Fall 2023

14.12 Game Theory

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Problem Set 9

Due: Monday, December 4 (10:00am EST)

You are encouraged to work together on the problem sets, but you must write up your own solutions. Consulting solutions from previous semesters (released by the instructor or written by other students) is prohibited. Problem sets must be submitted electronically through Canvas.

Late problem sets submitted within 24 hours of the deadline will be accepted with a 50% penalty. Problem sets more than 24 hours late will not be accepted. Make sure to allow yourself enough time to complete the submission process. (If you have technical difficulties, you may email your problem set to the TA by the deadline.)

Problem 1 (Modified first-price auction with reserve). Consider the following auction. If everyone bids strictly below r, the good is not allocated and no one pays anything. If exactly one bidder bids weakly above r, then that bidder gets the good and pays r. If more than one bidder bids weakly above r, then the highest bidder gets the good and pays her own bid. (No one else pays anything.) Ties are broken by a fair coin flip.

Suppose there are two bidders $i = 1, 2$. Their valuations v_1, v_2 for the good are drawn independently and uniformly from $[0, 1]$. For each reserve price r in $[0, 1]$, find a symmetric Bayes Nash equilibrium with piecewise-linear bidding strategies, and compute the revenue from this equilibrium. Solve for the revenue-maximizing reserve price r^*.

Hint: Guess that both players use the following bidding strategy for some parameter $\beta \in [0, 1]$:

$$b_i^*(v_i) = \begin{cases} v_i & \text{if } v_i < r, \\ r + \beta(v_i - r) & \text{if } v_i \geq r. \end{cases}$$

You should solve for β.

Optional: Repeat this exercise with n bidders, for arbitrary $n \geq 1$. How does the revenue-maximizing reserve price depend on n?

Problem 2 (First-price auction with non-uniform distributions). Exercise 20.7.

Problem 3 (Average-price auction). Exercise 20.10. (Hint: Look for a symmetric BNE with linear bidding strategies.)

Problem 4 (VCG). Consider a special case of the ad auction setting from class. There are three ad slots, with click-through rates $\alpha_1 > \alpha_2 > \alpha_3$. You are one of three bidders in the auction. Suppose you have valuation v per-click, and your two opponents bid b_1 and b_2, where $b_1 > b_2$.[1] Compute your expected utility from bidding b, in each of the following three cases:

1. $b > b_1 > b_2$;

2. $b_1 > b > b_2$;

3. $b_1 > b_2 > b$.

Denote these three expected utilities by U_1, U_2, U_3, respectively. Show that U_1 is highest if $v > b_1$; U_2 is highest if $b_1 > v > b_2$; and U_3 is highest if $b_2 > v$.

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[1]This is a thought experiment. In reality, you wouldn't know your opponents' bids.