Core Design Concepts Discussed:

(Analysis Evaluation)

# Performance and Optimization of Recursive Functions

Harley Eades III

Consider evaluating the following recursive function:

```
1. let rec mult m n =
   if m == 0
    then 0
    else if n == 0
5.
         then 0
6.
      else let rc = mult m (n - 1) in
              let ret = m + rc in
8.
              ret
9.
10. let main =
11.
      let m = 1 in
12.
      let n = 2 in
13.
         let answ = mult m n in
14.
             answ
15.
16. main;;
```

#### Core Design Concepts:



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Frame	Symbol	Value
init line: 16	ackermann main	<fun><fun></fun></fun>

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Frame	Symbol	Value
init line: 16	ackermann main	<fun><fun></fun></fun>
main line: 13	m n	1 2

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Frame	Symbol	Value
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main line: 13	m n	1 2
mult: line 7	m n rc	1 2 ?

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Frame	Symbol	Value	
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main line: 13	m n	1 2	
mult line: 8	m n rc	1 2 ?	
mult line: 8	m n rc	1 1 ?	
mult line: 5	m n	1 0	

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- Bad for performance: making a recursive call in an argument position (line 7).
- This results in the bindings of an activation record depending on the return value of a new activation record.
- Thus, the compiler will create lots of activation records that cannot be popped off of the stack until the end of evaluation.
- This results in a bad use of memory.

# Tail Recursion using the accumulator pattern

#### Non-tail recursive:

```
    let rec mult m n =
    if m == 0
    then 0
    else if n == 0
    then 0
    else m + (mult m (n - 1))
```

#### Tail recursive:

```
1. let rec mult_helper acc m n =
2.    if m == 0
3.    then 0
4.    else if n == 0
5.         then acc
6.         else mult_helper (m + acc) m (n - 1)
7.
8. let mult m n = mult_helper 0 m n
```

Frame	Symbol	Value
init	mult	<fun></fun>
line: 16	main	<fun></fun>

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let mult m n =
     let rec mult_helper acc n' =
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       then 0
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            else mult_helper (m + acc) (n - 1)
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Frame	Symbol	Value
init	mult	<fun></fun>
line: 16	main	<fun></fun>
main	m	1
line: 13	n	2

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let mult m n =
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init line: 16	mult main	<fun><fun></fun></fun>
main line: 13	m n	1 2
mult line: 8	m n	1 2

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Frame	Symbol	Value
init	mult	<fun></fun>
line: 16	main	<fun></fun>
main	m	1
line: 13	n	2
mult	m	1
line: 8	n	2
	m	1
mult_helper	n	2
line: 7	acc	0
	n'	2

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let mult m n =
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line: 7	acc	1
	n'	1

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line: 7	acc	0
	n'	2
	m	1
mult_helper	n	2
line: 7	acc	1
	n'	1
	m	1
mult_helper	n	2
line: 7	acc	2
	n'	0

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init	mult	<fun></fun>
line: 16	main	<fun></fun>

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mult_helper	n	2
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line: 7	acc	1
	n'	1

```
1. let mult m n =
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Frame	Symbol	Value
init	mult	<fun></fun>
line: 16	main	<fun></fun>
	m	1
mult_helper	n	2
line: 7	acc	2
	n'	0

# Tail Call Optimization

- Tail calls do not require any modifications to the activation frame. Thus, we do not need to keep them around.
- Compiler can detect tail recursion, and then optimize its stack usage by discarding each activation frame during evaluation.
  - Constant space usage!
  - The same performance as loops!
- Not all PLs offer this tail call optimization!

# Tail Call Optimization

PL	Tail Call Optimized	Compiler
C/C++	Yes	GCC
Swift	Yes	All
Python	No	All
C#	No	All
Java	Partially	JVM
OCaml	Yes	All
Haskell	Yes	GHC
javascript	Yes	ES6