Comonadic Matter Meets Monadic Anti-Matter: An Adjoint Model of Bi-Intuitionistic Logic

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Abstract

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1 Introduction

TODO [**?**]

References

$$\frac{G,(w,w);\Theta \vdash_{\mathsf{I}} w : Y}{G;\Theta \vdash_{\mathsf{I}} w : Y}\ \mathrm{I\_RL} \qquad \frac{w_1 G w_2 \quad w_2 G w_3 \quad G,(w_1,w_3);\Theta \vdash_{\mathsf{I}} w : Y}{G;\Theta \vdash_{\mathsf{I}} w : Y}\ \mathrm{I\_TS}$$

$$\frac{}{G; w : Y \vdash_{\mathsf{I}} w : Y}\ \mathrm{I\_ID} \qquad \frac{G;\Theta_2 \vdash_{\mathsf{I}} w : X \quad G;\Theta_1, w : X \vdash_{\mathsf{I}} w : Z}{G;\Theta_1,\Theta_2 \vdash_{\mathsf{I}} w : Z}\ \mathrm{I\_CUT}$$

$$\frac{G;\Theta \vdash_{\mathsf{I}} w : Y}{G;\Theta, w : X \vdash_{\mathsf{I}} w : Y}\ \mathrm{I\_WK} \qquad \frac{G;\Theta, w : X, w : X \vdash_{\mathsf{I}} w : Y}{G;\Theta, w : X \vdash_{\mathsf{I}} w : Y}\ \mathrm{I\_CR}$$

$$\frac{G; w : R \vdash_{\mathsf{C}} \Psi_1, w_1 : S, w_2 : T, \Psi_2}{G; w : R \vdash_{\mathsf{C}} \Psi_1, w_2 : T, w_1 : S, \Psi_2}\ \mathrm{C\_EX} \qquad \frac{w_1 G w_2 \quad G;\Theta, w_1 : X, w_2 : X \vdash_{\mathsf{I}} w : Y}{G;\Theta, w_1 : X \vdash_{\mathsf{I}} w : Y}\ \mathrm{I\_ML}$$

$$\frac{w_2 G w_1 \quad G;\Theta \vdash_{\mathsf{I}} w_2 : Y}{G;\Theta \vdash_{\mathsf{I}} w_1 : Y}\ \mathrm{I\_MR} \qquad \frac{G;\Theta \vdash_{\mathsf{I}} w : Y}{G;\Theta, w : \top \vdash_{\mathsf{I}} w : Y}\ \mathrm{I\_TL} \qquad \frac{}{G;\Theta \vdash_{\mathsf{I}} w : \top}\ \mathrm{I\_TR}$$

$$\frac{G;\Theta, w_1 : X, w_1 : Y \vdash_{\mathsf{I}} w_2 : Z}{G;\Theta, w_1 : X \times Y \vdash_{\mathsf{I}} w_2 : Z}\ \mathrm{I\_AL} \qquad \frac{G;\Theta_1 \vdash_{\mathsf{I}} w : X \quad G;\Theta_2 \vdash_{\mathsf{I}} w : Y}{G;\Theta_1,\Theta_2 \vdash_{\mathsf{I}} w : X \times Y}\ \mathrm{I\_AR}$$

$$\frac{w_1 G w_2 \quad G;\Theta_2 \vdash_{\mathsf{I}} w_2 : X \quad G;\Theta_1, w_2 : Y \vdash_{\mathsf{I}} w : Z}{G;\Theta_1,\Theta_2, w_1 : X \to Y \vdash_{\mathsf{I}} w : Z}\ \mathrm{I\_IL}$$

$$\frac{w_2 \notin |G|, |\Theta| \quad G,(w_1,w_2);\Theta, w_2 : X \vdash_{\mathsf{I}} w_2 : Y}{G;\Theta \vdash_{\mathsf{I}} w_1 : X \to Y}\ \mathrm{I\_IR} \qquad \frac{G;\Theta \mid \cdot \vdash_{\mathsf{LL}} w : A \mid \cdot}{G;\Theta \vdash_{\mathsf{I}} w : GA}\ \mathrm{I\_GR}$$

Figure 1 Intuitionistic Fragment of L

$$\frac{G,(w,w);w:S\vdash_{\mathsf{C}}\Psi}{G;w:S\vdash_{\mathsf{C}}\Psi}\ \mathrm{C\_RL}\qquad\qquad \frac{\begin{array}{c}w_1Gw_2\qquad w_2Gw_3\\ G,(w_1,w_3);w:S\vdash_{\mathsf{C}}\Psi\end{array}}{G;w:S\vdash_{\mathsf{C}}\Psi}\ \mathrm{C\_TS}$$

$$\frac{}{G;w:S\vdash_{\mathsf{C}}w:S}\ \mathrm{C\_ID}\qquad\qquad \frac{G;w:S\vdash_{\mathsf{C}}w:T,\Psi_2\qquad G;w:T\vdash_{\mathsf{C}}\Psi_1}{G;w:S\vdash_{\mathsf{C}}\Psi_1,\Psi_2}\ \mathrm{C\_CUT}$$

$$\frac{G;w:S\vdash_{\mathsf{C}}\Psi}{G;w:S\vdash_{\mathsf{C}}w:T,\Psi}\ \mathrm{C\_WK}\qquad\qquad \frac{G;w:S\vdash_{\mathsf{C}}w:T,w:T,\Psi}{G;w:S\vdash_{\mathsf{C}}w:T,\Psi}\ \mathrm{C\_CR}$$

$$\frac{G;w:R\vdash_{\mathsf{C}}\Psi_1,w_1:S,w_2:T,\Psi_2}{G;w:R\vdash_{\mathsf{C}}\Psi_1,w_2:T,w_1:S,\Psi_2}\ \mathrm{C\_EX}\qquad\qquad \frac{\begin{array}{c}w_1Gw_2\\ G;w_2:S\vdash_{\mathsf{C}}\Psi\end{array}}{G;w_1:S\vdash_{\mathsf{C}}\Psi}\ \mathrm{C\_ML}$$

$$\frac{\begin{array}{c}w_2Gw_1\\ G;w:S\vdash_{\mathsf{C}}w_2:T,w_1:T,\Psi\end{array}}{G;w:S\vdash_{\mathsf{C}}w_1:T,\Psi}\ \mathrm{C\_MR}\qquad\qquad \frac{}{G;w:\perp\vdash_{\mathsf{C}}\Psi}\ \mathrm{C\_FL}$$

$$\frac{G;w:S\vdash_{\mathsf{C}}\Psi}{G;w:S\vdash_{\mathsf{C}}w:\perp,\Psi}\ \mathrm{C\_FR}\qquad\qquad \frac{G;w:S\vdash_{\mathsf{C}}\Psi_1\qquad G;w:T\vdash_{\mathsf{C}}\Psi_2}{G;w:S+T\vdash_{\mathsf{C}}\Psi_1,\Psi_2}\ \mathrm{C\_DL}$$

$$\frac{G;w:R\vdash_{\mathsf{C}}w:S,w:T,\Psi}{G;w:R\vdash_{\mathsf{C}}w:S+T,\Psi}\ \mathrm{C\_DR}\qquad\qquad \frac{\begin{array}{c}w_2\notin|G|,|\Psi|\\ G,(w_2,w_1);w_2:S\vdash_{\mathsf{C}}w_2:T,\Psi\end{array}}{G;w_1:S-T\vdash_{\mathsf{C}}\Psi}\ \mathrm{C\_SL}$$

$$\frac{\begin{array}{c}w_2Gw_1\\ G;w:R\vdash_{\mathsf{C}}w_2:S,\Psi_2\qquad G;w_2:T\vdash_{\mathsf{C}}\Psi_1\end{array}}{G;w:R\vdash_{\mathsf{C}}w_1:S-T,\Psi_1,\Psi_2}\ \mathrm{C\_SR}\qquad\qquad \frac{G;\cdot\mid w:A\vdash_{\mathsf{LL}}\cdot\mid\Psi}{G;w:HA\vdash_{\mathsf{C}}\Psi}\ \mathrm{C\_HL}$$

Figure 2 Co-intuitionistic Fragment of L

$$\frac{G,(w,w);\Theta \mid \Gamma \vdash_{\mathsf{LL}} \Delta \mid \Psi}{G;\Theta \mid \Gamma \vdash_{\mathsf{LL}} \Delta \mid \Psi}\ \text{LL\_RL} \qquad \frac{w_1 G w_2 \quad w_2 G w_3 \quad G,(w_1,w_3);\Theta \mid \Gamma \vdash_{\mathsf{LL}} \Delta \mid \Psi}{G;\Theta \mid \Gamma \vdash_{\mathsf{LL}} \Delta \mid \Psi}\ \text{LL\_TS}$$

$$\frac{w_1 G w_2 \quad G;\Theta \mid \Gamma, w_1 : A, w_2 : A \vdash_{\mathsf{LL}} \Delta \mid \Psi}{G;\Theta \mid \Gamma, w_1 : A \vdash_{\mathsf{LL}} \Delta \mid \Psi}\ \text{LL\_ML}$$

$$\frac{w_2 G w_1 \quad G;\Theta \mid \Gamma \vdash_{\mathsf{LL}} w_2 : A, w_1 : A, \Delta \mid \Psi}{G;\Theta \mid \Gamma \vdash_{\mathsf{LL}} w_1 : A, \Delta \mid \Psi}\ \text{LL\_MR}$$

$$\frac{w_1 G w_2 \quad G;\Theta, w_1 : X, w_2 : X \mid \Gamma \vdash_{\mathsf{LL}} \Delta \mid \Psi}{G;\Theta, w_1 : X \mid \Gamma \vdash_{\mathsf{LL}} \Delta \mid \Psi}\ \text{LL\_IML}$$

$$\frac{w_2 G w_1 \quad G;\Theta \mid \Gamma \vdash_{\mathsf{LL}} \Delta \mid w_2 : T, w_1 : T, \Psi}{G;\Theta \mid \Gamma \vdash_{\mathsf{LL}} \Delta \mid w_1 : T, \Psi}\ \text{LL\_CMR}$$

Figure 3 Inference Rules for BiLNL Logic: Abstract Kripke Graph Rules

$$\frac{G;\Theta \mid \Gamma \vdash_{\mathsf{LL}} \Delta \mid \Psi}{G;\Theta, w : X \mid \Gamma \vdash_{\mathsf{LL}} \Delta \mid \Psi}\ \text{LL\_WKL} \qquad \frac{G;\Theta \mid \Gamma \vdash_{\mathsf{LL}} \Delta \mid \Psi}{G;\Theta \mid \Gamma \vdash_{\mathsf{LL}} \Delta \mid \Psi, w : S}\ \text{LL\_WKR}$$

$$\frac{G;\Theta, w : X, w : X \mid \Gamma \vdash_{\mathsf{LL}} \Delta \mid \Psi}{G;\Theta, w : X \mid \Gamma \vdash_{\mathsf{LL}} \Delta \mid \Psi}\ \text{LL\_CTRL}$$

$$\frac{G;\Theta \mid \Gamma \vdash_{\mathsf{LL}} \Delta \mid \Psi, w : S, w : S}{G;\Theta \mid \Gamma \vdash_{\mathsf{LL}} \Delta \mid \Psi, w : S}\ \text{LL\_CTRR}$$

$$\frac{G;\Theta \mid \Gamma_1, w_1 : A, w_2 : B, \Gamma_2 \vdash_{\mathsf{LL}} \Delta \mid \Psi}{G;\Theta \mid \Gamma_1, w_2 : B, w_1 : A, \Gamma_2 \vdash_{\mathsf{LL}} \Delta \mid \Psi}\ \text{LL\_EXL}$$

$$\frac{G;\Theta \mid \Gamma \vdash_{\mathsf{LL}} \Delta_1, w_1 : A, w_2 : B, \Delta_2 \mid \Psi}{G;\Theta \mid \Gamma \vdash_{\mathsf{LL}} \Delta_1, w_2 : B, w_1 : A, \Delta_2 \mid \Psi}\ \text{LL\_EXR}$$

$$\frac{G;\Theta_1, w_1 : X, w_2 : Y, \Theta_2 \mid \Gamma \vdash_{\mathsf{LL}} \Delta \mid \Psi}{G;\Theta_1, w_2 : Y, w_1 : X, \Theta_2 \mid \Gamma \vdash_{\mathsf{LL}} \Delta \mid \Psi}\ \text{LL\_ILEXL}$$

$$\frac{G;\Theta \mid \Gamma \vdash_{\mathsf{LL}} \Delta \mid \Psi_1, w_1 : S, w_2 : T, \Psi_2}{G;\Theta \mid \Gamma \vdash_{\mathsf{LL}} \Delta \mid \Psi_1, w_2 : T, w_1 : S, \Psi_2}\ \text{LL\_CLEXL}$$

Figure 4 Inference Rules for BiLNL Logic: Structural Rules

$$\frac{}{G; \cdot \mid w : A \vdash_{\mathsf{LL}} w : A \mid \cdot}\ \text{LL\_ID}$$

$$\frac{G; \Theta_1 \mid \Gamma_1 \vdash_{\mathsf{LL}} w : A, \Delta_2 \mid \Psi_1 \qquad G; \Theta_2 \mid w : A, \Gamma_2 \vdash_{\mathsf{LL}} \Delta_1 \mid \Psi_2}{G; \Theta_1, \Theta_2 \mid \Gamma_1, \Gamma_2 \vdash_{\mathsf{LL}} \Delta_1, \Delta_2 \mid \Psi_1, \Psi_2}\ \text{LL\_CUT}$$

$$\frac{G; \Theta_2 \vdash_{\mathsf{I}} w : X \qquad G; \Theta_1, w : X \mid \Gamma \vdash_{\mathsf{LL}} \Delta \mid \Psi}{G; \Theta_1, \Theta_2 \mid \Gamma \vdash_{\mathsf{LL}} \Delta \mid \Psi}\ \text{LL\_ILCUT}$$

$$\frac{G; \Theta \mid \Gamma \vdash_{\mathsf{LL}} \Delta \mid \Psi_1, w : S \qquad G; w : S \vdash_{\mathsf{C}} \Psi_2}{G; \Theta \mid \Gamma \vdash_{\mathsf{LL}} \Delta \mid \Psi_1, \Psi_2}\ \text{LL\_CLCUT}$$

Figure 5 Inference Rules for BiLNL Logic: Identity and Cut Rules

$$\frac{G; \Theta \mid \Gamma \vdash_{\mathsf{LL}} \Delta \mid \Psi}{G; \Theta \mid \Gamma, w : I \vdash_{\mathsf{LL}} \Delta \mid \Psi}\ \text{LL\_IL} \qquad \frac{}{G; \cdot \mid \cdot \vdash_{\mathsf{LL}} w : I \mid \cdot}\ \text{LL\_IR}$$

$$\frac{G; \Theta_1, w : X, w : Y, \Theta_2 \mid \Gamma \vdash_{\mathsf{LL}} \Delta \mid \Psi}{G; \Theta_1, w : X \times Y, \Theta_2 \mid \Gamma \vdash_{\mathsf{LL}} \Delta \mid \Psi}\ \text{LL\_cL}$$

$$\frac{G; \Theta \mid \Gamma_1, w : A, w : B, \Gamma_2 \vdash_{\mathsf{LL}} \Delta \mid \Psi}{G; \Theta \mid \Gamma_1, w : A \otimes B, \Gamma_2 \vdash_{\mathsf{LL}} \Delta \mid \Psi}\ \text{LL\_TL}$$

$$\frac{G; \Theta_1 \mid \Gamma_1 \vdash_{\mathsf{LL}} w : A, \Delta_1 \mid \Psi_1 \qquad G; \Theta_2 \mid \Gamma_2 \vdash_{\mathsf{LL}} w : B, \Delta_2 \mid \Psi_2}{G; \Theta_1, \Theta_2 \mid \Gamma_1, \Gamma_2 \vdash_{\mathsf{LL}} w : A \otimes B, \Delta_1, \Delta_2 \mid \Psi_1, \Psi_2}\ \text{LL\_TR}$$

Figure 6 Inference Rules for BiLNL Logic: Conjunction and Tensor Rules

$$\frac{}{G; \cdot \mid w : J \vdash_{\mathsf{LL}} \cdot \mid \cdot}\ \text{LL\_JL} \qquad \frac{G; \Theta \mid \Gamma \vdash_{\mathsf{LL}} \Delta \mid \Psi}{G; \Theta \mid \Gamma \vdash_{\mathsf{LL}} w : J, \Delta \mid \Psi}\ \text{LL\_JR}$$

$$\frac{G; \Theta \mid \Gamma \vdash_{\mathsf{LL}} \Delta \mid \Psi_1, w : S, w : T, \Psi_2}{G; \Theta \mid \Gamma \vdash_{\mathsf{LL}} \Delta \mid \Psi_1, w : S + T, \Psi_2}\ \text{LL\_DR}$$

$$\frac{G; \Theta_1 \mid \Gamma_1, w : A \vdash_{\mathsf{LL}} \Delta_1 \mid \Psi_1 \qquad G; \Theta_2 \mid \Gamma_2, w : B \vdash_{\mathsf{LL}} \Delta_2 \mid \Psi_2}{G; \Theta_1, \Theta_2 \mid \Gamma_1, \Gamma_2, w : A \oplus B \vdash_{\mathsf{LL}} \Delta_1, \Delta_2 \mid \Psi_1, \Psi_2}\ \text{LL\_PL}$$

$$\frac{G; \Theta \mid \Gamma \vdash_{\mathsf{LL}} \Delta_1, w : A, w : B, \Delta_2 \mid \Psi}{G; \Theta \mid \Gamma \vdash_{\mathsf{LL}} \Delta_1, w : A \oplus B, \Delta_2 \mid \Psi}\ \text{LL\_PR}$$

Figure 7 Inference Rules for BiLNL Logic: Disjunction and Par Rules

$$\frac{\begin{array}{l} w_1 G w_2 \\ G; \Theta_1 \mid \Gamma_1 \vdash_{\mathsf{LL}} w_2 : A, \Delta_1 \mid \Psi_1 \qquad G; \Theta_2 \mid \Gamma_2, w_2 : B \vdash_{\mathsf{LL}} \Delta_2 \mid \Psi_2 \end{array}}{G; \Theta_1, \Theta_2 \mid \Gamma_1, \Gamma_2, w_1 : A \multimap B \vdash_{\mathsf{LL}} \Delta_1, \Delta_2 \mid \Psi_1, \Psi_2}\ \text{LL\_IL}$$

$$\frac{\begin{array}{l} w_2 \notin |G|, |\Theta|, |\Gamma|, |\Delta|, |\Psi| \\ G, (w_1, w_2); \Theta \mid \Gamma, w_2 : A \vdash_{\mathsf{LL}} w_2 : B, \Delta \mid \Psi \end{array}}{G; \Theta \mid \Gamma \vdash_{\mathsf{LL}} w_1 : A \multimap B, \Delta \mid \Psi}\ \text{LL\_IR}$$

$$\frac{\begin{array}{l} w_1 G w_2 \\ G; \Theta_1 \vdash_{\mathsf{I}} w_2 : X \qquad G; \Theta_2, w_2 : Y \mid \Gamma \vdash_{\mathsf{LL}} \Delta \mid \Psi \end{array}}{G; \Theta_1, \Theta_2, w_1 : X \to Y \mid \Gamma \vdash_{\mathsf{LL}} \Delta \mid \Psi}\ \text{LL\_ILIL}$$

Figure 8 Inference Rules for BiLNL Logic: Implication Rules

$$\frac{\begin{array}{l} w_2 \notin |G|, |\Theta|, |\Gamma|, |\Delta|, |\Psi| \\ G, (w_2, w_1); \Theta \mid \Gamma, w_2 : A \vdash_{\mathsf{LL}} w_2 : B, \Delta \mid \Psi \end{array}}{G; \Theta \mid \Gamma, w_1 : A \mathbin{\bullet\!\!-} B \vdash_{\mathsf{LL}} \Delta \mid \Psi}\ \text{LL\_sL}$$

$$\frac{\begin{array}{l} w_2 G w_1 \\ G; \Theta_1 \mid \Gamma_1 \vdash_{\mathsf{LL}} w_2 : A, \Delta_1 \mid \Psi_1 \qquad G; \Theta_2 \mid \Gamma_2, w_2 : B \vdash_{\mathsf{LL}} \Delta_2 \mid \Psi_2 \end{array}}{G; \Theta_1, \Theta_2 \mid \Gamma_1, \Gamma_2 \vdash_{\mathsf{LL}} w_2 : A \mathbin{\bullet\!\!-} B, \Delta_1, \Delta_2 \mid \Psi_1, \Psi_2}\ \text{LL\_sR}$$

$$\frac{\begin{array}{l} w_2 G w_1 \\ G; \Theta \mid \Gamma \vdash_{\mathsf{LL}} \Delta \mid w_2 : S, \Psi_1 \qquad G; w_2 : T \vdash_{\mathsf{C}} \Psi_2 \end{array}}{G; \Theta \mid \Gamma \vdash_{\mathsf{LL}} \Delta \mid w_1 : S - T, \Psi_1, \Psi_2}\ \text{LL\_CLsR}$$

Figure 9 Inference Rules for BiLNL Logic: Co-implication Rules

$$\frac{G; \Theta, w : X \mid \Gamma \vdash_{\mathsf{LL}} \Delta \mid \Psi}{G; \Theta \mid \Gamma, w : F\,X \vdash_{\mathsf{LL}} \Delta \mid \Psi}\ \text{LL\_FL} \qquad \frac{G; \Theta \vdash_{\mathsf{I}} w : X}{G; \Theta \mid \cdot \vdash_{\mathsf{LL}} w : F\,X \mid \cdot}\ \text{LL\_FR}$$

$$\frac{G; \Theta \mid \Gamma, w : A \vdash_{\mathsf{LL}} \Delta \mid \Psi}{G; \Theta, w : G\,A \mid \Gamma \vdash_{\mathsf{LL}} \Delta \mid \Psi}\ \text{LL\_GL} \qquad \frac{G; w : S \vdash_{\mathsf{C}} \Psi}{G; \cdot \mid w : J\,S \vdash_{\mathsf{LL}} \cdot \mid \Psi}\ \text{LL\_JL}$$

$$\frac{G; \Theta \mid \Gamma \vdash_{\mathsf{LL}} \Delta \mid w : S, \Psi}{G; \Theta \mid \Gamma \vdash_{\mathsf{LL}} \Delta, w : J\,S \mid \Psi}\ \text{LL\_JR} \qquad \frac{G; \Theta \mid \Gamma \vdash_{\mathsf{LL}} \Delta, w : A \mid \Psi}{G; \Theta \mid \Gamma \vdash_{\mathsf{LL}} \Delta \mid w : H\,A, \Psi}\ \text{LL\_HR}$$

Figure 10 Inference Rules for BiLNL Logic: Adjoint Functors Rules

$$\frac{G,(w,w);\Gamma \vdash_{\mathsf{L}} \Delta}{G;\Gamma \vdash_{\mathsf{L}} \Delta}\ \textsc{rl} \qquad \frac{w_1 G w_2 \quad w_2 G w_3 \quad G,(w_1,w_3);\Gamma \vdash_{\mathsf{L}} \Delta}{G;\Gamma \vdash_{\mathsf{L}} \Delta}\ \textsc{ts}$$

$$\frac{G;\Gamma \vdash_{\mathsf{L}} w : A,\Delta \qquad G;\Gamma, w : A \vdash_{\mathsf{L}} \Delta}{G;\Gamma \vdash_{\mathsf{L}} \Delta}\ \textsc{cut} \qquad \frac{}{G;\Gamma, w : A \vdash_{\mathsf{L}} w : A,\Delta}\ \textsc{id}$$

$$\frac{w_1 G w_2 \quad G;\Gamma, w_1 : A, w_2 : A \vdash_{\mathsf{L}} \Delta}{G;\Gamma, w_1 : A \vdash_{\mathsf{L}} \Delta}\ \textsc{m}\mathrm{L} \qquad \frac{w_2 G w_1 \quad G;\Gamma \vdash_{\mathsf{L}} w_2 : A, w_1 : A,\Delta}{G;\Gamma \vdash_{\mathsf{L}} w_1 : A,\Delta}\ \textsc{m}\mathrm{R}$$

$$\frac{G;\Gamma \vdash_{\mathsf{L}} \Delta}{G;\Gamma, w : \top \vdash_{\mathsf{L}} \Delta}\ \textsc{t}\mathrm{L} \qquad \frac{}{G;\Gamma \vdash_{\mathsf{L}} w : \top,\Delta}\ \textsc{t}\mathrm{R} \qquad \frac{}{G;\Gamma, w : \perp \vdash_{\mathsf{L}} \Delta}\ \textsc{f}\mathrm{L}$$

$$\frac{G;\Gamma \vdash_{\mathsf{L}} \Delta}{G;\Gamma \vdash_{\mathsf{L}} w : \perp,\Delta}\ \textsc{f}\mathrm{R} \qquad \frac{G;\Gamma, w : A, w : B \vdash_{\mathsf{L}} \Delta}{G;\Gamma, w : A \times B \vdash_{\mathsf{L}} \Delta}\ \textsc{a}\mathrm{L}$$

$$\frac{G;\Gamma \vdash_{\mathsf{L}} w : A,\Delta \qquad G;\Gamma \vdash_{\mathsf{L}} w : B,\Delta}{G;\Gamma \vdash_{\mathsf{L}} w : A \times B,\Delta}\ \textsc{a}\mathrm{R}$$

$$\frac{G;\Gamma, w : A \vdash_{\mathsf{L}} \Delta \qquad G;\Gamma, w : B \vdash_{\mathsf{L}} \Delta}{G;\Gamma, w : A + B \vdash_{\mathsf{L}} \Delta}\ \textsc{d}\mathrm{L} \qquad \frac{G;\Gamma \vdash_{\mathsf{L}} w : A, w : B,\Delta}{G;\Gamma \vdash_{\mathsf{L}} w : A + B,\Delta}\ \textsc{d}\mathrm{R}$$

$$\frac{w_1 G w_2 \quad G;\Gamma \vdash_{\mathsf{L}} w_2 : A,\Delta \qquad G;\Gamma, w_2 : B \vdash_{\mathsf{L}} \Delta}{G;\Gamma, w_1 : A \rightarrow B \vdash_{\mathsf{L}} \Delta}\ \textsc{i}\mathrm{L} \qquad \frac{w_2 \notin |G|, |\Gamma|, |\Delta| \quad G,(w_1,w_2);\Gamma, w_2 : A \vdash_{\mathsf{L}} w_2 : B,\Delta}{G;\Gamma \vdash_{\mathsf{L}} w_1 : A \rightarrow B,\Delta}\ \textsc{i}\mathrm{R}$$

$$\frac{w_2 \notin |G|, |\Gamma|, |\Delta| \quad G,(w_2,w_1);\Gamma, w_2 : A \vdash_{\mathsf{L}} w_2 : B,\Delta}{G;\Gamma, w_1 : A - B \vdash_{\mathsf{L}} \Delta}\ \textsc{s}\mathrm{L} \qquad \frac{w_2 G w_1 \quad G;\Gamma \vdash_{\mathsf{L}} w_2 : A,\Delta \qquad G;\Gamma, w_2 : B \vdash_{\mathsf{L}} \Delta}{G;\Gamma \vdash_{\mathsf{L}} w_1 : A - B,\Delta}\ \textsc{s}\mathrm{R}$$

Figure 11 Inference Rules for L