14.12 Week 4 Recitation!!

Margaret Zheng (mzheng01@mit.edu)
Fridays 3-4pm, E51-361

Logistics :O

- No pset due next week!
- PSET 5 not out yet, due Monday Oct 23 :D have a good weekend!!
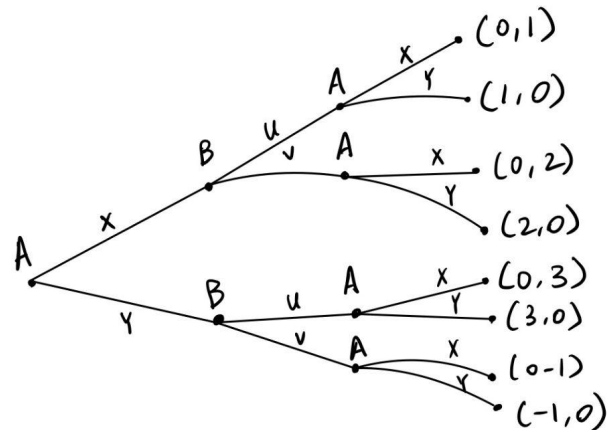- In class midterm in ~2 weeks: Thursday, October 26th, 10:30-12:00, 4-237

Topics

- Multi-stage game
- SPNE (Subgame-perfect Nash Equilibrium)
    - What is a subgame?
    - SPNE
    - Examples

Multi-stage Games

… is a game where one does not finish the
game in 1 round!

- At least 1 player has more than 1 round
  to play; like the example here ->
- More examples:
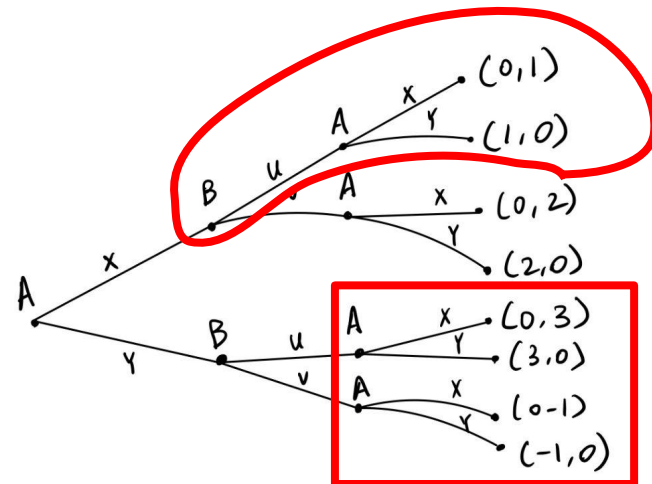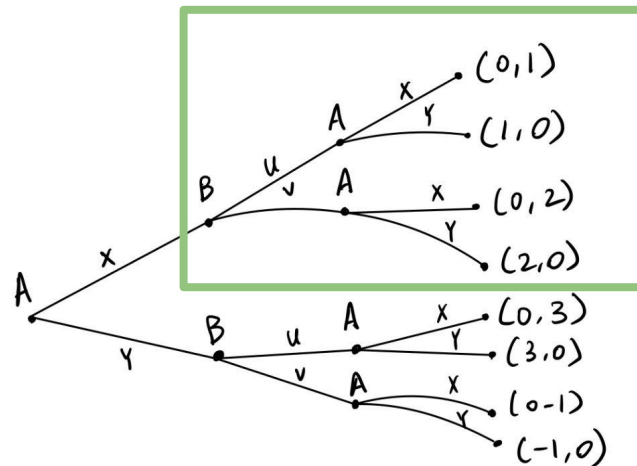 - Many backward induction games
  - Cartels
  - Negotiation

Subgames (Formally)

A subgame of a well-defined game satisfy the following 2 criterias:

- It has a unique "initial node"
- All the moves and information sets from the node must be in this subgame.

Subgames (Less Formally)

- In extensive form game trees, one can identify smaller trees within the big tree
  - The small tree you identify must start from 1 unique "initial node" and you can't cut branches from them
- Examples:
  - The green boxed part of this game is a subgame
  - The red enclosed parts of this game are NOT subgames

Subgame properties

- The full game is also a subgame!
- A <u>proper subgame</u> is a subgame that is formed from a strictly smaller set of nodes than the set in the original game
- A subgame must start at a trivial information set - that is, no other nodes can share an information set with your starting node.

Previously, on Nash Equilibria:

- Given something like this ->
- Find pure or mixed strategies of NEs
- Each player has 1 move (action) only

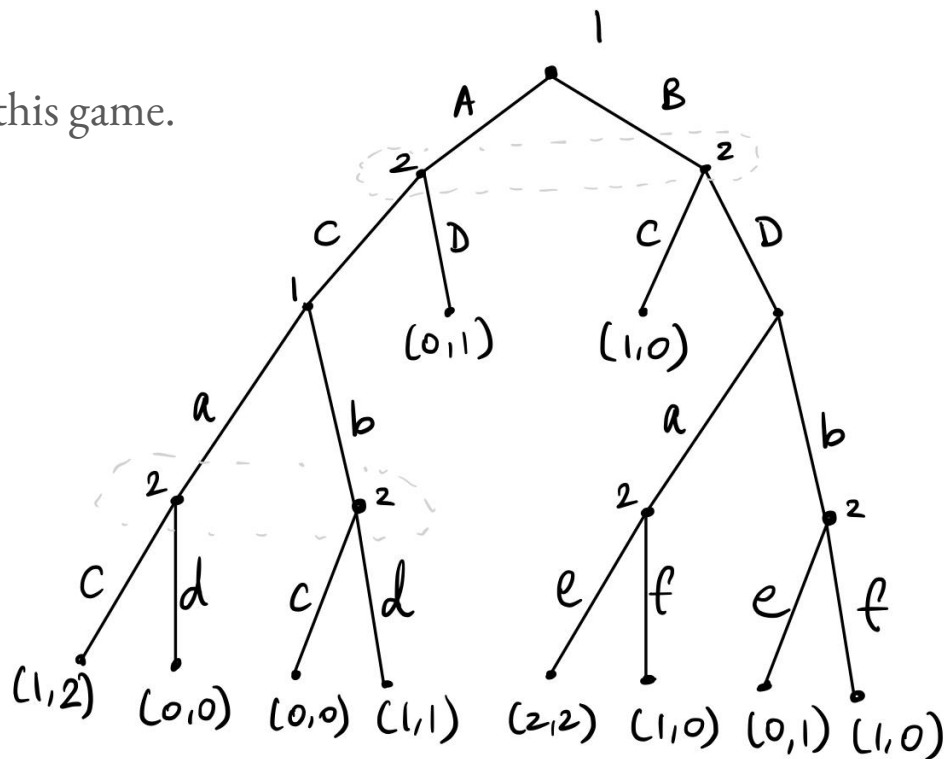|	L	M	R
U	(2,1)	(1,1)	(0,0)
C	(1,2)	(3,3)	(2,1)
D	(2,-2)	(1,0)	(-1,-1)

Subgame Perfect Nash Equilibrium

- A <u>Nash equilibrium</u> is said to be <u>subgame perfect</u> if it is a Nash equilibrium in every subgame of the game.
- A SPNE is an NE because the entire game is also a subgame - but the converse isn't true!
- You can have a nash equilibrium that isn't subgame perfect.

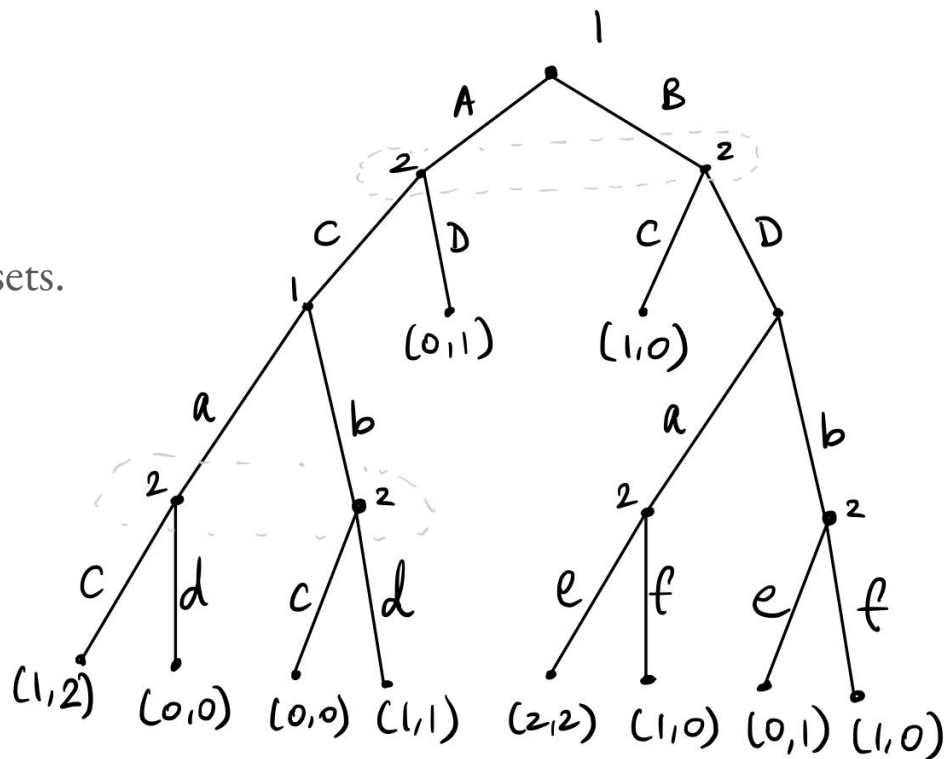Let's walk through a problem!

Example problem - SPNE

Find all the pure strategy SPNEs of this game.
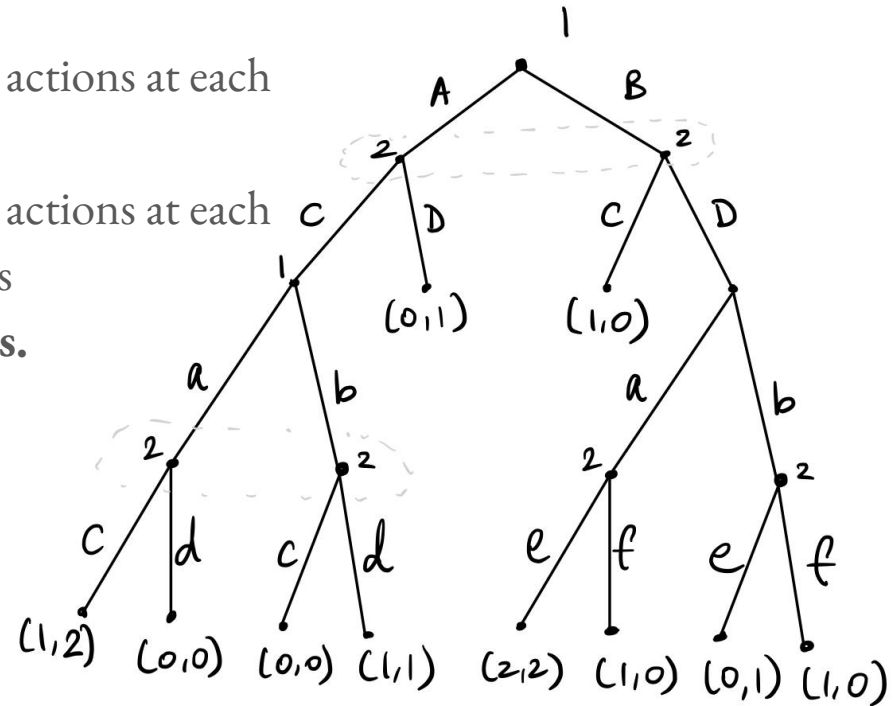
Example problem - SPNE

Part 1. How many possible strategy profiles are there?

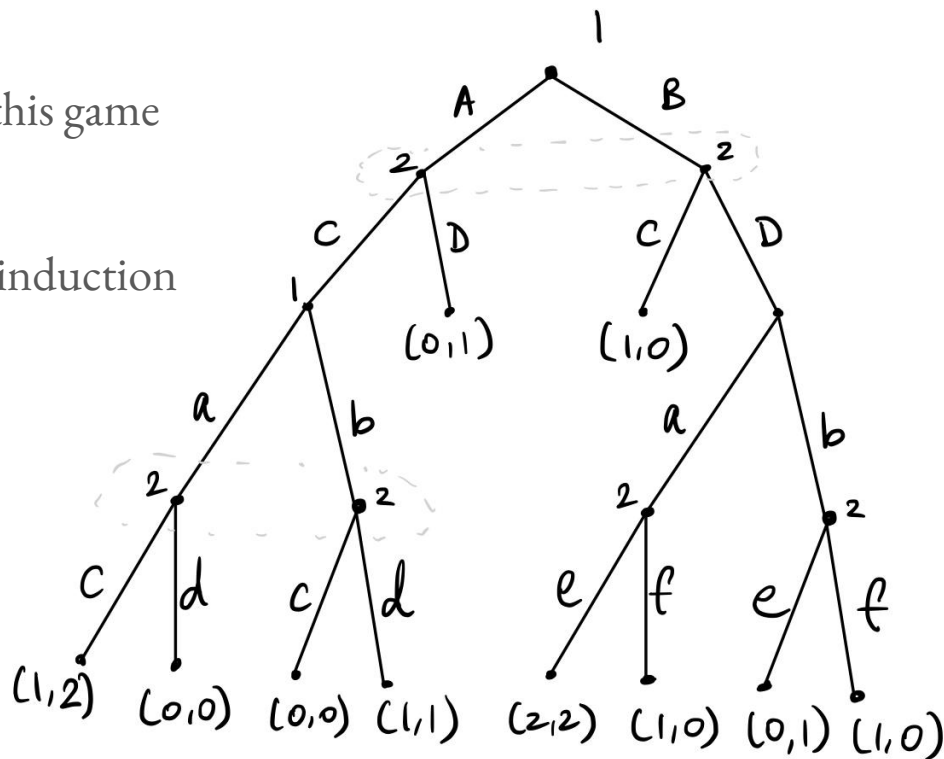Hint: pay attention to information sets.

Answer - Part 1

- Player 1 has 3 information sets and 2 actions at each information set - > 2^3 = 8 strategies
- Player 2 has 4 information sets and 2 actions at each information set -> 2^4 = 16 strategies
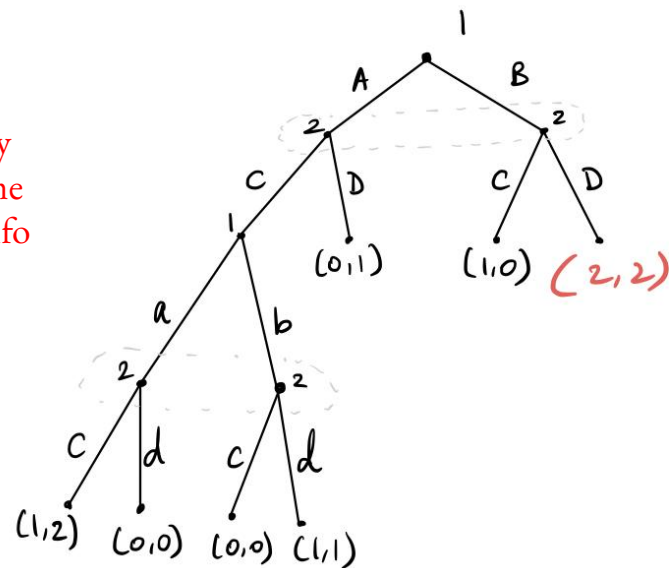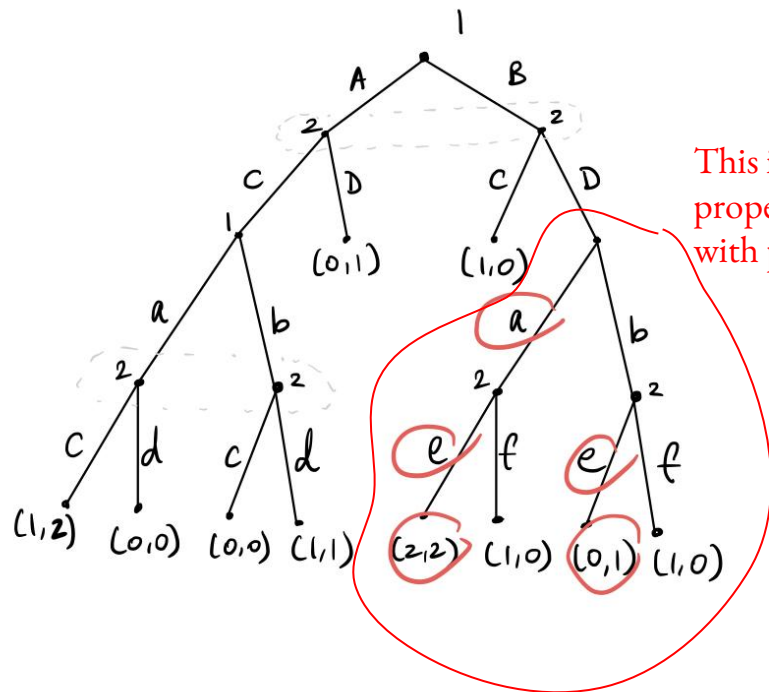- **8 * 16 = 128 total strategy profiles.**

Example problem - SPNE

Part 2: Can you reduce anything in this game using backward induction?

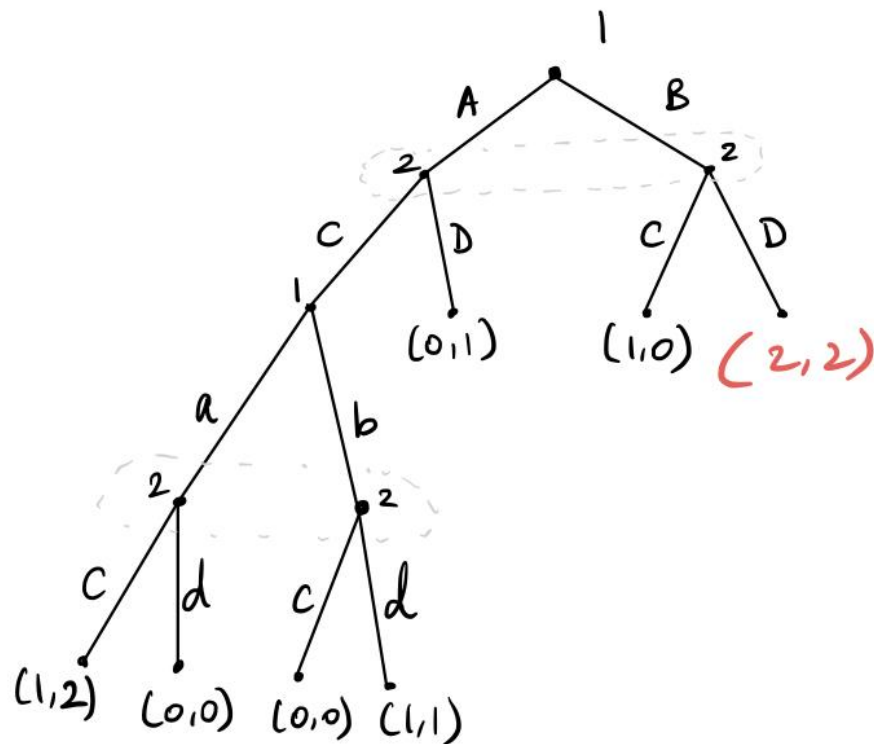Hint: you can only apply backward induction when there is perfect information.

Answer - Part 2

This is the only proper subgame with perfect info

Example problem - SPNE

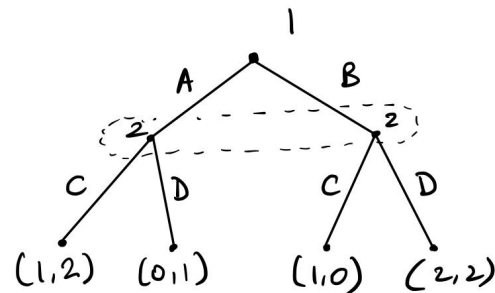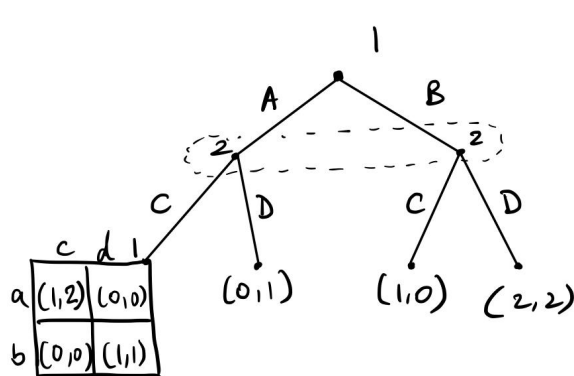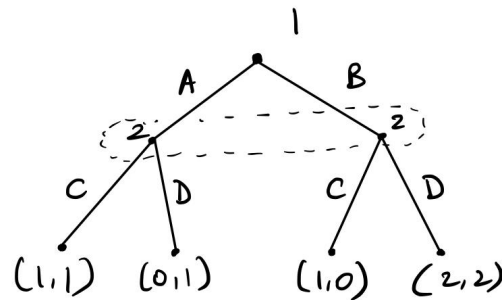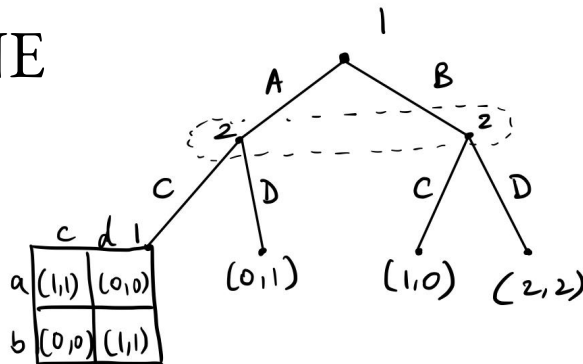Part 3. Let's solve for all the SPNEs!

Example problem - SPNE

Part 3. Let's solve for all the SPNEs!

There are 2 cases:

1. The NE on the left side is (1,1)
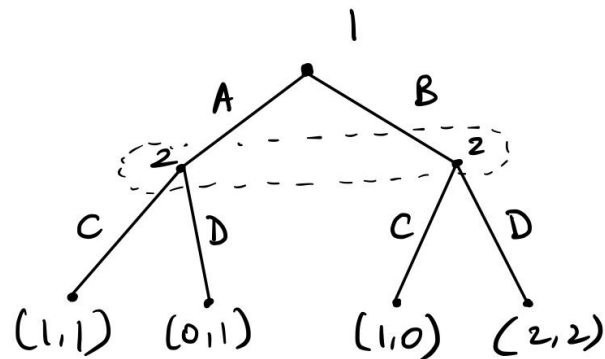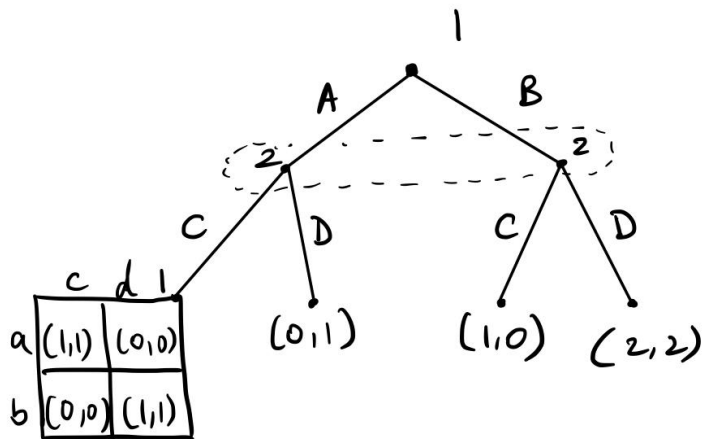2. The NE on the right side is (1,2)

We need to treat those 2 cases separately!

Example problem

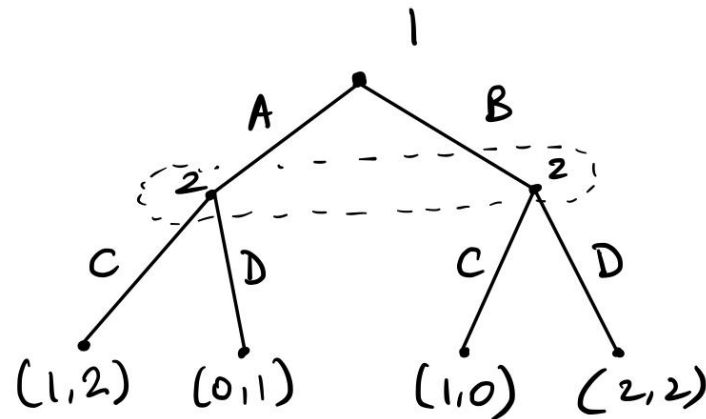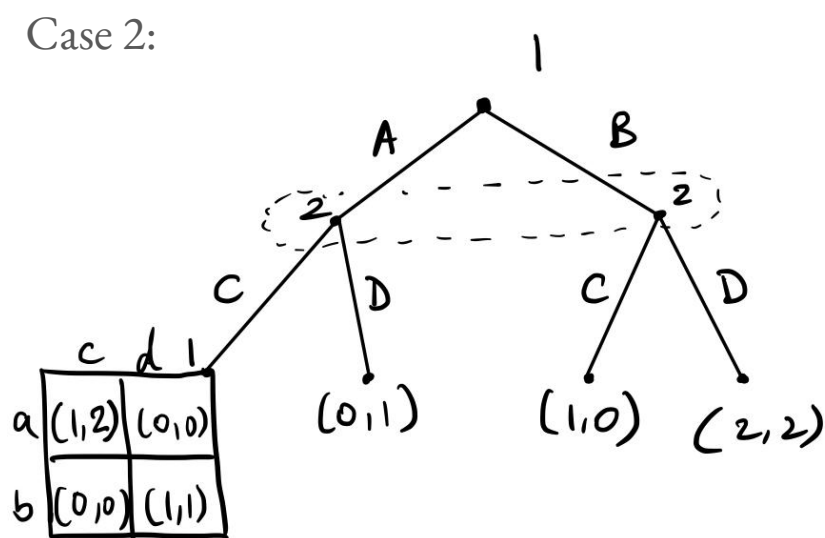Part 4. We are now ready to put everything together!

Case 1:

If the subgame NE ended up being (1,1), then we solve the game on the right. The NE of this game are (A,C) and (B,D).

	C	D
A	(1,1)	(0,1)
B	(1,0)	(2,2)

Example problem

Case 2:

If the subgame NE ended up being (1,1), then we solve the game on the right. The NE of this game are also (A,C) and (B,D).

	C	D
A	(1,2)	(0,1)
B	(1,0)	(2,2)

Example problem: all together now!

We summarize what we have done before:

- Step 1: in the backward induction step (right part of game tree), we concluded that player 2's strategy will be **e** for both scenarios and player 1's strategy will be **a**.
- Step 2: in the subgame on the left, we found 2 possible pure strategy NE, which are (a,c) and (b,d). They have different payoffs and we need to separately plug the payoff values in to solve the rest of the game.
- Step 3: In both versions of the game, (A,C) and (B,D) are the pure strategy NEs.
- Step 4 (Conclusion): There are **2*2 = 4 pure Nash Equilibria:**

{(Aaa,Ccee), (Aba, Cdee), (Baa, Dcee), (Bba, Ddee)}

- You can represent this in whatever order you would like but pls make sure that they are consistent!!

Problem Solving Strategies