Homework 1: Regular Languages and DFAs
Theory of Computation (CSCI 3500)
Total Points: 4

Due: Wednesday, Feb. 3 by 5:15pm

Write the solution to each question on its own page.
All papers must be stapled in order.
Your name must be on each page. All assignments not adhering to this will not be graded.

0. (4 pt): Define a DFA that recognizes your first name, and only your first name. Give both the formal, and informal definitions.

1. (4 pt): Construct a DFA for the following:

$$L = \{w \in \{0, 1, *\}^* \mid |w|_1 = 4 \text{ and } |w|_0 = 5\}$$

where $|w|_c = n$ for $c \in \Sigma$ means that the alphabet symbol, c, appears in the word, w, exactly n times.

2. (4 pt): Construct a DFA for the following:

$$L = \{!^n \mid n \in \mathbb{N} \text{ and } n \text{ is a multiple of 3, but not 7}\}$$

3. (4 pt): A language is called **regular** if it is the language of a DFA. The following algorithm describes how to construct the intersection of two regular languages by constructing a new DFA from the DFAs of the input languages:

> Suppose L_1 and L_2 are regular languages on the same alphabet. Then by regularity we know that there must exist two **DFAs** $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ and $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ such that $L(M_1) = L_1$ and $L(M_2) = L_2$. We construct a **DFA**, $M_\cap = (Q_\cap, \Sigma_\cap, \delta_\cap, q_\cap, F_\cap)$, where $L(M_\cap) = L_1 \cap L_2$ as follows:
>
> $$Q_\cap = Q_1 \times Q_2$$
> $$\Sigma_\cap = \Sigma$$
> $$\delta_\cap((r_1, r_2), a) = (\delta_1(r_1, a), \delta(r_2, a))$$
> $$q_\cap = (q_1, q_2)$$
> $$F_\cap = F_1 \times F_2$$

Consider the language $L = \{w \mid w \in \{a, b\}^* \text{ and } |w|_a = 3 \text{ and } w \text{ has odd length }\}$.

The language L is an intersection of two regular languages. Complete the following:

i. Identify the two regular languages L_1 and L_2 such that $L = L_1 \cap L_2$, and define DFAs for both of them.

ii. Using your solution of part i and the intersection construction given above define the **DFA** that recognizes $L_1 \cap L_2 = L$. You must draw the diagram of the DFA.

4. (4 pt): Using the mathematical definition of a DFA, prove that any finite language, L, over an alphabet Σ is regular.