

Fall 2023

14.12 Game Theory

Tomasz Sadzik

Problem Set 5

Due: Monday October 23 (10:00am EST)

You are encouraged to work together on the problem sets, but you must write up your own solutions. Consulting solutions from previous semesters (released by the instructor or written by other students) is prohibited. Problem sets must be submitted electronically through Canvas.

Late problem sets submitted within 24 hours of the deadline will be accepted with a 50% penalty. Problem sets more than 24 hours late will not be accepted. Make sure to allow yourself enough time to complete the submission process. (If you have technical difficulties, you may email your problem set to the TA by the deadline.)

Problem 1 (Pretrial negotiation). Consider the following single-proposal protocol for pretrial negotiation between Plaintiff and Defendant. There are five commonly known parameters $J, \hat{c}_P, \hat{c}_D, c_P, c_D$, which are all strictly positive. The negotiation proceeds in three phases.

- I. *Invitation to pretrial negotiation.* The Plaintiff chooses whether to invite the Defendant to pretrial negotiation. If the Defendant is invited, he can accept or reject the invitation. If the invitation is made and accepted, play proceeds to stage II, which costs c_P for the Plaintiff and c_D for the Defendant. Otherwise, the game proceeds to stage III.
- II. *Pretrial negotiation.* The Proposer (to be specified below) chooses a settlement $s \in \mathbf{R}$ to offer to the Respondent (the other player). The Respondent chooses to accept or reject this proposed settlement. If the settlement s is accepted, then the Defendant pays s to the Plaintiff and the game ends. Otherwise, play proceeds to stage III.

III. *Trial.* The Plaintiff chooses whether to bring the suit to trial. If she does not bring the suit to trial, the game ends. If she brings the suit to trial, then the Plaintiff and the Defendant pay trial costs \hat{c}_P and \hat{c}_D respectively. At the end of the trial, the Defendant pays J to the Plaintiff.

As in class, there is no discounting and both players are risk-neutral. For this protocol, answer the following questions.

- (a) Assuming $\hat{c}_P > J$, apply backward induction to stage III. (That is, for this part only, you may assume that the game consists only of Stage III.)

For the remaining questions, assume $\hat{c}_P < J$.

- (b) Suppose the Plaintiff is the Proposer. Write out the extensive-form game and apply backward induction.
- (c) Suppose the Defendant is the Proposer. Write out the extensive-form game and apply backward induction.
- (d) Now suppose that at the beginning of stage II, a fair coin is flipped to determine which party is the Proposer. Write out the extensive-form game. For which cost parameters is there a backward induction solution in which the case is settled out of court (with certainty).
- (e) Finally, suppose the coin can be biased so that the Plaintiff is the Proposer with commonly known probability $\alpha \in (0, 1)$. We want to find a probability α such that there is a backward induction solution in which the case is settled out of court (with certainty). For which cost parameters does such an α exist? Interpret your condition on the cost parameters.

Problem 2 (Subgame perfection). Exercise 10.6. (The four strategies in the bottom right corner of Figure 10.12 are missing labels; you can label them from left to right as v, w, v, w .)

Problem 3 (Subgame perfection, revisited). Alice and Bob are competing to play a game against Casey. Alice and Bob simultaneously bid p_A and p_B , respectively. The one who bids higher wins; if $p_A = p_B$, the winner is determined by a fair coin toss. The winner pays her bid to Casey and then plays the following game with Casey:

		Casey
	<i>L</i>	<i>R</i>
Winner	<i>T</i>	3, 1 0, 0
	<i>B</i>	0, 0 1, 3

Find two pure strategy subgame-perfect equilibria of this game. (Remember to specify the complete strategy profiles.) Which of the equilibria makes more sense to you?