

Fall 2023

14.12 Game Theory

Tomasz Sadzik

Problem Set 2

Due: Monday, September 25 (10:00am EST)

You are encouraged to work together on the problem sets, but you must write up your own solutions. Consulting solutions from previous semesters (released by the instructor or written by other students) is prohibited. Problem sets must be submitted electronically through Canvas.

Late problem sets submitted within 24 hours of the deadline will be accepted with a 50% penalty. Problem sets more than 24 hours late will not be accepted. Make sure to allow yourself enough time to complete the submission process. (If you have technical difficulties, you may email your problem set to the TA by the deadline.)

Problem 1 (Dominance and best responses).

1. Consider a strategic form game with $N = \{1, 2\}$, $S_1 = \{T, B\}$, and $S_2 = \{L, R\}$. Fill in the payoffs so that:¹
 - (i) For player 1, strategy T weakly dominates B , but B is a best response to some belief about player 2's strategy.
 - (ii) For player 2, strategy L does not weakly dominate strategy R , and strategy R does not weakly dominate strategy L .
2. Consider a two-player strategic form game. Suppose player 1 has a strategy s'_1 that is weakly dominated by another strategy s_1 . Show that s'_1 cannot be a best response to any belief β_2 that puts positive probability on each of player 2's strategies.

¹There are many correct answers.

Problem 2 (CEO compensation). Exercise 3.7.

Problem 3 (Insider trading). Intel, the leading chip maker, has a problem: it can only produce 10 nanometer chips while its competitors sell superior 7 nanometer chips. It is trying to develop 7 nanometer chip capability, but there is a 50% chance that the production of such chips will be delayed, in which case Intel will lose some of its market share.² This problem asks you to analyze insider trading of Intel stocks.

There are two players, named Executive and Investor. Executive works at Intel and knows whether the production will be delayed. She has one unit of Intel stock that she is considering selling at some fixed price p . Investor does not know whether the production will be delayed, but he knows that Executive knows it. He is considering buying the stock from Executive (at price p) if she sells it.

The timeline is as follows. First, Nature chooses between (production is) On Track and (production will be) Delayed, each with probability 1/2. Then, observing Nature's move, Executive decides whether to Sell or Keep the stock. If Executive decides to Sell the stock, then Investor decides whether to Buy the stock or Pass. If Executive Sells and Investor Buys, then the stock will be traded; there will be no trade otherwise. The payoffs of Executive and Investor are as in the following table:

	On Track	Delayed
Trade	$p, v_I - p$	$p, \delta v_I - p$
No Trade	$v_E, 0$	$\delta v_E, 0$

where the first entry is the payoff of Executive; $\delta \in (0, 1)$; $v_I > v_E > 0$; and $\min \{v_E, \frac{1+\delta}{2}v_I\} > p > \delta v_I$.

- (a) Write this as an extensive-form game.
- (b) Write this as a normal-form game.
- (c) Check if any player has a dominant strategy.
- (d) Compute the set of rationalizable strategies for each player.

²In 2020, Intel announced that the production will indeed be delayed, and the Intel stock dropped 15% immediately. The question considers the trade before the announcement.

Problem 4 (Beauty contest). Consider the beauty contest game from Section 4.2, in which each player i chooses a real number x_i from $[0, 100]$. Assume that there are only two players and that the payoff functions of the two players are given by

$$u_1(x_1, x_2) = -\frac{1}{2}(x_1 - \theta_1)^2 - \frac{1}{2}(x_1 - x_2)^2,$$

$$u_2(x_1, x_2) = -\frac{1}{2}(x_2 - \theta_2)^2 - \frac{1}{2}(x_2 - x_1)^2,$$

for some known numbers $\theta_1, \theta_2 \in [0, 100]$. Compute the set of rationalizable strategies for each player.