Midterm Exam: Rise out of the Fire! (45 pt)
Theory of Computation (CSCI 3500), Spring 2021

Name: ____________________

Final Score: ____________________

Midterm Grade: ____________________

1 Multiple Choice (15 pt)

The following are several multiple choice questions. Please circle the correct answers. Note: There may be more than one possible answer for each question, but there is always at least one answer.

0. (5 pt): Which of the following is the correct type (signature) of the NFA transition function:
   a) $(Q \times (\Sigma \cup \{\varepsilon\})) \to Q$
   b) $(Q \times (\Sigma \cup \{\varepsilon\})) \to \mathcal{P}(Q)$
   c) $(Q \times \Sigma) \to Q$
   d) $(Q \times (\Sigma^* \cup \{\varepsilon\})) \to \mathcal{P}(Q)$

1. (5 pt): A language is regular if and only if
   a) it is the language of a DFA.
   b) it is the language of a NFA.
   d) all of the above.

2. (5 pt): Instead of this exam, I would rather be doing:

   ____

2 Regular Languages (30 pt)

The following are several long answer questions. Please write legibly, and clearly mark your solution.

0. (10 pt): Prove that the following language is regular by constructing a DFA that accepts it:

$$L = \{w \in \{!, \square\}^* \mid w = vw'v \text{ where } v, w' \in \{!, \square\}^*, |w'| = 2, \text{ and } |v| = 3\}$$

1. (10 pt): Suppose we have two industrial control systems S_1 and S_2 that are working concurrently. We need to forbid some sequence of actions from taking place between the two systems. One such action is allowing S_1 and S_2 to be on at the same time. Consider the alphabet:

$$\Sigma = \{\mathsf{On}(S_1), \mathsf{On}(S_2), \mathsf{Off}(S_1), \mathsf{Off}(S_2)\}$$

A sequence of actions in a word over Σ^*.

   (a) Convert the property on sequences given above into a language, and then define an NFA that accepts it.

   (b) A second property is that S_2 must power off after S_1, but S_2 must power on before S_1. Define a language which captures this property, and then define a NFA that accepts it.

2. (10 pt) Suppose the language, L, over an alphabet Σ_0 is regular, and $f : \Sigma_0 \to \Sigma_1$, is a function from the alphabet Σ_0 to the alphabet Σ_1.

The function f can be lifted to words producing the function:

$$\begin{aligned}
&\hat{f} : \Sigma_0^* \to \Sigma_1^* \\
&\hat{f}(\varepsilon) = \varepsilon \\
&\hat{f}(aw) = f(a)\hat{f}(w)
\end{aligned}$$

Show that the language:

$$L_f = \{\hat{f}(w) \in \Sigma_1^* \mid w \in \Sigma_0^*\}$$

is regular.

Hint: You have to use the formal definition of DFAs.