Final Exam: Burn, Automata, Burn (52 pt)
Theory of Computation (CSCI 3500), Fall 2020

Name: _____

Final Score: _____

0. (20 pt) Peano numbers encode natural numbers as words over the alphabet $\{S, Z\}$. The following table describes each Peano number:

Peano Number	$\mathbb{N}$
Z	0
SZ	1
SSZ	2
$SSSZ$	3
$SSSSZ$	4
$\vdots$	$\vdots$
S^nZ	n

Thus, Peano numbers are elements of the language:

$$\mathbb{P} = \{S^nZ \mid n \in \mathbb{N}\}$$

Complete the following:

- Define a push-down automata that accepts the language $\mathbb{P}$.
- Define a context-free grammar that generates the language $\mathbb{P}$.

1. (20 pt) Surprisingly, push-down automata can be used to compute data as well as accept it. We can use the stack an the output data and the input word as the input data. Two peano numbers $S^{n_1}Z, S^{n_2}Z \in \mathbb{P}$ can be summed $S^{n_1}Z + S^{n_2}Z = S^{n_1+n_2}Z$ by combining successor markings S in both inputs and then following it by a Z. Define a deterministic push-down automata that takes as input a word $p_1 \# p_2$ for any $p_1, p_2 \in \mathbb{P}$ that only accepts when $S^{n_1+n_2}Z$ is on the tape.

2. (10 pt) Consider the following context-free grammar:

$$t \to x \mid \mathsf{fun}\, x \ \to t \mid \mathsf{apply}\, t\, t \mid \mathsf{if}\, b\, \mathsf{then}\, t\, \mathsf{else}\, t \mid b \mid (t)$$
$$b \to x \mid \mathsf{True} \mid \mathsf{False} \mid b \wedge b \mid b \vee b \mid \neg b \mid (b)$$

Complete the following:

(a) Derive the word $\mathsf{apply}\,(\mathsf{fun}\, x \to \mathsf{if}\, x\, \mathsf{then}\, \mathsf{False}\, \mathsf{else}\, \mathsf{True})\,(\mathsf{True} \vee \mathsf{False})$.

(b) Show that grammar is ambiguous.

3. (2 pt) Describe the differences between each model of computation that we have discussed this semester. That is, discuss the differences between regular languages and context-free languages. For full credit your answer should be rigorous and make reference to the properties of each type of model.