Chapter 13

Application: Implicit Cartels

> To combat falling revenue from oil sales, in 1982 Saudi Arabia pressed OPEC for audited national production quotas in an attempt to limit output and boost prices. When other OPEC nations failed to comply, Saudi Arabia first slashed its own production from 10 million barrels daily in 1979–1981 to just one-third of that level in 1985. When even this proved ineffective, Saudi Arabia reversed course and flooded the market with cheap oil, causing prices to fall below US$10 per barrel and higher-cost producers to become unprofitable. Faced with increasing economic hardship (which ultimately contributed to the collapse of the Soviet bloc in 1989), the "free-riding" oil exporters that had previously failed to comply with OPEC agreements finally began to limit production to shore up prices, based on painstakingly negotiated national quotas that sought to balance oil-related and economic criteria since 1986.
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> Wikipedia

OPEC (Organization of the Petroleum Exporting Countries) is formed by 15 oil producing countries, such as Saudi Arabia, Iran, Iraq, and Venezuela, that account for 44% of oil production (as of 2018) and 81% of proven oil reserves in the world; the United States and Russia account for another 28% of the global oil production. The members meet at least twice a year in its headquarters in Vienna to "coordinate" their petroleum policies, and their decisions have large impact on global oil market. As their first major

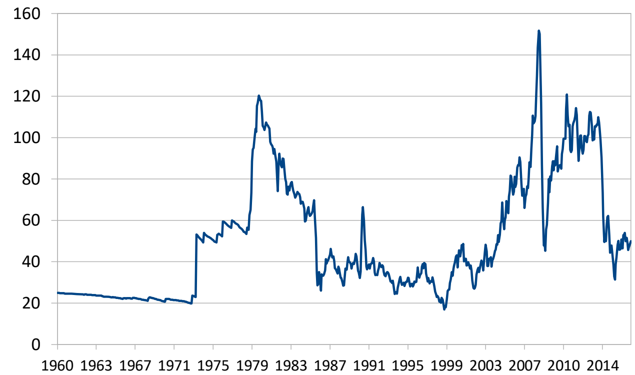

Figure 13.1: The oil prices in the United States (West Texas Intermediate, in 2017 constant dollars)

decision, in 1973, its Arab members cut their oil production and applied oil embargo to the western countries for their support of Israel. The global oil prices quadrupled as a result, causing major disruptions in the world.[1] In response, the other countries increased their oil production and reduced their energy consumption and oil dependence, many power generators switching from oil to coal and nuclear energy. By 1982, there was an abundance of oil in the market, and many cash strapped countries kept supplying more oil than OPEC would want, inducing lower prices. In response, as described in the quote above, Saudi Arabia, the leading OPEC member, started a punishing price war that persuaded the other countries eventually to cut their production.

Oil industry is not the only market that is dominated by a few powerful players. Many markets are dominated by a couple firms. For example, there are only a couple of grocery stores in a typical suburban neighborhood, a couple of hospitals in a typical medium size city, and a couple of airlines flying directly between most cities. These powerful players are often in a long term relationship, adjusting their prices and production levels as they see the market price and possibly the price and production levels set by the other firms. It would be naive then to think that these firms will set their prices or quantities myopically ignoring how the other firms would react to these prices. The other players' reactions would rather loom large in these players' decision making. Then, extrapolations from

[1] See Figure 13.1 for the oil prices in the United States; the price is only tripled in this figure because of the price caps introduced in the United States.

Cournot and Bertrand oligopoly models in Chapter 6 would be misleading. Moreover, it is in these players' best interest to set the prices as a monopoly would do and share the demand among them creatively, effectively forming (a possibly implicit) cartel.

Cartels face a fundamental challenge however: such collusive agreements are typically not enforceable by courts. In the case of firms, most countries have anti-trust laws that prohibit such agreements, and bringing such an agreement to the court will be detrimental to all of the parties involved. Likewise, it is hard to enforce agreements between sovereign nations because states are immune from civil or criminal prosecution, a legal doctrine known as sovereign immunity. Therefore, the collusive agreements must be self enforcing; they must also be implicit in the case of firms to avoid exposure. The colluding players must have a plan that prescribes what each party is to do at each contingency—not only when everything goes as planned but also when somebody deviates from the plan. And the plan must be a subgame-perfect Nash equilibrium.

This chapter is devoted to implicit cartels. It considers a repeated game in which the stage game is the linear Cournot oligopoly studied in Chapter 6. It presents many subgame-perfect Nash equilibria that can be viewed as self-enforcing cartel agreements. The first equilibrium uses a simple trigger strategy that switches to the myopic Nash equilibrium forever after any deviation. This is viewed as a simple cartel agreement in which each firm is assigned a quota, and the cartel breaks down permanently when a firm exceeds its quota. One can easily characterize the range of discount factors under which the monopoly prices can be supported by such a subgame-perfect Nash equilibrium. This is the case when the firms are sufficiently patient, where the required cutoff for the discount factor is increasing with the number of firms. When the discount factor is small or there are a lot of firms, this will not be a subgame-perfect Nash equilibrium because the temptation to undercut the other firms will be overwhelming. One can still find a larger quota—with smaller temptation to undercut—for each firm. The next equilibrium finds the optimal production supported by switching to myopic Nash permanently for a given number of firms and a given discount factor.

Cartel agreements are enforced by switching to an undesirable regime for the deviation firms. Switching strategies above are attractive in that the punishment lasts forever, deterring the firms from undercutting the other firms when the cartel agreement is in place. However, in order for such a switching strategy to be a subgame-perfect Nash equilibrium, they must be switching to a myopic Nash equilibrium. Unfortunately for the cartel members, the Nash equilibrium of Cournot oligopoly yields high profits relative to minmax payoffs, the theoretical limit of punishments available in self-enforcing contracts. The next class of subgame-perfect Nash equilibria employs shorter but more painful punishments. These equilibria can be viewed as models of price wars, where the firms resist undercutting others in order to avoid triggering a price war. One of these equilibria is in Carrot & Stick strategies that reward the good behavior by switching to Carrot state and punish the bad behavior by switching to the Stick state. Here, in the Stick state, the firms can inflict painful punishments, which can be costly to themselves, by fearing that the failure to punish will prolong the punishment and delay the reward at the end. The Stick state can be viewed as a price war: in that state, the firms flood the market with goods and sell their goods below the marginal cost. The general class of equilibria employ arbitrarily long spell of such states with prices below marginal costs, where any deviation from the plan restarts a price war. The only constraint on price war is that the firms' continuation payoffs at the beginning exceeds their minmax payoff, a payoff obtained by closing the firm for good.

13.1 Infinitely Repeated Cournot Oligopoly

Repeated linear Cournot oligopoly will serve as the main model of a cartel. There are n firms, each with marginal cost $c \in (0,1)$. In the stage game, each firm i simultaneously produces q_i units of a good and sells it at price

$$P = \max\{1 - Q, 0\}$$

where

$$Q = q_1 + \cdots + q_n$$

is the total supply. In the repeated game, all the past production levels of all firms are publicly observable, and each firm's utility function is the discounted sum of its stage profits, where the discount factor is δ:

$$u_i = \sum_{t=0}^{\infty} \delta^t q_{i,t} \left(P\left(Q_t\right) - c\right),$$

where

$$Q_t = q_{1,t} + \cdots + q_{n,t}$$

is the total supply in period t, and $q_{j,t}$ is the production level of firm j at time t. Sometimes it will be more convenient to use the discounted average value, which is $(1-\delta)\, u_i$.

For any q, write

$$f(q) = q\,(P(nq) - c) = q\,(\max\{1 - nq, 0\} - c) \tag{13.1}$$

for the per-period profit of a firm when each firm produces q and

$$g(q) = \max_{q'} q'\,(P(q' + (n-1)\,q) - c) = \max\left\{(1 - (n-1)\,q - c)^2/4, 0\right\} \tag{13.2}$$

for the maximum profit of a firm from best responding when all the other firms produce q.

13.2 Monopoly Production with Patient Firms

If it is possible to enforce, it is in the firms' best interest to produce the monopoly production level

$$Q^M = (1-c)/2$$

in total and divide the revenues according to their favored division rule, which could be attained by assigning some production quotas to the firms that add up to Q^M. For the sake of simplicity, let us assume that they would like to divide it equally. Then, the above outcome is attained by simply each firm producing

$$q^M = Q^M/n = \frac{1-c}{2n}.$$

In comparison, each firm produces

$$q^{NE} = \frac{1-c}{n+1}$$

in the unique myopic Nash equilibrium. The monopolistic quota is lower than Nash equilibrium production. The ratio becomes large as the number of firms goes to zero: the total monopolistic quota, Q^M, remains at $(1-c)/2$, while the total Nash equilibrium production, nq^{NE}, approaches to the competitive equilibrium supply, $1-c$, where the total profit is zero.

As it has been established by the Folk Theorem, when the discount factor is high, monopolistic quotas can be an outcome of a subgame-perfect Nash equilibrium. In that case, the firms can make some tacit informal plans that form a subgame-perfect Nash equilibrium and yield the desired outcome. Since the plan is a subgame-perfect Nash equilibrium, they may hope that everybody will follow through in the absence of an official enforcement mechanism, such as courts.

A simple strategy profile that leads to the above outcome is as follows:

Grim Trigger with Monopolistic Quotas Each firm is to produce q^M until somebody deviates, and produce q^{NE} thereafter.

The grim-trigger strategy profile has a specific meaning in this context. The firms have formed an implicit cartel, each producing q^M units so that they enjoy monopoly profits collectively. They are afraid that if any of the firms exceeds its quota, the cartel will break down, and each firm will be for itself with the understanding that they will never form another cartel again. After the cartel breaks down, they will play myopically the Nash equilibrium of the stage game forever.

When the cartel is in place, each firm produces q^M every period, and the average discounted value is

$$f\left(q^M\right) = \frac{(1-c)^2}{4n}.$$

After any deviation, the average discounted value drops to

$$f\left(q^{NE}\right) = \frac{(1-c)^2}{(n+1)^2},$$

regardless of the nature of the deviation.

The above strategy profile may or may not be a subgame-perfect Nash equilibrium, depending on the discount factor. This section is devoted to determine the range of discount factors under which it is indeed a subgame-perfect Nash equilibrium. Once a firm deviates and the cartel breaks down, the firms play the stage-game Nash equilibrium regardless of what happens after the initial deviation. This is a subgame-perfect Nash equilibrium of the subgame after the breakdown. Hence, by the One-Shot Deviation

Principle, it suffices to check whether a firm has an incentive to deviate while the cartel in place (i.e., no firm has deviated from producing q^M).

While the cartel is in place, the average discounted value of producing according to quota q^M is $f\left(q^M\right)$. If a firm unilaterally deviates and produces $q \neq q^M$, its profit will be $q\left(1-(n-1)\,q^M - q - c\right)$ in the current period and $f\left(q^{NE}\right)$ in each of the subsequent periods. This yields the average discounted value of

$$(1-\delta)\,q\left(1-(n-1)\,q^M - q - c\right) + \delta f\left(q^{NE}\right),$$

where the one time profit in the current period is multiplied by $(1-\delta)$ and the average value starting from the next period is multiplied by δ. Since the average value after deviation does not depend on q, the best possible deviation is obtained by best replying to $(n-1)\,q^M$, obtaining the profit of

$$g\left(q^M\right) = \left(\frac{n+1-2nc}{4n}\right)^2$$

in that period. Therefore, the best possible average discounted value from a deviation is

$$(1-\delta)\,g\left(q^M\right) + \delta f\left(q^{NE}\right).$$

No firm has an incentive to deviate if and only if this deviation payoff does not exceed the average discounted value of cartel:

$$f\left(q^M\right) \geq (1-\delta)\,g\left(q^M\right) + \delta f\left(q^{NE}\right).$$

This inequality holds if and only if the discount factor δ is weakly above the following critical threshold:

$$\hat{\delta} \equiv \frac{g\left(q^M\right) - f\left(q^M\right)}{g\left(q^M\right) - f\left(q^{NE}\right)}.$$

When $\delta \geq \hat{\delta}$, the above inequality holds, and no player has an incentive to deviate. In that case, the grim trigger strategy profile above is a subgame-perfect Nash equilibrium. Conversely, when $\delta < \hat{\delta}$, the players have incentive to deviate at the beginning, and the above strategy profile is not a Nash equilibrium. The critical threshold is strictly between 0 and 1.

The intuition for the critical threshold is as follows. Here, $g\left(q^M\right) - f\left(q^M\right)$ is the short-term gain from optimal deviation, enjoyed in the period of deviation. On the other

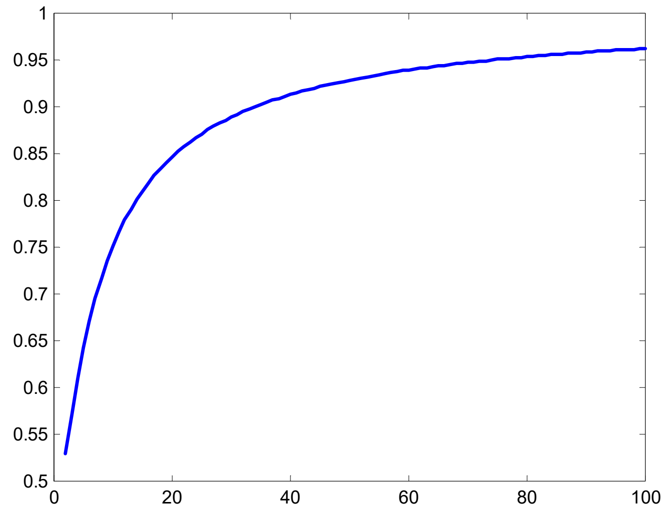

Figure 13.2: The threshold $\hat{\delta}$ as a function of n.

hand,

$$g\left(q^M\right) - f\left(q^{NE}\right) = \underbrace{g\left(q^M\right) - f\left(q^M\right)}_{\text{short-term gain}} + \underbrace{f\left(q^M\right) - f\left(q^{NE}\right)}_{\text{long-term loss}}$$

is the sum of this short-term gain and the per-period long-term loss that will be incurred every period starting from the next period on. The critical discount factor $\hat{\delta}$ is the short-term gain as a fraction of the sum, measuring the strength of the temptation in terms of the cost. As shown in Figure 13.2, for small n, $\hat{\delta}$ is reasonably small, and the monopoly prices are maintained in the grim trigger strategy equilibrium for reasonable values of δ. On the other hand, $\hat{\delta}$ is increasing in n, and $\hat{\delta} \to 1$ as $n \to \infty$. Hence, for any given discount factor, as the number of firms becomes very large, the grim trigger strategy profile fails to be an equilibrium.

13.3 Optimal Cartel with Switching Strategies

For a fixed δ and n with $\delta < \hat{\delta}$, the grim trigger strategy with monopolistic quotas is not an equilibrium. The firms are tempted to exceed their quotas and break the cartel as a result. The firms may still maintain a cartel by imposing less ambitious quotas, quotas larger than the monopolistic quota q^M, with the understanding that exceeding quota will break the cartel and lead to perpetual production of myopic Nash equilibrium level q^{NE}. This section derives the optimal quota for the cartel under this scenario.

Fix a discount factor δ and a number n of firms. Consider the following strategy profile:

Grim Trigger (q^*) Each firm is to produce q^* until somebody deviates, and produce q^{NE} thereafter.

On the path of this strategy profile, each firm produces q^* at each period, yielding the average discounted value of

$$f(q^*) = q^*(1 - nq^* - c) \tag{13.3}$$

to each firm. The main question is: Which q^* maximizes the firms' profits $f(q^*)$ subject to the constraint that the grim trigger strategy profile is a subgame-perfect Nash equilibrium?

Once again, since the myopic Nash equilibrium is played after the breakdown of the cartel, it suffices to check that there is no incentive to deviate on the path, in which all firms produced q^* at all times. At any such history, any unilateral deviation $q \neq q^*$ yields the average discounted value of

$$V_D(q) = (1-\delta)\, q\,(1 - (n-1)\, q^* - q - c) + \delta f\left(q^{NE}\right)$$

to the deviating firm. This differs from the previous section only in what the deviating firm gets at the period of deviation. That payoff is $q\,(1 - (n-1)\, q^* - q - c)$ as it produces q and all the other firms produce q^*. As before, the largest value for $V_D(q)$ is

$$(1-\delta)\, g(q^*) + \delta f\left(q^{NE}\right),$$

obtained by playing a best response to $(n-1)\, q^*$. The grim trigger strategy profile above is a subgame perfect Nash equilibrium if and only if

$$f(q^*) \geq (1-\delta)\, g(q^*) + \delta f\left(q^{NE}\right). \tag{13.4}$$

Hence, the objective in this section is to maximize $f(q^*)$ in (13.3) subject to the constraint $f(q^*) \geq (1-\delta)\, g(q^*) + \delta f\left(q^{NE}\right)$ in (13.4).

When $\delta \geq \hat{\delta}$, the monopoly production q^M is an equilibrium value for q^*. (After all, it has been shown in the previous section that the grim trigger strategy for $q^* = q^M$ is a subgame-perfect Nash equilibrium if and only if $\delta \geq \hat{\delta}$.) In that case, the optimal value for q^* is q^M. When $\delta < \hat{\delta}$, q^M is not an equilibrium value for q^*. In that case, the minimum allowable value for q^* is optimal, which is given by the equality

$$f(q^*) = (1-\delta)\, g(q^*) + \delta f\left(q^{NE}\right). \tag{13.5}$$

This is a quadratic equation for q^*. One of the solutions is clearly q^{NE} (because $f\left(q^{NE}\right) = g\left(q^{NE}\right)$ by definition). The optimal quota is the other solution:

$$q^* = q^{NE} \frac{4\delta + (n+1)^2 (1-\delta)}{4\delta n + (n+1)^2 (1-\delta)}.$$

It scales the myopic Nash equilibrium by a factor that depends on the discount factor and the number of firms. Any quota in between q^* and q^{NE} renders the grim-trigger strategy profile above a subgame-perfect Nash equilibrium, and hence it is feasible for the cartel. The optimal quota obtained as the lowest feasible quota.

As the number of firms gets larger, the scaling factor q^*/q^{NE} increases towards 1. The cartel sets larger (and less ambitious) total production, nq^*, in order to reduce the temptation to exceed the quota and break the cartel. The range of feasible cartel agreements shrinks. The feasible quotas are similar to the myopic Nash equilibrium production q^{NE}, and the outcome is approximately competitive, where the total profit is nearly zero.

The scaling factor q^*/q^{NE} is also decreasing in the discount factor δ. When δ is nearly zero, the scaling value is nearly one, and the cartel produces nearly the myopic Nash equilibrium levels. As δ gets larger, the optimal cartel production q^* decreases and reaches to the monopoly production q^M at $\delta = \hat{\delta}$. This effect can be gleaned directly from the equation (13.5). For any fixed q^*, the value of cartel on the left-hand side is independent of the discount factor, while the value of optimal deviation on the right-hand side is decreasing in δ—as the optimal deviation profit $g(q^*)$ is larger than the myopic Nash equilibrium profit $f\left(q^{NE}\right)$. Therefore, as the firms get more patient, it becomes easier to fight temptation, and the optimal quota for the cartel gets smaller, leading to higher prices and higher profits—in the expense of the consumers.[2]

[2] Formally, as the discount factor increases, the right-hand side goes down for all fixed q^*. Since the right-hand side is an increasing function of q^*, the q^* that solves the above equality decreases.

13.4 Carrot & Stick Strategies

In previous sections, cartel agreements are enforced by switching to myopic Nash equilibrium. This limits the scope of enforceable agreements substantially. The minmax payoff of a firm is substantially lower than its Nash equilibrium payoff in a Cournot oligopoly, and there can be subgame-perfect Nash equilibria in which a firm's average discounted payoff is lower than its myopic Nash equilibrium payoff. A cartel can enforce a wider range of equilibrium outcomes by switching to such subgame-perfect Nash equilibria. When the marginal cost is sufficiently small, it can implement its most-profitable self-enforcing cartel agreement using relatively simple Carrot & Stick strategies.

Carrot & Stick There are two states: Carrot and Stick. Each player plays q_C in Carrot state and q_S in Stick state. The game starts in Carrot state. At any t, if all players play what they are supposed to play, they go to Carrot state at $t+1$; they go to Stick state at $t+1$ otherwise.

At the Carrot state, the firms are expected to produce q_C units in the remainder of the game, and hence the average discounted payoff from the Carrot state is

$$V_C = f(q_C) . \tag{13.6}$$

At the Stick state, the firms are expected to produce q_S units at the current period and switch to the Carrot state in the next period. Hence, the average discounted payoff from the Stick state is

$$V_S = (1-\delta) f(q_S) + \delta V_C = (1-\delta) f(q_S) + \delta f(q_C) , \tag{13.7}$$

a convex combination of the profits in Stick and Carrot states.

Under the switching strategies employed the previous sections, the firms had to be given incentive to follow the plan only on the path, as they did not have an incentive to deviate after a deviation by construction. Under the Carrot & Stick strategies here, they must be given incentive to follow the plan not only in the Carrot state but also in the Stick state. Punishment in the Stick state can be costly for everyone including the other players who are punishing the deviant player. They may then forgive the deviant in order to avoid the cost. In order to deter them from failing to punish the deviant, equilibrium prescribes that they, too, will be punished the next period if they fail to punish. In both states, the firms are rewarded for honoring the agreement by switching to Carrot state in the next period, and they are punished for their deviation by switching to the Stick state in the next period. Accordingly, the profit $f(q_C)$ in the Carrot state is higher than the profit $f(q_S)$ in the Stick state, rendering the average discounted payoff V_C in the Carrot state higher than the average discounted payoff V_S in the Stick state.

One-Shot Deviation Principle yields two constraints under which the Carrot & Stick strategy profile above is a subgame-perfect Nash equilibrium. First, no player has an incentive to deviate unilaterally in the Carrot state:

$$V_C \geq (1-\delta)\, g(q_C) + \delta V_S. \tag{13.8}$$

Here, the first term $g(q_C)$ is the profit from the most-profitable deviation when the other firms produce q_C units each. This term is multiplied by $1-\delta$ as it is obtained only at the current period. If a firm deviates, they switch to the Stick state in the next period. The second term V_S is the average discounted payoff from the Stick state. This term is multiplied by δ because they switch in the next period. By substituting the values of V_C and V_S in (13.6) and (13.7) to (13.8), one can simplify (13.8) as

$$f(q_C) \geq \frac{1}{1+\delta} g(q_C) + \frac{\delta}{1+\delta} f(q_S). \tag{13.9}$$

This condition finds a lower bound on the average discounted payoff V_C from Carrot: it has to be at least as high as the daily profit from deviation, multiplied by $1/(1+\delta)$, and the daily profit at the Stick state, multiplied by $\delta/(1+\delta)$.

The second constraint is that no firm has an incentive to deviate unilaterally in the Stick state:

$$V_S \geq \max_q (1-\delta)\, qP(q+(n-1)\,q_S - c) + \delta V_S = (1-\delta)\, g(q_S) + \delta V_S. \tag{13.10}$$

That is, applying the possibly painful punishment at the Stick state must be at least as good as deviating from this for one period and postponing it to the next period. This constraint simplifies to

$$V_S \geq g(q_S). \tag{13.11}$$

That is, the average discounted payoff in the stick state is at least as high as the daily profit from deviation at that state. By substituting the value of V_S from (13.7), one can write this directly, again, as a lower bound on the equilibrium profit:

$$f(q_C) \geq g(q_S)/\delta - f(q_S)(1-\delta)/\delta. \tag{13.12}$$

The Carrot & Stick profile above is a subgame-perfect Nash equilibrium if and only if the constraints (13.8) and (13.10) are satisfied, as in the Carrot & Stick strategies in Section 12.2.3. In the current setup, these constraints can be written as simple lower bounds on the average payoff at the carrot state as in (13.9) and (13.12).

What is the best subgame-perfect Nash equilibrium in Carrot & Stick strategies for the cartel? To answer this question, one chooses production levels q_C and q_S in order to maximize $f(q_C)$ subject to the constraints (13.8) and (13.10). It is easier to satisfy the incentive constraint (13.8) for the Carrot state when the average value V_S in the Stick state is smaller. One can then enforce highly profitable production levels in equilibrium by choosing a low V_S, using a very low profit $f(q_S)$ as the punishment in the Stick state. There are two constraints on how low one can choose V_S. First, since a firm can choose to produce zero, the minmax value in the Cournot oligopoly is zero. Hence, V_S cannot be lower than zero. This lower bound applies uniformly for all histories in all subgame-perfect Nash equilibria. In Carrot & Stick, one can obtain this lower bound of $V_S = 0$ by setting q_S as the unique solution to

$$f(q_S) = -\frac{\delta}{1-\delta} f(q_C). \tag{13.13}$$

Second, the constraint (13.10) imposes yet another lower bound on V_S: $V_S \geq g(q_S)$. Here, the lower bound $g(q_S)$ is the highest deviation payoff in the Stick state. It is a decreasing function of q_S and it becomes zero when q_S reaches $(1-c)/(n-1)$. Now suppose that the marginal cost is sufficiently small, so that q_S in (13.13) exceeds $(1-c)/(n-1)$, i.e.,

$$f\left(\frac{1-c}{n-1}\right) \geq -\frac{\delta}{1-\delta}(n-1) f(q_C).$$

Then, one can pick q_S as in (13.13) and obtain the lowest value $V_S = 0$ that satisfies the constraint (13.10). In that case, the incentive constraint (13.8) in the Carrot state reduces to

$$f(q_C) \geq (1-\delta) g(q_C). \tag{13.14}$$

Therefore, when the discount factor δ exceeds the critical threshold

$$\delta_{CS} \equiv 1 - f\left(q^M\right)/g\left(q^M\right),$$

an optimal Carrot & Stick strategy is given by

$$q_C = q^M \text{ and } f(q_S) = -\frac{\delta}{1-\delta} f\left(q^M\right).$$

The firms supply monopoly quantity $Q^M = 1/2$ collectively, and any deviation leads to the production of q_S that offsets the gain from optimal deviation.

When $\delta < \delta_{CS}$, the constraint in (13.14) is binding, and the production q_C in the optimal Carrot & Stick strategy is the smallest solution to the quadratic equation

$$f(q_C) = (1-\delta)\, g(q_C).$$

The solution to this equation is

$$q_C = q^{NE} \frac{1-\sqrt{\delta}}{1-\frac{n-1}{n+1}\sqrt{\delta}}.$$

The optimal Carrot & Stick production is the myopic Nash equilibrium production q^{NE} when $\delta = 0$. When the firms are more patient, they impose lower quotas and enjoy higher prices and profits in the expense of the consumer. When the discount factor exceeds δ_{CS}, they impose the monopolistic quota, obtaining the monopolistic profit collectively and sharing it among them. For a fixed discount factor δ, as n gets large, the scaling factor q_C/q^{NE} approaches 1, and the optimal Carrot & Stick profit is approximately the myopic Nash equilibrium profit. When the number of firms is very large, the outcome is competitive despite the cartel's efforts to implement higher prices.

What is the most profitable self-enforcing cartel agreement? The above analysis provides an answer when the marginal cost c is sufficiently low. The lowest equilibrium payoff of zero can be obtained by using the Carrot and Stick strategies where one sets q_S as in (13.13). If a vector of quotas can be enforced by a subgame-perfect equilibrium, it can also be enforced by switching to the Stick state of the Carrot & Stick strategies above. In particular, if the cartel were to choose identical quotas on the path, the most profitable enforceable quotas are as in the Carrot & Stick strategies described above. If the cartel were to set any lower quota, so that $f(q_C) < (1-\delta)\, g(q_C)$, then at the beginning of the game, a firm could improve its payoff by best responding to $(n-1)\, q_C$ in that period and producing zero thereafter, obtaining the higher payoff of $(1-\delta)\, g(q_C)$—regardless of how the other firms react to this deviation. Such a quota cannot be supported by any Nash equilibrium.

In the optimal Carrot & Stick equilibrium, when a firm deviates, the firms inflict highly painful punishments on the deviating firm by producing large amounts, fearing that failure of punishment only delay the punishment and the subsequent reward one more period. They produce so much that the price becomes zero after a deviation. This can viewed as a price war.

13.5 Price Wars

The price wars in Carrot & Stick strategies above last only one period. Such a one-period price war may be sufficient for deterring any firm from deviation as long as the firms can burn arbitrarily large profits by producing arbitrarily large amounts of quantities and giving them away for free. In real life applications, the firms often face capacity constraints and producing such large amounts may not be feasible. In that case, a cartel can achieve higher profits by imposing more restrictive quotas and enforcing them with the fear of longer price wars. This section is devoted to analysis of subgame-perfect Nash equilibria with multiple Stick states, corresponding to multi-period price wars.

Price War There are $K+1$ states: Cartel, $W_0, \ldots, W_{K-1}$. Each firm produces q_C in Cartel state and q_W in states $W_0, \ldots, W_{K-1}$. The game starts at Cartel state. If each firm produces the above amounts (q_C in Cartel state and q_W in other states), then Cartel and W_{K-1} transition to Cartel and W_k transitions to W_{k+1} for all $k < K-1$. They go to W_0 in the next period otherwise.

On the path of the above strategy profile, the firms produce the cartel production q_C everyday. Any deviation from this production level starts a price war that lasts K periods. During the price war, the price is 0. If a firm is to deviate at any period during the punishment, the punishment starts all over again in order to punish the newly deviating firm.

Note that the average discounted profit at the cartel state is

$$V_C = f\left(q_C\right),$$

and the average discounted profit at W_k state is

$$V_k = \left(1-\delta^{K-k}\right) f\left(q_W\right) + \delta^{K-k} f\left(q_C\right), \tag{13.15}$$

where c is the marginal cost. Note that, assuming $f\left(q_C\right) \geq 0$, the situation improves as they leave more war dates in the past and get closer to the start date of the cartel with positive payoffs:

$$V_K \geq V_{K-1} \geq \cdots \geq V_0.$$

In order to check that this is a subgame-perfect Nash equilibrium, one needs to apply the One-Shot Deviation Test at each state, leading to $K+1$ constraints. First, the one-shot deviation test at the cartel state requires that the firms do not have incentive to deviate in the cartel state and start a price war:

$$f(q_C) \geq (1-\delta) g(q_C) + \delta V_0, \tag{13.16}$$

i.e., the value of cartel is higher than one period optimal deviation and the value of starting a war next period. As in the previous section, by substituting the value of V_0 from (13.15), one simplifies this constraint to

$$f(q_C) \geq \frac{1-\delta}{1-\delta^{K+1}} g(q_C) + \frac{\delta\left(1-\delta^K\right)}{1-\delta^{K+1}} f(q_W). \tag{13.17}$$

In any war state W_k, the one-shot deviation test requires that a firm does not have an incentive to deviate and start the war all over again:

$$V_k \geq (1-\delta) g(q_W) + \delta V_0.$$

That is, the value of being in the kth day of war is at least as good as best responding to $(n-1) q_W$ for that period and starting the war all over again in the next period. Since $V_k \geq V_0$ for each k, this constraint is satisfied at each war period W_k if it is satisfied at the first day of the war, i.e.,

$$V_0 \geq (1-\delta) g(q_W) + \delta V_0.$$

Therefore, the one-shot deviation test in the war states yields a single constraint:

$$V_0 \geq g(q_W), \tag{13.18}$$

as in the Carrot & Stick strategies. The Price-War strategy profile is a subgame-perfect Nash equilibrium if and only if conditions (13.16) and (13.18). These are the same conditions as in the case of Carrot & Stick strategies.

If the marginal cost c is sufficiently low, as in the case of Carrot & Stick strategies, one can select

$$q_W = \frac{\delta^K}{1-\delta^K} f(q_C) / c,$$

and this value would exceed $1/(n-1)$, yielding $V_0 = g(q_W) = 0$. As the length of price war gets longer, the amount produced during the price war gets smaller, but the amount is assumed to be large enough to pull the prices to zero. The constraint (13.17) reduces to:

$$f(q_C) \geq (1-\delta)\, g(q_C).$$

This is the same constraint as the optimal Carrot & Stick equilibrium. As in there, in the optimal price war equilibrium, one selects

$$q_C = \begin{cases} q^M & \text{if } \delta \geq 1 - f\left(q^M\right)/g\left(q^M\right) \\ q^{NE}\frac{1-\sqrt{\delta}}{1-\frac{n-1}{n+1}\sqrt{\delta}} & \text{otherwise.} \end{cases}$$

The equilibrium quota is the same as the Carrot and Stick strategies, but it is enforced by using less severe but longer price wars.

13.6 Cartel Agreements with Price Competition

Now assume that the firms are in a repeated price competition, instead of quantity competition assumed in the Cournot oligopoly. In the stage game, each firm i sets price $p_i \geq 0$ (simultaneously) and sells quantity $Q_i(p_1, \ldots, p_n)$ where $Q_i(p_1, \ldots, p_n) = (1-p_i)/m$ if $p_i = \min\{p_1, \ldots, p_n\}$ and m is the number of firms who charge the minimum price and $Q_i(p_1, \ldots, p_n) = 0$ if $p_i > \min\{p_1, \ldots, p_n\}$. The marginal cost is c, which is non-negative and strictly smaller than 1. This section explores the optimal self-enforcing cartel agreements for the cartel members under this setup.

Towards this goal, recall from Chapter 6 that a monopolist would set the price as

$$p^M = (1+c)/2$$

and sell the quantity

$$Q^M = (1-c)/2,$$

obtaining profit $(1-c)^2/4$. There is no point of charging prices higher than the monopoly price, and the focus will be on prices p in between marginal cost c and monopoly price p^M. For any price $p^M \in \left[c, p^M\right]$ with $c < p \leq p^M$, if all firms set price p, each will have profit

$$\tilde{f}(p) = (p-c)(1-p)/n.$$

If a firm deviates and sets a slightly lower price, it can get a profit arbitrarily close to

$$\tilde{g}(p) = (p - c)(1 - p) = n\tilde{f}(p);$$

there is no optimal deviation when $p > c$. In the stage game, in every Nash equilibrium at least two firms charge marginal cost c as the price. All those equilibria are payoff-equivalent to the Nash equilibrium $\left(p^{NE}, \ldots, p^{NE}\right)$ where

$$p^{NE} = c.$$

In any Nash equilibrium, the firms obtain zero profit; $\tilde{f}\left(p^{NE}\right) = \tilde{g}\left(p^{NE}\right) = 0$. Note that, since the firms can ensure zero profit by setting price equal to the marginal cost, the minmax value is also zero.

Since firms charge competitive prices in any Nash equilibrium, some may argue that the equilibrium outcome is already competitive in contestable markets, markets in which at least two firm can supply the demand, ruling out any room for policy intervention such as anti-trust laws. They would have been mistaken when the firms are in a repeated interaction. In that case, the firms can enforce highly collusive prices by switching to the myopic Nash equilibrium, as soon as a firm deviates from the agreement. Indeed, this is the most effective way to enforce cartel agreements. Therefore, towards identifying the most profitable cartel agreements it suffices to consider the following class of switching strategies.

Grim Trigger Each firm is to set price p until some firm deviates and set price p^{NE} thereafter.

Since the firms play the myopic Nash equilibrium after a deviation, this strategy profile is a subgame-perfect Nash equilibrium if and only if no player has an incentive to deviate at the beginning. Since the payoffs are all zero after a deviation, this is the case if and only if

$$\tilde{f}(p) \geq (1 - \delta)\tilde{g}(p) = (1 - \delta)n\tilde{f}(p).$$

Of course, this inequality is satisfied by the Nash equilibrium price $p = c$. For any $p \in (c, p^M]$, the inequality is satisfied if and only if $(1 - \delta)n \leq 1$, i.e.,

$$\delta \geq 1 - 1/n.$$

The optimal enforceable cartel price is

$$p_C = \begin{cases} p^M & \text{if } \delta \geq 1 - 1/n \\ c & \text{otherwise.} \end{cases}$$

If the discount factor is above $(n-1)/n$, the firms can charge the monopoly price p^M and share the monopoly profit among them, and enforce such an implicit agreement by switching to the competitive equilibrium when somebody undercuts the others. If the discount factor is below $(n-1)/n$, they cannot enforce any price above the marginal cost, and they must charge the marginal cost at any self enforcing agreement. Indeed in any Nash equilibrium, the price must be equal to the marginal cost every period on the path. If the minimum price were greater than c at any period, a firm could improve its payoff by slightly undercutting the others in the current period and charging marginal cost thereafter. Once again, for any fixed δ, the scope of cartel agreements depends on the number of firms. The firms can charge monopoly price when $n \leq 1/(1-\delta)$, and must sell at the marginal cost when $n > 1/(1-\delta)$.

13.7 Exercises with Solutions

13.8 Exercises

Exercise 13.1. Consider the infinitely repeated linear Cournot oligopoly in Section 13.1 with $c = 0$. Construct a Grim-Trigger equilibrium and a Carrot-Stick equilibrium, such that each firm produces q_C on the path and q_S in a punishment state. For each strategy profile you constructed, find the parameter values under which the strategy profile is a subgame-perfect Nash equilibrium.

Exercise 13.2. Consider the infinitely repeated linear Bertrand duopoly in Section 13.6 with two firms. Construct a Price-War equilibrium in which the firms charge p^* on the path and price wars last K periods during which the firms charge p_W. Find the range of parameters under which the strategy profile is a subgame-perfect Nash equilibrium.

Exercise 13.3. Consider the infinitely repeated game with discount factor $\delta = 0.99$ and with the following stage game. There are two firms, 1 and 2. Simultaneously, each firm i selects a price $p_i \in \{0.01, 0.02, \ldots, 0.99, 1\}$. If $p_1 = p_2$, then each firm sells 1 unit of the good; otherwise, the cheaper firm sells 2 units and the more expensive firm sells 0 units. Producing the good does not cost anything to the firms. Find a subgame-perfect Nash equilibrium in which the *average value* of Firm 1 is at least 1.4. (Check that the strategy profile you construct is indeed subgame-perfect Nash equilibrium.)

Exercise 13.4. Consider the infinitely repeated linear Cournot oligopoly in Section 13.1 with $n > 2$ firms and with zero marginal cost. For each strategy profile below, find the range of δ under which the strategy profile is a subgame-perfect Nash equilibrium. (The range may be empty.)

1. At each t, each firm produces $1/(2n)$ until some firm produces another amount; each firm produces $1/n$ thereafter.

2. At each t, firms $1, \ldots, n$ produce $1/2$, $1/4, \ldots$, $1/2^n$, respectively, until some firm deviates (by not producing the amount that it is supposed to produce); they all produce $1/(n+1)$ thereafter.

Exercise 13.5. Consider the infinitely repeated game with discount rate δ and the following stage game. There are two players: a seller and a customer. Simultaneously, the seller chooses quality $q \in [0, \infty)$ of the product and the customer decides whether to buy the product. If she buys, the seller and the customer get $p - q^2/2$ and $vq - p$, respectively; if she does not buy, the seller and the customer get $-q^2/2$ and 0, respectively, where $p \geq 0$ and $v > 0$ are constants.

1. Find the highest price p for which there is a SPNE such that customer buys on the path everyday.

2. Find the set of parameters $\hat{q}$, p, n and δ for which the following is a SPNE. There are a Trade state and n Waste states $(W_1, W_2, \ldots, W_n)$. In Trade state, the seller chooses quality $q = v$, and the buyer buys. In any Waste state, the seller chooses quality level $\hat{q}$ and the buyer does not buy. If everybody does what she is supposed to do, in the next period Trade leads to Trade, W_1 leads to W_2, W_2 leads to W_3, $\ldots$, W_{n-1} leads to W_n, and W_n leads to Trade. Any deviation leads to W_1. The game starts at Trade state.

Exercise 13.6. Consider the infinitely repeated game with discount factor $\delta \in (0,1)$ and with the following stage game. There are two profit-maximizing firms, A and B, which produce applesauce and banana puree, respectively. (Applesauce and banana puree are imperfect substitutes.) Simultaneously, A and B sets the prices $p_A \in [0,1]$ and $p_B \in [0,1]$ of applesauce and banana puree, respectively. Firms A and B sell

$$q_A(p_A, p_B) = 1 - p_A - \gamma(1 - p_B) \text{ and } q_B(p_A, p_B) = 1 - p_B - \gamma(1 - p_A)$$

units of applesauce and banana puree, respectively, where $0 < \gamma < 1$. (The amounts can be negative.) The marginal cost is $c = 0$ for each firm.

1. For each strategy profile below, find the set of parameters $(\delta, \gamma, p^*, p^{**}, K)$ for which the strategy profile is a subgame-perfect Nash equilibrium in the repeated game. (The set may be empty.)

   Price War There are two states: Collusion, and War. The game starts in the Collusion state. In the Collusion state, each player sets price p^*. If everybody sets price p^*, then they remain in the Collusion state; otherwise they switch to the War state, which lasts K periods. During the War, each sets price p^{**} for K periods, and they switch back to Collusion state at the end of K periods regardless of what happens during the War.

   Carrot & Stick There are two states: Collusion, and War. The game starts in the Collusion state. In the Collusion state, each player sets price p^*. If everybody sets price p^*, then they remain in the Collusion state; otherwise they switch to the War state. In the War state, each sets price p^{**}. If everybody sets price p^{**} then they switch back to Collusion state; otherwise they remain in War state.

2. Fix $\delta = 0.8$ and $\gamma = 1/2$. For each strategy profile above, by varying the parameters (p^*, p^{**}, K) find the subgame-perfect Nash equilibrium with the highest profit.

Exercise 13.7. Consider the infinitely repeated linear Cournot duopoly in Section 13.1 with discount factor $\delta = 0.99$ and marginal cost zero; there are two firms. For each strategy profile below check if it is a subgame-perfect Nash equilibrium.

1. There are two states: Cartel and Competition. The game starts at Cartel state. In Cartel state, each supplies $q_i = 1/4$. In Cartel state, if each supplies $q_i = 1/4$, they remain in Cartel state in the next period; otherwise they switch to Competition state in the next period. In Competition state, each supplies $q_i = 1/2$. In Competition state, they automatically switch to Cartel state in the next period.

2. There are two states: Cartel and Competition. The game starts at Cartel state. In Cartel state, each supplies $q_i = 1/4$. In Cartel state, if each supplies $q_i = 1/4$, they remain in Cartel state in the next period; otherwise they switch to Competition state in the next period. In Competition state, each supplies $q_i = 1/2$. In Competition state, they switch to Cartel state in the next period if and only if both supply $q_i = 1/2$; otherwise they remain in Competition state in the next period, too.

Exercise 13.8. Consider the infinitely repeated game with discount factor δ and the stage game in Exercise 10.12. Take $n = 3$. For sufficiently large δ, construct a subgame-perfect Nash equilibrium of the repeated game in which each firm chooses the advertisement level $\hat{a}$ and the production level $\hat{q}$ at every period on the path. Determine the range of δ under which your strategy profile is a subgame-perfect Nash equilibrium. (Hint: make sure that you specify a **stage-game strategy profile** for each history of past advertisement and production levels; the production levels may differ from $\hat{q}$ when the current advertisement levels differ from $\hat{a}$.)

Exercise 13.9. Redo the previous Exercise for arbitrary $n \geq 3$, and briefly discuss how the range of δ varies with n.

Exercise 13.10. Consider the infinitely repeated game with discount factor $\delta \in (0, 1)$ and the following stage game, which is a linear Cournot duopoly with stochastic demand. There are two firms. First, Nature chooses $\theta \in \{1, 2\}$, each with probability 1/2. Then, observing θ, each firm i simultaneously produces q_i units of a good at zero marginal cost and sells it at price

$$P = \theta - q_1 - q_2,$$

obtaining stage-game payoff $q_i P$.

1. For each $\theta \in \{1,2\}$, compute the symmetric production level $q^*(\theta)$ that maximizes the sum of the profits of the two firms. For each $\theta \in \{1,2\}$, compute also the Nash equilibrium $(\hat{q}(\theta), \hat{q}(\theta))$.

2. Assuming δ is sufficiently high, construct a subgame perfect Nash equilibrium of the repeated game in which each player produces $q^*(\theta)$ (as a function of θ) every period on the path. Determine the range of discount factors under which your strategy profile is a SPNE and verify that it is a subgame-perfect Nash equilibrium for those discount factors.

3. Take $\delta = 1/2$, and determine the range of values $(\bar{q}(1), \bar{q}(2))$ under which the following strategy profile is a subgame-perfect Nash equilibrium:

   Grim Trigger Each firm produces $\bar{q}(\theta)$—as a function of θ—until a firm deviates; each produces $\hat{q}(\theta)$—as a function of θ—thereafter.

   (You only need to write down the inequalities $\bar{q}(1)$ and $\bar{q}(2)$ must satisfy.) What is the profit maximizing pair $(\bar{q}(1), \bar{q}(2))$ in this range? Briefly discuss your findings.

Exercise 13.11. Consider the infinitely repeated game with the tariff-setting game described in Section 10.4.4 as its stage game. For each strategy profile s^* below, find the range of parameters under which s^* is a subgame-perfect Nash Equilibrium. (In each part, determine the range of δ, $\hat{x}$, and the functions $q_{ii}(x_i)$ and $q_{ij}(x_j)$.)

1. (Free-Trade Agreement) There are two states: Agreement and Competition. The game starts at the Agreement state. At the Agreement state, each government sets tariff to zero (i.e., $x_A = x_B = 0$) and each firm i produces $q_{ii}(x_i)$ and $q_{ij}(x_j)$ as functions of the tariff rates set at that period. If any government sets non-zero tariff, then the state switches to Competition in the next period and remains as Competition forever. In the Competition state, each government sets tariff rate equal to $\hat{x}$ (i.e., $x_A = x_B = \hat{x}$) and each firm i produces $q_{ii}(x_i)$ and $q_{ij}(x_j)$ as functions of the tariff rates set at that period.

2. (Market Segmentation) There are two states: Segmentation and Competition. The game starts at the Segmentation state. In both states, each government sets tariff rate equal to $\hat{x}$ (i.e., $x_A = x_B = \hat{x}$). In the Segmentation state, each firm i chooses

$q_{ii} = 1/2$ and $q_{ij} = 0$ (irrespective of the tariff rate). If any firm exports to the other country by choosing $q_{ij} > 0$, then the state switches to Competition in the next period and remains as Competition forever. In the Competition state, each firm i produces $q_{ii}(x_i)$ and $q_{ij}(x_j)$ as functions of the tariff rates set at that period.

Exercise 13.12. Consider the infinitely repeated game in which the stage game is the game in Exercise 10.10 and the discount factor is $\delta \in (0,1)$. Take $n = 2$. For each outcome path below, for sufficiently large δ, find a subgame-perfect Nash equilibrium with that outcome. Specify the range of δ under which your answer is valid. (It suffices to find the inequalities that δ must satisfy.)

1. On each date, the supplier sets $c = 1/2$ and each firm i chooses quantity $q_i = 1/8$.

2. On each date, the supplier sets $c = 0$ and each firm i chooses quantity $q_i = 1/4$.

Exercise 13.13. Consider a repeated game with the following stage game. A unit mass of kids are uniformly located on a street, denoted by the $[0,1]$ interval. There are two ice cream parlors, namely 1 and 2, located at 0 and 1, respectively. Each ice cream parlor i sets a price $p_i \leq \bar{p}$ for its own ice cream, simultaneously, where $\bar{p} > 0$.[3] A kid located in w is to pay cost $c|w-y|$ to go to a store located at y, where $c \in (0, \bar{p}/3)$. Given the prices p_1 and p_2, each kid buys one unit of ice cream from the store with the lowest total cost, which is the sum of the price and the cost to go to the store. (If the total cost is the same, she flips a coin to choose the store to buy.)

1. Assume that the above game is repeated 100 times, and find the subgame-perfect Nash equilibria.

2. Assume that the above game is repeated infinitely many times and the discount rate is $\delta \in (1/3, 1)$. For each of the following strategy profile, find the highest p^* under which the strategy profile is a subgame-perfect Nash equilibrium. (Here, p^* may be a function of δ. You need to choose both p^* and $\hat{p}$ to make the strategy profile a subgame-perfect Nash equilibrium.)

[3] The price can be negative (e.g. ice cream parlor can give a gift to the kids who buy) and bounded from above (e.g. the kids cannot pay more than a fixed amount).

(a) At the beginning each parlor i chooses $p_i = p^*$ and continues to do so until some player sets a different price; each i selects price $p_i = \hat{p}$ thereafter.

(b) There are two states: Collusion and War. The game starts at the state Collusion. In Collusion state, each player i chooses $p_i = p^*$, and in War state each player i chooses $p_i = \hat{p}$. If both players set the price prescribed for the state, then the state in the next round is Collusion; the state in the next round is War otherwise.

(c) In part (b), assume that $\hat{p} \geq 0$ but war can last multiple periods. State such a strategy profile formally and answer the above question for such a strategy profile.