Comonadic Matter Meets Monadic Anti-Matter: An Adjoint Model of Bi-Intuitionistic Logic

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| — Abstract — |
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| 1 Introduction |
| TODO [?] |
| References |

$$\frac{G, (w, w); \Theta \vdash_{\vdash} w : Y}{G; \Theta \vdash_{\vdash} w : Y} \quad I_{_RL} \qquad \frac{w_1 G w_2 \quad w_2 G w_3}{G, (w_1, w_3); \Theta \vdash_{\vdash} w : Y}}{G; \Theta \vdash_{\vdash} w : Y} \quad I_{_TS}$$

$$\frac{G; \Theta \vdash_{\vdash} w : Y}{G; \Theta, w : X \vdash_{\vdash} w : Y} \quad I_{_DD} \qquad \frac{G; \Theta \vdash_{\vdash} w : Y}{G; \Theta, w : X \vdash_{\vdash} w : Y} \quad I_{_WK}$$

$$\frac{G; \Theta, w : X, w : X \vdash_{\vdash} w : Y}{G; \Theta, w : X \vdash_{\vdash} w : Y} \quad I_{_CR} \qquad \frac{w_1 G w_2}{G; \Theta, w_1 : X, w_2 : X \vdash_{\vdash} w : Y} \quad I_{_ML}$$

$$\frac{W_2 G w_1}{G; \Theta \vdash_{\vdash} w_2 : Y} \quad I_{_MR} \qquad \frac{G; \Theta \vdash_{\vdash} w : Y}{G; \Theta, w : T \vdash_{\vdash} w : Y} \quad I_{_TL} \qquad \frac{G; \Theta \vdash_{\vdash} w : Y}{G; \Theta \vdash_{\vdash} w : Y} \quad I_{_TR}$$

$$\frac{G; \Theta, w_1 : X, w_1 : Y \vdash_{\vdash} w_2 : Z}{G; \Theta, w_1 : X \times Y \vdash_{\vdash} w_2 : Z} \quad I_{_AL} \qquad \frac{G; \Theta \vdash_{\vdash} w : X}{G; \Theta_1, \Theta_2 \vdash_{\vdash} w : X \times Y} \quad I_{_AR}$$

$$\frac{G; \Theta, w_1 : X \times Y \vdash_{\vdash} w_2 : Z}{G; \Theta_1, \Theta_2, w_1 : X \to Y \vdash_{\vdash} w : Z} \quad I_{_IL}$$

$$\frac{w_1 G w_2}{G; \Theta_1, \Theta_2, w_1 : X \to Y \vdash_{\vdash} w : Z} \quad I_{_IL}$$

$$\frac{w_1 G w_2}{G; \Theta_1, \Theta_2, w_1 : X \to Y \vdash_{\vdash} w : Z} \quad I_{_IL}$$

$$\frac{w_2 \notin_{\vdash} |G|, |\Theta|}{G, (w_1, w_2); \Theta, w_2 : X \vdash_{\vdash} w_2 : Y}}{G; \Theta_1, \Theta_2 \vdash_{\vdash} w : X \to Y} \quad I_{_IR}$$

$$\frac{G; \Theta_2 \vdash_{\vdash} w : X}{G; \Theta_1, \Theta_2 \vdash_{\vdash} w : Z} \quad I_{_IL}$$

Figure 1 Intuitionistic Fragment of L

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$$\frac{G, (w, w); w : S \vdash_{\mathbb{C}} \Psi}{G; w : S \vdash_{\mathbb{C}} \Psi} \quad C_{_RL} \qquad \frac{w_1 G w_2 \qquad w_2 G w_3}{G; w : S \vdash_{\mathbb{C}} \Psi} \quad C_{_TS}$$

$$\frac{G; w : S \vdash_{\mathbb{C}} W : S \vdash_{\mathbb{C}} \Psi}{G; w : S \vdash_{\mathbb{C}} W : S \vdash_{\mathbb{C}} W : S \vdash_{\mathbb{C}} \Psi} \quad C_{_TS}$$

$$\frac{G; w : S \vdash_{\mathbb{C}} w : T, \Psi}{G; w : S \vdash_{\mathbb{C}} w : T, \Psi} \quad C_{_WK}$$

$$\frac{G; w : S \vdash_{\mathbb{C}} w : T, \Psi}{G; w : S \vdash_{\mathbb{C}} w : T, \Psi} \quad C_{_CR} \qquad \frac{w_1 G w_2}{G; w_2 : S \vdash_{\mathbb{C}} \Psi} \quad C_{_ML}$$

$$\frac{w_2 G w_1}{G; w : S \vdash_{\mathbb{C}} w : T, \Psi} \quad C_{_MR} \qquad \frac{G; w_2 : S \vdash_{\mathbb{C}} \Psi}{G; w : S \vdash_{\mathbb{C}} \Psi} \quad C_{_L}$$

$$\frac{G; w : S \vdash_{\mathbb{C}} w}{G; w : S \vdash_{\mathbb{C}} w : \bot, \Psi} \quad C_{_FR} \qquad \frac{G; w : S \vdash_{\mathbb{C}} \Psi_1 \quad G; w : \bot \vdash_{\mathbb{C}} \Psi}{G; w : S \vdash_{\mathbb{C}} \Psi_1, \Psi_2} \quad C_{_DL}$$

$$\frac{G; w : R \vdash_{\mathbb{C}} w : S, w : T, \Psi}{G; w : R \vdash_{\mathbb{C}} w : S \vdash_{\mathbb{C}} \Psi} \quad C_{_SL}$$

$$\frac{w_2 G w_1}{G; w : R \vdash_{\mathbb{C}} w : S \vdash_{\mathbb{C}} \Psi_2} \quad C_{_SL}$$

$$\frac{w_2 G w_1}{G; w : R \vdash_{\mathbb{C}} w : S \vdash_{\mathbb{C}} \Psi_1 : S \vdash_{\mathbb{C}} \Psi_1} \quad C_{_SR}$$

$$\frac{G; w : R \vdash_{\mathbb{C}} w : S \vdash_{\mathbb{C}} w_1 : S \vdash_{\mathbb{C}} \Psi_1}{G; w : S \vdash_{\mathbb{C}} \Psi_1, \Psi_2} \quad C_{_SR}$$

$$\frac{G; w : S \vdash_{\mathbb{C}} w : T, \Psi_2}{G; w : S \vdash_{\mathbb{C}} \Psi_1, \Psi_2} \quad C_{_SR}$$

$$\frac{G; w : S \vdash_{\mathbb{C}} w : T, \Psi_2}{G; w : S \vdash_{\mathbb{C}} \Psi_1, \Psi_2} \quad C_{_CUT}$$

Figure 2 Co-intuitionistic Fragment of L

$$\frac{G,(w,w);\Gamma\vdash_{\vdash}\Delta}{G;\Gamma\vdash_{\vdash}\Delta} \quad \text{RL} \qquad \frac{w_1Gw_2 \quad w_2Gw_3}{G,(w_1,w_3);\Gamma\vdash_{\vdash}\Delta} \quad \text{TS}$$

$$\frac{G;\Gamma\vdash_{\vdash}w:A,\Delta}{G;\Gamma\vdash_{\vdash}\Delta} \quad \text{RL} \qquad \frac{G;\Gamma\vdash_{\vdash}w:A,\Delta}{G;\Gamma\vdash_{\vdash}\Delta} \quad \text{TS}$$

$$\frac{G;\Gamma\vdash_{\vdash}w:A,\Delta}{G;\Gamma\vdash_{\vdash}\Delta} \quad \text{TD}$$

$$\frac{w_1Gw_2}{G;\Gamma\vdash_{\vdash}\Delta} \quad \text{GCT} \qquad \frac{G;\Gamma\vdash_{\vdash}w:A,\Delta}{G;\Gamma\vdash_{\vdash}w:A,\Delta} \quad \text{ID}$$

$$\frac{w_2Gw_1}{G;\Gamma\vdash_{\vdash}w:A,\Delta} \quad \text{MR}$$

$$\frac{G;\Gamma\vdash_{\vdash}\Delta}{G;\Gamma\vdash_{\vdash}w:A,\Delta} \quad \text{TL} \qquad \frac{G;\Gamma\vdash_{\vdash}w:T,\Delta}{G;\Gamma\vdash_{\vdash}w:T,\Delta} \quad \text{TR} \qquad \frac{G;\Gamma\vdash_{\vdash}w:A,\Delta}{G;\Gamma\vdash_{\vdash}w:A,\Delta} \quad \text{FL}$$

$$\frac{G;\Gamma\vdash_{\vdash}\Delta}{G;\Gamma\vdash_{\vdash}w:A,\Delta} \quad \text{FR} \qquad \frac{G;\Gamma\vdash_{\vdash}w:B,\Delta}{G;\Gamma\vdash_{\vdash}w:A\times B\vdash_{\vdash}\Delta} \quad \text{AL}$$

$$\frac{G;\Gamma\vdash_{\vdash}w:A,\Delta}{G;\Gamma\vdash_{\vdash}w:A\times B,\Delta} \quad \text{AR}$$

$$\frac{G;\Gamma\vdash_{\vdash}w:A,\Delta}{G;\Gamma\vdash_{\vdash}w:A+B\vdash_{\vdash}\Delta} \quad \text{DL} \qquad \frac{G;\Gamma\vdash_{\vdash}w:A,w:B,\Delta}{G;\Gamma\vdash_{\vdash}w:A+B,\Delta} \quad \text{DR}$$

$$\frac{w_1Gw_2}{G;\Gamma\vdash_{\vdash}w:A+B\vdash_{\vdash}\Delta} \quad \text{DL} \qquad \frac{G;\Gamma\vdash_{\vdash}w:A+B,\Delta}{G;\Gamma\vdash_{\vdash}w:A+B,\Delta} \quad \text{DR}$$

$$\frac{w_1Gw_2}{G;\Gamma\vdash_{\vdash}w:A+B\vdash_{\vdash}\Delta} \quad \text{IL} \qquad \frac{w_2\notin[G],|\Gamma|,|\Delta|}{G;\Gamma\vdash_{\vdash}w:A+B,\Delta} \quad \text{DR}$$

$$\frac{w_2\notin[G],|\Gamma|,|\Delta|}{G;\Gamma\vdash_{\vdash}w:A+B\vdash_{\vdash}\Delta} \quad \text{SL} \qquad \frac{w_2Gw_1}{G;\Gamma\vdash_{\vdash}w:A+B,\Delta} \quad \text{SR}$$

Figure 3 Inference Rules for L