termvar, x, y, z, fsemiringEl, r, stype, T, Smathcals, $C, \mathcal{M}, \mathcal{G}$ mathsft, F, Go, i, j, k, lmode, m

```
\mathcal{R}
                                                               Semiring
                                                                   Element
                                                                   Additive Unit
                                                                   Multiplimathcalive Unit
                                  m
                                  \mathcal{R}_1\circledast\mathcal{R}_2
                                                                   \\ Multiplimath calion
                                  \mathcal{R}_1 \oplus \mathcal{R}_2
                                                                   Addition
A, B, C, D
                                                               Linear Formulas
                                  1
                                                                   Unit
                                  A\boxtimes B
                                                                   Tensor Product
                                  (A\odot\mathcal{R})\to B
                                                                   Implication
                                  \square_{\mathcal{R}} A
                                                                   Graded Necessity Modality
                                  (A)
                                                         S
                                                                   Parentheses
\Gamma, \Delta
                                                               Context
                                  \emptyset
                                                                   Empty Context
                                  x:A
                                  \Gamma_1, \Gamma_2
                                                                   Context Extension
                                                         S
                                  (\Gamma)
p
                                  \boldsymbol{x}
                                  (p_1,p_2)
                                  \Box p
t
                                                               Terms
                                  \boldsymbol{x}
                                  (t_1, t_2)
                                  let p = t_3 in t_4
                                  \lambda x.t
                                  t_1 t_2
                                  \Box t
                                                               Vectors
\gamma
                          ::=
                                  \emptyset
                                  \mathcal{R}
```

$$\begin{vmatrix} \gamma_1, \dots, \gamma_i \\ (\gamma) & \mathsf{S} \end{vmatrix}$$

 $\gamma_1 \le \gamma_2$ Context Order

$$\frac{\overline{\emptyset} \leq \emptyset}{\emptyset} \text{SCTX_EMPTY}$$

$$\frac{\gamma_1 \leq \gamma_2 \quad r_1 \leq r_2}{(\gamma_1, r_1) \leq (\gamma_2, r_2)} \text{SCTX_EXT}$$

 $\gamma \odot \Gamma \vdash t : A$ Graded Term Assignment

$$\overline{\mathfrak{m} \odot x : A \vdash x : A} \text{ T-Id}$$

$$\overline{\mathfrak{m} \odot x : A \vdash x : A} \text{ T-UNITI}$$

$$\gamma_{1} \odot \Gamma_{1} \vdash t_{1} : A$$

$$\gamma_{2} \odot \Gamma_{2} \vdash t_{2} : B$$

$$\overline{(\gamma_{1}, \gamma_{2})} \odot (\Gamma_{1}, \Gamma_{2}) \vdash (t_{1}, t_{2}) : A \boxtimes B} \text{ T-TenI}$$

$$\gamma_{2} \odot \Gamma_{2} \vdash t_{1} : A \boxtimes B$$

$$\overline{(\gamma_{1}, r_{1}, r_{1}, \gamma_{3})} \odot (\Gamma_{1}, x : A, y : B, \Gamma_{3}) \vdash t_{2} : C}$$

$$\overline{(\gamma_{1}, r_{2} \circledast \gamma_{2}, \gamma_{3})} \odot (\Gamma_{1}, \Gamma_{2}, \Gamma_{3}) \vdash \text{let } (x, y) = t_{1} \text{ in } t_{2} : C} \text{ T-TenE}$$

$$\frac{(\gamma, r_{1}) \odot (\Gamma, x : A) \vdash t : B}{\gamma \odot \Gamma \vdash \lambda x . t : (A \odot r_{1}) \to B} \text{ T-FunI}$$

$$\gamma_{2} \odot \Gamma_{2} \vdash t_{2} : A$$

$$\gamma_{1} \odot \Gamma_{1} \vdash t_{1} : (A \odot r_{1}) \to B$$

$$\overline{(\gamma_{1}, r_{2} \circledast \gamma_{2})} \odot (\Gamma_{1}, \Gamma_{2}) \vdash t_{1} t_{2} : B} \text{ T-FunE}$$

$$\frac{\gamma \odot \Gamma \vdash t : A}{(r \circledast \gamma) \odot \Gamma \vdash \Box t : \Box_{r} A} \text{ T-BoxI}$$

$$\frac{\gamma_{2} \odot \Gamma_{2} \vdash t_{1} : \Box_{r} A \quad (\gamma_{1}, r, \gamma_{3}) \odot (\Gamma_{1}, x : A, \Gamma_{3}) \vdash t_{2} : B}{(\gamma_{1}, \gamma_{2}, \gamma_{3}) \odot (\Gamma_{1}, \Gamma_{2}, \Gamma_{3}) \vdash \text{let } \Box x = t_{1} \text{ in } t_{2} : B}} \text{ T-BoxE}$$

$$\frac{\gamma_{1} \odot \Gamma_{1} \vdash t : A \quad \gamma_{1} \leq \gamma_{2}}{\gamma_{2} \odot \Gamma_{2} \vdash t : A} \text{ T-SuB}$$

$$\frac{(\gamma_{1}, \gamma_{2}) \odot (\Gamma_{1}, \Gamma_{2}) \vdash t : B}{(\gamma_{1}, \alpha_{2}, \gamma_{2}) \odot (\Gamma_{1}, x : A, \Gamma_{2}) \vdash t : B}} \text{ T-Weak}$$

$$\frac{(\gamma_{1}, r_{1}, r_{2}, \gamma_{2}) \odot (\Gamma_{1}, x : A, y : A, \Gamma_{2}) \vdash t : B}{(\gamma_{1}, (r_{1} \oplus r_{2}), \gamma_{2}) \odot (\Gamma_{1}, x : A, \Gamma_{2}) \vdash t : B}} \text{ T-Cont}$$

$$\frac{(\gamma_1, r_1, r_2, \gamma_2) \odot (\Gamma_1, x: A, y: B, \Gamma_2) \vdash t: C}{(\gamma_1, r_2, r_1, \gamma_2) \odot (\Gamma_1, y: B, x: A, \Gamma_2) \vdash t: C} \text{T_Ex}$$