Core Design Concepts Discussed:

Conceptual Models

Analysis

Evaluation

Recursive Functions and their Evaluation

Harley Eades III

Core Design Concepts:

Recursive Functions in OCaml

Syntax

```
# let rec f x = e;;
```

Core Design Concepts:

Syntax

Recursive Functions in OCaml

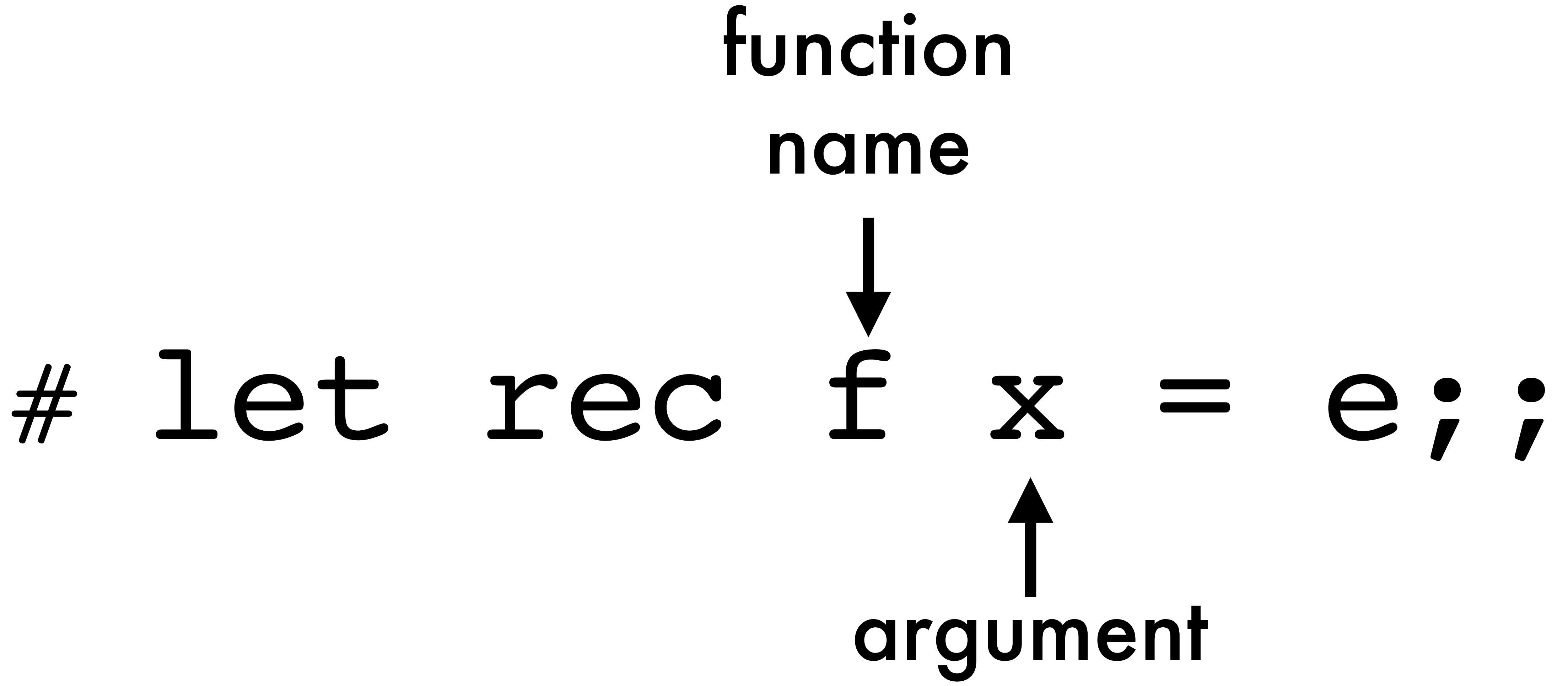
function
name

```
# let rec f x = e;;
```

Core Design Concepts:

Recursive Functions in OCaml

Syntax



function name

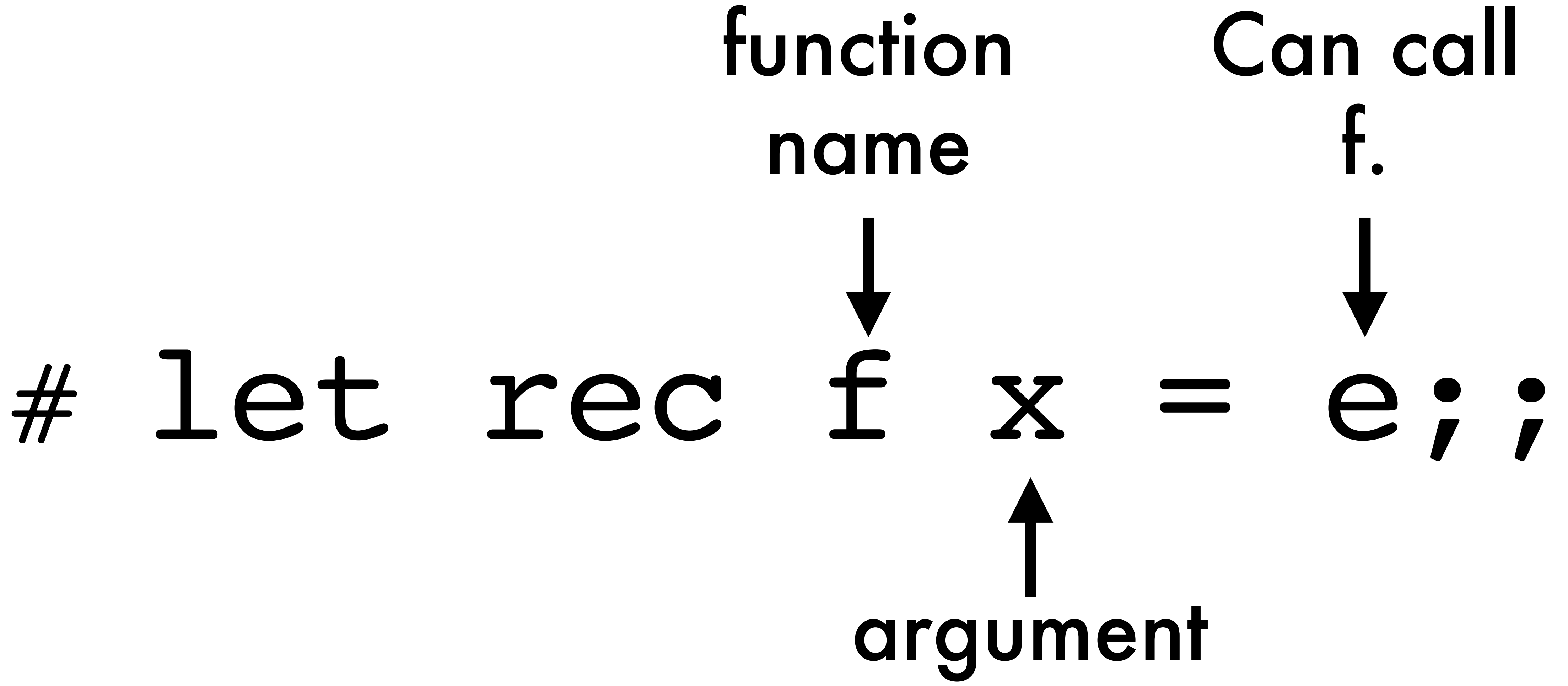
```
# let rec f x = e;;
```

argument

Core Design Concepts:

Syntax

Recursive Functions in OCaml



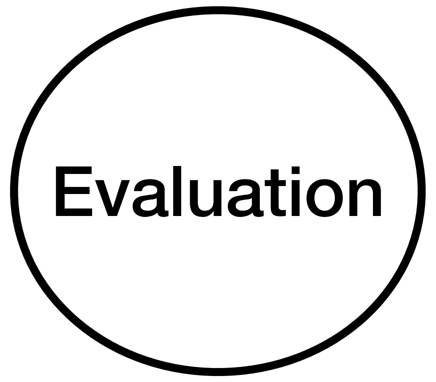
function name

Can call f.

```
# let rec f x = e;;
```

argument

Core Design Concepts:



But, what about performance?

Core Design Concepts:

Evaluation

The Function Call Stack

Activation Record:

The location in memory where an executing function stores its bindings.

Activation records are sometimes referred to as frames.

Consider evaluating the following function:

```
let cube n =
  let c = n*n*n in
      c;;
let main =
  let n = 5 in
  let ans = cube n in
      ans;;
main;;
```

Core Design Concepts:

Analysis

Evaluation

The Function Call Stack

Activation Record:

The location in memory where an executing function stores its binding.

Activation records are sometimes referred to as frames.

Activation Record: program initialization

Frame	Symbol	Value
init line: 8	`cube` `main`	`<fun>` `<fun>`

Consider evaluating the following function:

```
1: let cube n =
2:   let c = n*n*n in
3:       c;;
4: let main =
5:   let n = 5 in
6:   let ans = cube n in
7:       ans;;
8: main;;
```

Core Design Concepts:

Analysis

Evaluation

The Function Call Stack

Activation Record:

The location in memory where an executing function stores its binding.

Activation records are sometimes referred to as frames.

Consider evaluating the following function:

```
1: let cube n =
2:   let c = n*n*n in
3:       c;;
4: let main =
5:   let n = 5 in
6:   let ans = cube n in
7:       ans;;
8: main;;
```

Activation Record: program initialization

Frame	Symbol	Value
init line: 8	cube main	<fun> <fun>

Activation Record: after calling main

Frame	Symbol	Value
init line: 8	cube main	<fun> <fun>
main line: 6	n ans	5 ?

Core Design Concepts:

Analysis

Evaluation

The Function Call Stack

Activation Record:

The location in memory where an executing function stores its binding.

Activation records are sometimes referred to as frames.

Consider evaluating the following function:

```
1: let cube n =
2:   let c = n*n*n in
3:       c;;
4: let main =
5:   let n = 5 in
6:   let ans = cube n in
7:       ans;;
8: main;;
```

Activation Record: after calling `main`

Frame	Symbol	Value
init line: 8	cube main	<fun> <fun>
main line: 6	n ans	5 ?
cube line: 2	n c	5 125

Core Design Concepts:

Analysis

Evaluation

The Function Call Stack

Activation Record:

The location in memory where an executing function stores its binding.

Activation records are sometimes referred to as frames.

Consider evaluating the following function:

```
1: let cube n =
2:   let c = n*n*n in
3:       c;;
4: let main =
5:   let n = 5 in
6:   let ans = cube n in
7:       ans;;
8: main;;
```

Activation Record: after calling `main`

Frame	Symbol	Value
init line: 8	`cube` `main`	`<fun>` `<fun>`

`main`
line: 6
	`n`
	`ans`
		`5`
		`125`

`cube`
line: 2
	`n`
	`c`
		`5`
		`125`

Evaluating Recursive Functions

Core Design Concepts:

Evaluation

Consider evaluating the following recursive function:

```
let rec ackermann m n =
 if m == 0
 then let ret = n + 1 in ret
 else if n == 0
      then let ack  = ackermann (m - 1) 1 in ack
      else let ack1 = ackermann m (m - 1) in
           let ack2 = ackermann (m - 1) ack1 in
             ack2

let main =
  let m = 1 in
   let n = 0 in
     let ans1 = ackermann m n in
       let ans2 = ackermann m m in
         ans1 + ans2

main;;
```

Evaluating Recursive Functions

Core Design Concepts:

Evaluation

Consider evaluating the following recursive function:

```
1. let rec ackermann m n =
2.  if m == 0
3.  then let ret = n + 1 in ret
4.  else if n == 0
5.       then let ack  = ackermann (m - 1) 1 in ack
6.       else let ack1 = ackermann m (m - 1) in
7.            let ack2 = ackermann (m - 1) ack1 in
8.              ack2
9.
10. let main =
11.   let m = 1 in
12.    let n = 0 in
13.      let ans1 = ackermann m n in
14.        let ans2 = ackermann m m in
15.          ans1 + ans2
16.
17. main;;
```

Frame	Symbol	Value
init line: 17	ackermann main	<fun> <fun>

Evaluating Recursive Functions

Core Design Concepts:

Evaluation

Consider evaluating the following recursive function:

```
1. let rec ackermann m n =
2.  if m == 0
3.  then let ret = n + 1 in ret
4.  else if n == 0
5.       then let ack  = ackermann (m - 1) 1 in ack
6.       else let ack1 = ackermann m (m - 1) in
7.            let ack2 = ackermann (m - 1) ack1 in
8.              ack2
9.
10. let main =
11.   let m = 1 in
12.    let n = 0 in
13.      let ans1 = ackermann m n in
14.        let ans2 = ackermann m m in
15.          ans1 + ans2
16.
17. main;;
```

Frame	Symbol	Value
init line: 17	ackermann main	<fun> <fun>
main line: 14	m n ans1 ans2	1 0 ? ?

Evaluating Recursive Functions

Core Design Concepts:

Evaluation

Consider evaluating the following recursive function:

```
1. let rec ackermann m n =
2.  if m == 0
3.  then let ret = n + 1 in ret
4.  else if n == 0
5.       then let ack  = ackermann (m - 1) 1 in ack
6.       else let ack1 = ackermann m (m - 1) in
7.            let ack2 = ackermann (m - 1) ack1 in
8.              ack2
9.
10. let main =
11.   let m = 1 in
12.    let n = 0 in
13.      let ans1 = ackermann m n in
14.        let ans2 = ackermann m m in
15.          ans1 + ans2
16.
17. main;;
```

Frame	Symbol	Value
init line: 17	ackermann main	<fun> <fun>
main line: 14	m n ans1 ans2	1 0 ? ?
ackermann line: 5	m n ack	1 0 ?

Evaluating Recursive Functions

Core Design Concepts:

Evaluation

Consider evaluating the following recursive function:

```
1. let rec ackermann m n =
2.  if m == 0
3.  then let ret = n + 1 in ret
4.  else if n == 0
5.       then let ack  = ackermann (m - 1) 1 in ack
6.       else let ack1 = ackermann m (m - 1) in
7.            let ack2 = ackermann (m - 1) ack1 in
8.              ack2
9.
10. let main =
11.   let m = 1 in
12.    let n = 0 in
13.      let ans1 = ackermann m n in
14.        let ans2 = ackermann m m in
15.          ans1 + ans2
16.
17. main;;
```

Frame	Symbol	Value
init line: 17	ackermann main	<fun> <fun>
main line: 13	m n ans1 ans2	1 0 ? ?
ackermann line: 5	m n ack	1 0 ?
ackermann line: 3	m n ret	0 1 2

Evaluating Recursive Functions

Core Design Concepts:

Evaluation

Consider evaluating the following recursive function:

```
1. let rec ackermann m n =
2.  if m == 0
3.  then let ret = n + 1 in ret
4.  else if n == 0
5.       then let ack  = ackermann (m - 1) 1 in ack
6.       else let ack1 = ackermann m (m - 1) in
7.            let ack2 = ackermann (m - 1) ack1 in
8.              ack2
9.
10. let main =
11.   let m = 1 in
12.    let n = 0 in
13.      let ans1 = ackermann m n in
14.        let ans2 = ackermann m m in
15.          ans1 + ans2
16.
17. main;;
```

Frame	Symbol	Value
init line: 17	ackermann main	<fun> <fun>
main line: 13	m n ans1 ans2	1 0 ? ?
ackermann line: 5	m n ack	1 0 2
ackermann line: 3	m n ret	0 1 2

Evaluating Recursive Functions

Core Design Concepts:

Evaluation

Consider evaluating the following recursive function:

```
1. let rec ackermann m n =
2.  if m == 0
3.  then let ret = n + 1 in ret
4.  else if n == 0
5.       then let ack  = ackermann (m - 1) 1 in ack
6.       else let ack1 = ackermann m (m - 1) in
7.            let ack2 = ackermann (m - 1) ack1 in
8.              ack2
9.
10. let main =
11.   let m = 1 in
12.    let n = 0 in
13.      let ans1 = ackermann m n in
14.        let ans2 = ackermann m m in
15.          ans1 + ans2
16.
17. main;;
```

Frame	Symbol	Value
init line: 17	ackermann main	<fun> <fun>
main line: 14	m n ans1 ans2	1 0 2 ?
ackermann line: 5	m n ack	1 0 2
ackermann line: 3	m n ret	0 1 2

Evaluating Recursive Functions

Core Design Concepts:

Evaluation

Consider evaluating the following recursive function:

```
1. let rec ackermann m n =
2.  if m == 0
3.  then let ret = n + 1 in ret
4.  else if n == 0
5.       then let ack  = ackermann (m - 1) 1 in ack
6.       else let ack1 = ackermann m (m - 1) in
7.            let ack2 = ackermann (m - 1) ack1 in
8.              ack2
9.
10. let main =
11.   let m = 1 in
12.    let n = 0 in
13.      let ans1 = ackermann m n in
14.        let ans2 = ackermann m m in
15.          ans1 + ans2
16.
17. main;;
```

Frame	Symbol	Value
main line: 14	m n ans1 ans2	1 0 2 ?
ackermann line: 7	m n ack1 ack2	1 1 ? ?

Core Design Concepts:

Evaluation

Evaluating Recursive Functions

Consider evaluating the following recursive function:

```
1. let rec ackermann m n =
2.  if m == 0
3.  then let ret = n + 1 in ret
4.  else if n == 0
5.       then let ack  = ackermann (m - 1) 1 in ack
6.       else let ack1 = ackermann m (m - 1) in
7.            let ack2 = ackermann (m - 1) ack1 in
8.              ack2
9.
10. let main =
11.   let m = 1 in
12.    let n = 0 in
13.      let ans1 = ackermann m n in
14.        let ans2 = ackermann m m in
15.          ans1 + ans2
16.
17. main;;
```

Frame	Symbol	Value
main line: 14	m n ans1 ans2	1 0 2 ?
ackermann line: 7	m n ack1 ack2	1 1 ? ?
ackermann line: 5	m n ack	1 0 ?

Evaluating Recursive Functions

Core Design Concepts:

Evaluation

Consider evaluating the following recursive function:

```
1. let rec ackermann m n =
2.  if m == 0
3.  then let ret = n + 1 in ret
4.  else if n == 0
5.       then let ack  = ackermann (m - 1) 1 in ack
6.       else let ack1 = ackermann m (m - 1) in
7.            let ack2 = ackermann (m - 1) ack1 in
8.              ack2
9.
10. let main =
11.   let m = 1 in
12.    let n = 0 in
13.      let ans1 = ackermann m n in
14.        let ans2 = ackermann m m in
15.          ans1 + ans2
16.
17. main;;
```

Frame	Symbol	Value
main line: 14	m n ans1 ans2	1 0 2 ?
ackermann line: 7	m n ack1 ack2	1 1 ? ?
ackermann line: 5	m n ack	1 0 ?
ackermann line: 3	m n ret	0 1 2

Core Design Concepts:

Evaluation

Evaluating Recursive Functions

Consider evaluating the following recursive function:

```
1. let rec ackermann m n =
2.  if m == 0
3.  then let ret = n + 1 in ret
4.  else if n == 0
5.       then let ack  = ackermann (m - 1) 1 in ack
6.       else let ack1 = ackermann m (m - 1) in
7.            let ack2 = ackermann (m - 1) ack1 in
8.              ack2
9.
10. let main =
11.   let m = 1 in
12.    let n = 0 in
13.      let ans1 = ackermann m n in
14.        let ans2 = ackermann m m in
15.          ans1 + ans2
16.
17. main;;
```

Frame	Symbol	Value
main line: 14	m n ans1 ans2	1 0 2 ?
ackermann line: 7	m n ack1 ack2	1 1 ? ?
ackermann line: 5	m n ack	1 0 2
ackermann line: 3	m n ret	0 1 2

Evaluating Recursive Functions

Core Design Concepts:

Evaluation

Consider evaluating the following recursive function:

```
1. let rec ackermann m n =
2.  if m == 0
3.  then let ret = n + 1 in ret
4.  else if n == 0
5.       then let ack  = ackermann (m - 1) 1 in ack
6.       else let ack1 = ackermann m (m - 1) in
7.            let ack2 = ackermann (m - 1) ack1 in
8.              ack2
9.
10. let main =
11.   let m = 1 in
12.    let n = 0 in
13.      let ans1 = ackermann m n in
14.        let ans2 = ackermann m m in
15.          ans1 + ans2
16.
17. main;;
```

Frame	Symbol	Value
main line: 14	m n ans1 ans2	1 0 2 ?
ackermann line: 7	m n ack1 ack2	1 1 2 ?
ackermann line: 5	m n ack	1 0 2
ackermann line: 3	m n ret	0 1 2

Evaluating Recursive Functions

Core Design Concepts:

Evaluation

Consider evaluating the following recursive function:

```
1. let rec ackermann m n =
2.  if m == 0
3.  then let ret = n + 1 in ret
4.  else if n == 0
5.       then let ack  = ackermann (m - 1) 1 in ack
6.       else let ack1 = ackermann m (m - 1) in
7.            let ack2 = ackermann (m - 1) ack1 in
8.              ack2
9.
10. let main =
11.   let m = 1 in
12.    let n = 0 in
13.      let ans1 = ackermann m n in
14.        let ans2 = ackermann m m in
15.          ans1 + ans2
16.
17. main;;
```

Frame	Symbol	Value
main line: 14	m n ans1 ans2	1 0 2 ?
ackermann line: 7	m n ack1 ack2	1 1 2 ?
ackermann line: 5	m n ack	1 0 2
ackermann line: 3	m n ret	0 1 2
ackermann line: 3	m n ret	0 2 3

Evaluating Recursive Functions

Core Design Concepts:

Evaluation

Consider evaluating the following recursive function:

```
1. let rec ackermann m n =
2.  if m == 0
3.  then let ret = n + 1 in ret
4.  else if n == 0
5.       then let ack  = ackermann (m - 1) 1 in ack
6.       else let ack1 = ackermann m (m - 1) in
7.            let ack2 = ackermann (m - 1) ack1 in
8.              ack2
9.
10. let main =
11.   let m = 1 in
12.    let n = 0 in
13.      let ans1 = ackermann m n in
14.        let ans2 = ackermann m m in
15.          ans1 + ans2
16.
17. main;;
```

Frame	Symbol	Value
main line: 14	m n ans1 ans2	1 0 2 ?
ackermann line: 7	m n ack1 ack2	1 1 2 3
ackermann line: 5	m n ack	1 0 2
ackermann line: 3	m n ret	0 1 2
ackermann line: 3	m n ret	0 2 3

Core Design Concepts:

Evaluation

Evaluating Recursive Functions

Consider evaluating the following recursive function:

```
1. let rec ackermann m n =
2.  if m == 0
3.  then let ret = n + 1 in ret
4.  else if n == 0
5.       then let ack  = ackermann (m - 1) 1 in ack
6.       else let ack1 = ackermann m (m - 1) in
7.            let ack2 = ackermann (m - 1) ack1 in
8.              ack2
9.
10. let main =
11.   let m = 1 in
12.    let n = 0 in
13.      let ans1 = ackermann m n in
14.        let ans2 = ackermann m m in
15.          ans1 + ans2
16.
17. main;;
```

Frame	Symbol	Value
main line: 14	m n ans1 ans2	1 0 2 3
ackermann line: 7	m n ack1 ack2	1 1 2 3
ackermann line: 5	m n ack	1 0 2
ackermann line: 3	m n ret	0 1 2
ackermann line: 3	m n ret	0 2 3