

termvar, name, f, x, y, z, xs

indecies, i, j, k

<i>exp, e</i>	$::=$	Expressions
		x
		$\text{num}[n]$
		$\text{str}[s]$
		$\text{plus}(e_1; e_2)$
		$\text{mult}(e_1; e_2)$
		$\text{cat}(e_1; e_2)$
		$\text{len}(e)$
		$\text{let}(e_1; x.e_2)$
		$\text{fun}[T_1; T_2](x.e_1; f.e_2)$
		$\text{call}[f](e)$

<i>type, T</i>	$::=$	Types
		Str
		Num
		(T)

S

Γ	$::=$	Typing Contexts
		\emptyset
		$x : T$
		$f(T_1) : T_2$
		Γ_1, Γ_2

\mathcal{E}	$::=$	Evaluation Contexts
		\square
		$\text{plus}(\mathcal{E}; e_2)$
		$\text{plus}(e_1; \mathcal{E})$
		$\text{mult}(\mathcal{E}; e_2)$
		$\text{mult}(e_1; \mathcal{E})$
		$\text{cat}(\mathcal{E}; e_2)$
		$\text{cat}(e_1; \mathcal{E})$
		$\text{len}(\mathcal{E})$
		$\text{let}(\mathcal{E}; x.e_2)$
		$\text{let}(e_1; x.\mathcal{E})$
		$\mathcal{E}[e]$

S

<i>nat, n</i>	$::=$	Nats
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<i>char, c</i>	$::=$	Character
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<i>strings, s</i>	$::=$
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$\llbracket x.e_1/f \rrbracket e_2$

$$\llbracket x.e_1/f \rrbracket \text{call}[f](e_2) \equiv \text{let}(\llbracket x.e_1/f \rrbracket e_2; x.e_1)$$

$\Gamma \vdash f(T_1) : T_2$	Function Header
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$$\overline{\Gamma, f(T_1) : T_2 \vdash f(T_1) : T_2} \quad \text{FUNH}$$

$\boxed{\Gamma \vdash e : T}$ Typing

$$\begin{array}{c}
\frac{}{\Gamma, x : T \vdash x : T} \text{VAR} \\
\\
\frac{}{\Gamma \vdash \text{str}[s] : \text{Str}} \text{STR} \\
\\
\frac{}{\Gamma \vdash \text{num}[n] : \text{Num}} \text{NUM} \\
\\
\frac{\Gamma \vdash e_1 : \text{Num} \quad \Gamma \vdash e_2 : \text{Num}}{\Gamma \vdash \text{plus}(e_1; e_2) : \text{Num}} \text{PLUS} \\
\\
\frac{\Gamma \vdash e_1 : \text{Num} \quad \Gamma \vdash e_2 : \text{Num}}{\Gamma \vdash \text{mult}(e_1; e_2) : \text{Num}} \text{MULT} \\
\\
\frac{\Gamma \vdash e : \text{Str}}{\Gamma \vdash \text{len}(e) : \text{Num}} \text{LENGTH} \\
\\
\frac{\Gamma \vdash e_1 : \text{Str} \quad \Gamma \vdash e_2 : \text{Str}}{\Gamma \vdash \text{cat}(e_1; e_2) : \text{Str}} \text{CAT} \\
\\
\frac{\Gamma \vdash e_1 : T_1 \quad \Gamma, x : T_1 \vdash e_2 : T_2}{\Gamma \vdash \text{let}(e_1; x.e_2) : T_2} \text{LET} \\
\\
\frac{\Gamma, x : T_1 \vdash e_1 : T_2 \quad \Gamma, f(T_1) : T_2 \vdash e_2 : T}{\Gamma \vdash \text{fun}[T_1; T_2](x.e_1; f.e_2) : T} \text{FUN} \\
\\
\frac{\Gamma \vdash f(T_1) : T_2 \quad \Gamma \vdash e : T_1}{\Gamma \vdash \text{call}[f](e) : T_2} \text{CALL}
\end{array}$$

$\boxed{e \text{ val}}$ Values

$$\begin{array}{c}
\frac{}{\text{num}[n] \text{ val}} \text{V_NUM} \\
\\
\frac{}{\text{str}[s] \text{ val}} \text{V_STR}
\end{array}$$

$\boxed{e_1 \mapsto e_2}$ Evaluation

$$\begin{array}{c}
\frac{n_1 + n_2 = n \text{ nat}}{\text{plus}(\text{num}[n_1]; \text{num}[n_2]) \mapsto \text{num}[n]} \text{PLUSVAL} \\
\\
\frac{e_4 \mapsto e_4''}{\text{plus}(e_4; e_5) \mapsto \text{plus}(e_4''; e_5)} \text{PLUS1} \\
\\
\frac{e_1 \text{ val} \quad e_2 \mapsto e_2'}{\text{plus}(e_1; e_2) \mapsto \text{plus}(e_1; e_2')} \text{PLUS2} \\
\\
\frac{e_1 \mapsto e_1'}{\text{mult}(e_1; e_2) \mapsto \text{mult}(e_1'; e_2)} \text{MULT1} \\
\\
\frac{e_1 \text{ val} \quad e_2 \mapsto e_2'}{\text{mult}(e_1; e_2) \mapsto \text{mult}(e_1; e_2')} \text{MULT2} \\
\\
\frac{n_1 * n_2 = n \text{ nat}}{\text{mult}(\text{num}[n_1]; \text{num}[n_2]) \mapsto \text{num}[n]} \text{MULTVAL} \\
\\
\frac{s_1 s_2 = s \text{ str}}{\text{cat}(\text{str}[s_1]; \text{str}[s_2]) \mapsto \text{str}[s]} \text{CATVAL}
\end{array}$$

$$\begin{array}{c}
\frac{e_1 \mapsto e'_1}{\text{cat}(e_1; e_2) \mapsto \text{cat}(e'_1; e_2)} \quad \text{CAT1} \\
\\
\frac{e_1 \text{ val} \quad e_2 \mapsto e'_2}{\text{cat}(e_1; e_2) \mapsto \text{cat}(e_1; e'_2)} \quad \text{CAT2} \\
\\
\frac{|s| = n \text{ num}}{\text{len}(\text{str}[s]) \mapsto \text{num}[n]} \quad \text{LENGTHVAL} \\
\\
\frac{e \mapsto e'}{\text{len}(e) \mapsto \text{len}(e')} \quad \text{LENGTH1} \\
\\
\frac{e_1 \text{ val}}{\text{let}(e_1; x.e_2) \mapsto [e_1/x]e_2} \quad \text{LETVAL} \\
\\
\frac{e_1 \mapsto e'_1}{\text{let}(e_1; x.e_2) \mapsto \text{let}(e'_1; x.e_2)} \quad \text{LET1} \\
\\
\frac{}{\text{let}(e_1; x.e_2) \mapsto [e_1/x]e_2} \quad \text{LETL} \\
\\
\frac{}{\text{fun}[T_1; T_2](x.e_1; f.e_2) \mapsto \llbracket x.e_1/f \rrbracket e_2} \quad \text{FUNVAL}
\end{array}$$

$\boxed{e_1 \mapsto^* e_2}$ Multistep Evaluation

$$\begin{array}{c}
\frac{}{e \mapsto^* e} \quad \text{REFL} \\
\\
\frac{e_1 \mapsto e_2 \quad e_2 \mapsto^* e_3}{e_1 \mapsto^* e_3} \quad \text{STEP}
\end{array}$$

$\boxed{e = \mathcal{E}}$ Contextual Equality

$$\begin{array}{c}
\frac{}{e = \square[e]} \quad \text{C_HOLE} \\
\\
\frac{e_1 = \mathcal{E}_1[e]}{\text{plus}(e_1; e_2) = \text{plus}(\mathcal{E}_1; e_2)[e]} \quad \text{C_PLUS1} \\
\\
\frac{e_1 \text{ val} \quad e_2 = \mathcal{E}_2[e]}{\text{plus}(e_1; e_2) = \text{plus}(e_1; \mathcal{E}_2)[e]} \quad \text{C_PLUS2} \\
\\
\frac{e_1 = \mathcal{E}_1[e]}{\text{mult}(e_1; e_2) = \text{mult}(\mathcal{E}_1; e_2)[e]} \quad \text{C_MULT1} \\
\\
\frac{e_1 \text{ val} \quad e_2 = \mathcal{E}_2[e]}{\text{mult}(e_1; e_2) = \text{mult}(e_1; \mathcal{E}_2)[e]} \quad \text{C_MULT2} \\
\\
\frac{e_1 = \mathcal{E}_1[e]}{\text{cat}(e_1; e_2) = \text{cat}(\mathcal{E}_1; e_2)[e]} \quad \text{C_CAT1} \\
\\
\frac{e_1 \text{ val} \quad e_2 = \mathcal{E}_2[e]}{\text{cat}(e_1; e_2) = \text{cat}(e_1; \mathcal{E}_2)[e]} \quad \text{C_CAT2} \\
\\
\frac{e' = \mathcal{E}[e]}{\text{len}(e') = \text{len}(\mathcal{E})[e]} \quad \text{C_LENGTH} \\
\\
\frac{e_1 = \mathcal{E}[e]}{\text{let}(e_1; x.e_2) = \text{let}(\mathcal{E}; x.e_2)[e]} \quad \text{C_LET1}
\end{array}$$

$\boxed{e_1 \mapsto_S e_2}$ Single Step Evaluation

$$\begin{array}{c}
\frac{n_1 + n_2 = n \text{ nat}}{\text{plus}(\text{num}[n_1]; \text{num}[n_2]) \mapsto_S \text{num}[n]} \quad \text{S_PLUSVAL} \\
\frac{n_1 * n_2 = n \text{ nat}}{\text{mult}(\text{num}[n_1]; \text{num}[n_2]) \mapsto_S \text{num}[n]} \quad \text{S_MULTVAL} \\
\frac{s_1 s_2 = s \text{ str}}{\text{cat}(\text{str}[s_1]; \text{str}[s_2]) \mapsto_S \text{str}[s]} \quad \text{S_CATVAL} \\
\frac{|s| = n \text{ num}}{\text{len}(\text{str}[s]) \mapsto_S \text{num}[n]} \quad \text{S_LENGTHVAL} \\
\frac{e_1 \text{ val}}{\text{let}(e_1; x.e_2) \mapsto_S [e_1/x]e_2} \quad \text{S_LETVAL}
\end{array}$$

$e_1 \mapsto e_2$ Evaluation Contexts

$$\frac{e = \mathcal{E}[e_1] \quad e_1 \mapsto_S e_2 \quad e' = \mathcal{E}[e_2]}{e \mapsto e'} \quad \text{C_STEP}$$

$\Gamma \vdash e_1 \equiv e_2 : T$ Equality

$$\begin{array}{c}
\frac{}{\Gamma \vdash e \equiv e : T} \quad \text{E_REFL} \\
\frac{\Gamma \vdash e_2 \equiv e_1 : T}{\Gamma \vdash e_1 \equiv e_2 : T} \quad \text{E_SYM} \\
\frac{\Gamma \vdash e_1 \equiv e_2 : T \quad \Gamma \vdash e_2 \equiv e_3 : T}{\Gamma \vdash e_1 \equiv e_3 : T} \quad \text{E_TRANS} \\
\frac{n_1 + n_2 = n \text{ nat}}{\Gamma \vdash \text{plus}(\text{num}[n_1]; \text{num}[n_2]) \equiv \text{num}[n] : \text{Num}} \quad \text{E_PLUSVAL} \\
\frac{\Gamma \vdash e_1 \equiv e'_1 : \text{Num} \quad \Gamma \vdash e_2 \equiv e'_2 : \text{Num}}{\Gamma \vdash \text{plus}(e_1; e_2) \equiv \text{plus}(e'_1; e'_2) : \text{Num}} \quad \text{E_PLUS} \\
\frac{n_1 * n_2 = n \text{ nat}}{\Gamma \vdash \text{mult}(\text{num}[n_1]; \text{num}[n_2]) \equiv \text{num}[n] : \text{Num}} \quad \text{E_MULTVAL} \\
\frac{\Gamma \vdash e_1 \equiv e'_1 : \text{Num} \quad \Gamma \vdash e_2 \equiv e'_2 : \text{Num}}{\Gamma \vdash \text{mult}(e_1; e_2) \equiv \text{mult}(e'_1; e'_2) : \text{Num}} \quad \text{E_MULT} \\
\frac{s_1 s_2 = s \text{ str}}{\Gamma \vdash \text{cat}(\text{str}[s_1]; \text{str}[s_2]) \equiv \text{str}[s] : \text{Str}} \quad \text{E_CATVAL} \\
\frac{\Gamma \vdash e_1 \equiv e'_1 : \text{Str} \quad \Gamma \vdash e_2 \equiv e'_2 : \text{Str}}{\Gamma \vdash \text{cat}(e_1; e_2) \equiv \text{cat}(e'_1; e'_2) : \text{Str}} \quad \text{E_CAT} \\
\frac{|s| = n \text{ num}}{\Gamma \vdash \text{len}(\text{str}[s]) \equiv \text{num}[n] : \text{Num}} \quad \text{E_LENGTHVAL} \\
\frac{\Gamma \vdash e \equiv e' : \text{Str}}{\Gamma \vdash \text{len}(e) \equiv \text{len}(e') : \text{Num}} \quad \text{E_LENGTH1} \\
\frac{\Gamma \vdash e_1 \equiv e'_1 : T_1 \quad \Gamma, x : T_1 \vdash e_2 \equiv e'_2 : T_2}{\Gamma \vdash \text{let}(e_1; x.e_2) \equiv \text{let}(e'_1; x.e_2) : T_2} \quad \text{E_LET} \\
\frac{}{\Gamma \vdash \text{let}(e_1; x.e_2) \equiv [e_1/x]e_2 : T} \quad \text{E_LETL}
\end{array}$$

Definition rules: 58 good 0 bad
Definition rule clauses: 104 good 0 bad