

<i>typevar</i> , X, Y, Z		
<i>termvar</i> , <i>name</i> , <i>double</i> , $f, g, x, y, z, rf, p, r, n, xs$		
<i>indecies</i> , i, j, k		
<i>pattern</i> , p	$::=$ <code>left</code> (x) <code>right</code> (x) <code>prod</code> ($x; y$)	Patterns
<i>patterns</i>	$::=$ $p_1 \rightarrow e_1; \dots; p_i \rightarrow e_i$	
<i>tyop</i>	$::=$ $X.T$	Type Operator
<i>exp</i> , e	$::=$ x <code>map</code> [<i>tyop</i>]($x.e_1; e_2$) <code>match</code> ($e_1; patterns$) <code>rec</code> [T]($r.e$) <code>let</code> ($e_1; x.e_2$) <code>lam</code> [T]($x.e$) <code>app</code> ($e_1; e_2$) <code>triv</code> <code>prod</code> ($e_1; e_2$) <code>left</code> (e) <code>right</code> (e) <code>abort</code> (e)	Expressions
<i>type</i> , T	$::=$ X <code>Void</code> <code>Unit</code> <code>Sum</code> ($T_1; T_2$) <code>Prod</code> ($T_1; T_2$) <code>Arrow</code> ($T_1; T_2$)	Types
Δ	$::=$ empty X type Δ_1, Δ_2	Kinding Contexts
Γ	$::=$ \emptyset $x : T$ Γ_1, Γ_2	Typing Contexts
<i>char</i> , c	$::=$	Character
<i>strings</i> , s	$::=$	
$\boxed{\Delta \vdash T \text{ type}}$		Kinding

$$\begin{array}{c}
\frac{}{X \text{ type} \vdash X \text{ type}} \quad \text{K_VAR} \\
\frac{}{X \text{ type} \vdash \text{Void type}} \quad \text{K_VOID} \\
\frac{}{X \text{ type} \vdash \text{Unit type}} \quad \text{K_UNIT} \\
\frac{X \text{ type} \vdash T_1 \text{ type} \quad X \text{ type} \vdash T_2 \text{ type}}{X \text{ type} \vdash \text{Sum}(T_1; T_2) \text{ type}} \quad \text{K_SUM} \\
\frac{X \text{ type} \vdash T_1 \text{ type} \quad X \text{ type} \vdash T_2 \text{ type}}{X \text{ type} \vdash \text{Prod}(T_1; T_2) \text{ type}} \quad \text{K_PROD} \\
\frac{X \notin \text{FV}(T_1) \quad X \text{ type} \vdash T_2 \text{ type}}{X \text{ type} \vdash \text{Arrow}(T_1; T_2) \text{ type}} \quad \text{K_ARROW}
\end{array}$$

$\boxed{\Gamma \vdash e : T}$ Typing

$$\begin{array}{c}
\frac{}{\Gamma, x : T \vdash x : T} \quad \text{VAR} \\
\frac{}{\Gamma \vdash \text{triv} : \text{Unit}} \quad \text{TRIV} \\
\frac{\Gamma \vdash e_1 : T_1 \quad \Gamma \vdash e_2 : T_2}{\Gamma \vdash \text{prod}(e_1; e_2) : \text{Prod}(T_1; T_2)} \quad \text{PROD} \\
\frac{\Gamma \vdash e : \text{Prod}(T_1; T_2) \quad \Gamma, x : T_1, y : T_2 \vdash e_2 : T}{\Gamma \vdash \text{match}(e; \text{prod}(x; y) \rightarrow e_2) : T} \quad \text{MATCHPROD} \\
\frac{\Gamma, r : T \vdash e : T}{\Gamma \vdash \text{rec}[T](r.e) : T} \quad \text{REC} \\
\frac{\Gamma \vdash e_1 : T_1 \quad \Gamma, x : T_1 \vdash e_2 : T_2}{\Gamma \vdash \text{let}(e_1; x.e_2) : T_2} \quad \text{LET} \\
\frac{\Gamma, x : T_1 \vdash e : T_2}{\Gamma \vdash \text{lam}[T_1](x.e) : \text{Arrow}(T_1; T_2)} \quad \text{LAM} \\
\frac{\Gamma \vdash e_1 : \text{Arrow}(T_1; T_2) \quad \Gamma \vdash e_2 : T_1}{\Gamma \vdash \text{app}(e_1; e_2) : T_2} \quad \text{APP} \\
\frac{\Gamma \vdash e : T_1}{\Gamma \vdash \text{left}(e) : \text{Sum}(T_1; T_2)} \quad \text{INJECTLEFT} \\
\frac{\Gamma \vdash e : T_2}{\Gamma \vdash \text{right}(e) : \text{Sum}(T_1; T_2)} \quad \text{INJECTRIGHT} \\
\frac{\Gamma \vdash e : \text{Sum}(T_1; T_2) \quad \Gamma, x : T_1 \vdash e_1 : T \quad \Gamma, y : T_2 \vdash e_2 : T}{\Gamma \vdash \text{match}(e; \text{left}(y) \rightarrow e_1; \text{right}(y) \rightarrow e_2) : T} \quad \text{MATCHSUM} \\
\frac{\Gamma \vdash e : \text{Void}}{\Gamma \vdash \text{abort}(e) : T} \quad \text{ABORT} \\
\frac{X \text{ type} \vdash T \text{ type} \quad \Gamma, x : T_1 \vdash e_2 : T_2 \quad \Gamma \vdash e_1 : [T_1/X]T}{\Gamma \vdash \text{map}[X.T](x.e_2; e_1) : [T_2/X]T} \quad \text{MAP} \\
\frac{}{\Gamma \vdash \text{True} : \text{Bool}} \quad \text{TRUE} \\
\frac{}{\Gamma \vdash \text{False} : \text{Bool}} \quad \text{FALSE}
\end{array}$$

$$\begin{array}{c}
\frac{\Gamma \vdash e : \text{Bool} \quad \Gamma \vdash e_1 : T \quad \Gamma \vdash e_2 : T}{\Gamma \vdash \text{if } (e; e_1; e_2) : T} \quad \text{IF} \\
\\
\frac{}{\Gamma \vdash \text{nothing} : \text{Maybe}(T)} \quad \text{NOTHING} \\
\\
\frac{\Gamma \vdash e : T}{\Gamma \vdash \text{just}(e) : \text{Maybe}(T)} \quad \text{JUST} \\
\\
\frac{\Gamma \vdash e : \text{Maybe}(T) \quad \Gamma \vdash e_1 : T' \quad \Gamma, x : T \vdash e_2 : T'}{\Gamma \vdash \text{ifNothing}[T](e; e_1; x.e_2) : T'} \quad \text{IFN}
\end{array}$$

$e \text{ val}$ Values

$$\begin{array}{c}
\frac{}{\text{triv val}} \quad \text{V_TRIV} \\
\\
\frac{}{\text{lam}[T](x.e) \text{ val}} \quad \text{V_LAM} \\
\\
\frac{e_1 \text{ val} \quad e_2 \text{ val}}{\text{prod}(e_1; e_2) \text{ val}} \quad \text{V_PROD} \\
\\
\frac{e \text{ val}}{\text{left}(e) \text{ val}} \quad \text{V_LEFT} \\
\\
\frac{e \text{ val}}{\text{right}(e) \text{ val}} \quad \text{V_RIGHT}
\end{array}$$

$e_1 \mapsto e_2$ Evaluation

$$\begin{array}{c}
\frac{}{\text{map}[X.X](x.e_2; e_1) \mapsto [e_1/x]e_2} \quad \text{E_MAPID} \\
\\
\frac{}{\text{map}[X.\text{Unit}](x.e_2; e_1) \mapsto \text{triv}} \quad \text{E_MAPUNIT} \\
\\
\frac{}{\text{map}[X.\text{Void}](x.e_2; e_1) \mapsto \text{abort}(e_1)} \quad \text{E_MAPVOID} \\
\\
\frac{}{\text{map}[X.\text{Prod}(T_1; T_2)](x.e_2; e_1) \mapsto \text{prod}(\text{map}[X.T_1](x.e_1; \text{fst}(e_1)); \text{map}[X.T_2](x.e_1; \text{snd}(e_1)))} \quad \text{E_MAPPROD} \\
\\
\frac{}{\text{map}[X.\text{Sum}(T_1; T_2)](x.e_2; e_1) \mapsto \text{match}(e_1; \text{left}(x) \rightarrow \text{map}[X.T_1](x.e_2; x); \text{right}(y) \rightarrow \text{map}[X.T_2](x.e_2; y))} \quad \text{E_MATCH} \\
\\
\frac{}{\text{map}[X.\text{Arrow}(T_1; T_2)](x.e_2; e_1) \mapsto \text{lam}[T_1](y.\text{map}[X.T_2](x.e_2; \text{app}(e_1; y)))} \quad \text{E_MAPARROW} \\
\\
\frac{e \mapsto e'}{\text{abort}(e) \mapsto \text{abort}(e')} \quad \text{E_ABORT} \\
\\
\frac{e \mapsto e'}{\text{left}(e) \mapsto \text{left}(e')} \quad \text{E_LEFT} \\
\\
\frac{e \mapsto e'}{\text{right}(e) \mapsto \text{right}(e')} \quad \text{E_RIGHT} \\
\\
\frac{}{\text{match}(\text{left}(e); \text{left}(x) \rightarrow e_1; \text{right}(y) \rightarrow e_2) \mapsto [e/x]e_1} \quad \text{E_MATCHLEFT} \\
\\
\frac{}{\text{match}(\text{right}(e); \text{left}(x) \rightarrow e_1; \text{right}(y) \rightarrow e_2) \mapsto [e/y]e_2} \quad \text{E_MATCHRIGHT} \\
\\
\frac{e_1 \mapsto e'_1}{\text{prod}(e_1; e_2) \mapsto \text{prod}(e'_1; e_2)} \quad \text{E_PROD1}
\end{array}$$

$$\begin{array}{c}
\frac{e_1 \text{ val} \quad e_2 \mapsto e'_2}{\text{prod}(e_1; e_2) \mapsto \text{prod}(e_1; e'_2)} \quad \text{E_PROD2} \\
\\
\frac{e_1 \text{ val} \quad e_2 \text{ val}}{\text{match}(\text{prod}(e_1; e_2); \text{prod}(x; y) \rightarrow e_3) \mapsto [e_1/x][e_2/y]e_3} \quad \text{E_MATCHP} \\
\\
\frac{e \mapsto e'}{\text{match}(e; \text{patterns}) \mapsto \text{match}(e'; \text{patterns})} \quad \text{E_MATCH} \\
\\
\frac{}{\text{rec}[T](r.e) \mapsto [\text{rec}[T](r.e)/r]e} \quad \text{E_REC} \\
\\
\frac{e_1 \text{ val}}{\text{let}(e_1; x.e_2) \mapsto [e_1/x]e_2} \quad \text{E_LETVAL} \\
\\
\frac{e_1 \mapsto e'_1}{\text{let}(e_1; x.e_2) \mapsto \text{let}(e'_1; x.e_2)} \quad \text{E_LET1} \\
\\
\frac{}{\text{let}(e_1; x.e_2) \mapsto [e_1/x]e_2} \quad \text{E_LETL} \\
\\
\frac{e_2 \text{ val}}{\text{app}(\text{lam}[T](x.e_2); e_1) \mapsto [e_1/x]e_2} \quad \text{E_APPVAL} \\
\\
\frac{e_1 \mapsto e'_1}{\text{app}(e_1; e_2) \mapsto \text{app}(e'_1; e_2)} \quad \text{E_APP1} \\
\\
\frac{e_1 \text{ val} \quad e_2 \mapsto e'_2}{\text{app}(e_1; e_2) \mapsto \text{app}(e_1; e'_2)} \quad \text{E_APP2}
\end{array}$$

$e_1 \mapsto^* e_2$

Multistep Evaluation

$$\begin{array}{c}
\frac{}{e \mapsto^* e} \quad \text{REFL} \\
\\
\frac{e_1 \mapsto e_2 \quad e_2 \mapsto^* e_3}{e_1 \mapsto^* e_3} \quad \text{STEP}
\end{array}$$

Definition rules: 54 good 0 bad
Definition rule clauses: 87 good 0 bad