

# Theory of Computation

## Midterm Exam

Due: Thursday, Feb. 24th by 11:59pm

### 1 Multiple Choice Questions

1. Which of the following is the correct type (signature) of the NFA transition function:
  - a.  $(Q \times (\Sigma \cup \{\varepsilon\})) \rightarrow Q$
  - b.  $(Q \times (\Sigma \cup \{\varepsilon\})) \rightarrow \mathcal{P}(Q)$
  - c.  $(Q \times \Sigma) \rightarrow Q$
  - d.  $(Q \times (\Sigma^* \cup \{\varepsilon\})) \rightarrow \mathcal{P}(Q)$
2. A language is regular if and only if
  - a) it is the language of a DFA.
  - b) it is the language of a NFA.
  - c) all of the above.
3. Instead of this exam, I would rather be doing:

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## 2 Regular Languages

4. Prove that the following language is regular by constructing a DFA that accepts it:

$$L = \{w \in \{!, \square\}^* \mid w = vw'v \text{ where } v, w' \in \{!, \square\}^*, |w'| = 2, \text{ and } |v| = 3\}$$

5. Suppose we have two industrial control systems  $S_1$  and  $S_2$  that are working concurrently. We need to forbid some sequence of actions from taking place between the two systems. One such action is allowing  $S_1$  and  $S_2$  to be on at the same time. Consider the alphabet:

$$\Sigma = \{\text{On}(S_1), \text{On}(S_2), \text{Off}(S_1), \text{Off}(S_2)\}$$

A sequence of actions is a word over  $\Sigma^*$ .

- i. Convert the property on sequences given above into a language, and then define an NFA that accepts it.
- ii. A second property is that  $S_2$  must power off after  $S_1$ , but  $S_2$  must power on before  $S_1$ . Define a language which captures this property (and this property only), and then define a NFA that accepts it.

6. Suppose the language,  $L$ , over an alphabet  $\Sigma_0$  is regular, and  $f : \Sigma_0 \rightarrow \Sigma_1$ , is a function from the alphabet  $\Sigma_0$  to the alphabet  $\Sigma_1$ .

The function  $f$  can be lifted to words producing the function:

$$\begin{aligned}\hat{f} : \Sigma_0^* &\rightarrow \Sigma_1^* \\ \hat{f}(\varepsilon) &= \varepsilon \\ \hat{f}(aw) &= f(a)\hat{f}(w)\end{aligned}$$

Show that the language  $L_f = \{\hat{f}(w) \in \Sigma_1^* \mid w \in \Sigma_0^*\}$  is regular.

Hint: You have to use the formal definition of DFAs.

7. Convert the following NFA into its equivalent DFA using the NFA-to-DFA algorithm:

