

Introducing a New Project on The Combination of Substructural Logics and Dependent Type Theory

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Proof Assistants have been extremely useful for formalizing large scale resource dependent systems

CompCert

seL4

But, there is a limitation!

“...managing assumption contexts does not work for the substructural separation logic and therefore needs to be done manually.”

Gerwin Klein et al. “Mechanised separation algebra.”

I propose that we mix substructural logics with dependent types at the foundational level, rather than, as an add on.

Two Main Problems

- How to modularly support many different substructural logics?
- How to integrate the substructural logical framework with dependent types?

A Basic Substructural Logic

A magmoidal category with a unit has the following data:

- $\odot : \mathcal{M} \times \mathcal{M} \rightarrow \mathcal{M}$

- $I \in \text{Obj}(\mathcal{M})$

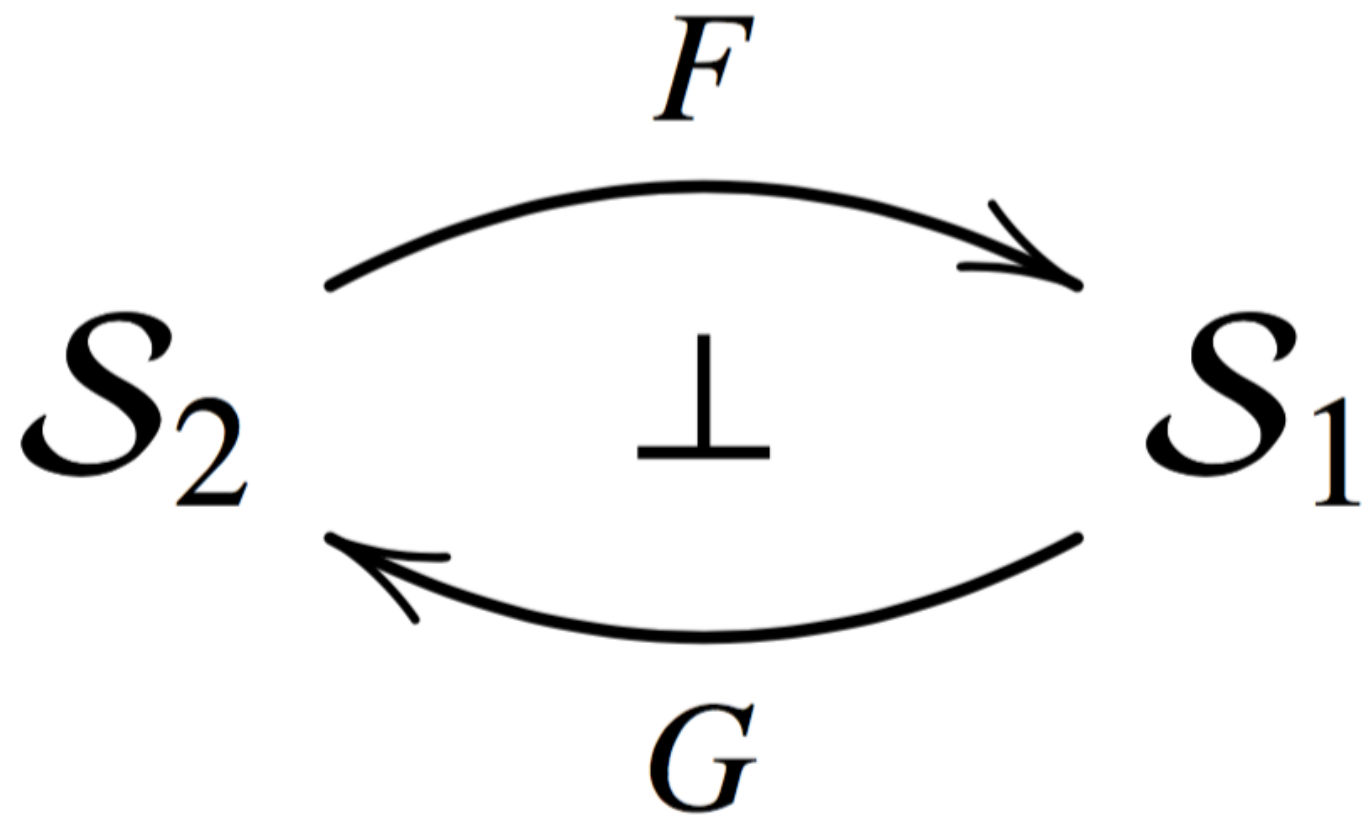
- $\lambda_A : A \odot I \rightarrow A$

- $\rho_A : I \odot A \rightarrow A$

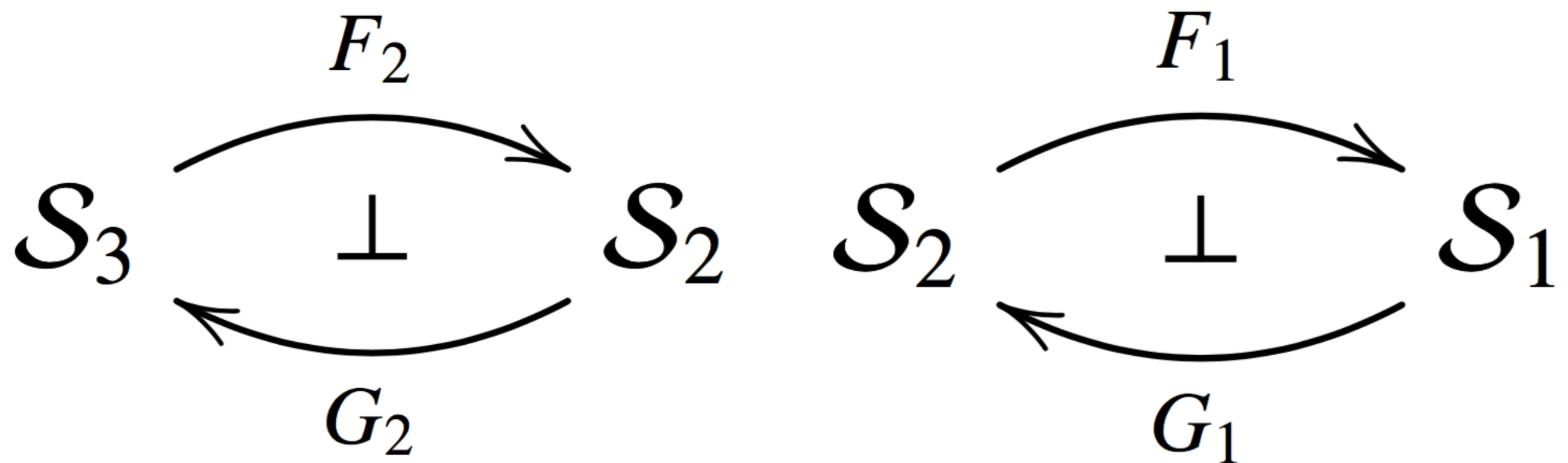
The Five Basic Substructural Logics

- \mathcal{M} : None
- \mathcal{A} : Associative
- \mathcal{E} : Commutative
- \mathcal{W} : Affine
- \mathcal{C} : Contractive

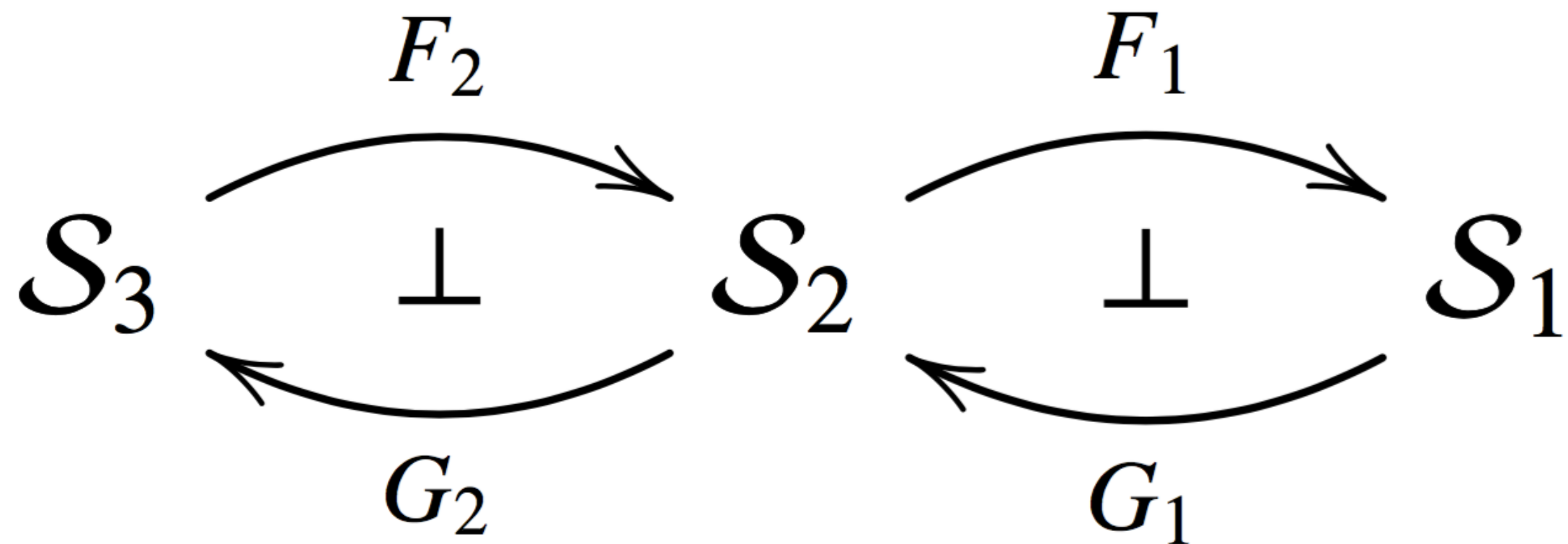
Adjoint Models



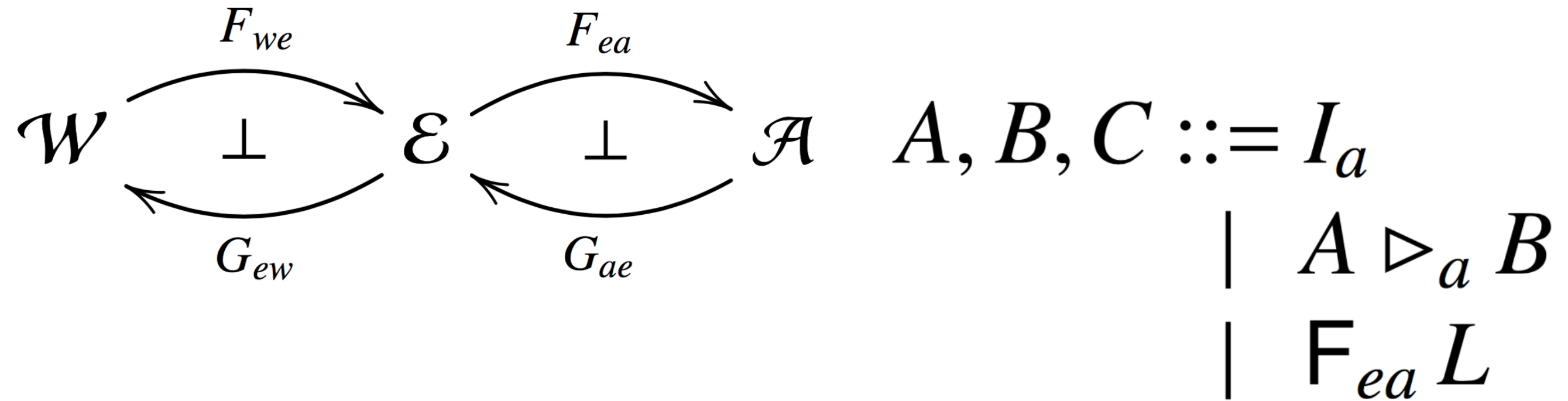
Composition of Substructural Logics



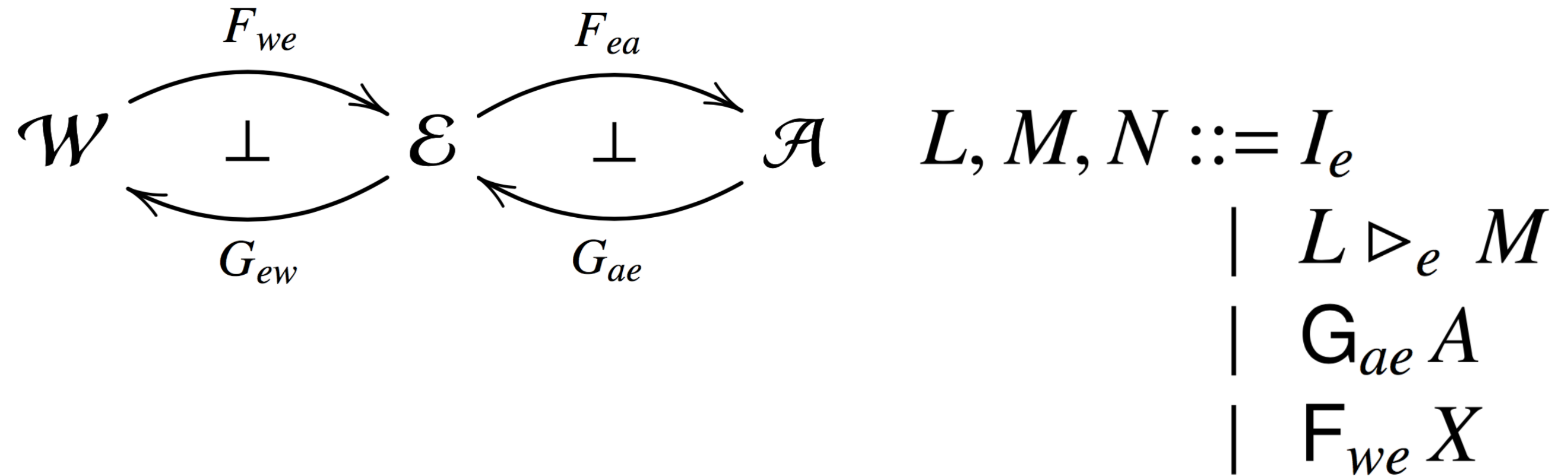
Composition of Substructural Logics



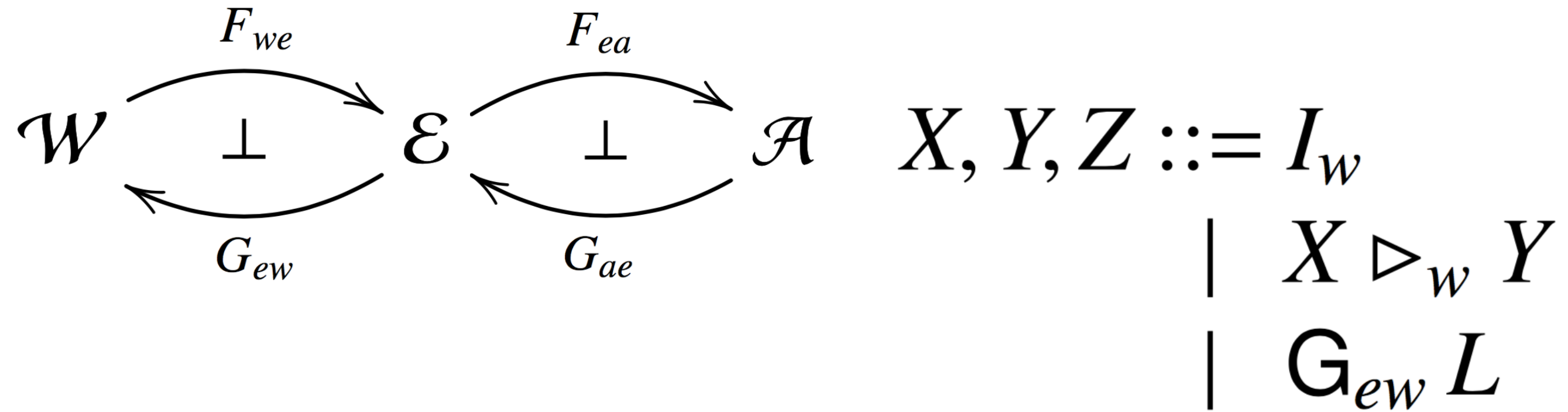
A Concrete Example



A Concrete Example



A Concrete Example



A Concrete Example

$$\boxed{\Phi \vdash_{\mathcal{W}} X}$$

$$\Phi ::= \cdot$$

$$| X$$

$$| \Phi_1; \Phi_2$$

$$\boxed{\Delta \vdash_{\mathcal{E}} L}$$

$$\Delta ::= \cdot$$

$$| L$$

$$| X$$

$$| \Delta_1; \Delta_2$$

$$\boxed{\Gamma \vdash_{\mathcal{A}} A}$$

$$\Gamma ::= \cdot$$

$$| A$$

$$| L$$

$$| X$$

$$| \Gamma_1; \Gamma_2$$

A Concrete Example

$$\frac{}{A \vdash_{\mathcal{A}} A} \text{id}$$

$$\frac{}{\cdot \vdash_{\mathcal{A}} I_a} I_i$$

A Concrete Example

$$\frac{\Gamma_1 \vdash_{\mathcal{A}} A \quad \Gamma_2 \vdash_{\mathcal{A}} B}{\Gamma_1; \Gamma_2 \vdash_{\mathcal{A}} A \triangleright_a B} T_i$$

$$\frac{\Gamma_2 \vdash_{\mathcal{A}} A \triangleright_a B \quad \Gamma_1; A; B; \Gamma_3 \vdash_{\mathcal{A}} C}{\Gamma_1; \Gamma_2; \Gamma_3 \vdash_{\mathcal{A}} C} T_e$$

A Concrete Example

$$\frac{\Delta \vdash_{\mathcal{E}} L}{\Delta \vdash_{\mathcal{A}} \mathbf{F}_{ea} L} F_i \quad \frac{\Gamma_2 \vdash_{\mathcal{A}} \mathbf{F}_{ea} L \quad \Gamma_1; L; \Gamma_3 \vdash_{\mathcal{A}} A}{\Gamma_1; \Gamma_2; \Gamma_3 \vdash_{\mathcal{A}} A} F_e$$

$$\frac{\Delta \vdash_{\mathcal{E}} \mathbf{G}_{ae} A}{\Delta \vdash_{\mathcal{A}} A} G_e$$

A Concrete Example

$$\frac{\Delta_1; L; M; \Delta_2 \vdash_{\varepsilon} N}{\Delta_1; M; L; \Delta_2 \vdash_{\varepsilon} N} E$$

A Concrete Example

$$\frac{\Phi \vdash_I X}{\Phi \vdash_{\mathcal{E}} \mathbf{F}_{we} X} F_i \quad \frac{\Delta_2 \vdash_{\mathcal{E}} \mathbf{F}_{we} X \quad \Delta_1; X; \Delta_3 \vdash_{\mathcal{E}} L}{\Delta_1; \Delta_2; \Delta_3 \vdash_{\mathcal{E}} L} F_e$$

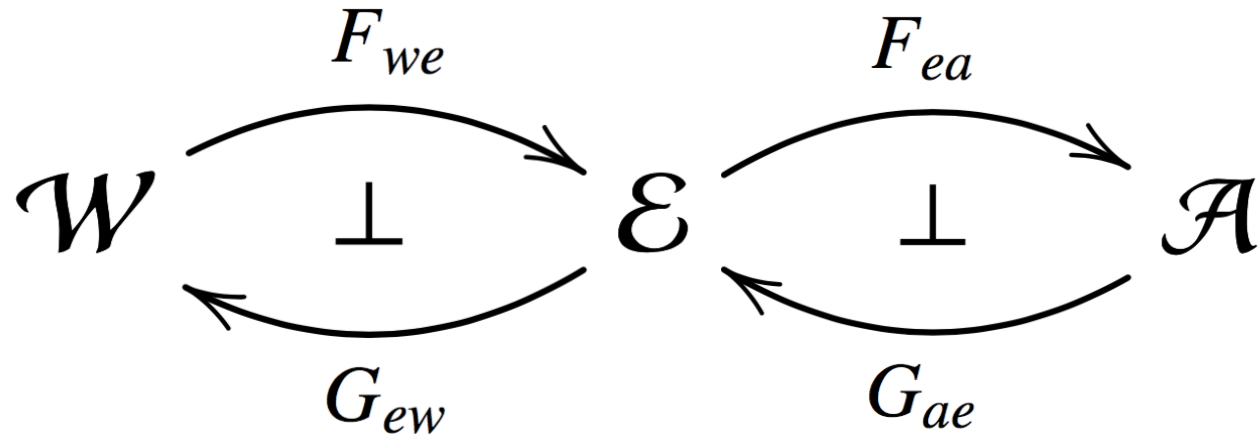
$$\frac{\Phi \vdash_I \mathbf{G}_{ew} L}{\Phi \vdash_{\mathcal{E}} L} G_e \quad \frac{\Delta \vdash_{\mathcal{A}} A}{\Delta \vdash_{\mathcal{E}} \mathbf{G}_{ae} A} G_i$$

A Concrete Example

$$\frac{\Phi_1; \Phi_2 \vdash_{\mathcal{I}} Y}{\Phi_1; X; \Phi_2 \vdash_{\mathcal{I}} Y} W$$

$$\frac{\Phi \vdash_{\mathcal{E}} L}{\Phi \vdash_{\mathcal{I}} G_{ew} L} G_i$$

A Concrete Example



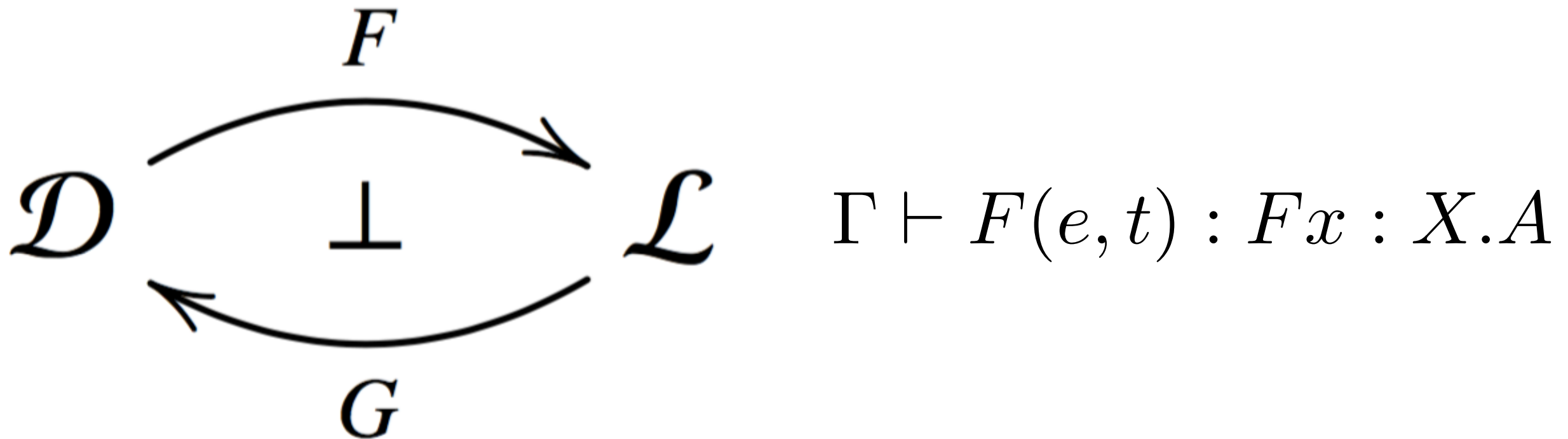
$$\text{AF } A \vdash_{\mathcal{A}} I_a$$

$$\text{WL} \vdash_{\mathcal{E}} I_e$$

$$(\text{AF } A) \triangleright_a (\text{AF } B) \vdash_{\mathcal{A}} (\text{AF } B) \triangleright_a (\text{AF } A)$$

$$(\text{LIN } A) \triangleright_a (\text{LIN } B) \vdash_{\mathcal{A}} (\text{LIN } B) \triangleright_a (\text{LIN } A)$$

Dependent Types



See: Krishnaswami, Pradic, and Benton's, "Integrating Linear and Dependent Types", POPL'15

The Tenli Proof Assistant

