Monoidal-Annex

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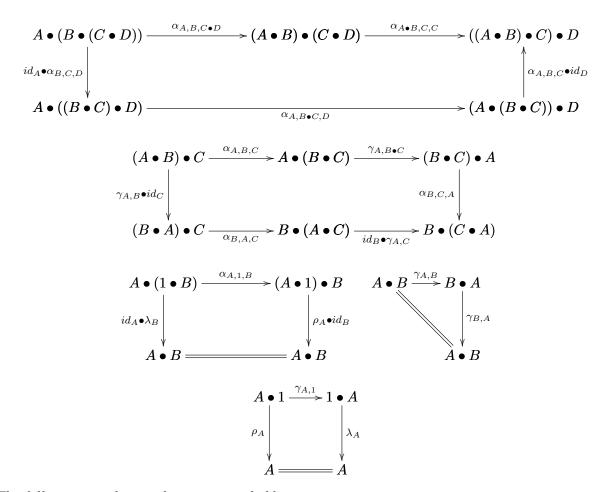
 $which \ make \ the \ following \ 'coherence' \ diagrams \ commute.$

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1.	1 5	Symmetric Monoidal Category		
Definition 1. A symmetric monoidal catergory (SMC), $(\mathbb{C}, \bullet, 1, \alpha, \lambda, \rho, \gamma)$, is a category \mathbb{C} equippped with a bifunctor $\bullet : \mathbb{C} \times \mathbb{C} \to \mathbb{C}$ with a neutral element 1 and natural isomorphisms α, λ, ρ , and γ :				
	1. α	$A,B,C:A \bullet (B \bullet C) \xrightarrow{\sim} (A \bullet B) \bullet C$		
	2. \(\lambda\)	$A: 1 \bullet A \xrightarrow{\sim} A$		
	β . ρ	$A:A \bullet 1 \xrightarrow{\sim} A$		
	4. γ ₂	$A,B:Aullet B\xrightarrow{\sim} Bullet A$		



The following equality is also require to hold:

$$\lambda_1 = \rho_1 : 1 \bullet 1 \to 1$$

1.2 Symmetric Monoidal Closed Category

Definition 2. A symmetric monoidal closed category (SMCC), $(\mathbb{C}, \bullet, \multimap, 1, \alpha, \lambda, \rho, \gamma)$, is a SMC such that for all objects A in \mathbb{C} , the functor $-\otimes A$ has a specified right adjoint $A \multimap -$.

2 Symmetric Monoidal Categories

Definition 3. A symmetric monoidal catergory (SMC), $(\mathbb{C}, \otimes, I, \alpha, \lambda, \rho, \gamma)$, is a category \mathbb{C} equippped with a bifunctor $\otimes : \mathbb{C} \times \mathbb{C} \to \mathbb{C}$ with a neutral element I and natural isomorphisms α, λ, ρ , and γ :

1.
$$\alpha_{A,B,C}: A \otimes (B \otimes C) \xrightarrow{\sim} (A \otimes B) \otimes C$$

2.
$$\lambda_A: I \otimes A \xrightarrow{\sim} A$$

3.
$$\rho_A: A \otimes I \xrightarrow{\sim} A$$

4.
$$\gamma_{A,B}: A \otimes B \xrightarrow{\sim} B \otimes A$$

which make the following 'coherence' diagrams commute.

$$A \otimes (B \otimes (C \otimes D)) \xrightarrow{\alpha_{A,B,C \otimes D}} (A \otimes B) \otimes (C \otimes D) \xrightarrow{\alpha_{A \otimes B,C,C}} ((A \otimes B) \otimes C) \otimes D$$

$$\downarrow_{id_{A} \otimes \alpha_{B,C,D}} \downarrow \qquad \qquad \downarrow_{\alpha_{A,B,C} \otimes id_{D}} \downarrow \qquad \qquad \downarrow_{\alpha_{B,C,A}} \downarrow \qquad$$

The following equality is also require to hold:

$$\lambda_I = \rho_I : I \otimes I \to I$$

2.1 Symmetric Monoidal Closed Category

Definition 4. A symmetric monoidal closed category (SMCC), $(\mathbb{C}, \otimes, \neg, I, \alpha, \lambda, \rho, \gamma)$, is a SMC such that for all objects A in \mathbb{C} , the functor $-\otimes A$ has a specified right adjoint $A \multimap -$.