| | | | | |
|---|---|---|---|---|
| *termvar*, $x$, $y$ | | | | |
| *index*, $i$, $j$, $k$ | | | | |
| $d$ | ::= | | | |
| | \| | 1 | | |
| | \| | 2 | | |
| *pol*, $p$ | ::= | | | |
| | \| | $+$ | | |
| | \| | $-$ | | |
| *type*, $S$, $T$, $R$ | ::= | | | Cointuitionistic types |
| | \| | $0$ | | |
| | \| | $S + T$ | | |
| | \| | $S - T$ | | |
| | \| | $(S)$ | S | |
| *term*, $t$ | ::= | | | Cointuitionistic terms |
| | \| | $x$ | | |
| | \| | $\mathsf{unit}$ | | |
| | \| | $(t_1, t_2)$ | | |
| | \| | $\mathsf{in}_d\, x.t$ | | |
| | \| | $\langle t_1, t_2 \rangle$ | | |
| | \| | $\lambda x.t$ | | |
| | \| | $\mathsf{cut}\, x.t_1 \bullet t_2$ | | |
| | \| | $(t)$ | S | |
| $\Gamma$ | ::= | | | Cointuitionistic contexts |
| | \| | $\emptyset$ | | |
| | \| | $x : p\, S$ | | |
| | \| | $\Gamma, \Gamma'$ | | |
| | \| | $(\Gamma)$ | S | |

$\boxed{\Gamma\, \mathsf{Neg}}$

$$\frac{}{\emptyset\, \mathsf{Neg}}\ \textsc{neg\_empty}$$

$$\frac{\Gamma\, \mathsf{Neg}}{(\Gamma, x : -S)\, \mathsf{Neg}}\ \textsc{neg\_neg}$$

$\boxed{\Gamma\, \mathsf{Ok}}$

$$\frac{}{\emptyset\ \mathsf{Ok}}\ \textsc{ok\_empty}$$

$$\frac{\Gamma\ \mathsf{Ok}}{(\Gamma, x : -S)\ \mathsf{Ok}}\ \textsc{ok\_neg}$$

$$\frac{\Gamma\ \mathsf{Neg}}{(\Gamma, x : +S)\ \mathsf{Ok}}\ \textsc{ok\_pos}$$

$\boxed{\Gamma \vdash t : p\, S}$

$$\frac{(\Gamma, x : p\, S)\ \mathsf{Ok}}{\Gamma, x : p\, S \vdash x : p\, S}\ \textsc{ty\_var}$$

$$\frac{\Gamma\ \mathsf{Neg}}{\Gamma \vdash \mathsf{unit} : -0}\ \textsc{ty\_1}\mathrm{L}$$

$$\frac{\Gamma \vdash t_1 : -S \quad \Gamma \vdash t_2 : -T \quad \Gamma\ \mathsf{Neg}}{\Gamma \vdash (t_1, t_2) : -(S+T)}\ \textsc{ty\_or}\mathrm{L}$$

$$\frac{\Gamma, x : -T \vdash t : +S}{\Gamma \vdash \mathsf{in}_1\, x.t : +(S+T)}\ \textsc{ty\_or}\mathrm{R1}$$

$$\frac{\Gamma, x : -S \vdash t : +T}{\Gamma \vdash \mathsf{in}_2\, x.t : +(S+T)}\ \textsc{ty\_or}\mathrm{R2}$$

$$\frac{\Gamma, x : -S \vdash t : -T \quad \Gamma\ \mathsf{Neg}}{\Gamma \vdash \lambda x.t : -(S-T)}\ \textsc{ty\_sub}\mathrm{L}$$

$$\frac{\Gamma_1 \vdash t_1 : -S \quad \Gamma_2 \vdash t_2 : +T \quad \Gamma_1\ \mathsf{Neg}}{\Gamma_1, \Gamma_2 \vdash \langle t_1, t_2\rangle : +(S-T)}\ \textsc{ty\_sub}\mathrm{R}$$

$$\frac{\Gamma_1, x : -S \vdash t_1 : -T \quad \Gamma_2, x : -S \vdash t_2 : +T \quad \Gamma_1\ \mathsf{Neg}}{\Gamma_1, \Gamma_2 \vdash \mathsf{cut}\, x.t_1 \bullet t_2 : +S}\ \textsc{ty\_cut}$$

$\boxed{t_1 \rightsquigarrow t_2}$

$$\frac{}{\mathsf{cut}\, y.\lambda x.t \bullet \langle t_1, t_2\rangle \rightsquigarrow \mathsf{cut}\, y.[t_1/x]t \bullet t_2}\ \textsc{rw\_cb}$$

$$\frac{}{\mathsf{cut}\, y.(t_1, t_2) \bullet \mathsf{in}_1\, x.t \rightsquigarrow \mathsf{cut}\, y.t_1 \bullet [t_2/x]t}\ \textsc{rw\_or}1$$

$$\frac{}{\mathsf{cut}\, y.(t_1, t_2) \bullet \mathsf{in}_2\, x.t \rightsquigarrow \mathsf{cut}\, y.t_2 \bullet [t_1/x]t}\ \textsc{rw\_or}2$$

$$\frac{x \notin \mathsf{FV}(t)}{\mathsf{cut}\, x.t \bullet x \rightsquigarrow t}\ \textsc{rw\_return}$$

$$\frac{}{\mathsf{cut}\, y.t \bullet (\mathsf{cut}\, x.t_1 \bullet t_2) \rightsquigarrow \mathsf{cut}\, x.[t/y]t_1 \bullet [t/y]t_2}\ \textsc{rw\_cut}$$