(Simplified) Bounded Linear Logic

$$\frac{\Gamma_1, \Gamma_2 \vdash B}{\Gamma_1, !_0 A, \Gamma_2 \vdash B}\, W \qquad \frac{\Gamma_1, !_{p_1} A, !_{p_2} A, \Gamma_2 \vdash B}{\Gamma_1, !_{p_1+p_2} A, \Gamma_2 \vdash B}\, C$$

The precursor to graded comonads.

$$\frac{!_{\overrightarrow{p}}\Gamma \vdash B}{!_{p*\overrightarrow{p}}\Gamma \vdash !_p B}\, P \qquad \frac{\Gamma, A \vdash B}{\Gamma, !_1 A \vdash B}\, D$$

Bounded Linear Logic in a Semiring

- **Data-usage annotations are from a semiring**
- **<u>Externally graded</u>: no modality, all hypothesis are give a grade**