Example: Peano Arithmetic in Agda

$$_++_ : \mathbb{N} \to \mathbb{N} \to \mathbb{N}$$

 $0 + n = n$
 $S(m) + n = S (m + n)$

Example: Peano Arithmetic in Agda

```
+comm : \forall (x y : \mathbb{N}) \rightarrow x + y \equiv y + x
+comm Z y rewrite +0 y = refl
+comm (S x) y rewrite +suc y x | +comm x y = refl
```

```
+suc : \forall (x y : \mathbb{N}) \rightarrow x + (S y) \equiv S(x + y)
+suc Z y = refl
+suc (S x) y rewrite +suc x y = refl
```

$$+0: \forall (x: \mathbb{N}) \rightarrow x + Z \equiv x$$

+0 Z = refl
+0 (S x) rewrite +0 x = refl