General (non-exhaustive) Outline of Topics

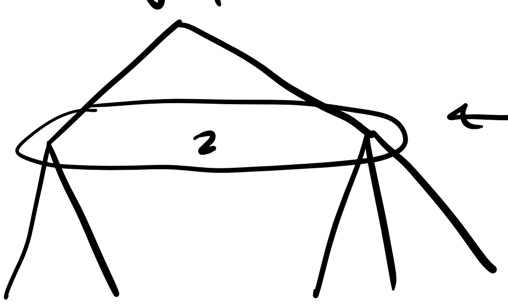
Topic 1: Representation of Games

- extensive form vs normal form (or strategic form)

players, tree,
players @ non-terminal nodes
information sets,
payoffs @ terminal nodes

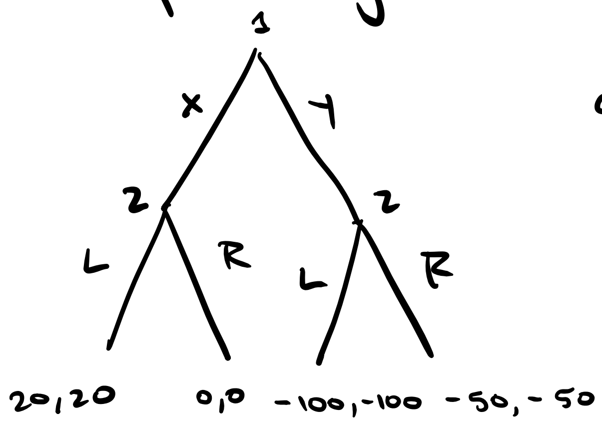
→ way to represent (players, strategies, utilities) often matrix form

info sets: same player moves, w/ same available moves
* w/ multiple nodes in an info set, player cannot distinguish (does not know where she is)



← not a valid info set (why?)

- expressing the info sets & strategies



a strategy for P2 should be:

L L

if @ X node
if @ Y node

- calculating payoffs (use probability weights,) see PSet 1 for more details

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Dominance & Rationalizability

strategy s_i^* strictly <u>dominates</u> a $s_i \in S_i$ if:

$$u_i(s_i^*, s_{-i}) > u_i(s_i, s_{-i}) \quad \forall s_{-i} \in S_{-i}$$

(in words, no matter what is done by other player i comparison b/w strategies s_i^*, $s_i - s_i^*$ is strictly better)

weakly dominates: similar to above, but " $\geq$ " ; inequality is strict for some s_{-i}

- write out expected payoffs as fn of beliefs

- can be dominated by mixed strategy

Thm finitely many strategies ("non-weird" games)
A strategy s_i is a BR to some belief iff s_i is not strictly dominated

also:
(playing s_i is never rational iff s_i is dominated by some strategy)

→ iterated elimation of strictly dominated strategies

(U, L : DSE; but ISD doesn't eliminate anything)

	L	R
U	1,1	0,0
D	0,0	0,0

(rationalizability: may have many left & cannot rule any out)

Does a dominant-strategy equilibrium always exist? (no)

- if s^* a DSE, then a NE

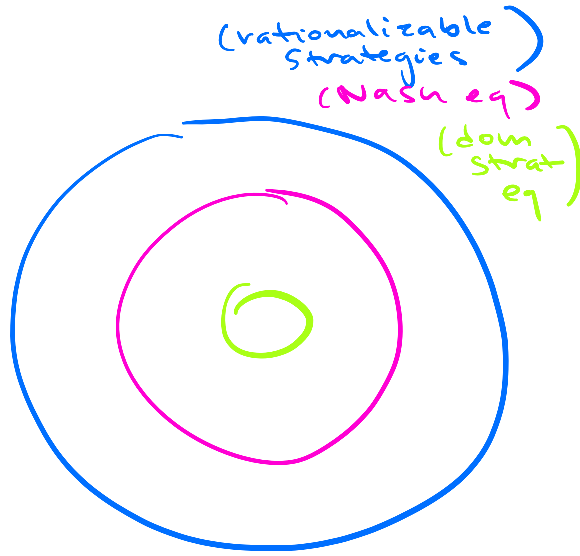
(in DSE, players all playing weakly dominant strategy; implies BR :))

NE : $s^* = (s_1^*, \dots, s_n^*)$ is a NE if

$$u_i(s_i^*, s_{-i}') \geq u_i(s_i, s_{-i}') \quad \forall s_i \in S_i$$

(s_i^* a BR to s_{-i}' for each i)

(but, can have NE that is not DSE)



Thm: if s^* a NE, s_i^* rationalizable $\forall$ player i

↳ in words, all players in NE are playing rationalizable strategies

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Next topic: MSNE

σ^* is a MSNE $\quad \sigma^* = (\sigma_i^*, \sigma_{-i}^*)$

if $\quad u_i(\sigma_i^*, \sigma_{-i}^*) \geq u_i(\sigma_i', \sigma_{-i}^*)$

$$\forall i, \sigma_i' \in \Delta(S_i)$$

[Set of probability distributions on S_i]

How to find? S_1 : strategies for P1] restrict to those that survive iterated dominance ↳ simplifies!
S_2 : " " P2

1) choose all pairs (T_1, T_2) where

$T_1 \in S_1$ & $T_2 \in S_2$

2) create system of equations for indifference conditions & solve
($P1\ IC \Rightarrow \sigma_2^*$; $P2\ IC \Rightarrow \sigma_1^*$)

3) σ_1^* + σ_2^* as found in 2) says what σ^* will be if $\exists$ a NE over supports T_1 & T_2 → still need to check for profitable deviations.

<u>Question</u>: for profitable deviations, do you need to check against other mixtures? or can you just check pure strategies?

turns out, a mixture cannot be a profitable deviation unless at least one of the pure strategies in its support is a profitable deviation ⇒ all you need to check is pure strategies

(in practice, this process is computationally difficult) finding Nash equilibria –

PSNE : underlining approach

MSNE : algorithm outlined above

- rule out anything not ruled out by iterated deletion
- check all combinations of what is left

finding rationalizable strategies:

a strategy is rationalizable if it survives iterated elimination of strictly dominated strategies

↳ order doesn't matter

↳ not so for elimination of weakly dominated strategies (e.g.)

	L	R
U	2,0	2,0
D	0,2	0,0

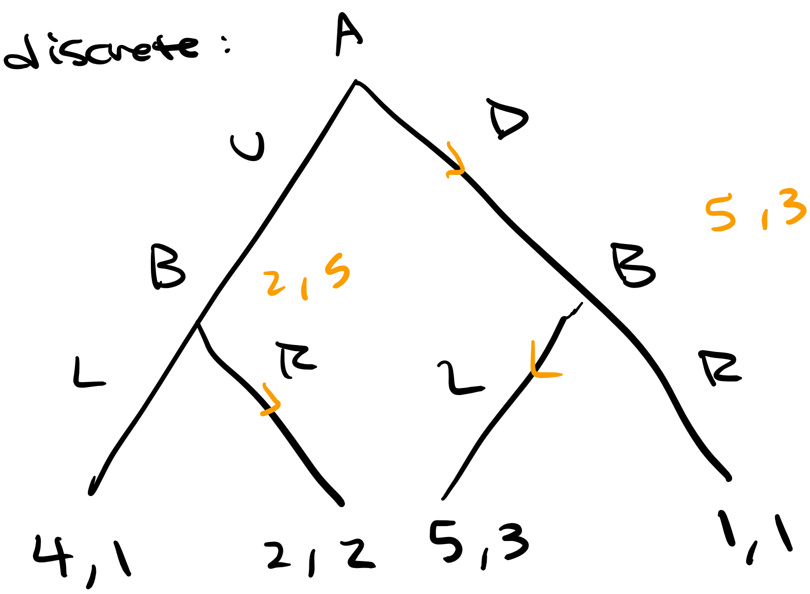
($D^{\text{first}} \to$ U,L ; U,R left)

(R first → only U,L left)

skipping ahead...

NE vs SPNE (sequential move games) (+ backwards induction)

discrete:



SPNE → found via backwards induction

A: D

B: (R, L)

	LL	LR	RL	RR
U	4,1	$\underline{4}$,1	2,$\underline{2}$	$\underline{2}$,$\underline{2}$
D	$\underline{5}$,$\underline{3}$	1,1	$\underline{5}$,$\underline{3}$	1,1

(3 NE)

- 2 do not survive backwards induction

(U,RR - a NE where B has lower payoff than SPNE)

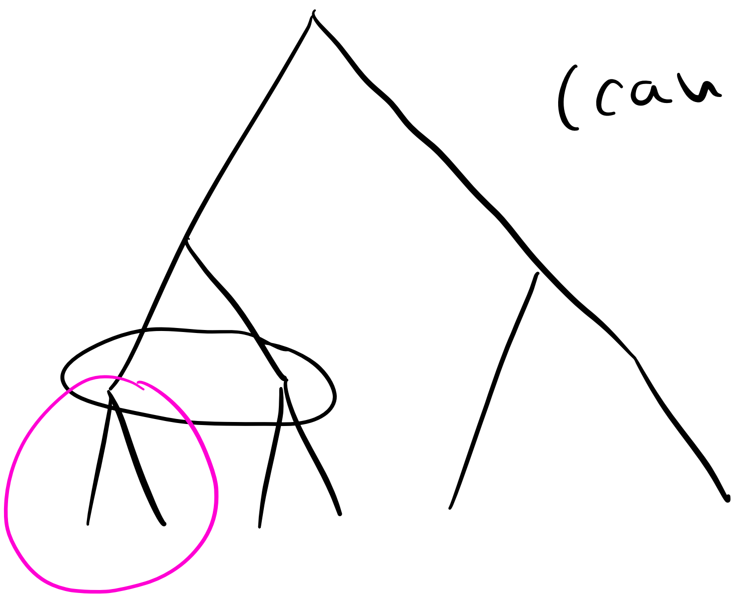
SPNE: a <u>refinement</u> of NE (in dynamic games)

↳ must be a Nash equilibrium in every subgame of the game

(all SPNE are NE; converse is not true)

Recall: what is a subgame?

↳ must have "unique initial node" & all moves and info sets following it must remain in the subgame



(can't cut an info set in order to create a subgame)

↳ not a subgame

SPNE: still must specify what is done @ every info set!!

- generally, always be careful when writing out equilibrium strategies — a strategy is a complete contingent plan!

Some applications :

Ch. 6 - Imperfect Competition

- market w/ multiple firms (overall production levels impact market price)

Cournot (quantities) vs Bertrand
↳ undercutting prices

$Q(P)$ (maps prices to demand)

$C_i(q)$ cost of producing q units for firm i

$C_i'(q)$ marginal cost

each firm: maximizing profit

$Pq - C_i(q)$

Cournot

$P = \max\{1 - \sum_i q_i, 0\}$ (marginal cost $c \geq 0$)

$\pi_i = q_i P_i(q) - c_i(q_i)$ → be careful w/ setup!

(each firm can impact price, depending on how much they produce)

calculate BR of each firm i in response to $Q_{-i} = \sum_{j \neq i} q_i$

(if $Q_{-i} > 1$, best to set quantity to 0)

$\pi_i(q_i, Q_{-i}) = q_i(1 - Q_{-i} - q_i - c)$ ← given assumptions in text

$\frac{\partial}{\partial q_i}$: $1 - Q_{-i} - 2q_i - c$

duopoly - $Q_{-i} = q_j$

(price same across firms)

• both firms have same BR function

$$BR_1(q_2) = \frac{1 - q_2 - c}{2} \Rightarrow q_i^* = \frac{1-c}{3}$$

$$BR_2(q_1) = \frac{1 - q_1 - c}{2}$$

(2 players: can plot these & find intersection)

Rationalizability -

w/ BR fn, q_2 must be ≥ 0

so: anything $> \frac{1-c}{2}$ is not a BR (both players)

(would require q_2 to be < 0)

$(0, \frac{1-c}{2})$

$$\frac{1 - \left(\frac{1-c}{2}\right) - c}{2} \Rightarrow \frac{\frac{1-c}{2}}{2} \quad \frac{1-c}{4} \text{ (lower bound)}$$

q_j never above $\frac{1-c}{2}$; so, anything below

$\frac{1-c}{4}$ not a BR (is a BR to sthg above $\frac{1-c}{2}$)

use $\left(\frac{1-c}{4}, \frac{1-c}{2}\right)$; apply similar process

...

oligopoly - system of equations

↳ multiple firms

Bertrand competition (undercutting prices)

(choosing prices, not quantities)

(all firms price @ marginal cost)

perfect competition

Other applications we have seen —
Bargaining, Negotiations (Pset 5)
(generally: backwards
induction approach

Infinite Horizon games: one shot
deviation principle (for games
that are continuous @ infinity)
↳ defined @ end of Ch. 10

OSD principle: used to check whether
a subgame perfect equilibria

Zero - sum games:
Worst gain / worst loss;
minimax theorem