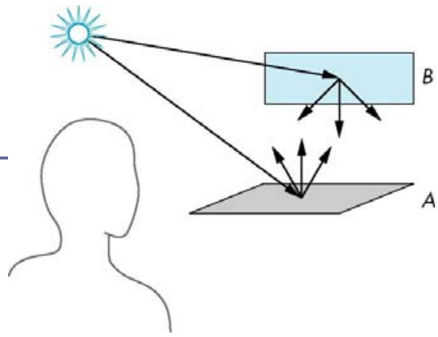
Chapters 1~13



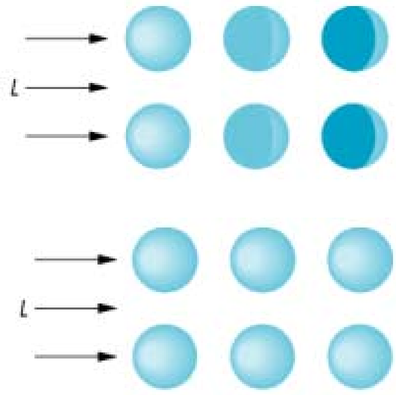
Lighting and Shading (1)

May 10, 2016

Light and Matter



- Rendering equation
  — very complex
- Ray tracing / Radiocity
  - global model
  - Not suitable for the graphics pipeline
- **Phong reflection model**
  - local model
  - A point on the surface is independent of the other points



Phong Reflection Model

- 3 types of material-light interactions

Reflection ⊗ **Illumination** ⟶ intensity

  - Ambient : same at every point
  - Diffuse : Lambert's law
  - Specular : shininess
- 3 color model (R, G, B)

Phong Reflection Model

- **Light source**
  - L : illumination
  - For each light source i

$$L_i = \begin{array}{c} \\ \\ \\ \\ \end{array}\begin{array}{ccc} \text{red} & \text{green} & \text{blue} \\ \end{array}$$

$$\begin{array}{l} \\ L_i = \end{array} \begin{array}{cc} \begin{matrix} \text{red} & \text{green} & \text{blue} \end{matrix} & \\ \begin{pmatrix} L_{ira} & L_{iga} & L_{iba} \\ L_{ird} & L_{igd} & L_{ibd} \\ L_{irs} & L_{igs} & L_{ibs} \end{pmatrix} & \begin{matrix} \textit{ambient} \\ \textit{diffuse} \\ \textit{specular} \end{matrix} \end{array}$$

In fixed graphics rendering (OpenGL)…

Each source has separate diffuse, specular and ambient RGB parameters.

A real light source has but one color and cannot be characterized as being both a blue diffuse source and a white ambient source.

Because we cannot do global lighting in OpenGL, we can use this added flexibility to give better approximations.

Phong Reflection Model

- **Material model**
  - R : reflection (how much of each of the incident lights is reflected at the point of interest)
  - At a point, it has the reflection for each light source

$$R_i = \begin{pmatrix} R_{ira} & R_{iga} & R_{iba} \\ R_{ird} & R_{igd} & R_{ibd} \\ R_{irs} & R_{igs} & R_{ibs} \end{pmatrix}$$

  - In OpenGL
    - Materials are modeled in a complementary manner.
    - For each surface, we must give separate ambient, diffuse, and specular components or use default values. These parameters are the fraction of the incoming light of each type that is reflected.

Phong Reflection Model

- Light source
  - L : illumination
  - For each light source i

$$L_i = \begin{array}{cccc} \text{red} & \text{green} & \text{blue} & \\ \left(\begin{matrix} L_{ira} & L_{iga} & L_{iba} \\ L_{ird} & L_{igd} & L_{ibd} \\ L_{irs} & L_{igs} & L_{ibs} \end{matrix}\right) & & & \begin{matrix} \textit{ambient} \\ \textit{diffuse} \\ \textit{specular} \end{matrix} \end{array}$$

- Material model
  - R : reflection (how much of each of the incident lights is reflected at the point of interest)
  - At a point, it has the reflection for each light source

$$R_i = \left(\begin{matrix} R_{ira} & R_{iga} & R_{iba} \\ R_{ird} & R_{igd} & R_{ibd} \\ R_{irs} & R_{igs} & R_{ibs} \end{matrix}\right)$$

Intensity at **a** **point** *p*: **I = Reflection ⊗ Illumination**

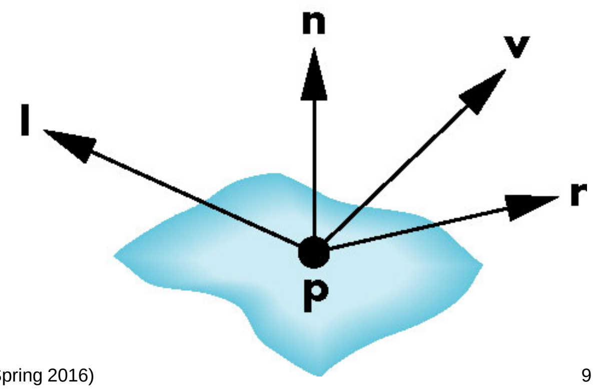
Phong Reflection Model

- For each light source
  - For each color component
    - Ambient + Diffuse + Specular
- Illumination for the i^{th} light source L_i
- Reflection term for each color component r,g,b (e.g., R_{ira})
  - depends on
    - the material properties
    - the orientation of the surface
    - the direction of the light source
    - and the distance between the light source and the viewer
- $I = I_a + I_d + I_s = R_aL_a + R_dL_d + R_sL_s$

Phong Reflection Model

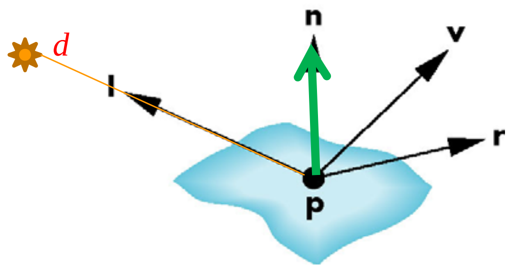
- Efficient, and close enough to physical reality
- Supports ambient, diffuse and specular
  (material-light interactions)

- To compute a color at a point **p** on the surface, use 4 vectors
  - Surface normal
  - Direction from p to the viewer
  - Direction of a line from p to a light source
  - Direction of reflection



Computation of Vectors

- Normal Vectors
- Angle of Reflection
- Halfway Vector
- Transmitted Light

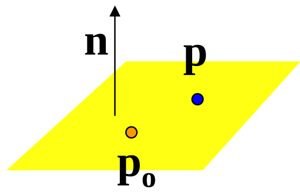


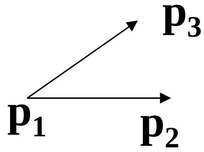
Phong Reflection Model

- $I = \dfrac{1}{a + bd + cd^2} \left(k_d L_d (\mathbf{l} \bullet \mathbf{n}) + k_s L_s (\mathbf{r} \bullet \mathbf{v})^{\alpha}\right) + k_a L_a$

Normal Vectors

- Plane
  - $ax + by + cz + d = 0$
  - $\mathbf{n} \cdot (p-p_o) = 0, \quad \mathbf{n}=[a,b,c]^T$
  - Given p_1, p_2, p_3 on the plane
  - $\mathbf{n} = (p_2 - p_1) \times (p_3 - p_1)$
  - Be careful with the order



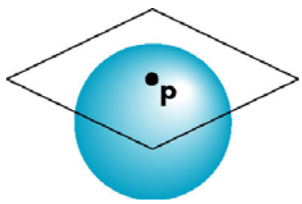


- Curved surfaces
  - Sphere
    - $f(x,y,z) = x^2 + y^2 + z^2 - 1 = 0$ ← Implicit
    - $x(u,v) = \cos u \sin v$
      $y(u,v) = \cos u \cos v$ ← Parametric
      $z(u,v) = \sin u$

Normal Vectors

- Curved surface
  - Implicit form $f(x,y,z) = 0$
    - The normal is given by the gradient vector

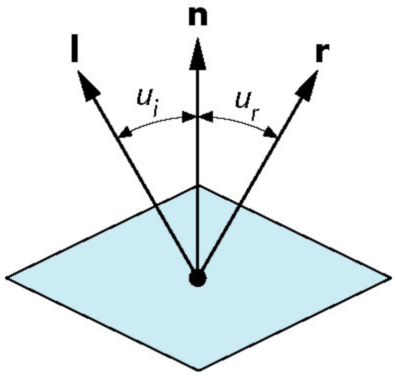
$$\mathbf{n} = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)^T$$



  - Explicit form $(x,y,z)^T$
    $x=g(u,v)$, $y=h(u,v)$, $z=k(u,v)$
    - We obtain the normal from the tangent plane at a point $p(u,v) = (x(u,v), y(u,v), z(u,v))^T$ on the surface
    - Lines in the directions of vectors $(\partial \mathbf{p}/\partial u)$, $(\partial \mathbf{p}/\partial v)$ lie in the tangent plane.
    - $n = (\partial p/\partial u) \times (\partial p/\partial v)$

Angle of Reflection

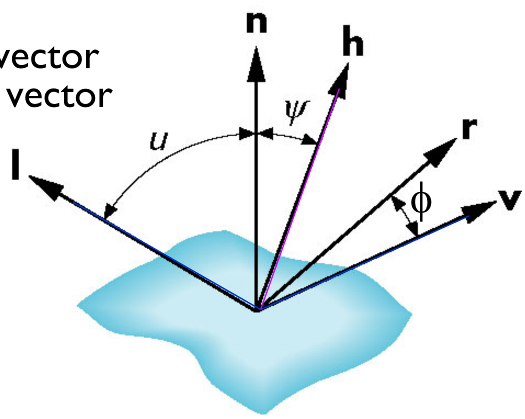
- u_i: angle of incidence
  u_r: angle of reflection
  $u_i = u_r$
- $|\mathbf{l}| = |\mathbf{n}| = |\mathbf{r}| = 1$
- $\mathbf{l} \bullet \mathbf{n} = \mathbf{n} \bullet \mathbf{r}$
- $\mathbf{r} = \alpha\, \mathbf{l} + \beta\, \mathbf{n}$
  (coplanar condition)



- $\mathbf{r} = 2\,(\mathbf{l} \bullet \mathbf{n})\mathbf{n} - \mathbf{l}$

Halfway Vector

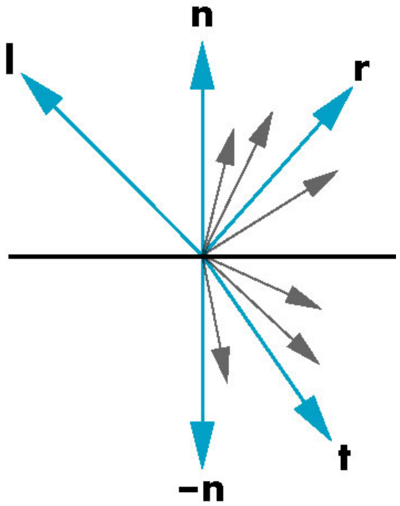
- For specualr reflection, we need to compute r•v
  $$I = (k_d L_d (\mathbf{l} \bullet \mathbf{n}) + k_s L_s (\mathbf{r} \bullet \mathbf{v})^{\alpha}) + k_a L_a$$
- Half way vector h between the viewer vector and the light-source vector
  $\mathbf{h} = (\mathbf{l} + \mathbf{v}) / |\mathbf{l} + \mathbf{v}|$
- $2\,\psi = \phi$
- Replace **r•v** with **n•h**
  Adjust α with α'



- If the surface is oriented so that its normal were in the same direction as h, the viewer would see the brightest specular highlight, since **r** and v would also point in the same direction.

(used in Ray Tracer)

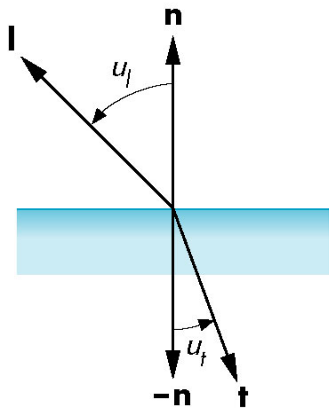
Transmitted Light



- Refraction
  - Snell's Law

$$\frac{\sin u_l}{\sin u_t} = \frac{\eta_t}{\eta_l} = \eta$$

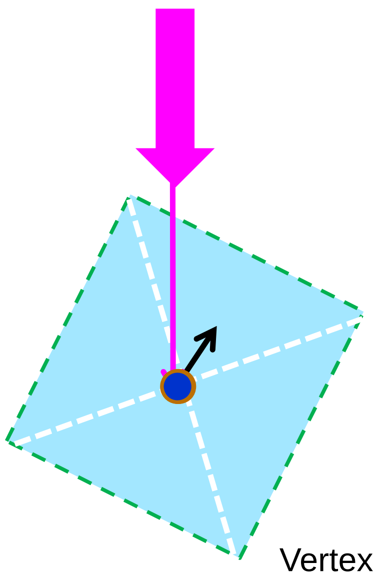
    - Indices of refraction (measure of the relative speed of light in the two materials)
  - $t = \alpha\, n + \beta\, l$
  - $t = \frac{1}{\eta} l - (\cos u_t - \frac{1}{\eta} \cos u_l)\, n$



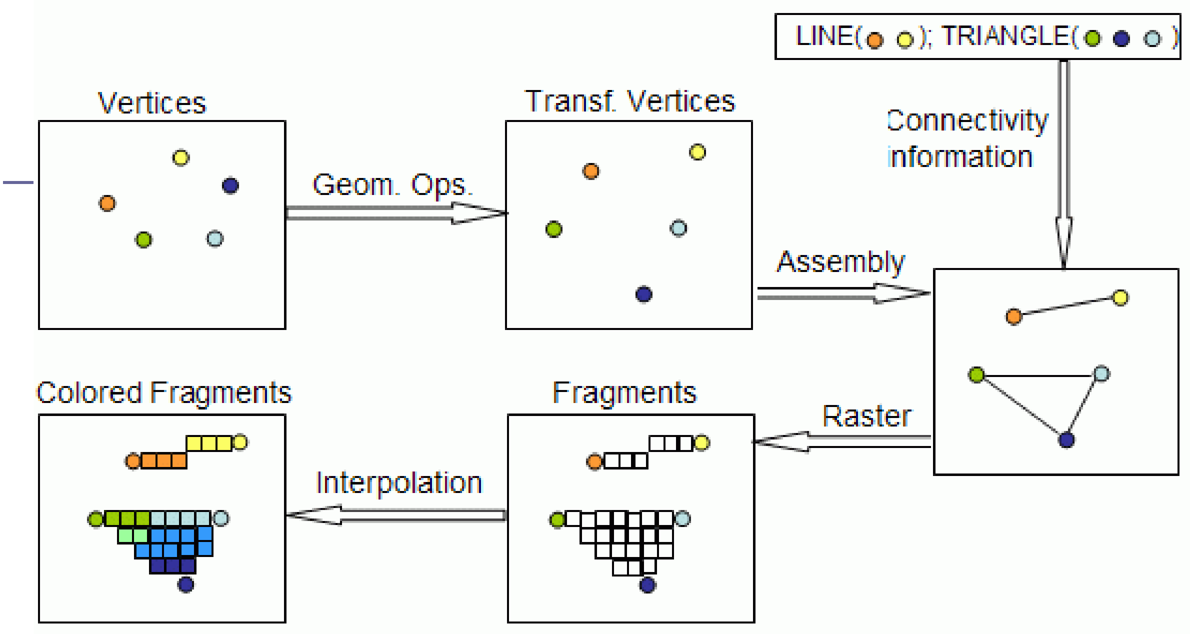
So far …

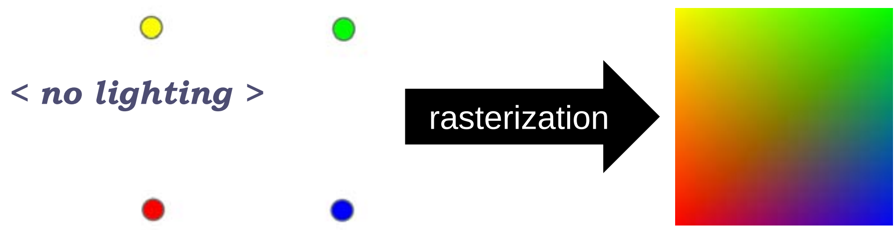
- Phong Reflection Model
  - Ambient reflection
  - Diffuse reflection
  - Specular reflection
- Computation of Vectors

- ***Light Sources***
  - Ambient light
  - Point sources
  - Spotlight
  - Distant light sources



Vertex attributes
- position
- color
- normal





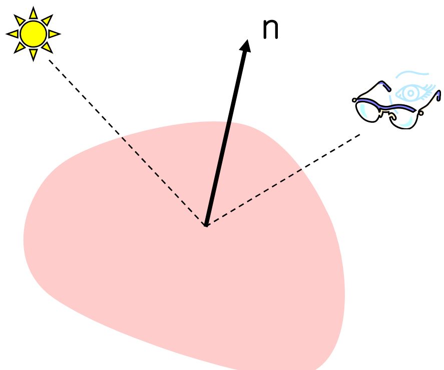
Now … with lighting

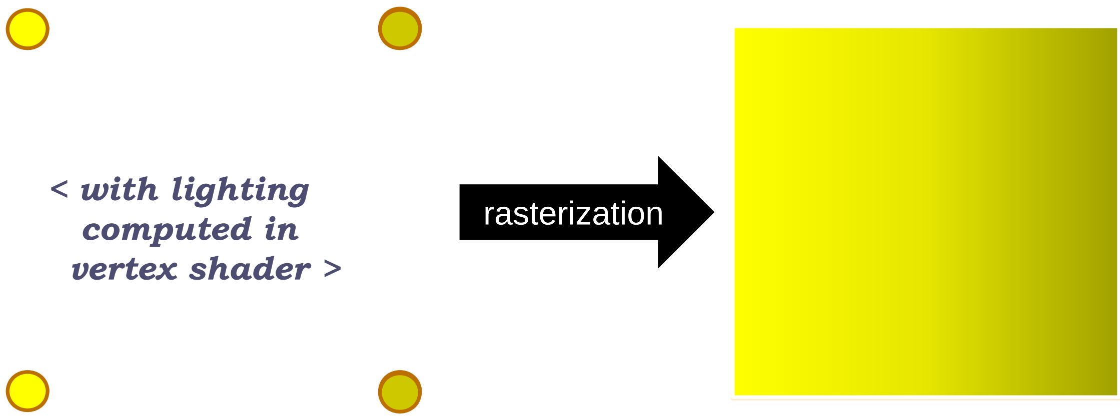
- Phong illumination model

$$I = k_a L_a + \sum \frac{1}{a + bd + cd^2} [k_d (\ell \cdot n) L_d + k_s (r \cdot v)^{\alpha} L_s]$$

f (distance)

  - Computes a color for the plane defined by the normal vector
  - → The normal vector is associated with a vertex
  - → computes a color for a vertex





Now … with lighting

- Polygonal **Shading**
  - Flat shading
  - Gouraud shading
  - Phong shading

  - Toon shading
  - …