Towards a Formal Theory of Graded Monads
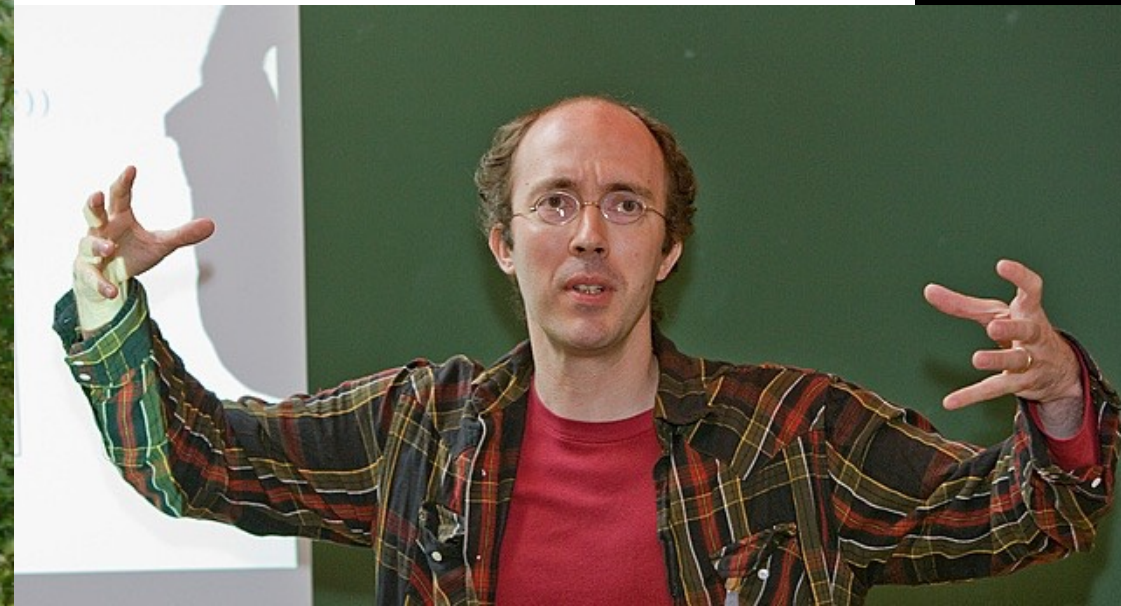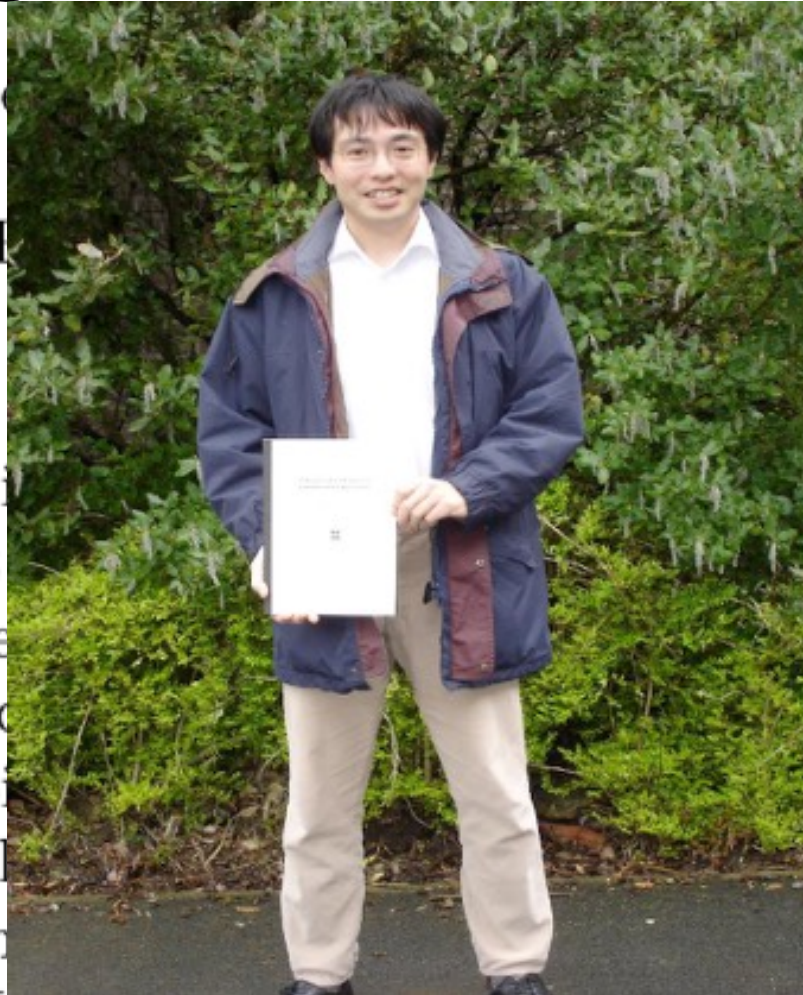
Soichiro Fujii , Shin-ya Katsumata[2], and Paul-André Melliès[3]

[1] Departm

[3] Laboratoire

Abstract. We
pose is to adapt
by Street in the e   articular that every graded
monad can be fac   a strict action transported
along a left adjo   n what sense the first con-
struction general   struction while the second
construction gen   ion. Finally, we illustrate
the Eilenberg-Moore construction on the graded state monad induced
by any object V in a symmetric monoidal closed category $\mathscr{C}$.

Typing for Graded Monads

Given: $(E, \top, \otimes, \leq)$

$$\frac{\Gamma \vdash t : B}{\Gamma \vdash \langle t \rangle : M_\top B} \; \eta$$

$$\frac{\Gamma_1 \leq \Gamma_2 \qquad A \leq B \qquad \Gamma_1 \vdash t : A}{\Gamma_2 \vdash t : B} \; \leq$$

$$\frac{\Gamma_2 \vdash t_1 : M_{e_1} A \qquad \Gamma_1, x : A \vdash t_2 : M_{e_2} B}{\Gamma_1, \Gamma_2 \vdash \mathsf{let}\ \langle x \rangle = t_1\ \mathsf{in}\ t_2 : M_{e_1 \otimes e_2} B} \; \mu$$