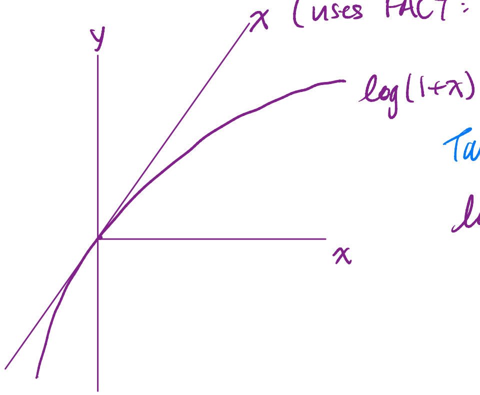
differences in logs
Chebyshev's inequality
$\longrightarrow$ LLN

$A, B.$

$\log(A) - \log(B) \approx \%$ change $\left(\frac{A-B}{B}\right)$

$= \log\left(\frac{A}{B}\right) = \log\left(1 + \frac{A-B}{B}\right) \approx \frac{A-B}{B}.$

(uses FACT: $\log(1+x) \approx x$ for small x).



Taylor expansion of $\log(1+x)$ around $x=0$

$$\log(1+x) \approx \log(1) + (x-0)\left.\frac{d\log(1+x)}{dx}\right|_{x=0} + \dots$$

$$= 0 + x \underbrace{\left.\frac{1}{1+x}\right|_{x=0}}_{=1}$$

$$= x.$$

Chebyshev's Inequality

$$\Pr(|X_i - \mu| \geq k) \leq \frac{\sigma^2}{k^2}$$

$$\sigma^2 = \underbrace{E[(X_i-\mu)^2]}_{\geq k^2}$$

$$= E[(X_i-\mu)^2 \mid |X_i-\mu| \geq k] \Pr(|X_i-\mu| \geq k) \quad \text{(LIE)}$$

$$+ \underbrace{E[(X_i-\mu)^2 / |X_i-\mu| < k]}_{\geq 0} \underbrace{\Pr(|X_i-\mu| < k)}_{\geq 0}.$$

$$\sigma^2 \geq k^2 \Pr(|X_i-\mu| \geq k)$$

$$\Pr(|X_i-\mu| \geq k) \leq \frac{\sigma^2}{k^2}. \quad \square.$$

Law of Large Numbers.

$$\lim_{n\to\infty} Pr(|\bar{X}_n - \mu_x| > \varepsilon) = 0. \quad \forall\ \varepsilon > 0.$$

Recall: $Var(\bar{X}_n) = Var\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) = \frac{1}{n^2} Var(\sum X_i)$

$$= \frac{1}{n^2} n\sigma^2 = \frac{\sigma^2}{n}.$$

Now use Chebyshev.

$$Pr(|X_i - \mu| \geq k) \leq \frac{\sigma^2}{k^2}.$$

$$\rightarrow Pr(|\bar{X}_n - \mu| \geq \varepsilon) \leq \frac{Var(\bar{X}_n)}{\varepsilon^2} = \frac{1}{n}\frac{\sigma^2}{\varepsilon^2}.$$

$$\lim_{n\to\infty} Pr(|\bar{X}_n - \mu| \geq \varepsilon) = \lim_{n\to\infty} \frac{1}{n}\frac{\sigma^2}{\varepsilon^2} = 0. \quad \square.$$