

typevar, *X*, *Y*, *Z*
termvar, *name*, *double*, *f*, *g*, *x*, *y*, *z*, *rf*, *p*, *r*, *n*, *xs*
indecies, *i*, *j*, *k*

$pattern, p$	$::=$ $ \text{left}(x)$ $ \text{right}(x)$ $ \text{prod}(x; y)$	Patterns
$patterns$	$::=$ $ p_1 \rightarrow e_1; \dots; p_i \rightarrow e_i$	
$tyop$	$::=$ $ X.T$	Type Operator
exp, e	$::=$ $ x$ $ \mathbf{tlam}(X.e)$ $ \mathbf{tapp}[T](e)$ $ \mathbf{map}[tyop](x.e_1; e_2)$ $ \mathbf{match}(e_1; patterns)$ $ \mathbf{rec}[T](r.e)$ $ \mathbf{let}(e_1; x.e_2)$ $ \mathbf{lam}[T](x.e)$ $ \mathbf{app}(e_1; e_2)$ $ \mathbf{triv}$ $ \mathbf{prod}(e_1; e_2)$ $ \mathbf{left}(e)$ $ \mathbf{right}(e)$ $ \mathbf{abort}(e)$	Expressions
$type, T$	$::=$ $ X$ $ \mathbf{Void}$ $ \mathbf{Unit}$ $ \mathbf{Sum}(T_1; T_2)$ $ \mathbf{Prod}(T_1; T_2)$ $ \mathbf{Arrow}(T_1; T_2)$ $ \mathbf{Forall}(X.T)$	Types
Δ	$::=$ $ \mathbf{empty}$ $ X \text{ type}$ $ \Delta_1, \Delta_2$	Kinding Contexts
Γ	$::=$ $ \emptyset$ $ x : T$ $ \Gamma_1, \Gamma_2$	Typing Contexts
$char, c$	$::=$	Character
$strings, s$	$::=$	

$\Delta \vdash T \text{ type}$ Kinding

$$\begin{array}{c}
\frac{}{\Delta, X \text{ type} \vdash X \text{ type}} \text{K_VAR} \\
\frac{}{\Delta \vdash \text{Void type}} \text{K_VOID} \\
\frac{}{\Delta \vdash \text{Unit type}} \text{K_UNIT} \\
\frac{\Delta \vdash T_1 \text{ type} \quad \Delta \vdash T_2 \text{ type}}{\Delta \vdash \text{Sum}(T_1; T_2) \text{ type}} \text{K_SUM} \\
\frac{\Delta \vdash T_1 \text{ type} \quad \Delta \vdash T_2 \text{ type}}{\Delta \vdash \text{Prod}(T_1; T_2) \text{ type}} \text{K_PROD} \\
\frac{\Delta \vdash T_1 \text{ type} \quad \Delta \vdash T_2 \text{ type}}{\Delta \vdash \text{Arrow}(T_1; T_2) \text{ type}} \text{K_ARROW} \\
\frac{\Delta, X \text{ type} \vdash T \text{ type}}{\Delta \vdash \text{Forall}(X.T) \text{ type}} \text{K_FORALL}
\end{array}$$

$X \text{ type}; \Delta \vdash T \text{ typeop}$ Type Operators

$$\begin{array}{c}
\frac{}{X \text{ type}; \Delta \vdash X \text{ typeop}} \text{TO_ID} \\
\frac{}{X \text{ type}; \Delta, Y \text{ type} \vdash Y \text{ typeop}} \text{TO_VAR} \\
\frac{}{X \text{ type}; \Delta \vdash \text{Void typeop}} \text{TO_VOID} \\
\frac{}{X \text{ type}; \Delta \vdash \text{Unit typeop}} \text{TO_UNIT} \\
\frac{X \text{ type}; \Delta \vdash T_1 \text{ typeop} \quad X \text{ type}; \Delta \vdash T_2 \text{ typeop}}{X \text{ type}; \Delta \vdash \text{Sum}(T_1; T_2) \text{ typeop}} \text{TO_SUM} \\
\frac{X \text{ type}; \Delta \vdash T_1 \text{ typeop} \quad X \text{ type}; \Delta \vdash T_2 \text{ typeop}}{X \text{ type}; \Delta \vdash \text{Prod}(T_1; T_2) \text{ typeop}} \text{TO_PROD} \\
\frac{X \text{ type}; \Delta, Y \text{ type} \vdash T \text{ typeop}}{X \text{ type}; \Delta \vdash \text{Forall}(Y.T) \text{ typeop}} \text{TO_FORALL} \\
\frac{X \notin \text{FV}(T_1) \quad X \text{ type}; \Delta \vdash T_2 \text{ typeop}}{X \text{ type}; \Delta \vdash \text{Arrow}(T_1; T_2) \text{ typeop}} \text{TO_ARROW}
\end{array}$$

$\Delta; \Gamma \vdash e : T$ Typing

$$\begin{array}{c}
\frac{\Delta \vdash T \text{ type}}{\Delta; \Gamma, x : T \vdash x : T} \text{VAR} \\
\frac{}{\Delta; \Gamma \vdash \text{triv} : \text{Unit}} \text{TRIV} \\
\frac{\Delta; \Gamma \vdash e_1 : T_1 \quad \Delta; \Gamma \vdash e_2 : T_2}{\Delta; \Gamma \vdash \text{prod}(e_1; e_2) : \text{Prod}(T_1; T_2)} \text{PROD} \\
\frac{\Delta; \Gamma \vdash e : \text{Prod}(T_1; T_2) \quad \Delta; \Gamma, x : T_1, y : T_2 \vdash e_2 : T}{\Delta; \Gamma \vdash \text{match}(e; \text{prod}(x; y) \rightarrow e_2) : T} \text{MATCHPROD}
\end{array}$$

$$\begin{array}{c}
\frac{\Delta; \Gamma, r : T \vdash e : T}{\Delta; \Gamma \vdash \text{rec}[T](r.e) : T} \text{ REC} \\
\\
\frac{\Delta; \Gamma \vdash e_1 : T_1 \quad \Delta; \Gamma, x : T_1 \vdash e_2 : T_2}{\Delta; \Gamma \vdash \text{let}(e_1; x.e_2) : T_2} \text{ LET} \\
\\
\frac{\Delta; \Gamma, x : T_1 \vdash e : T_2}{\Delta; \Gamma \vdash \text{lam}[T_1](x.e) : \text{Arrow}(T_1; T_2)} \text{ LAM} \\
\\
\frac{\Delta, X \text{ type}; \Gamma \vdash e : T}{\Delta; \Gamma \vdash \text{tlam}(X.e) : \text{Forall}(X.T)} \text{ TLAM} \\
\\
\frac{\Delta; \Gamma \vdash e : \text{Forall}(X.T_2) \quad \Delta \vdash T_1 \text{ type}}{\Delta; \Gamma \vdash \text{tapp}[T_1](e) : [T_1/X]T_2} \text{ TAPP} \\
\\
\frac{\Delta; \Gamma \vdash e_1 : \text{Arrow}(T_1; T_2) \quad \Delta; \Gamma \vdash e_2 : T_1}{\Delta; \Gamma \vdash \text{app}(e_1; e_2) : T_2} \text{ APP} \\
\\
\frac{\Delta; \Gamma \vdash e : T_1}{\Delta; \Gamma \vdash \text{left}(e) : \text{Sum}(T_1; T_2)} \text{ INJECTLEFT} \\
\\
\frac{\Delta; \Gamma \vdash e : T_2}{\Delta; \Gamma \vdash \text{right}(e) : \text{Sum}(T_1; T_2)} \text{ INJECTRIGHT} \\
\\
\frac{\Delta; \Gamma \vdash e : \text{Sum}(T_1; T_2) \quad \Delta; \Gamma, x : T_1 \vdash e_1 : T \quad \Delta; \Gamma, y : T_2 \vdash e_2 : T}{\Delta; \Gamma \vdash \text{match}(e; \text{left}(y) \rightarrow e_1; \text{right}(y) \rightarrow e_2) : T} \text{ MATCHSUM} \\
\\
\frac{\Delta; \Gamma \vdash e : \text{Void}}{\Delta; \Gamma \vdash \text{abort}(e) : T} \text{ ABORT} \\
\\
\frac{X \text{ type}; \Delta \vdash T \text{ typeop} \quad \text{empty}; \Gamma, x : T_1 \vdash e_2 : T_2 \quad \Delta; \Gamma \vdash e_1 : [T_1/X]T}{\Delta; \Gamma \vdash \text{map}[X.T](x.e_2; e_1) : [T_2/X]T} \text{ MAP} \\
\\
\frac{}{\Delta; \Gamma \vdash \text{True} : \text{Bool}} \text{ TRUE} \\
\\
\frac{}{\Delta; \Gamma \vdash \text{False} : \text{Bool}} \text{ FALSE} \\
\\
\frac{\Delta; \Gamma \vdash e : \text{Bool} \quad \Delta; \Gamma \vdash e_1 : T \quad \Delta; \Gamma \vdash e_2 : T}{\Delta; \Gamma \vdash \text{if}(e; e_1; e_2) : T} \text{ IF} \\
\\
\frac{}{\Delta; \Gamma \vdash \text{nothing} : \text{Maybe}(T)} \text{ NOTHING} \\
\\
\frac{\Delta; \Gamma \vdash e : T}{\Delta; \Gamma \vdash \text{just}(e) : \text{Maybe}(T)} \text{ JUST} \\
\\
\frac{\Delta; \Gamma \vdash e : \text{Maybe}(T) \quad \Delta; \Gamma \vdash e_1 : T' \quad \Delta; \Gamma, x : T \vdash e_2 : T'}{\Delta; \Gamma \vdash \text{ifNothing}[T](e; e_1; x.e_2) : T'} \text{ IFN}
\end{array}$$

$e \text{ val}$ Values

$$\begin{array}{c}
\frac{}{\text{triv val}} \text{ V_TRIV} \\
\\
\frac{}{\text{lam}[T](x.e) \text{ val}} \text{ V_LAM} \\
\\
\frac{}{\text{tlam}(X.e) \text{ val}} \text{ V_TLAM} \\
\\
\frac{e_1 \text{ val} \quad e_2 \text{ val}}{\text{prod}(e_1; e_2) \text{ val}} \text{ V_PROD}
\end{array}$$

$$\frac{e \text{ val}}{\text{left } (e) \text{ val}} \quad \text{V_LEFT}$$

$$\frac{e \text{ val}}{\text{right } (e) \text{ val}} \quad \text{V_RIGHT}$$

$e_1 \mapsto e_2$ Evaluation

$$\frac{}{\mathbf{tapp} [T](\mathbf{tlam} (X.e)) \mapsto [T/X]e} \quad \text{E_TAPPVAL}$$

$$\frac{e_1 \mapsto e_2}{\mathbf{tapp} [T](e_1) \mapsto \mathbf{tapp} [T](e_2)} \quad \text{E_TAPP}$$

$$\frac{}{\mathbf{map} [X.X](x.e_2; e_1) \mapsto [e_1/x]e_2} \quad \text{E_MAPID}$$

$$\frac{}{\mathbf{map} [X.\text{Unit}](x.e_2; e_1) \mapsto \text{triv}} \quad \text{E_MAPUNIT}$$

$$\frac{}{\mathbf{map} [X.\text{Void}](x.e_2; e_1) \mapsto \mathbf{abort} (e_1)} \quad \text{E_MAPVOID}$$

$$\frac{}{\mathbf{map} [X.\text{Prod} (T_1; T_2)](x.e_2; e_1) \mapsto \mathbf{prod} (\mathbf{map} [X.T_1](x.e_1; \mathbf{fst} (e_1)); \mathbf{map} [X.T_2](x.e_1; \mathbf{snd} (e_1)))} \quad \text{E_MAPPROD}$$

$$\frac{}{\mathbf{map} [X.\text{Sum} (T_1; T_2)](x.e_2; e_1) \mapsto \mathbf{match} (e_1; \text{left } (x) \rightarrow \mathbf{map} [X.T_1](x.e_2; x); \text{right } (y) \rightarrow \mathbf{map} [X.T_2](x.e_2; y))} \quad \text{E_MAPSUM}$$

$$\frac{}{\mathbf{map} [X.\text{Arrow} (T_1; T_2)](x.e_2; e_1) \mapsto \mathbf{lam} [T_1](y.\mathbf{map} [X.T_2](x.e_2; \mathbf{app} (e_1; y)))} \quad \text{E_MAPARROW}$$

$$\frac{}{\mathbf{map} [X.\text{Forall} (Y.T)](x.e_2; e_1) \mapsto \mathbf{tlam} (Y.\mathbf{map} [X.T](x.e_2; \mathbf{tapp} [Y](e_1)))} \quad \text{E_MAPFORALL}$$

$$\frac{e \mapsto e'}{\mathbf{abort} (e) \mapsto \mathbf{abort} (e')} \quad \text{E_ABORT}$$

$$\frac{e \mapsto e'}{\text{left } (e) \mapsto \text{left } (e')} \quad \text{E_LEFT}$$

$$\frac{e \mapsto e'}{\text{right } (e) \mapsto \text{right } (e')} \quad \text{E_RIGHT}$$

$$\frac{}{\mathbf{match} (\text{left } (e); \text{left } (x) \rightarrow e_1; \text{right } (y) \rightarrow e_2) \mapsto [e/x]e_1} \quad \text{E_MATCHLEFT}$$

$$\frac{}{\mathbf{match} (\text{right } (e); \text{left } (x) \rightarrow e_1; \text{right } (y) \rightarrow e_2) \mapsto [e/y]e_2} \quad \text{E_MATCHRIGHT}$$

$$\frac{e_1 \mapsto e'_1}{\mathbf{prod} (e_1; e_2) \mapsto \mathbf{prod} (e'_1; e_2)} \quad \text{E_PROD1}$$

$$\frac{e_1 \text{ val} \quad e_2 \mapsto e'_2}{\mathbf{prod} (e_1; e_2) \mapsto \mathbf{prod} (e_1; e'_2)} \quad \text{E_PROD2}$$

$$\frac{e_1 \text{ val} \quad e_2 \text{ val}}{\mathbf{match} (\mathbf{prod} (e_1; e_2); \mathbf{prod} (x; y) \rightarrow e_3) \mapsto [e_1/x][e_2/y]e_3} \quad \text{E_MATCHP}$$

$$\frac{e \mapsto e'}{\mathbf{match} (e; \text{patterns}) \mapsto \mathbf{match} (e'; \text{patterns})} \quad \text{E_MATCH}$$

$$\frac{}{\mathbf{rec} [T](r.e) \mapsto [\mathbf{rec} [T](r.e)/r]e} \quad \text{E_REC}$$

$$\frac{e_1 \text{ val}}{\mathbf{let} (e_1; x.e_2) \mapsto [e_1/x]e_2} \quad \text{E_LETVAL}$$

$$\begin{array}{c}
\frac{e_1 \mapsto e'_1}{\text{let}(e_1; x.e_2) \mapsto \text{let}(e'_1; x.e_2)} \quad \text{E_LET1} \\
\frac{}{\text{let}(e_1; x.e_2) \mapsto [e_1/x]e_2} \quad \text{E_LETL} \\
\frac{e_2 \text{ val}}{\text{app}(\text{lam}[T](x.e_2); e_1) \mapsto [e_1/x]e_2} \quad \text{E_APPVAL} \\
\frac{e_1 \mapsto e'_1}{\text{app}(e_1; e_2) \mapsto \text{app}(e'_1; e_2)} \quad \text{E_APP1} \\
\frac{e_1 \text{ val} \quad e_2 \mapsto e'_2}{\text{app}(e_1; e_2) \mapsto \text{app}(e_1; e'_2)} \quad \text{E_APP2}
\end{array}$$

$e_1 \mapsto^* e_2$ Multistep Evaluation

$$\begin{array}{c}
\frac{}{e \mapsto^* e} \quad \text{REFL} \\
\frac{e_1 \mapsto e_2 \quad e_2 \mapsto^* e_3}{e_1 \mapsto^* e_3} \quad \text{STEP}
\end{array}$$

Definition rules: 69 good 0 bad
Definition rule clauses: 111 good 0 bad