Generic Types

Motivating Example

Type Operator:

$$X.\mathsf{Prod}\,(T; X)$$

$$\frac{A \to B}{\mathsf{Prod}(T; A) \to \mathsf{Prod}(T; B)}$$

Motivating Example

Type Operator:

$$X.\mathsf{Prod}\,(T; X)$$

Transformation

$$\boxed{\emptyset; x : A \vdash e : B}$$

$$\frac{\emptyset; \emptyset \vdash \mathsf{prod}\,(t; a) : \mathsf{Prod}\,(T; A) \qquad X\ \mathsf{type} \vdash \mathsf{Prod}\,(T; X)\ \mathsf{typeop}}{\emptyset; \emptyset \vdash \mathsf{map}\,[X.\mathsf{Prod}\,(T; X)](x.e; \mathsf{prod}\,(t; a)) : \mathsf{Prod}\,(T; B)}$$

Motivating Example

Type Operator:

$$X.\mathsf{Prod}\,(T; X)$$

Transformation

$$\dfrac{\boxed{\emptyset; x : A \vdash e : B} \qquad \boxed{\emptyset; \emptyset \vdash \mathsf{prod}\,(t; a) : \mathsf{Prod}\,(T; A)} \quad \textbf{Source} \qquad X\ \mathsf{type} \vdash \mathsf{Prod}\,(T; X)\ \mathsf{typeop}}{\emptyset; \emptyset \vdash \mathsf{map}\,[X.\mathsf{Prod}\,(T; X)](x.e; \mathsf{prod}\,(t; a)) : \mathsf{Prod}\,(T; B)}$$

Motivating Example

Type Operator:

$$X.\mathsf{Prod}\,(T; X)$$

Transformation

$\emptyset; x : A \vdash e : B$

Source

$\emptyset; \emptyset \vdash \mathsf{prod}\,(t; a) : \mathsf{Prod}\,(T; A)$

Type Operator

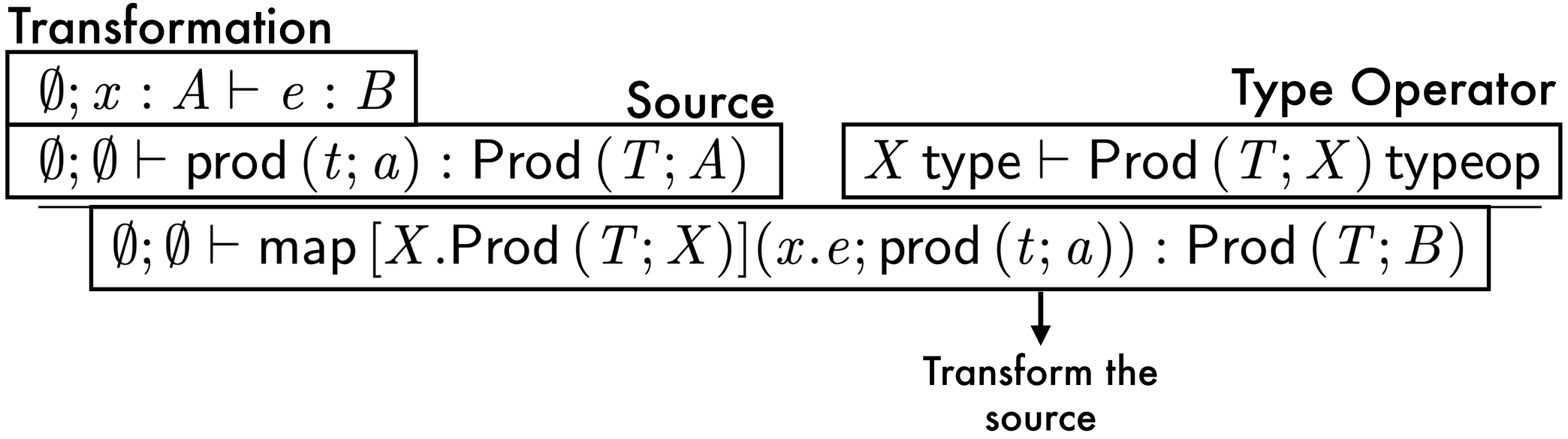
$X\ \mathsf{type} \vdash \mathsf{Prod}\,(T; X)\ \mathsf{typeop}$

$$\emptyset; \emptyset \vdash \mathsf{map}\,[X.\mathsf{Prod}\,(T; X)](x.e; \mathsf{prod}\,(t; a)) : \mathsf{Prod}\,(T; B)$$

Motivating Example

Type Operator:

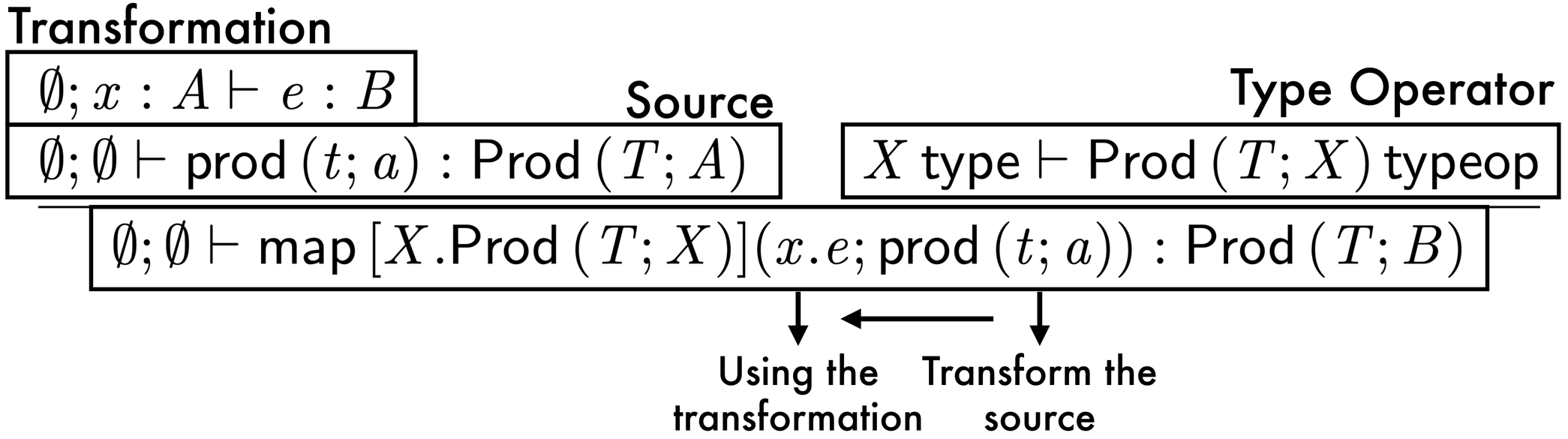
$$X.\mathsf{Prod}\,(T; X)$$



Motivating Example

Type Operator:

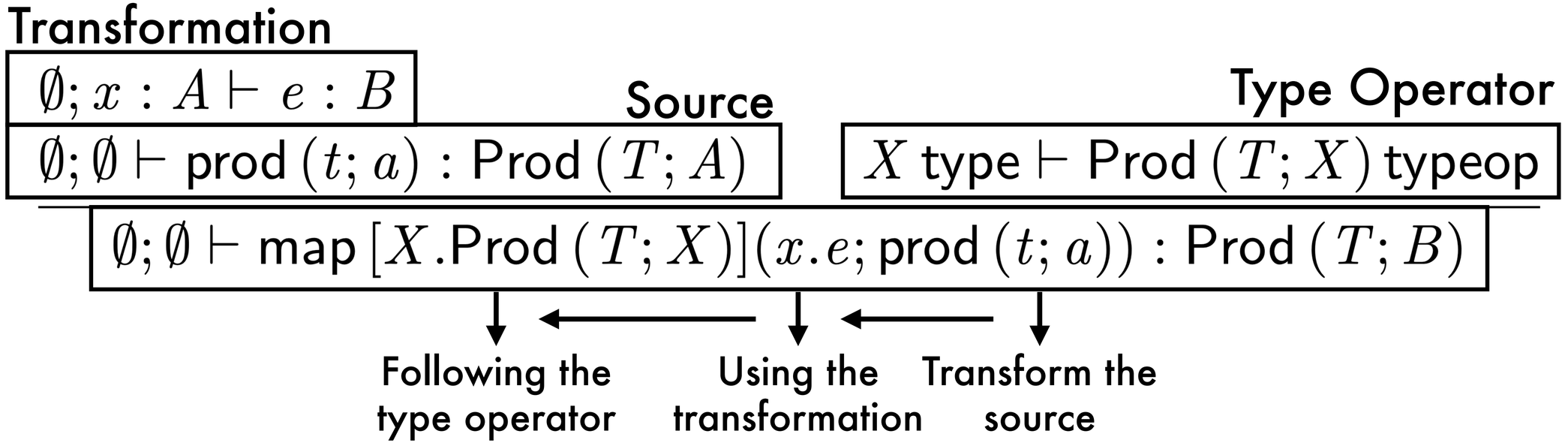
$$X.\mathsf{Prod}\,(T; X)$$



Motivating Example

Type Operator:

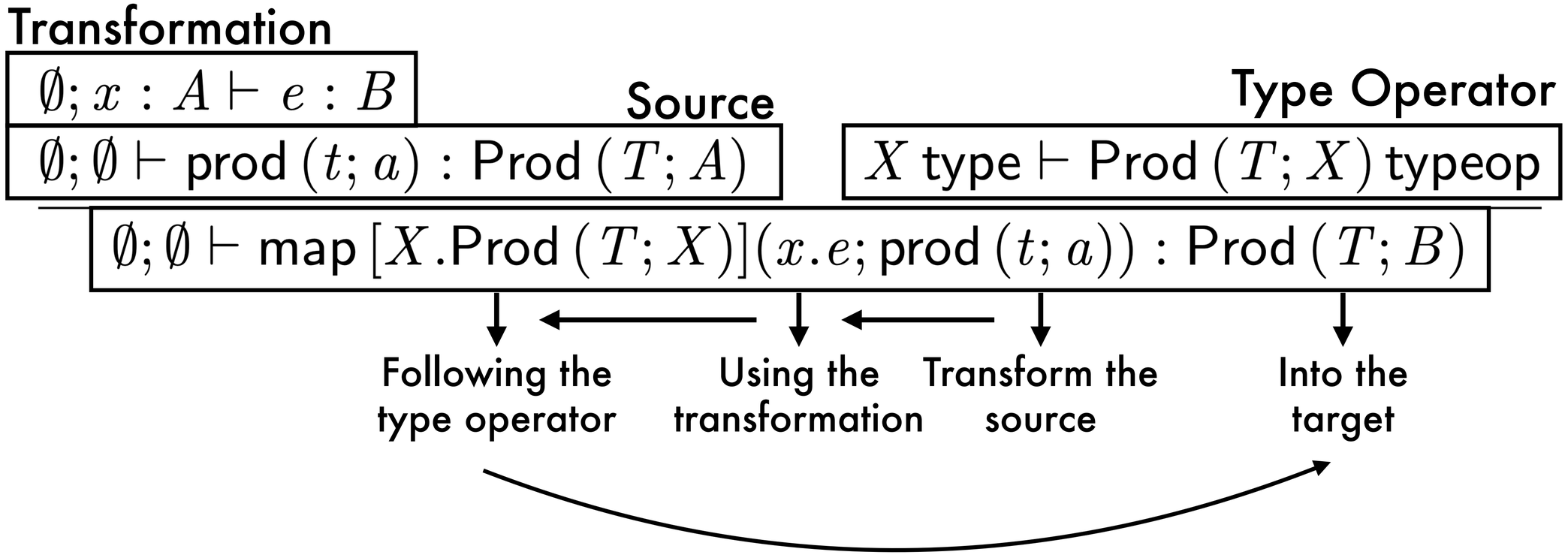
$$X.\mathsf{Prod}\,(T; X)$$



Motivating Example

Type Operator:

$$X.\mathsf{Prod}\,(T; X)$$



Statics: Expressions

$$
\begin{array}{rll}
e ::= & x \\
| & \mathsf{map}\,[tyop](x.e_1; e_2) \\
| & \mathsf{match}\,(e_1; patterns) \\
| & \mathsf{rec}\,[T](r.e) \\
| & \mathsf{let}\,(e_1; x.e_2) \\
| & \mathsf{lam}\,[T](x.e) \\
| & \mathsf{app}\,(e_1; e_2) \\
| & \mathsf{triv} \\
| & \mathsf{prod}\,(e_1; e_2) \\
| & \mathsf{left}\,(e) \\
| & \mathsf{right}\,(e) \\
| & \mathsf{abort}\,(e)
\end{array}
\qquad
\begin{array}{rll}
p ::= & \mathsf{left}\,(x) \\
& \mathsf{right}\,(x) \\
& \mathsf{prod}\,(x; y)
\end{array}
\qquad
patterns ::= \quad p_1 \to e_1; \ldots; p_i \to e_i
$$

Statics: Types

$$T ::= X$$
Void
Unit
Sum $(T_1; T_2)$
Prod $(T_1; T_2)$
Arrow $(T_1; T_2)$

$$tyop ::= X.T$$

Statics: Typing

$$\frac{X\ \mathsf{type}; \Delta \vdash T\ \mathsf{typeop} \quad \emptyset; \Gamma, x : T_1 \vdash e_2 : T_2 \quad \Delta; \Gamma \vdash e_1 : [T_1/X]\,T}{\Delta; \Gamma \vdash \mathsf{map}\,[X.\,T](x.e_2; e_1) : [T_2/X]\,T}\ \textsc{Map}$$

Statics: Positivity Checking

$$\frac{}{X \text{ type}; \Delta \vdash X \text{ typeop}} \textsc{To\_Id} \qquad \frac{}{X \text{ type}; \Delta \vdash \mathsf{Unit} \text{ typeop}} \textsc{To\_Unit}$$

$$\frac{X \text{ type}; \Delta \vdash T_1 \text{ typeop} \quad X \text{ type}; \Delta \vdash T_2 \text{ typeop}}{X \text{ type}; \Delta \vdash \mathsf{Sum}\,(T_1; T_2) \text{ typeop}} \textsc{To\_Sum}$$

$$\frac{X \text{ type}; \Delta \vdash T_1 \text{ typeop} \quad X \text{ type}; \Delta \vdash T_2 \text{ typeop}}{X \text{ type}; \Delta \vdash \mathsf{Prod}\,(T_1; T_2) \text{ typeop}} \textsc{To\_Prod}$$

Statics: Positivity Checking

$$\frac{X \notin \mathsf{FV}(T_1) \quad X\ \mathsf{type}; \Delta \vdash T_2\ \mathsf{typeop}}{X\ \mathsf{type}; \Delta \vdash \mathsf{Arrow}\,(T_1; T_2)\ \mathsf{typeop}}\ \textsc{To\_Arrow}$$

Dynamics

$$\frac{}{\mathsf{map}\,[X.X](x.e_2;\, e_1) \mapsto [e_1/x]e_2} \quad \textsc{E\_MapId}$$

Dynamics

$$\frac{}{\mathsf{map}\,[X.\mathsf{Unit}](x.e_2;\, e_1) \mapsto \mathsf{triv}}\quad \textsc{E\_MapUnit}$$

Dynamics

$$\frac{}{\mathsf{map}\,[X.\mathsf{Void}](x.e_2;\, e_1) \mapsto \mathsf{abort}\,(e_1)} \quad \textsc{E\_MapVoid}$$

Dynamics

$$\frac{}{\mathsf{map}\,[X.\mathsf{Prod}\,(T_1; T_2)](x.e_2; e_1) \mapsto \mathsf{prod}\,(\mathsf{map}\,[X.T_1](x.e_1; \mathsf{fst}\,(e_1)); \mathsf{map}\,[X.T_2](x.e_1; \mathsf{snd}\,(e_1)))}\ \textsc{E\_MapProd}$$

Dynamics

$$\mathsf{map}\,[X.\mathsf{Sum}\,(T_1;\,T_2)](x.e_2;\,e_1) \mapsto \mathsf{match}\,(e_1;\,\mathsf{left}\,(x) \to \mathsf{map}\,[X.\,T_1](x.e_2;\,x);\,\mathsf{right}\,(y) \to \mathsf{map}\,[X.\,T_2](x.e_2;\,y))$$

Dynamics

$$\frac{}{\mathsf{map}\,[X.\mathsf{Arrow}\,(T_1; T_2)](x.e_2; e_1) \mapsto \mathsf{lam}\,[T_1](y.\mathsf{map}\,[X.T_2](x.e_2; \mathsf{app}\,(e_1; y)))}\ \textsc{E\_MapArrow}$$

Examples: Products