

$vars, n, a, x, y, z, w, m, o$

$ivar, i, k, j, l$

$R, S, T ::=$
 $| 0$
 $| S + T$
 $| S - T$
 $| HA$

$A, B, C ::=$
 $| \perp$
 $| A \oplus B$
 $| A \bullet B$
 $| JS$

$s, t ::=$
 $| x$
 $| \text{connect}_w \text{ to } t$
 $| t_1 \cdot t_2$
 $| \text{false } t$
 $| x(t)$
 $| \text{mkc}(t, x)$
 $| \text{postp}(x \mapsto t_1, t_2)$
 $| \text{inl } t$
 $| \text{inr } t$
 $| \text{case } t_1 \text{ of } x.t_2, y.t_3$
 $| H e$
 $| \text{let } J x = e \text{ in } t_2$
 $| \text{let } H x = t_1 \text{ in } t_2$
 $| (t) \quad S$

$e, u ::=$
 $| x$
 $| \text{connect}_\perp \text{ to } e$
 $| \text{postp}_\perp e$
 $| \text{postp}(x \mapsto e_1, e_2)$
 $| \text{mkc}(e, x)$
 $| x(e)$
 $| e_1 \oplus e_2$
 $| \text{case l } e$
 $| \text{case r } e$
 $| J t$
 $| (e) \quad S$

$\Psi, \Pi ::=$
 $| \cdot$

$$\begin{array}{c}
| \quad T \\
| \quad t : T \\
| \quad \Psi, \Pi \\
| \quad (\Psi) \quad \text{S}
\end{array}$$

$$\begin{array}{c}
\Gamma, \Delta \quad ::= \\
| \quad \cdot \\
| \quad A \\
| \quad e : A \\
| \quad \Gamma, \Gamma' \\
| \quad (\Gamma) \quad \text{S}
\end{array}$$

$$\boxed{S \vdash_{\text{C}} \Psi}$$

$$\begin{array}{c}
\frac{}{S \vdash_{\text{C}} S} \text{C_ID} \\
\frac{S \vdash_{\text{C}} \Psi}{S \vdash_{\text{C}} T, \Psi} \text{C_WK} \\
\frac{S \vdash_{\text{C}} T, T, \Psi}{S \vdash_{\text{C}} T, \Psi} \text{C_CR} \\
\frac{R \vdash_{\text{C}} \Psi_1, S, T, \Psi_2}{R \vdash_{\text{C}} \Psi_1, T, S, \Psi_2} \text{C_EX} \\
\frac{}{0 \vdash_{\text{C}} \Psi} \text{C_FL} \\
\frac{T_1 \vdash_{\text{C}} \Psi_1 \quad T_2 \vdash_{\text{C}} \Psi_2}{T_1 + T_2 \vdash_{\text{C}} \Psi_1, \Psi_2} \text{C_DL} \\
\frac{R \vdash_{\text{C}} \Psi, T_1}{R \vdash_{\text{C}} \Psi, T_1 + T_2} \text{C_DR1} \\
\frac{R \vdash_{\text{C}} \Psi, T_2}{R \vdash_{\text{C}} \Psi, T_1 + T_2} \text{C_DR2} \\
\frac{T_1 \vdash_{\text{C}} T_2, \Psi}{T_1 - T_2 \vdash_{\text{C}} \Psi} \text{C_SL} \\
\frac{S \vdash_{\text{C}} \Psi_1, T_1 \quad T_2 \vdash_{\text{C}} \Psi_2}{S \vdash_{\text{C}} \Psi_1, \Psi_2, T_1 - T_2} \text{C_SR} \\
\frac{S \vdash_{\text{C}} \Psi_1, T \quad T \vdash_{\text{C}} \Psi_2}{S \vdash_{\text{C}} \Psi_1, \Psi_2} \text{C_CUT} \\
\frac{A \vdash_{\text{L}} \cdot; \Psi}{HA \vdash_{\text{C}} \Psi} \text{C_HL}
\end{array}$$

$$\boxed{A \vdash_{\text{L}} \Delta; \Psi}$$

$$\begin{array}{c}
\frac{}{A \vdash_{\text{L}} A; \cdot} \text{L_ID} \\
\frac{A \vdash_{\text{L}} \Delta; \Psi}{A \vdash_{\text{L}} \Delta; T, \Psi} \text{L_WK}
\end{array}$$

$$\begin{array}{c}
\frac{A \vdash_L \Delta; T, T, \Psi}{A \vdash_L \Delta; T, \Psi} \quad \text{L_CTR} \\
\frac{A \vdash_L \Delta_1, A, B, \Delta_2; \Psi}{A \vdash_L \Delta_1, B, A, \Delta_2; \Psi} \quad \text{L_EX} \\
\frac{A \vdash_L \Delta; \Psi_1, S, T, \Psi_2}{A \vdash_L \Delta; \Psi_1, T, S, \Psi_2} \quad \text{L_CEX} \\
\frac{A \vdash_L \Delta_1, B; \Psi_1 \quad B \vdash_L \Delta_2; \Psi_2}{A \vdash_L \Delta_1, \Delta_2; \Psi_1, \Psi_2} \quad \text{L_CUT} \\
\frac{A \vdash_L \Delta; \Psi_1, T \quad T \vdash_C \Psi_2}{A \vdash_L \Delta; \Psi_1, \Psi_2} \quad \text{L_CCUT} \\
\frac{}{\perp \vdash_L \cdot; \cdot} \quad \text{L_FLL} \\
\frac{A \vdash_L \Delta; \Psi}{A \vdash_L \perp, \Delta; \Psi} \quad \text{L_FLR} \\
\frac{A \vdash_L \Delta; \Psi, T_1}{A \vdash_L \Delta; \Psi, T_1 + T_2} \quad \text{L_DR1} \\
\frac{A \vdash_L \Delta; \Psi, T_2}{A \vdash_L \Delta; \Psi, T_1 + T_2} \quad \text{L_DR2} \\
\frac{B_1 \vdash_L \Delta_1; \Psi_1 \quad B_2 \vdash_L \Delta_2; \Psi_2}{B_1 \oplus B_2 \vdash_L \Delta_1, \Delta_2; \Psi_1, \Psi_2} \quad \text{L_PL} \\
\frac{A \vdash_L \Delta, B, C; \Psi}{A \vdash_L \Delta, B \oplus C; \Psi} \quad \text{L_PR} \\
\frac{B_1 \vdash_L B_2, \Delta; \Psi}{B_1 \bullet B_2 \vdash_L \Delta; \Psi} \quad \text{L_SL} \\
\frac{A \vdash_L B_1, \Delta_1; \Psi_1 \quad B_2 \vdash_L \Delta_2; \Psi_2}{A \vdash_L B \bullet C, \Delta_1, \Delta_2; \Psi_1, \Psi_2} \quad \text{L_sR} \\
\frac{A \vdash_L \Delta; \Psi_1, T_1 \quad T_2 \vdash_C \Psi_2}{A \vdash_L \Delta; \Psi_1, \Psi_2, T_1 - T_2} \quad \text{L_CsR} \\
\frac{T \vdash_C \Psi}{JT \vdash_L \cdot; \Psi} \quad \text{L_JL} \\
\frac{A \vdash_L \Delta; T, \Psi}{A \vdash_L \Delta, JT; \Psi} \quad \text{L_JR} \\
\frac{A \vdash_L \Delta, B; \Psi}{A \vdash_L \Delta; HB, \Psi} \quad \text{L_HR}
\end{array}$$

$$\boxed{x : S \vdash_C \Psi}$$

$$\begin{array}{c}
\frac{}{x : S \vdash_C x : S} \quad \text{C_VAR} \\
\frac{s : T' \in \Psi \quad x : S \vdash_C \Psi}{x : S \vdash_C \text{connect}_w \text{ to } s : T, \Psi} \quad \text{C_WEAK} \\
\frac{x : S \vdash_C t_1 : T, t_2 : T, \Psi}{x : S \vdash_C t_1 \cdot t_2 : T, \Psi} \quad \text{C_CONTR}
\end{array}$$

$$\begin{array}{c}
\frac{x : S \vdash_{\mathbf{C}} t : 0, \Psi \quad x_1 : S_1 \vdash_{\mathbf{C}} \Psi_1 \dots x_i : S_i \vdash_{\mathbf{C}} \Psi_i}{x : S \vdash_{\mathbf{C}} [\text{false } t/x_1] \Psi_1, \dots, [\text{false } t/x_i] \Psi_i, \Psi} \text{C_ZERO} \\
\\
\frac{x : S \vdash_{\mathbf{C}} t : T_1, \Psi_1 \quad y : T_2 \vdash_{\mathbf{C}} \Psi_2}{x : S \vdash_{\mathbf{C}} \Psi_1, \text{mkc}(t, y) : T_1 - T_2, [y(t)/y] \Psi_2} \text{C_SUBI} \\
\\
\frac{x : S \vdash_{\mathbf{C}} t_1 : T_1 - T_2, \Psi_1 \quad y : T_1 \vdash_{\mathbf{C}} t_2 : T_2, \Psi_2}{x : S \vdash_{\mathbf{C}} \text{postp}(y \mapsto t_2, t_1), \Psi_1, [y(t_1)/y] \Psi_2} \text{C_SUBE} \\
\\
\frac{x : S \vdash_{\mathbf{C}} t : T_1, \Psi}{x : S \vdash_{\mathbf{C}} \text{inl } t : T_1 + T_2, \Psi} \text{C_ORI1} \\
\\
\frac{x : S \vdash_{\mathbf{C}} t : T_2, \Psi}{x : S \vdash_{\mathbf{C}} \text{inr } t : T_1 + T_2, \Psi} \text{C_ORI2} \\
\\
\frac{y : T_1 \vdash_{\mathbf{C}} \Psi_2 \quad y : T_2 \vdash_{\mathbf{C}} \Psi_3 \quad x : S \vdash_{\mathbf{C}} t : T_1 + T_2, \Psi_1 \quad |\Psi_2| = |\Psi_3|}{x : S \vdash_{\mathbf{C}} \Psi_1, \text{case } t \text{ of } y. \Psi_2, y. \Psi_3} \text{C_ORE} \\
\\
\frac{x : S \vdash_{\mathbf{C}} t : \text{HA}, \Psi_1 \quad y : A \vdash_{\mathbf{L}} \cdot; \Psi_2 \quad |\Psi_1| = |\Psi_2|}{x : S \vdash_{\mathbf{C}} \Psi_1 \cdot (\text{let } H y = t \text{ in } \Psi_2)} \text{C_HE}
\end{array}$$

$$x : A \vdash_{\mathbf{L}} \Delta; \Psi$$

$$\begin{array}{c}
\frac{}{x : A \vdash_{\mathbf{L}} x : A; \Psi} \text{L_VAR} \\
\\
\frac{s : T' \in \Psi \quad x : A \vdash_{\mathbf{L}} \Delta; \Psi}{x : A \vdash_{\mathbf{L}} \Delta; \text{connect}_w \text{ to } s : T, \Psi} \text{L_WEAK} \\
\\
\frac{x : A \vdash_{\mathbf{L}} \Delta; t_1 : T, t_2 : T, \Psi}{x : A \vdash_{\mathbf{L}} \Delta; t_1 \cdot t_2 : T, \Psi} \text{L_CONTR} \\
\\
\frac{x : A \vdash_{\mathbf{L}} \Delta; \Psi \quad e : B \in \Delta}{x : A \vdash_{\mathbf{L}} \text{connect}_{\perp} \text{ to } e : \perp, \Delta; \Psi} \text{L_PERPI} \\
\\
\frac{x : A \vdash_{\mathbf{L}} e : \perp, \Delta; \Psi}{x : A \vdash_{\mathbf{L}} \text{postp}_{\perp} e, \Delta; \Psi} \text{L_PERPE} \\
\\
\frac{x : A \vdash_{\mathbf{L}} \Delta_1, e : B; \Psi_1 \quad y : C \vdash_{\mathbf{L}} \Delta_2; \Psi_2 \quad |\Psi_1| = |\Psi_2|}{x : A \vdash_{\mathbf{L}} \Delta_1, \text{mkc}(e, y) : B \bullet C, [y(e)/y] \Delta_2; \Psi_1 \cdot [y(e)/y] \Psi_2} \text{L_SUBI} \\
\\
\frac{x : A \vdash_{\mathbf{L}} \Delta_1, e_1 : B \bullet C; \Psi_1 \quad y : C \vdash_{\mathbf{L}} e_2 : B, \Delta_2; \Psi_2 \quad |\Psi_1| = |\Psi_2|}{x : A \vdash_{\mathbf{L}} \Delta_1, \text{postp}(y \mapsto e_2, e_1), [y(e_1)/y] \Delta_2; \Psi_1 \cdot [y(e_1)/y] \Psi_2} \text{L_SUBE} \\
\\
\frac{x : A \vdash_{\mathbf{L}} \Delta_1, e_1 : B, e_2 : C, \Delta_2; \Psi}{x : A \vdash_{\mathbf{L}} \Delta_1, e_1 \oplus e_2 : B \oplus C, \Delta_2; \Psi} \text{L_PARI} \\
\\
\frac{y : B \vdash_{\mathbf{L}} \Delta_2; \Psi_2 \quad |\Psi_2| = |\Psi_3| \quad z : C \vdash_{\mathbf{L}} \Delta_3; \Psi_3 \quad x : A \vdash_{\mathbf{L}} e : B \oplus C, \Delta_1; \Psi_1 \quad |\Psi_1| = |\Psi_2|}{x : A \vdash_{\mathbf{L}} \Delta_1, [\text{casel}(e)/y] \Delta_2, [\text{caser}(e)/z] \Delta_3; \Psi_1 \cdot [\text{casel}(e)/y] \Psi_2 \cdot [\text{caser}(e)/z] \Psi_3} \text{L_PARE} \\
\\
\frac{x : A \vdash_{\mathbf{L}} \Delta; t : T, \Psi}{x : A \vdash_{\mathbf{L}} \Delta, \text{J } t : \text{J } T; \Psi} \text{L_JI} \\
\\
\frac{x : A \vdash_{\mathbf{L}} \Delta, e : \text{J } T; \Psi_1 \quad y : T \vdash_{\mathbf{C}} \Psi_2 \quad |\Psi_1| = |\Psi_2|}{x : A \vdash_{\mathbf{L}} \Delta; \Psi_1 \cdot \text{let } J y = e \text{ in } \Psi_2} \text{L_JE}
\end{array}$$

$$\begin{array}{c}
\frac{x : A \vdash_{\mathcal{L}} \Delta, e : B; \Psi}{x : A \vdash_{\mathcal{L}} \Delta; \mathsf{H}e : \mathsf{H}B, \Psi} \quad \mathsf{L_HI} \\
\\
\boxed{x : T \vdash_{\mathcal{C}} \Psi_1 = \Psi_2} \\
\\
\frac{
\begin{array}{l}
|\Psi_1| = |\Psi'_1| \\
|\Psi_2| = |\Psi'_2| \quad x : T_1 \vdash_{\mathcal{C}} \Psi_2 = \Psi'_2 \\
|\Psi_3| = |\Psi'_3| \quad y : T_2 \vdash_{\mathcal{C}} \Psi_3 = \Psi'_3 \quad z : S \vdash_{\mathcal{C}} t_1 : T_1, \Psi_1 = t'_1 : T_1, \Psi'_1
\end{array}
}{z : S \vdash_{\mathcal{C}} \Psi_1, \text{case (inl } t_1) \text{ of } y. \Psi_2, y. \Psi_3 = [t'_1/y] \Psi'_2} \quad \mathsf{LEQ_OR1} \\
\\
\frac{
\begin{array}{l}
|\Psi_1| = |\Psi'_1| \\
|\Psi_2| = |\Psi'_2| \quad x : T_1 \vdash_{\mathcal{C}} \Psi_2 = \Psi'_2 \\
|\Psi_3| = |\Psi'_3| \quad y : T_2 \vdash_{\mathcal{C}} \Psi_3 = \Psi'_3 \quad z : S \vdash_{\mathcal{C}} t_2 : T_2, \Psi_1 = t'_2 : T_2, \Psi'_1
\end{array}
}{z : S \vdash_{\mathcal{C}} \Psi_1, \text{case (inr } t_2) \text{ of } x. \Psi_2, y. \Psi_3 = [t'_2/y] \Psi'_3} \quad \mathsf{LEQ_OR2} \\
\\
\frac{
\begin{array}{l}
|\Psi_1| = |\Psi'_1| \\
|\Psi_2| = |\Psi'_2| \quad y : T_2 \vdash_{\mathcal{C}} \Psi_2 = \Psi'_2 \\
|\Psi_3| = |\Psi'_3| \quad z : T_1 \vdash_{\mathcal{C}} t_2 : T_2, \Psi_3 = t'_2 : T_2, \Psi'_3 \quad x : S \vdash_{\mathcal{C}} t_1 : T_1, \Psi_1 = t'_1 : T_1, \Psi'_1
\end{array}
}{x : S \vdash_{\mathcal{C}} \Psi_1, [y(t_1)/y] \Psi_2, \text{postp}(z \mapsto t_2, \text{mkc}(t_1, y)), [z(\text{mkc}(t_1, y))/z] \Psi_3 = \Psi'_1, [[t'_1/z]t'_2/y] \Psi'_2, [t'_1/z] \Psi'_3} \quad \mathsf{LEQ_SUB} \\
\\
\boxed{x : A \vdash_{\mathcal{L}} \Delta_1; \Psi_1 = \Delta_2; \Psi_2} \\
\\
\frac{
\begin{array}{l}
|\Psi_1| = |\Psi_2| \\
|\Psi_1| = |\Psi'_1| \\
|\Psi_2| = |\Psi'_2| \quad y : S \vdash_{\mathcal{C}} \Psi_2 = \Psi'_2 \quad x : A \vdash_{\mathcal{L}} \Delta; s : S, \Psi_1 = \Delta'; s' : S, \Psi'_1
\end{array}
}{x : A \vdash_{\mathcal{L}} \Delta; \Psi_1, (\text{let } \mathsf{J} y = \mathsf{J} s \text{ in } \Psi_2) = \Delta'; (\Psi'_1, [s'/y] \Psi'_2)} \quad \mathsf{CEQ_LETJ} \\
\\
\frac{
\begin{array}{l}
|\Psi_1| = |\Psi_2| \\
|\Psi_1| = |\Psi'_1| \\
|\Psi_2| = |\Psi'_2| \quad x : B \vdash_{\mathcal{L}} \Delta, e : A; \Psi_1 = \Delta', e' : A; \Psi'_1 \quad y : A \vdash_{\mathcal{L}} \cdot; \Psi_2 = \cdot; \Psi'_2
\end{array}
}{x : B \vdash_{\mathcal{L}} \Delta; \Psi_1, \text{let } \mathsf{H} y = \mathsf{H} e \text{ in } \Psi_2 = \Delta'; (\Psi'_1, [e'/y] \Psi'_2)} \quad \mathsf{CEQ_LETH} \\
\\
\begin{array}{l}
e \equiv \text{postp}(z \mapsto e_2, \text{mkc}(e_1, y)) \\
e' \equiv z(\text{mkc}(e_1, y)) \\
\Delta \equiv [y(e_1)/y] \Delta_2, e, [e'/z] \Delta_3 \\
\Delta' \equiv [[e'_1/z]e'_2/y] \Delta'_2, [e'_1/z] \Delta'_3
\end{array}
\frac{
\begin{array}{l}
|\Psi_1| = |\Psi'_1| \quad |\Delta_1| = |\Delta'_1| \quad x : B \vdash_{\mathcal{L}} e_1 : A_1, \Delta_1; \Psi_1 = e'_1 : A_1, \Delta'_1; \Psi'_1 \\
|\Psi_2| = |\Psi'_2| \quad |\Delta_2| = |\Delta'_2| \quad y : A_2 \vdash_{\mathcal{L}} \Delta_2; \Psi_2 = \Delta'_2; \Psi'_2 \\
|\Psi_3| = |\Psi'_3| \quad |\Delta_3| = |\Delta'_3| \quad z : A_1 \vdash_{\mathcal{L}} e_2 : A_2, \Delta_3; \Psi_3 = e'_2 : A_2, \Delta'_3; \Psi'_3
\end{array}
}{x : C \vdash_{\mathcal{L}} \Delta_1, \Delta; \Psi_1, [y(e_1)/y] \Psi_2, [e'/z] \Psi_3 = \Delta'_1, \Delta'; \Psi'_1, [[e'_1/z]e'_2/y] \Psi'_2, [e'_1/z] \Psi'_3} \quad \mathsf{CEQ_SUB} \\
\\
\frac{
\begin{array}{l}
e \equiv \text{casel}(e_1 \oplus e_2) \quad |\Delta_1| = |\Delta'_1| \quad |\Psi_1| = |\Psi'_1| \quad x : A_1 \vdash_{\mathcal{L}} \Delta_2; \Psi_2 = \Delta'_2; \Psi'_2 \\
e' \equiv \text{caser}(e_1 \oplus e_2) \quad |\Delta_2| = |\Delta'_2| \quad |\Psi_2| = |\Psi'_2| \quad y : A_2 \vdash_{\mathcal{L}} \Delta_3; \Psi_3 = \Delta'_3; \Psi'_3 \\
|\Delta_3| = |\Delta'_3| \quad |\Psi_3| = |\Psi'_3| \quad z : B \vdash_{\mathcal{L}} e_1 : A_1, e_2 : A_2, \Delta_1; \Psi_1 = e'_1 : A_1, e'_2 : A_2, \Delta'_1; \Psi'_1
\end{array}
}{z : B \vdash_{\mathcal{L}} \Delta_1, [e/x] \Delta_2, [e'/x] \Delta_3; \Psi_1, [e/x] \Psi_2, [e'/x] \Psi_3 = \Delta'_1, [e'_1/x] \Delta'_2, [e'_2/x] \Delta'_3; \Psi'_1, [e'_1/x] \Psi'_2, [e'_2/x] \Psi'_3} \quad \mathsf{CEQ_PAR} \\
\\
\frac{x : A \vdash_{\mathcal{L}} \Delta; \Psi = \Delta'; \Psi' e : B \in \Delta}{x : A \vdash_{\mathcal{L}} \Delta, \text{postp}_{\perp}(\text{connect}_{\perp} \text{ to } e); \Psi = \Delta'; \Psi'} \quad \mathsf{CEQ_UNIT} \\
\\
\frac{|\Delta| = |\Delta'| \quad |\Psi| = |\Psi'| \quad z : B \vdash_{\mathcal{L}} \Delta; \Psi = \Delta'; \Psi'}{z : B \vdash_{\mathcal{L}} \text{postp}(x \mapsto y, e), \text{mkc}(x(e), y) : A_1 \bullet\!\!\!\leftarrow A_2, \Delta; \Psi = e : A_1 \bullet\!\!\!\leftarrow A_2, \Delta'; \Psi'} \quad \mathsf{CEQ_ETASUB} \\
\\
\frac{|\Delta| = |\Delta'| \quad |\Psi| = |\Psi'| \quad z : B \vdash_{\mathcal{L}} \Delta; \Psi = \Delta'; \Psi'}{z : B \vdash_{\mathcal{L}} (\text{casel } e) \oplus (\text{caser } e) : A_1 \oplus A_2, \Delta; \Psi = e : A_1 \oplus A_2, \Delta'; \Psi'} \quad \mathsf{CEQ_ETAPAR} \\
\\
\frac{|\Delta| = |\Delta'| \quad |\Psi| = |\Psi'| \quad z : B \vdash_{\mathcal{L}} \Delta; \Psi = \Delta'; \Psi'}{z : B \vdash_{\mathcal{L}} \text{connect}_{\perp} \text{ to } (\text{postp}_{\perp} e) : \perp, \Delta; \Psi = e : \perp, \Delta'; \Psi'} \quad \mathsf{CEQ_ETAUNIT}
\end{array}$$