Stochastic Processes and Forecasting Assignment

Nick Head

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Question 1

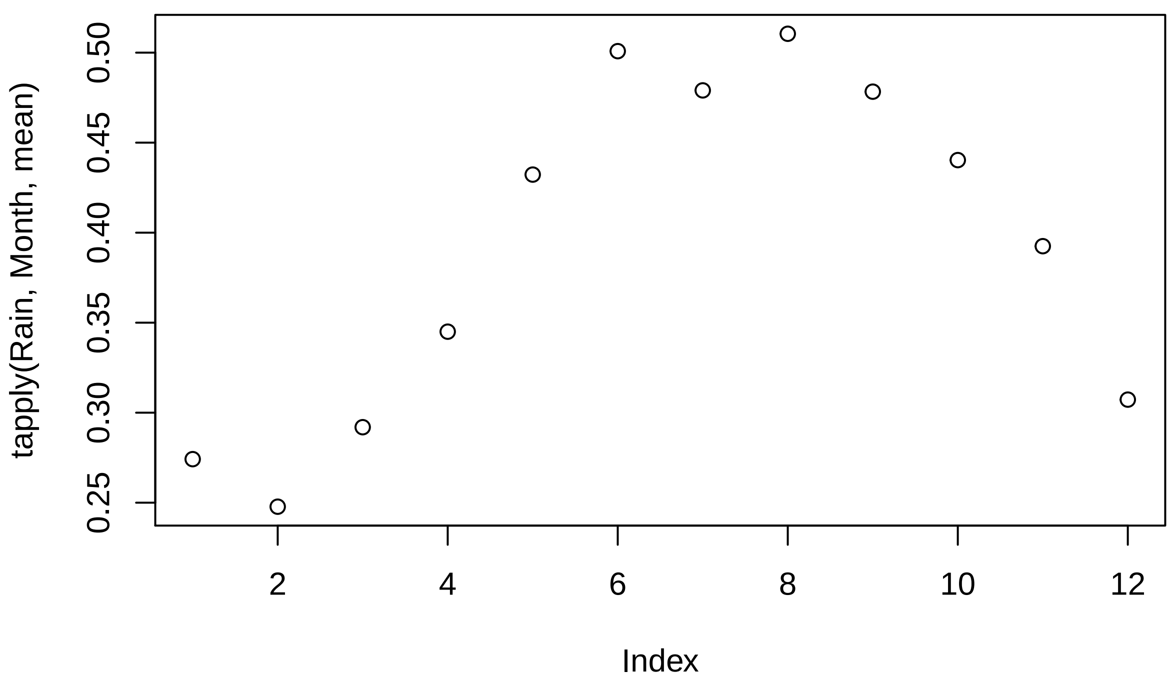
i)

The overall proportion of rainy days to sunny days is 0.392. The proportion of rainy days seems to increase over the (Southern Hemisphere) winter months and decreases over the summer months.

```
mean(Rain)
```

```
## [1] 0.392
```

```
plot(tapply(Rain,Month,mean))
```



ii)

For $m = 1$:

```
m <- 1
pi0 <- c(mean(Rain))
gamma0 <- matrix(c(1),nrow=m)
Melbourne.mle1 <- binary.HMM.mle(Rain,m,pi0,gamma0)
```

The fitted parameter values are:

$$\boldsymbol{\pi} = (\pi_1) = (0.392)$$

and

$$\boldsymbol{\Gamma} = (1)$$

For $m = 2$:

```
m <- 2
pi0 <- c(0.001,0.9)
gamma0 <- matrix(c(0.8,0.2,0.4,0.6),nrow=m,byrow=TRUE)
Melbourne.mle2 <- binary.HMM.mle(Rain,m,pi0,gamma0)
```

The fitted parameter values are:

$$\boldsymbol{\pi} = (\pi_1, \pi_2) = (0, 0.848)$$

and

$$\boldsymbol{\Gamma} = \begin{pmatrix} 0.702 & 0.298 \\ 0.346 & 0.654 \end{pmatrix}$$

For $m = 3$:

```
m <- 3
pi0 <- c(0.01,0.5,0.99)
gamma0 <-matrix(c(0.8,0.1,0.1,0.1,0.8,0.1,0.2,0.2,0.6),nrow=m,byrow=TRUE)
Melbourne.mle3 <- binary.HMM.mle(Rain,m,pi0,gamma0)
```

The fitted parameter values are:

$$\boldsymbol{\pi} = (\pi_1, \pi_2\, \pi_3) = \left(2.022 \times 10^{-15}, 0.581, 1\right)$$

and

$$\boldsymbol{\Gamma} = \begin{pmatrix} 0.747 & 1.112 \times 10^{-39} & 0.253 \\ 3.815 \times 10^{-75} & 0.967 & 0.033 \\ 0.457 & 0.017 & 0.526 \end{pmatrix}$$

iii)

	mllk	AIC	BIC
1	9785.50	19572.99	19580.58
2	9254.95	18517.89	18548.25
3	9208.61	18435.22	18503.52

As can be seen in the table, $m = 3$ minimises both the AIC and the BIC and therefore should be chosen as the most appropriate.

As $\pi_1 \approx 0$ and two of the γ_{ij} are also close to zero, three of the natural parameters are close to their boundary values and this could cause convergence issues. A modified HMM could be used where $\pi_1 = 0$.

iv)

```
m <- 3
pi <- Melbourne.mle3$pi
gamma <- Melbourne.mle3$gamma
Melbourne.mle3probs <- binary.HMM.state_probs(MelbourneRain$Rain,m,pi,gamma)
Melbourne.mle3local <- binary.HMM.local_decoding(MelbourneRain$Rain,m,pi,gamma)
Melbourne.mle3global <- binary.HMM.viterbi(MelbourneRain$Rain,m,pi,gamma)
p1 <- round(Melbourne.mle3probs[1,],3)
p2 <- round(Melbourne.mle3probs[2,],3)
p3 <- round(Melbourne.mle3probs[3,],3)
ld <- Melbourne.mle3local
gd <- Melbourne.mle3global
Melbournedecode <- cbind(MelbourneRain,p1,p2,p3,ld,gd)
Melbournedecode.txt <- write.table(Melbournedecode,file="Melbournedecode.txt",
                                   quote = FALSE,sep="\t", row.names = FALSE)

decode = read.table(file="Melbournedecode.txt", header = TRUE)
```

	Year	Month	Day	Rain	p1	p2	p3	ld	gd
1	1971	1	1	1	0.00	0.08	0.92	3	3
2	1971	1	2	1	0.00	0.07	0.94	3	3
3	1971	1	3	1	0.00	0.05	0.95	3	3
4	1971	1	4	1	0.00	0.03	0.97	3	3
5	1971	1	5	1	0.00	0.00	1.00	3	3
6	1971	1	6	0	1.00	0.00	0.00	1	1

v)

The following tables show that local and global decoding do largely agree, except in the decoding of state 2, where local decoding will identify it around twice as often as global decoding.

```
## ld
##    1    2    3 
## 8151 1916 4543
```

```
## gd
##    1    2    3 
## 8567  832 5211
```

```
##    gd
## ld     1    2    3
##   1 8119   32    0
##   2  448  743  725
##   3    0   57 4486
```

The following table shows the date periods which according to global decoding are in state 2:

	From	To
1	1971-06-15	1971-11-28
2	1980-06-29	1980-08-15
3	1981-05-24	1981-08-22
4	1986-08-16	1986-11-01
5	1989-05-23	1989-10-15
6	1991-06-03	1991-09-27
7	1992-08-10	1992-10-10
8	1996-06-05	1996-10-05

The following contingency tables show that (according to both local and global decoding) state 1 is strongly associated with rainy days and state 2 is strongly associated with dry days. State 3 can be characterised as having mixed periods.

```
##     gd
## Rain    1    2    3
##    0 8567  311    0
##    1    0  521 5211
```

```
##     ld
## Rain    1    2    3
##    0 8151  727    0
##    1    0 1189 4543
```

Question 2

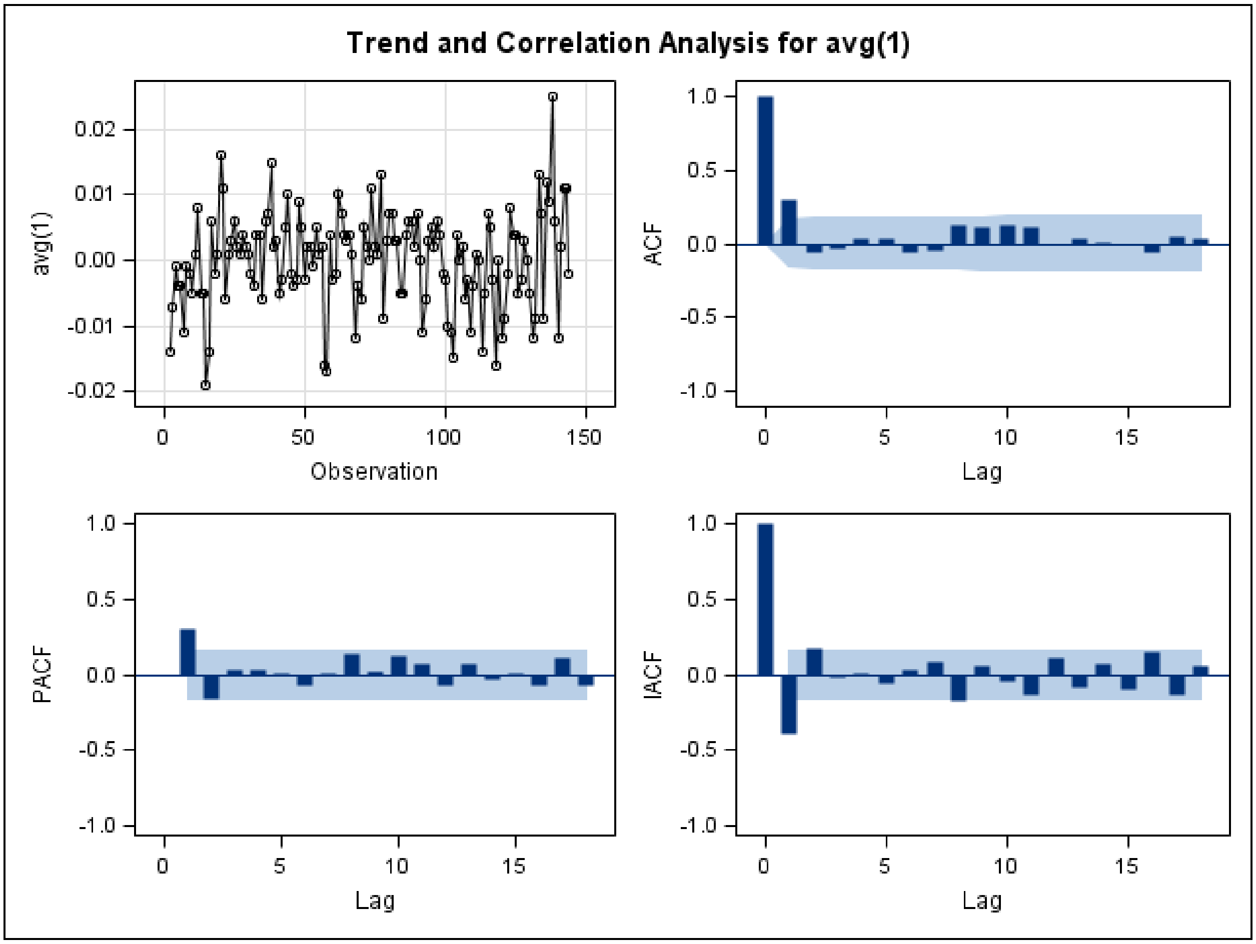
i)

The following SAS code was used to identify and estimate appropriate univariate ARIMA models for each of the avg and end exchange rate time series:

```
proc arima data=LatDol;
identify var=avg(1) nlag=18;
estimate q=1 noint;
identify var=end nlag=18;
estimate p=1;
run;
```

The diagnostics plots indicates that the first difference of avg has an ACF cut-off at lag 1, thereby leading us to believe the data follows an Arima(0,1,1) model with the following model equation:

$$\Delta Y_t = -0.36347\epsilon_{t-1} + \epsilon_t$$



In addition the Ljung-Box chi-square statistics do not have any significant p-values, giving further weight to the chosen model:

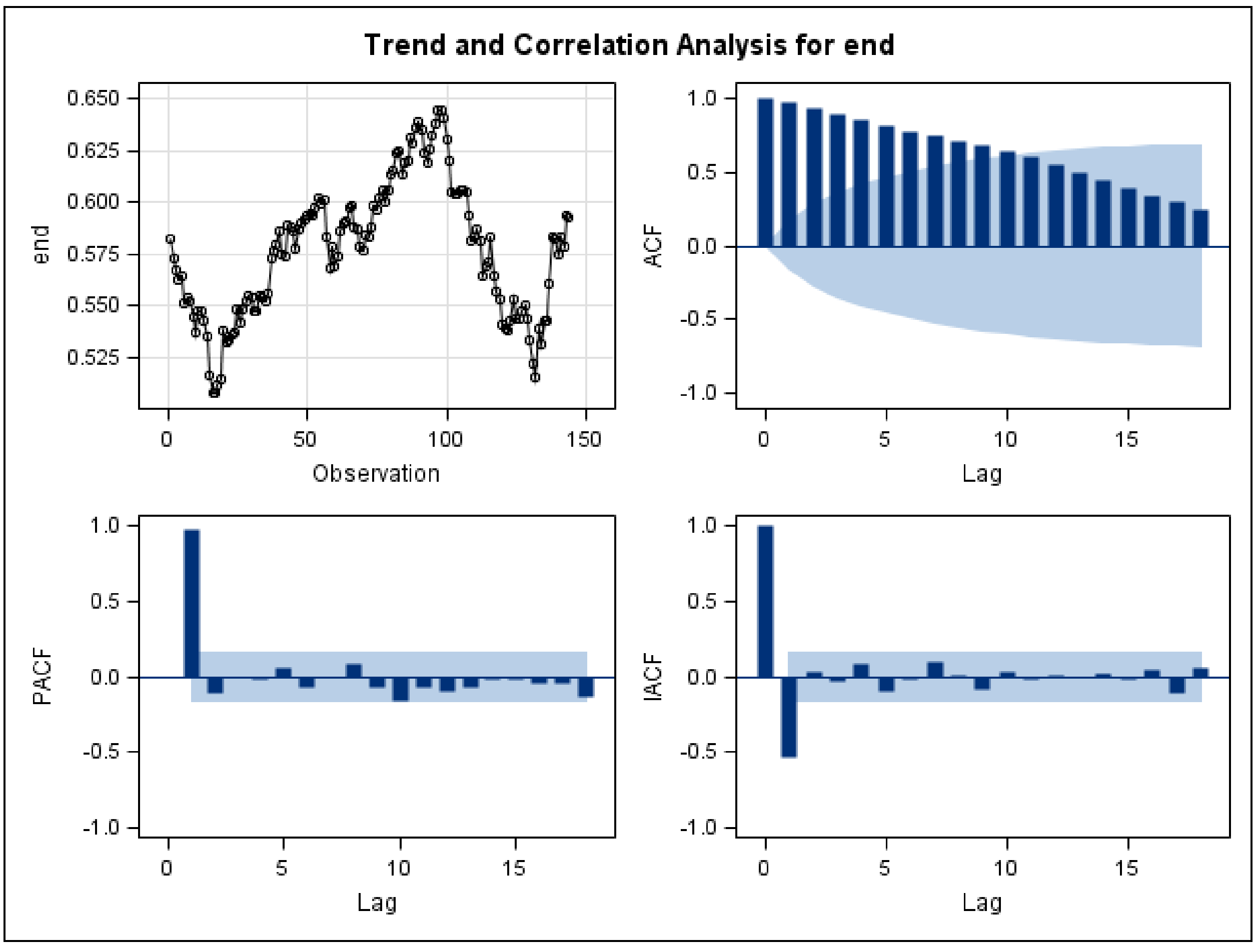
Autocorrelation Check of Residuals

To Lag	Chi-Square	DF	Pr > ChiSq	Autocorrelations					
6	1.30	5	0.9346	-0.010	-0.041	-0.034	0.032	0.046	-0.051
12	8.50	11	0.6681	-0.071	0.133	0.046	0.079	0.104	-0.066
18	11.49	17	0.8299	0.059	-0.022	0.030	-0.087	0.075	0.015
24	16.13	23	0.8497	-0.014	-0.121	-0.044	-0.074	-0.025	-0.066

`end` is found to follow an AR(1) model with the following model equation:

$$Y_t = 0.58167 + 0.96945Y_{t-1} + \epsilon_t$$

This model was identified by the lag 1 cut-off in the PACF as shown in the diagnostics plots below:



In addition the Ljung-Box chi-square statistics do not have any significant p-values, giving further weight to the chosen model:

Autocorrelation Check of Residuals

To Lag	Chi-Square	DF	Pr > ChiSq	--------------------Autocorrelations--------------------					
6	2.84	5	0.7239	0.116	0.043	0.007	-0.048	0.040	0.006
12	11.19	11	0.4279	-0.050	0.081	0.153	0.092	0.081	0.076
18	13.25	17	0.7192	-0.035	-0.019	-0.019	-0.022	0.095	-0.033
24	18.91	23	0.7064	-0.024	-0.076	-0.109	-0.044	-0.028	-0.108

ii)

The following SAS code identifies and fits a VAR(p) model to the bivariate data:

```
proc varmax data=LatDol;
model end avg /
minic = (type=sbc p=(0:10) q=0) noint dif=(end(1) avg(1)) PRINT=DIAGNOSE;
output lead=3;
run;
```

Which gives the following truncated output:

```
                          Model Parameter Estimates

                                                 Standard
Equation    Parameter        Estimate           Error    t Value    Pr > |t|    Variable

end         AR1_1_1           0.03600         0.11185       0.32      0.7480    end(t-1)
            AR1_1_2           0.11050         0.12356       0.89      0.3727    avg(t-1)
avg         AR1_2_1           0.70995         0.07553       9.40      0.0001    end(t-1)
            AR1_2_2          -0.22236         0.08344      -2.66      0.0086    avg(t-1)

                          Covariances of Innovations

                     Variable             end              avg

                     end              0.00007          0.00004
                     avg              0.00004          0.00003
```

Based off which the fitted model is as follows:

$$\begin{pmatrix} x_t \\ y_t \end{pmatrix} = \begin{pmatrix} .03600 & 0.11050 \\ .70995 & 0.22236 \end{pmatrix} \begin{pmatrix} x_{t-1} \\ y_{t-1} \end{pmatrix} + \epsilon_t$$

The following Portmanteau Statistics shows significant p-values at all lags indicating that this VAR(1) model does not fit adequatly:

```
                     Portmanteau Test for Cross
                      Correlations of Residuals
        Up To
        Lag            DF    Chi-Square    Pr > ChiSq

              2          4         23.29        0.0001
              3          8         23.46        0.0028
              4         12         26.37        0.0095
              5         16         34.09        0.0053
              6         20         35.06        0.0198
              7         24         43.30        0.0092
              8         28         44.79        0.0232
              9         32         51.56        0.0157
             10         36         58.37        0.0106
             11         40         63.56        0.0103
             12         44         64.87        0.0219
```

The following outputs show the forecasts of the average exchange rate for the next three months of 2006 are: 0.59273, 0.59261, 0.59261

Forecasts

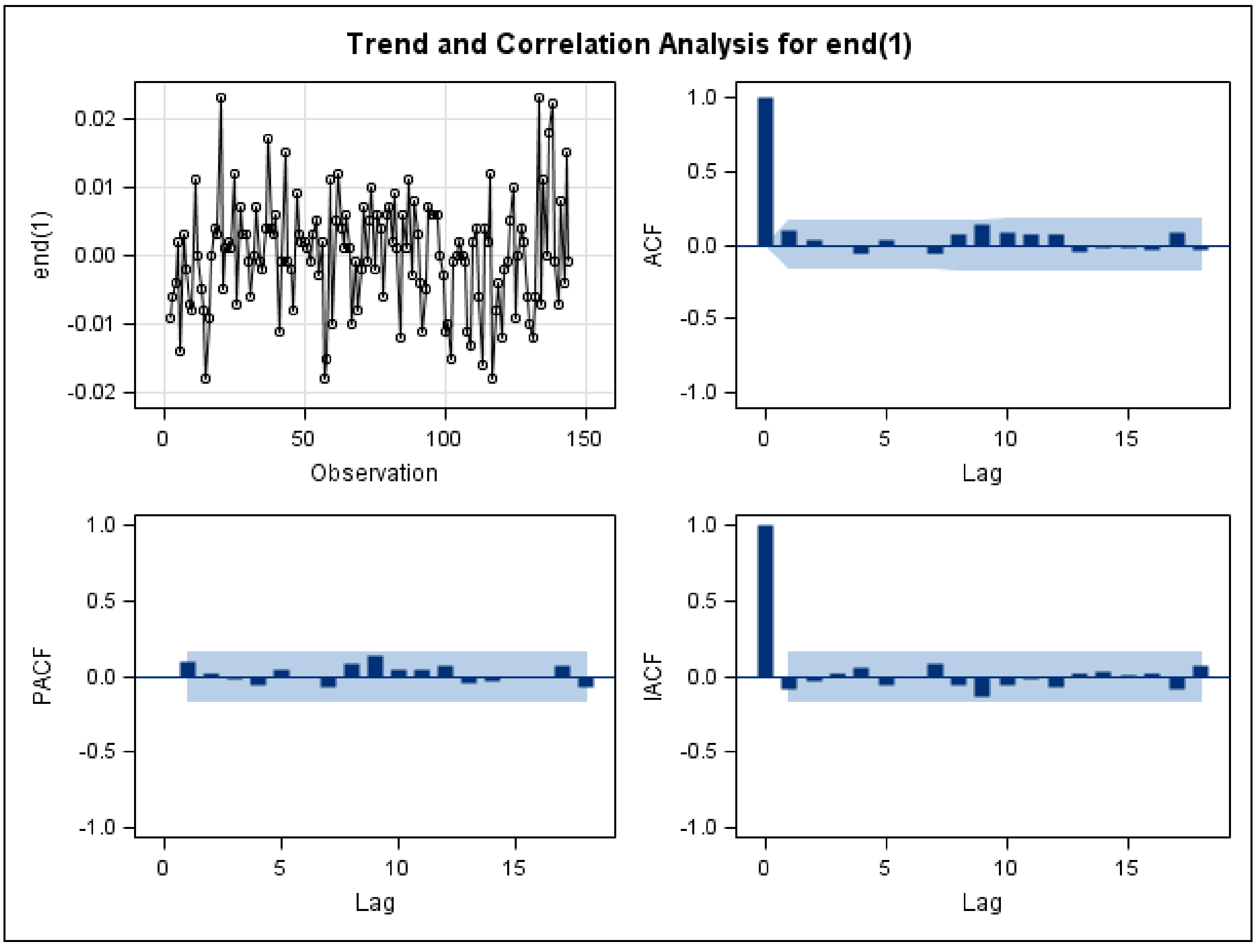
Variable	Obs	Forecast	Standard Error	95% Confidence Limits	
end	145	0.59274	0.00810	0.57686	0.60863
	146	0.59270	0.01204	0.56911	0.61630
	147	0.59269	0.01530	0.56270	0.62268
avg	145	0.59273	0.00547	0.58201	0.60346
	146	0.59261	0.01102	0.57101	0.61421
	147	0.59261	0.01435	0.56448	0.62074

iii)

The following SAS code was used to identify and estimate an appropriate transfer function model where `avg` was specified as the output variable and `end` the input variable:

```
proc arima data=LatDol;
identify var=end(1) nlag=18;
estimate q=1 noint;
identify var=avg(1) crosscor=end(1) nlag=18;
estimate q=1 input=(1 $ (1) /  end) noint;
forecast lead=3 out=results;
run;
```

The following diagnostics plots show from the lag 1 cutoff that an ARIMA(0,1,1) model is appropriate for the first difference of `end`:



Hence the estimation procedure shows the following model for `end`:

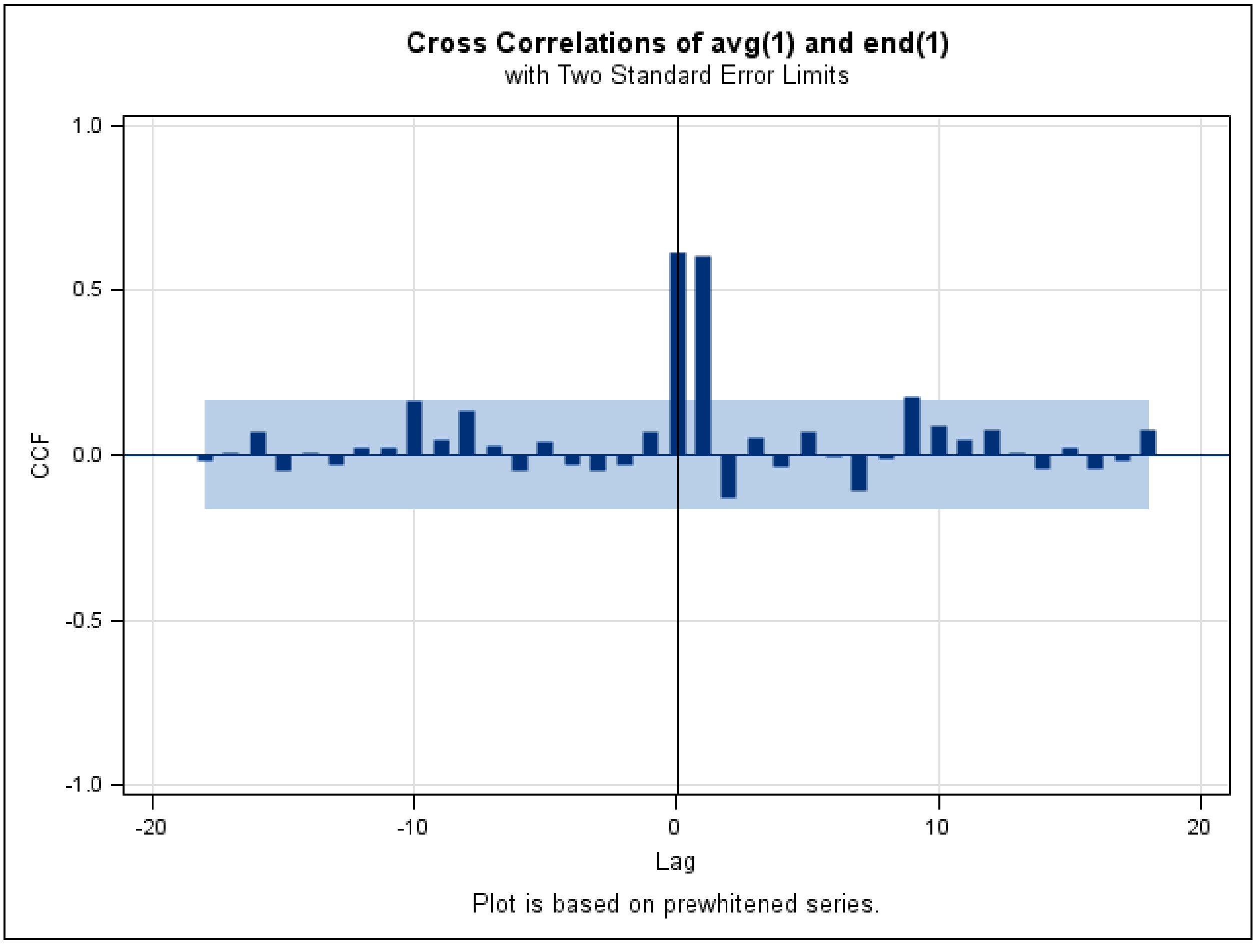
$$\Delta X_t = \eta_t + 0.09801\eta_{t-1}$$

The following output shows the p-values from the portmanteau statistics indicate the model fits well:

```
Autocorrelation Check of Residuals
```

To Lag	Chi-Square	DF	Pr > ChiSq	Autocorrelations					
6	0.95	5	0.9668	0.003	0.029	-0.003	-0.065	0.035	-0.003
12	7.14	11	0.7880	-0.069	0.066	0.134	0.066	0.062	0.069
18	9.75	17	0.9136	-0.045	-0.017	-0.018	-0.034	0.101	-0.044
24	14.42	23	0.9143	-0.015	-0.067	-0.102	-0.033	-0.013	-0.105

The next statement shows the output of the second `identify` statement which shows the following cross-correlation function:



This indicates that the transfer function is in the form of a lag-1 pulse function. There is a sharp (not decayed) response after lag 1 which indicates a transfer function of the form:

$$Y_t = \omega X_t + U_t$$

NB: The lack of intercept is due to the first differencing, and the lack of denominator terms due to the pulse intervention

With the subsequent estimation run, SAS provides us the estimated model of:

$$Y_t = (1.1118 - 0.14899L)X_{t-1} + U_t$$

Where

$$U_t = (1 - 0.95978L)\epsilon_t$$

```
                  The ARIMA Procedure

                Moving Average Factors

             Factor 1:  1 - 0.95978 B**(1)


                    Input Number 1

           Input Variable               end
           Shift                          1
           Period(s) of Differencing      1


                  Numerator Factors

           Factor 1:  1.1118 - 0.14899 B**(1)
```

The forecast values for `avg` are given as:

```
                         Forecasts for variable avg

      Obs       Forecast    Std Error       95% Confidence Limits

      145         0.5924       0.0051         0.5824         0.6024
      146         0.5922       0.0104         0.5719         0.6125
      147         0.5923       0.0135         0.5658         0.6188
```