# Function Types

# First-Order Functions: Statics

$$exp,\ e \quad ::=$$
$$| \quad x$$
$$| \quad \mathsf{num}[n]$$
$$| \quad \mathsf{str}[s]$$
$$| \quad \mathsf{plus}\,(e_1;e_2)$$
$$| \quad \mathsf{mult}\,(e_1;e_2)$$
$$| \quad \mathsf{cat}\,(e_1;e_2)$$
$$| \quad \mathsf{len}\,(e)$$
$$| \quad \mathsf{let}\,(e_1;x.e_2)$$
$$| \quad \mathsf{fun}\,[T_1;T_2](x.e_1;f.e_2)$$
$$| \quad \mathsf{call}\,[f](e)$$

$$\Gamma \quad ::=$$
$$| \quad \emptyset$$
$$| \quad x:T$$
$$| \quad f(T_1):T_2$$
$$| \quad \Gamma_1,\Gamma_2$$

# First-Order Functions: Statics

$$\text{fun}[T_1, T_2](x \cdot e_1; f \cdot e_2)$$

```
1. fun f(x:T1): T2 {
2.   e1
3. }
4.
5. e2
```

# First-Order Functions: Statics

$$\text{fun}[\text{Num}, \text{Num}](x \,.\, \text{plus}(x; x); \text{double} \,.\, \text{call}[\text{double}](\text{num}[2]))$$

```
1. fun double(x:Num): Num {
2.  plus(x,x)
3. }
4. double(2)
```

# First-Order Functions: Statics

$\text{fun[Num, Num]}(x \, . \, \text{plus}(x; x); \text{double} \, .$

$\qquad \text{fun[Str; Num]}(y \, . \, \text{call[double]}(\text{length}(y)); dl \, .$

$\qquad\qquad \text{plus}(\text{call}[dl](\text{str}[\textbf{"statics"}]); \text{num}[1]))$

```
1. fun double(x:Num): Num {
2.   plus(x,x)
3. }
4. fun dl(y:Str): Num {
5.    double(length(y))
6. }
4. plus(dl("statics");1)
```

# First-Order Functions: Statics

$$\Gamma \quad ::= \quad \begin{array}{l} \mid \quad \emptyset \\ \mid \quad x : T \\ \mid \quad f(T_1) : T_2 \\ \mid \quad \Gamma_1, \Gamma_2 \end{array}$$

$$\frac{\Gamma, x : T_1 \vdash e_1 : T_2 \quad \Gamma, f(T_1) : T_2 \vdash e_2 : T}{\Gamma \vdash \mathsf{fun}\,[T_1; T_2](x.e_1; f.e_2) : T} \;\; \text{FUN}$$

$$\boxed{\Gamma \vdash f(T_1) : T_2} \quad \text{Function Header}$$

$$\frac{}{\Gamma, f(T_1) : T_2 \vdash f(T_1) : T_2} \;\; \text{FUNH}$$

$$\frac{\Gamma \vdash f(T_1) : T_2 \quad \Gamma \vdash e : T_1}{\Gamma \vdash \mathsf{call}\,[f](e) : T_2} \;\; \text{CALL}$$

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# First-Order Functions: Dynamics

$$\frac{}{\mathsf{fun}\,[T_1;\,T_2](x.e_1;\,f.e_2) \mapsto [\![x.e_1/f]\!]e_2}\ \text{FUNVAL}$$

$$[\![x.e_1/f]\!]\mathsf{call}\,[f](e_2) \equiv \mathsf{let}\,([\![x.e_1/f]\!]e_2;\,x.e_1)$$

# Dynamics Example

$$\mathsf{fun}[\mathsf{Num}, \mathsf{Num}](x.\mathsf{plus}(x; x); \mathsf{double}.\mathsf{call}[\mathsf{double}](\mathsf{num}[2]))$$
$$\mapsto [\![x.\mathsf{plus}(x; x)/\mathsf{double}]\!](\mathsf{call}[\mathsf{double}](\mathsf{num}[2]))$$
$$\equiv \mathsf{let}(\mathsf{num}[2], x.\mathsf{plus}(x; x))$$
$$\mapsto \mathsf{plus}(\mathsf{num}[2]; \mathsf{num}[2])$$
$$\mapsto \mathsf{num}[4]$$

# First-Order Functions: Dynamics

$$\frac{}{\mathsf{fun}\,[T_1;\,T_2](x.e_1;f.e_2) \mapsto [\![x.e_1/f]\!]e_2}\;\;\text{FUNVAL}$$

$$[\![x.e_1/f]\!]\mathsf{call}\,[f](e_2) \equiv \mathsf{let}\,([\![x.e_1/f]\!]e_2;\,x.e_1)$$

# First-Order Functions

$$\frac{}{\Gamma, f(T_1) : T_2 \vdash f(T_1) : T_2} \text{FunH}$$

$$
\begin{array}{l}
\Gamma \quad\quad\quad ::= \\
\quad\quad\quad\quad | \quad \emptyset \\
\quad\quad\quad\quad | \quad x : T \\
\quad\quad\quad\quad | \quad f(T_1) : T_2 \\
\quad\quad\quad\quad | \quad \Gamma_1, \Gamma_2
\end{array}
$$

$$\frac{}{\Gamma, x : T \vdash x : T} \text{Var}$$

$$[\![ x.e_1 / f ]\!] e_2 \qquad [e_1 / x] e_2$$

# Higher-Order Functions