14.12 Week 2 Recitation!!

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Fridays 3-4pm, E51-361

Sign-in

(Old?) New Topics! Who dis?

- Rationalizability (last week)
- Nash Equilibrium (last week)
  - PSNE, MSNE
- Imperfect Competition (this week)

Iterated elimination of strictly dominated strategies

Initialization set $S_i^0 = S_i$ for each player i.

Elimination Round k For each $k = 1, 2, \ldots$ and for each player j, let S_j^{k-1} be the set of strategies of player j that survived the first $k-1$ rounds of elimination. For each player i and each strategy $s_i \in S_i$, eliminate strategy s_i if and only if there exists a mixed strategy σ_i (in the original game) with

$$u_i(\sigma_i, s_{-i}) > u_i(s_i, s_{-i}) \qquad \text{for all } s_{-i} \in S_{-i}^{k-1},$$

where $S_{-i}^{k-1} = \prod_{j \neq i} S_j^{k-1}$. Set

$$S_i^k = S_i^{k-1} \backslash \{s_i | s_i \text{ is eliminated in round } k\}$$

as the set of strategies that survives the first k rounds. Iterate.

Note that one considers only the strategies s_{-i} of other players that are available at any given round. In contrast, in looking for dominating strategies, one considers all possible strategies σ_i of player i, even those that are eliminated previously. When S_i is finite, one can ignore the previously eliminated strategies of player i, too.

Example: Iterated Elimination

What is S^0, S^1, ..., S^∞?

	L	R
T	(3,3)	(0,1)
M	(1,2)	(1,4)
B	(0,3)	(3,0)

Iterated elimination of strictly dominated strategies

Caution: 2 important things!

- You can only eliminate STRICTLY dominated strategies at each step.
- You should eliminate strategies that are eliminated by mixed strategies.

Example: Iterated Elimination

	L	R
T	(3,3)	(0,1)
M	(1,2)	(1,4)
B	(0,3)	(3,0)

S^0_1:{T,M,B}
S^0_2:{L,R}

S^0 ‘s are the strategy sets we started with.

Example: Iterated Elimination

	L	R
T	(3,3)	(0,1)
~~M~~	~~(1,2)~~	~~(1,4)~~
B	(0,3)	(3,0)

S^1_1:{T,B}
S^1_2:{L,R}

"What are the responses that are not eliminated for player 1, given that player 2 can play L or R?"

Example: Iterated Elimination

What is the dominant strategy equilibrium of the game below?

	L	R
T	(3,3)	(0,1)
M	(1,2)	(1,4)
B	(0,3)	(3,0)

S^2_1:{T,B}
S^2_2:{L}

Example: Iterated Elimination

	L	R
T	(3,3)	(0,1)
~~M~~	~~(1,2)~~	~~(1,4)~~
~~B~~	~~(0,3)~~	~~(3,0)~~

S^3_1:{T}
S^3_2:{L}

No more to eliminate! Those are the final S^∞ (i.e. rationalizable strategies).

Nash Equilibrium

- Compute each player’s best response
- Look for a cell where both (all) players are playing best responses
    - This means player 1 forms a belief of what player 2’s strategies are
- Nash equilibria are <u>strategy profiles</u>, NOT payoffs!!

In English: a NE occurs if all players have no incentive to <u>deviate</u> given the beliefs they have about other players’ moves.

The Classic Prisoner's Dilemma

	C	D
C	(1,1)	(-1,2)
D	(2,-1)	(0,0)

The Classic Prisoner’s Dilemma

	C	D
C	(1,1)	(-1,2)
D	(2,-1)	(0,0)

Why is (D,D) the only Nash Equilibrium here?

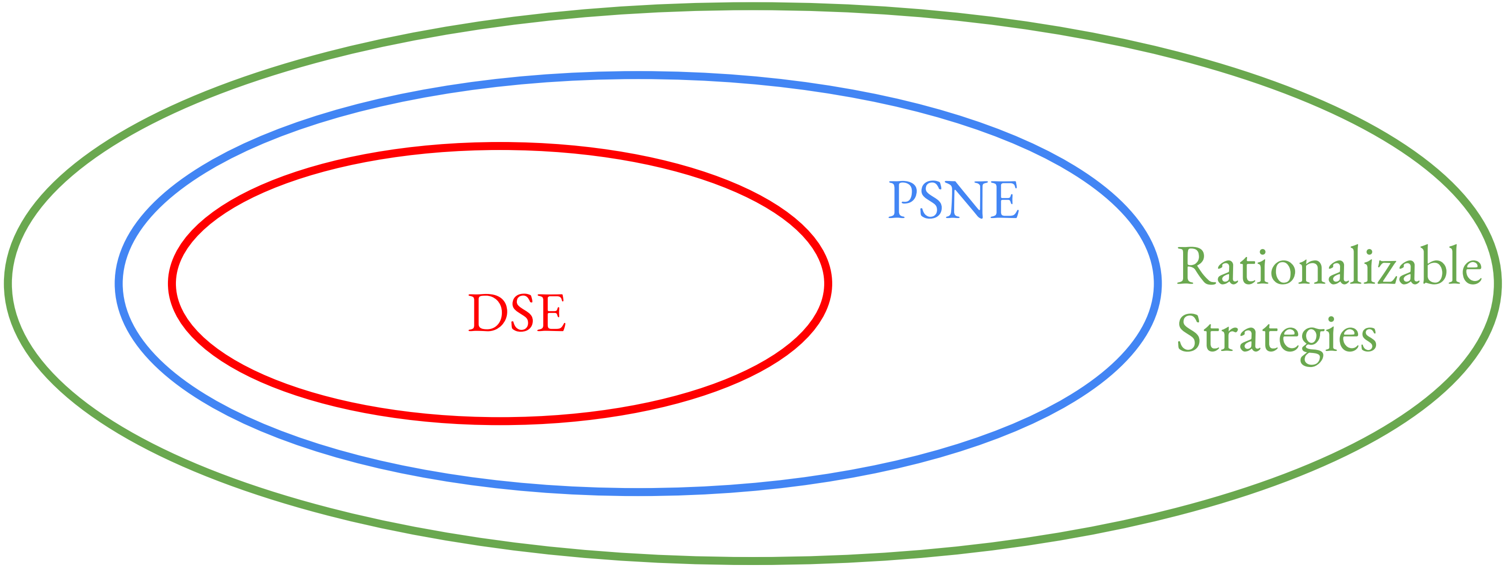
No matter what player 2 does, player 1 will still be better off by playing D (i.e., D is a dominant strategy), and vice versa. Therefore, both will choose to play D in a Nash Equilibrium.

More Equilibria Notes

- If a dominant strategy D exists for player **i**, then **i** will always play D in any nash equilibrium.
- If both players have a dominant strategy, the <u>unique</u> NE where both play their respective dominant strategies is called the <u>dominant strategy equilibrium</u>.

Those Equilibria Together

Theorem. In any game, a dominant strategy equilibrium is a pure strategy nash equilibrium, and a pure strategy nash equilibrium is rationalizable.



Finding NEs

- For PSNE: highlight the best responses for each player, and select the cells where all players are playing their best responses.
  - There may be more than 1 PSNE, or none.
- **In each game, there is at least one MSNE (a PSNE is a special case of MSNE).**
- In general, MSNE is a lot harder to compute, and may contain a continuum of strategies
- MSNE Problem-Solving Idea:
  - Assign probabilities to each strategy for a player, and then calculate the probabilities needed such that the other player is <u>indifferent</u> between each of their choices
  - You may need to consider situations where the probability of a player playing a certain move is 0.
  - Lots of casework!
  - This can be done in conjunction with certain smart ways of reducing the problem - see example.

Let's follow an example!

Example: Nash Equilibrium

Find all the Nash equilibria of this game (including PSNE and MSNE).

	L	M	R
U	(2,1)	(1,1)	(0,0)
C	(1,2)	(3,3)	(2,1)
D	(2,-2)	(1,0)	(-1,-1)

Example: Nash Equilibrium

PSNE: highlight the best responses.

	L	M	R
U	(2,1)	(1,1)	(0,0)
C	(1,2)	(3,3)	(2,1)
D	(2,-2)	(1,0)	(-1,-1)

Example: Nash Equilibrium

PSNE: highlight the best responses.

For player 1: we look at the situations where player 2 plays L, M, R separately.

	L	M	R
U	(<u>2</u>,1)	(1,1)	(0,0)
C	(1,2)	(3,3)	(2,1)
D	(<u>2</u>,-2)	(1,0)	(-1,-1)

Example: Nash Equilibrium

PSNE: highlight the best responses.

For player 1: we look at the situations where player 2 plays L, M, R separately.

	L	M	R
U	(<u>2</u>,1)	(1,1)	(0,0)
C	(1,2)	(<u>3</u>,3)	(2,1)
D	(<u>2</u>,-2)	(1,0)	(-1,-1)

Example: Nash Equilibrium

PSNE: highlight the best responses.

For player 1: we look at the situations where player 2 plays L, M, R separately.

	L	M	R
U	($\underline{2}$,1)	(1,1)	(0,0)
C	(1,2)	($\underline{3}$,3)	($\underline{2}$,1)
D	($\underline{2}$,-2)	(1,0)	(-1,-1)

Example: Nash Equilibrium

PSNE: highlight the best responses.

Do the same for player 2!

	L	M	R
U	$(\underline{2},\underline{1})$	$(1,\underline{1})$	$(0,0)$
C	$(1,2)$	$(\underline{3},\underline{3})$	$(\underline{2},1)$
D	$(\underline{2},-2)$	$(1,\underline{0})$	$(-1,-1)$

Example: Nash Equilibrium

Answer: there are 2 PSNEs: (U,L), and (C,M).

	L	M	R
U	**($\underline{2}$,$\underline{1}$)**	(1,$\underline{1}$)	(0,0)
C	(1,2)	**($\underline{3}$,$\underline{3}$)**	($\underline{2}$,1)
D	($\underline{2}$,-2)	(1,$\underline{0}$)	(-1,-1)

Example: Nash Equilibrium

MSNE:

First, eliminate the strictly dominated strategies

	L	M	R
U	(2,1)	(1,1)	(0,0)
C	(1,2)	(3,3)	(2,1)
D	(2,-2)	(1,0)	(-1,-1)

Example: Nash Equilibrium

MSNE:

First, eliminate the strictly dominated strategies

	L	M	R
U	(2,1)	(1,1)	(0,0)
C	(1,2)	(3,3)	(2,1)
D	(2,-2)	(1,0)	(-1,-1)

Example: Nash Equilibrium

MSNE:

Second: Assign probabilities to each strategy for a player, and then calculate the probabilities needed such that the other player is indifferent between each of their choices

	L	M
U	(2,1)	(1,1)
C	(1,2)	(3,3)
D	(2,-2)	(1,0)

Example: Nash Equilibrium

We assign p_L, p_M to be the probabilities for player 2 to play L and M, respectively. Note that $p_L + p_M = 1$.

- We need to find p_L, p_M such that player 1 is indifferent between U, C, and D.
- Calculate:
- $u_1(U)=u_1(D) = 2p_L + 1p_M = 1+p_L$
- $u_1(C) = 1p_L + 3p_M = 3\text{-}2p_L$

We see that p_L = ⅔ will make player 1 indifferent between U,C,D.

	L	M
U	(2,1)	(1,1)
C	(1,2)	(3,3)
D	(2,-2)	(1,0)

Example: Nash Equilibrium

We assign p_U, p_C, p_D to be the probabilities for player 2 to play U, C, and D, respectively.

- $p_2(L) = 1p_U + 2p_C - 2p_D$
- $p_2(M) = 1p_U + 3p_C$

Equating those 2 expressions, we get $1p_C = -2p_D$, which is only possible if both p_C and p_D are 0. Thus when $p_U = 1$, player 2 is indifferent across strategies L and M.

Answer: the MSNE is $\{p_U=1, p_L = ⅔\}$.

	L	M
U	(2,1)	(1,1)
C	(1,2)	(3,3)
D	(2,-2)	(1,0)

Example: Nash Equilibrium

We assign p_U, p_C, p_D to be the probabilities for player 2 to play U, C, and D, respectively.

… but we can work smarter in this case! Note that L is *weakly* dominated by M, which means that player 2 will only be indifferent to L and M if player 1 played U with 100% certainty. Therefore, $p_U = 1$.

	L	M
U	(2,1)	(1,1)
C	(1,2)	(3,3)
D	(2,-2)	(1,0)

Answer: the MSNE is {p_U=1, p_L = ⅔}.

Imperfect competition - TLDR definitions

- Monopoly: 1 seller
- Oligopoly: a few sellers, many buyers
  - Examples: Airlines, oil companies, big pharma companies
- Oligopsony: a few buyers, many sellers
  - Examples: fast food restaurants (buying burger patties, for instance), book publishers, coffee beans
- Bertrand competition: <u>N firms</u> make the <u>same good</u> and compete on <u>price</u>
- Cournot competition: N firms sell the same good at the same price but potentially each at a different <u>quantity</u> (that influences price)
- Collusion/cartel: when N firms make an agreement to produce at some price/quantity and there may be a punitive action taken by other firms when one firm acts not accordingly
  - E.g. Other firms could punish the deviating firm by producing at another quantity that minimizes the firm's sale
  - This will be more relevant very soon!

Bertrand Competition

- Firms produce the same good and set their price $\{p_1, p_2, \ldots, p_n\}$ simultaneously
- The firm with lowest price gets all the business, rest gets none
- If every firm sets the same price p then they will each sell quantity p/n
- The only Nash Equilibrium: everyone sets their price to 0!
- Why? (Because there is incentive to deviate to a lower price if price is not 0!)

Cournot Competition

- There are n firms; each firm i produces at q_i, determined simultaneously
- Then everyone's price is P = max{1-Q,0}, where Q = sum of prices for each firm
- You calculate the best response for each firm by taking partial derivatives
  - Assume production cost of each unit of good is c.
  - The best response is $(1-Q_{-i}-c)/(n+1)$
  - Plug this into every firm - in the unique NE, we can calculate the total supply to be (n* (1-c))/(n+1)

Solving Problems - General Strategy

- When question asks for normal form game: list players, strategies, and payoffs
- When question asks to solve for Nash Equilibrium:
  - Write down utility function for player i
  - Solve first order condition of utility with respect to price/quantity/whatever applicable variable
  - When payoff of player i involves decisions of player j: plug things in!

Bigger pictures/implications

Cournot competition model says:

- When an industry does not have many firms (e.g. airlines), price regulation can increase efficiency
- When an industry has many firms (e.g. restaurants), price regulation is not necessarily effective or necessary
- In practice, Cournot competition is closer to real markets

Bertrand competition model says:

- As long as there is more than 1 firm, they will compete over price until reaching the perfectly competitive outcome
- Therefore, price regulation is not effective or necessary at any time

However, remember that both models are oversimplified versions of the real world, which contains lots of noise (such as imperfect competition, search cost, repeated games, incomplete information,..., just to name a few).

Supermodular games

- A game is supermodular if one player's marginal utility is increasing in other players' actions.
- For 2 players: check if $$\frac{\partial^2 u_i}{\partial s_i \partial s_j} \geq 0$$

- It is enough to check the second derivatives only when one of the actions is yours. (For example, no need to check second derivative of 1's utility with respect to actions of 2 and 3)

Q & A