

Monoidal-Annex

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1 Monoidal Categories

Definition 1.

2 Symmetric Monoidal Categories

Definition 2. A symmetric monoidal category (SMC), $(\mathbb{C}, \otimes, I, \alpha, \lambda, \rho, \gamma)$, is a category \mathbb{C} equipped with a bifunctor $\otimes : \mathbb{C} \times \mathbb{C} \rightarrow \mathbb{C}$ with a neutral element I and natural isomorphisms α, λ, ρ , and γ :

1. $\alpha_{A,B,C} : A \otimes (B \otimes C) \xrightarrow{\sim} (A \otimes B) \otimes C$
2. $\lambda_A : I \otimes A \xrightarrow{\sim} A$
3. $\rho_A : A \otimes I \xrightarrow{\sim} A$
4. $\gamma_{A,B} : A \otimes B \xrightarrow{\sim} B \otimes A$

which make the following 'coherence' diagrams commute.

$$\begin{array}{ccccc}
A \otimes (B \otimes (C \otimes D)) & \xrightarrow{\alpha_{A,B,C \otimes D}} & (A \otimes B) \otimes (C \otimes D) & \xrightarrow{\alpha_{A \otimes B,C,C}} & ((A \otimes B) \otimes C) \otimes D \\
\downarrow id_A \otimes \alpha_{B,C,D} & & & & \uparrow \alpha_{A,B,C} \otimes id_D \\
A \otimes ((B \otimes C) \otimes D) & \xrightarrow{\alpha_{A,B \otimes C,D}} & & & (A \otimes (B \otimes C)) \otimes D
\end{array}$$

$$\begin{array}{ccccc}
(A \otimes B) \otimes C & \xrightarrow{\alpha_{A,B,C}} & A \otimes (B \otimes C) & \xrightarrow{\gamma_{A,B \otimes C}} & (B \otimes C) \otimes A \\
\downarrow \gamma_{A,B} \otimes id_C & & & & \downarrow \alpha_{B,C,A} \\
(B \otimes A) \otimes C & \xrightarrow{\alpha_{B,A,C}} & B \otimes (A \otimes C) & \xrightarrow{id_B \otimes \gamma_{A,C}} & B \otimes (C \otimes A)
\end{array}$$

$$\begin{array}{ccccc}
A \otimes (I \otimes B) & \xrightarrow{\alpha_{A,I,B}} & (A \otimes I) \otimes B & & A \otimes B \xrightarrow{\gamma_{A,B}} B \otimes A \\
\downarrow id_A \otimes \lambda_B & & \downarrow \rho_A \otimes id_B & & \downarrow \gamma_{B,A} \\
A \otimes B & \xlongequal{\quad} & A \otimes B & & A \otimes B
\end{array}$$

$$\begin{array}{ccc}
A \otimes I & \xrightarrow{\gamma_{A,I}} & I \otimes A \\
\downarrow \rho_A & & \downarrow \lambda_A \\
A & \xlongequal{\quad} & A
\end{array}$$

The following equality is also require to hold:

$$\lambda_I = \rho_I : I \otimes I \rightarrow I$$

[1]

Definition 3. A symmetric monoidal closed category (SMCC), $(\mathbb{C}, \otimes, \multimap, I, \alpha, \lambda, \rho, \gamma)$, is a SMC such that for all objects A in \mathbb{C} , the functor $- \otimes A$ has a specified right adjoint $A \multimap -$.

References

- [1] G.M. Bierman. *On Intuitionistic Linear Logic*. PhD thesis, University of Cambridge, 1993.