Solutions to Problem Set 5
14.12 Fall 2023

October 24, 2023

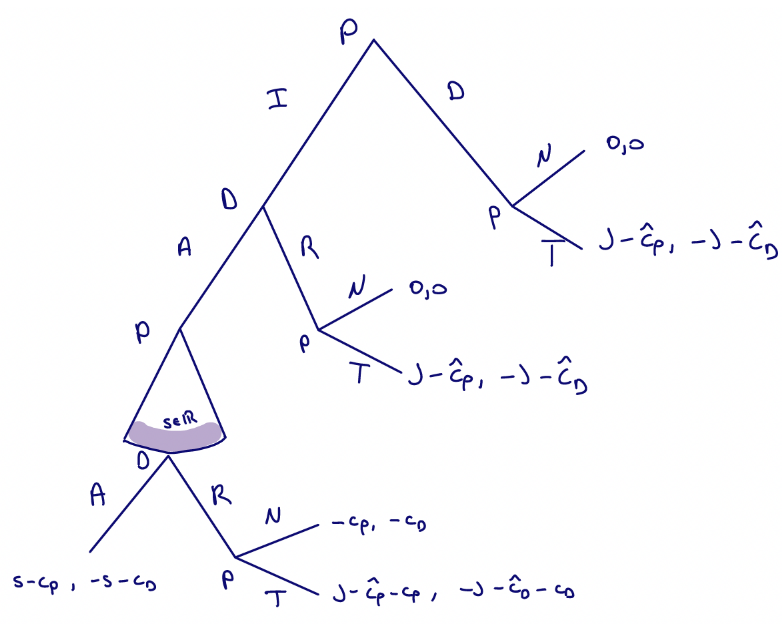
Problem 1

Part a

Note that in this case, the plaintiff is playing a single player game, where T (bringing the suit to the trial) gives a payoff of $J - \hat{c}_P$ while N (not bringing the suit to the trial) gives a payoff of 0. Since $J - \hat{c}_P < 0$ for this part, the SPE of this game can be denoted by $s_P = N$.

Part b

Here is the extensive form



First, let us give the SPE. First, note that the strategy space of the Plaintiff is

$$S_P = \{I, D\} \times \{N, T\}^3 \times \mathbb{R}$$

The Defendant acts twice. They either accept or reject the invitation and the settlement offer. Let $\mathscr{F}$ denote the space of all functions with domain $\mathbb{R}$ and range $\{A, R\}$.[1] Thus the defendant's strategy space is

$$S_D = \{A, R\} \times \mathscr{F}$$

Let

$$f(s) = \begin{cases} A & \text{if} \quad s \leq J + \hat{c}_D \\ R & \text{if} \quad s > J + \hat{c}_D \end{cases} \tag{1}$$

The following are SPE of this game

- $S_P = \{I, T, T, T, J + \hat{c}_D\}$, $S_D = \{R, f\}$
- $S'_P = \{D, T, T, T, J + \hat{c}_D\}$, $S'_D = \{R, f\}$

Now I will describe why these are indeed SPE. It is easy to see under our assumptions, plaintiff must choose T rather than N in all three nodes they are making that choice. Given this, in any SPE, the defendant must use f as the decision rule after an offer.[2] Then clearly the plaintiff offers $J + \hat{c}_D$ as the settlement. However, then it the Defendant get a strictly lower utility if they accept, and in any SPE they must reject. As a result, there are two equilibria where in one, the Plaintiff does not propose a pre-trial negotiation, while in the other they do.

[1] You do not need this formalism in the problem sets or exams, I am writing this because of a few questions I got from students about how to best express strategies in this case. But you must make precise that the strategy of the defendant is a complete contingent plan from the settlement offer to accept reject decisions, not only what they do for the offer they end up receiving in the equilibrium you are characterizing.

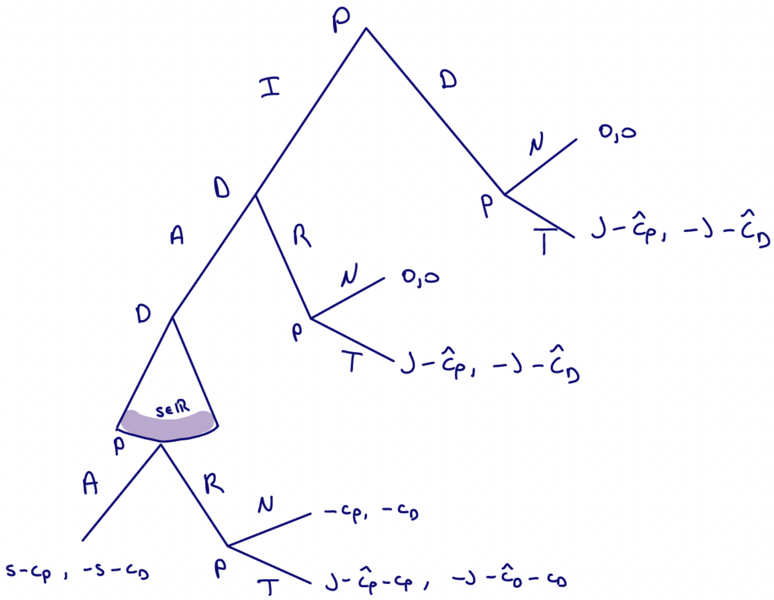
[2] One minor detail is that, the defendant can also optimally use

$$f'(s) = \begin{cases} A & \text{if} \quad s < J - \hat{c}_D \\ R & \text{if} \quad s \geq J - \hat{c}_D \end{cases} \tag{2}$$

However, this cannot be part of any equilibrium as there does not exists a settlement offer that is a best response to this. The reason is that the continuity of the utility of the plaintiff fails, as we covered in examples in the first weeks.

Part c

Extensive form here



The reasoning of the SPE is very similar to the previous case. Let

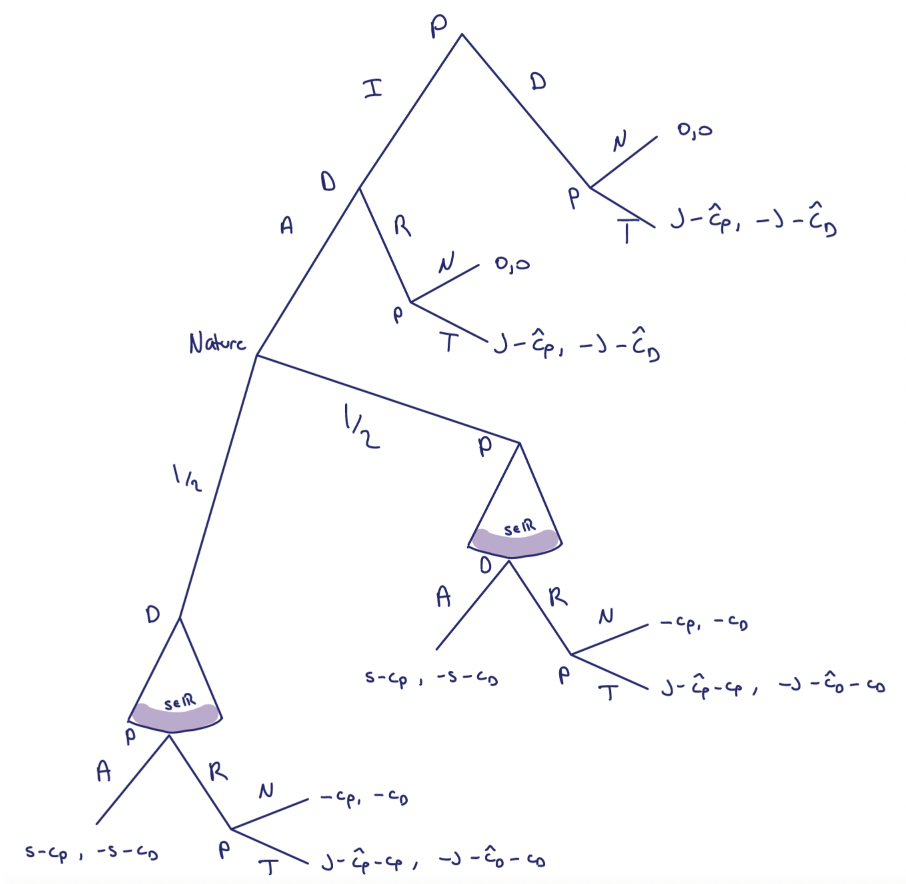
$$g(s) = \begin{cases} A & \text{if} \quad s \geq J - \hat{c}_P \\ R & \text{if} \quad s < J - \hat{c}_P \end{cases} \tag{3}$$

The following is the SPE of this game

- $S'_P = \{D, T, T, T, g\}$, $S'_D = \{A, J - \hat{c}_P\}$

Part d

Here is the extensive form game



We will calculate the parameter space where a SPE with pre-trial negotiation exists. Let V_P and V_D denote the expected payoffs the Plaintiff and the Defendant gets in case of a pretrial negotiation. We have already solved those subgames in previous parts, thus

$$V_P = \frac{1}{2}(J + \hat{c}_D - c_P) + \frac{1}{2}(J - \hat{c}_P - c_P) \tag{4}$$

$$V_D = \frac{1}{2}(-J - \hat{c}_D - c_D) + \frac{1}{2}(-J + \hat{c}_P - c_D) \tag{5}$$

Note that both players can make sure (unilaterally) that the case goes to the trial. Thus, to have a pre-trial negotiation, the following are necessary

$$V_P \geq J - \hat{c}_P \tag{6}$$

$$V_D \geq -J - \hat{c}_D \tag{7}$$

Moreover, you can see that they are also sufficient since if both equalities are satisfied, then there will be a SPE where the Plaintiff proposes the pre-trial negotation and the Defendant accepts it. Rearranging the equations above, we get

$$\max\{c_P, c_D\} \leq \frac{\hat{c}_D + \hat{c}_P}{2} \tag{8}$$

Part e

Rewriting the payoffs under $\alpha \in (0,1)$

$$V_P = \alpha(J + \hat{c}_D - c_P) + (1-\alpha)(J - \hat{c}_P - c_P) \tag{9}$$

$$V_D = \alpha(-J - \hat{c}_D - c_D) + (1-\alpha)(-J + \hat{c}_P - c_D) \tag{10}$$

For the Plaintiff to propose the pre-trial negotiation, we need

$$\alpha(J + \hat{c}_D - c_P) + (1-\alpha)(J - \hat{c}_P - c_P) \geq J + \hat{c}_P \tag{11}$$

Rearranging, we get $\alpha \geq \frac{c_P}{\hat{c}_P + \hat{c}_D}$. For the Defendant to accept the pre-trial negotiation,

$$\alpha(-J - \hat{c}_D - c_D) + (1-\alpha)(-J + \hat{c}_P - c_D) \geq -J - \hat{c}_D \tag{12}$$

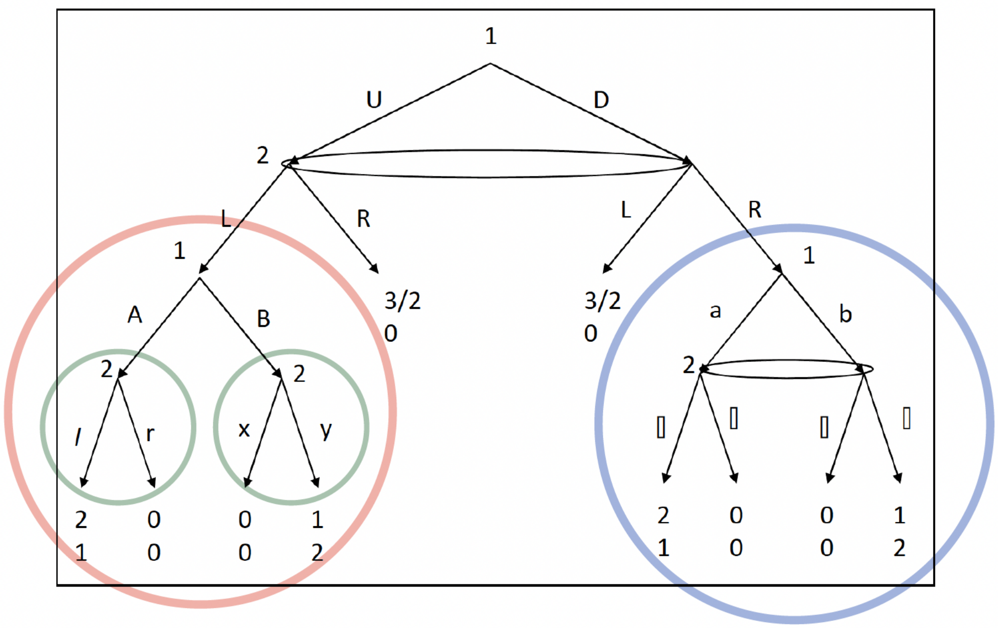
Rearranging, we get $\alpha \leq 1 - \frac{c_D}{\hat{c}_P + \hat{c}_D}$. Clearly, such an α exists whenever

$$1 - \frac{c_D}{\hat{c}_P + \hat{c}_D} \geq \frac{c_P}{\hat{c}_P + \hat{c}_D} \iff \hat{c}_P + \hat{c}_D \geq c_P + c_D \tag{13}$$

Note that this condition corresponds to total pre-trial negotiation costs being lower than the total court costs. This makes sense, since if this is the case, there is a way of allocating the surplus from not going to court (by choosing the right α) that makes both parties prefer pre-trial settlement to the court.

Problem 2

First, let us understand how many subgames are there in this game. The picture below illustrates all subgames apart from the game itself (which is also a subgame).



In any SPE, 2 must play l and y at the information sets in green subgames. Given this, in any SPE 1 must play A in the information set in the red subgame.

Moving to the blue subgame, there are two pure strategy equilibria here. One is where 1 plays a and 2 plays v, the other is where 1 plays b and 2 plays w. Assume the first case. In this case, there are two equilibria in the first game. We write down two SPE strategies (s and s') below (even though our descriptions until now are important, it is very important to give explicit strategies clearly to characterize SPE)

- $s_1 = \{U, A, a\}$, $s_2 = \{L, l, y, v\}$
- $s'_1 = \{D, A, a\}$, $s'_2 = \{R, l, y, v\}$

Next, assume the second case. The SPE stratgies s'' and s''' are

- $s''_1 = \{U, A, b\}$, $s''_2 = \{L, l, y, w\}$

Problem 3

First, lets write down Casey's strategy space. For simplicity, assume Casey observes who he is playing with when they are playing, but does not observe the winning bid until the end of the game. Then Casey has two information sets, denoted by C_A and C_B for the cases he is playing with Alice and Bob. The strategy of Casey is denoted by s_C.

Both Alice and Bob has two information sets. Let h_A and h_B denote their first information set where they are choosing the bids. In this case, their strategies can be denoted by $p_A : h_A \to \mathbb{R}_+$ and $p_B : h_B \to \mathbb{R}_+$. Let W_A and W_B denote the information sets where they win and play with Casey and let s_A and s_B denote their strategies.

First equilibrium is $s_C(C_A) = s_C(C_B) = L$, $s_A(W_A) = T$, $s_B(W_B) = T$ and $p_A(h_A) = p_B(h_B) = 3$. This is a SPE since the strategies when they are playing is a NE and given the outcome of the second stage, $p_A(h_A) = p_B(H_B) = 3$ are best responses to each other.

Second equilibrium is $s_C(C_A) = s_C(C_B) = R$, $s_A(W_A) = B$, $s_B(W_B) = B$ and $p_A(h_A) = p_B(h_B) = 1$. This is a SPE since the strategies when they are playing is a NE and given the outcome of the second stage, $p_A(h_A) = p_B(h_B) = 1$ are best responses to each other. Both equilibria are sensible as in both cases the winner pays all expected payoff to Casey.