Fall 2023
14.12 Game Theory
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Problem Set 4

Due: **UPDATED:** Wednesday, October 11 (10:00am EST)

You are encouraged to work together on the problem sets, but you must write up your own solutions. Consulting solutions from previous semesters (released by the instructor or written by other students) is prohibited. Problem sets must be submitted electronically through Canvas.

Late problem sets submitted within 24 hours of the deadline will be accepted with a 50% penalty. Problem sets more than 24 hours late will not be accepted. Make sure to allow yourself enough time to complete the submission process. (If you have technical difficulties, you may email your problem set to the TA by the deadline.)

Problem 1 (Weak dominance and security strategies)**.** Give an example of a two-player zero-sum game in strategic form in which player 1 has a security strategy that is weakly dominated.[1]

Problem 2 (Sequential poker variant)**.** Consider the following sequential variant of the simplified poker game from class. There are two known parameters: the ante α and fixed bet size β. There are two players. The timing is as follows:

1. Each player pays an ante α into the pot.

2. Each player is (privately) dealt either heads (H) or tails (T).

3. Player 1 chooses whether to bet (B) the amount β or not (N).

[1] To rule our trivial cases, each player should have at least two strategies.

4. If player 1 does not bet, then player 2 cannot bet (so player 2 has no choices to make). If player 1 bets, then player 2 chooses whether to bet (B) the amount β or not (N).

5. If exactly one player bets, that player takes the pot. Otherwise (if both bet or neither bets), the players show their "hands." The player with the better hand (H is better than T) takes the pot. In case of a tie, the players split the pot.

Suppose each player's utility equals her monetary gain or loss.

1. Write this game in extensive form.

2. Write this game in strategic form.

3. For this part, restrict attention to strategies Bp, in which a player always bets when H, and bets with probability p when T. Compute player 1's worst-case gain function WG, where

$$\mathrm{WG}(p) = \min_{s_2 \in S_2} u(p, s_2).$$

   Find the value of $p*$ that maximizes this worst case gain $\mathrm{WG}(p)$.

4. (Not graded!) You can also compute, for any strategy Bq for player 2 the worst-case loss

$$\mathrm{WL}(q) = \max_{s_1 \in S_1} u(s_1, q).$$

   You can find the value $q*$ that minimizes that worst-case loss. Verify that (with some abuse of notation) $(p*, q*)$ is a Nash Equilibrium of the game.

Problem 3 (Meet-up)**.** Suppose, as in class, that Alice and Bob are deciding whether to walk along the Charles (C) or go downtown (D). If Alice and Bob go to different locations, they both get utility 0. If they both go to the Charles, Alice gets utility 2 and Bob gets utility 1. If they both go downtown, Alice gets utility 1 and Bob gets utility 2.

Now we consider the following timing. First Alice chooses between the Charles and downtown. Bob observes Alice's choice and chooses between the Charles and downtown. Finally, Alice observes Bob's choice and chooses either to stay where she is or to travel to the other location.

(a) Write this as an extensive form game.

(b) Apply backward induction to this game.

Problem 4 (Meet-up with a cost). Reconsider the setting of Problem 3, but now suppose that Alice must pay a utility cost c if she switches locations after seeing Bob's choice.

(a) Apply backward induction to this game in each of the following cases: (i) $0 < c < 1$; (ii) $1 < c < 2$; (iii) $c > 2$.

(b) Discuss how the cost c affects the result of backward induction. Is Alice better off with a lower cost c? Interpret this finding.