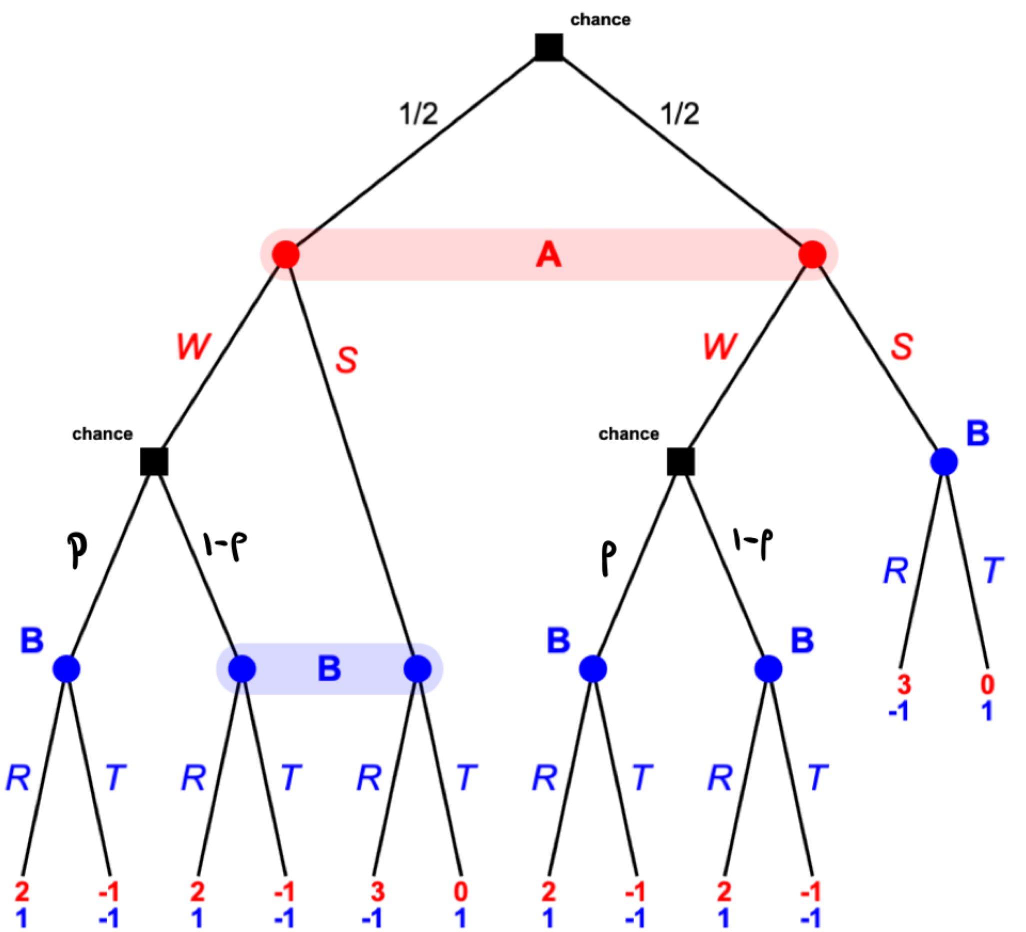
Solutions to Problem Set 1
14.12 Fall 2023

September 2023

Problem 1

Part 1



Part 2

Players: $N = \{Ann, Bob\}$. Ann has only one information set and two actions, so we can represent Ann's strategy space by $S_A = \{W, S\}$. Bob has 5 information sets and two actions available in each of them, so we can represent Bob's strategies as $S_B = \{R, T\}^5$.

To illustrate how payoffs are computed, we will compute the payoffs for a particular strategy profile: $s_a = W$ and $s_b = (R, T, R, R, T)$, in words, Ann works and Bob renews if (i) he cannot observe whether Ann works, and sees a success (ii) he can observe whether Ann works and Ann works, and terminates otherwise.

$$u_A(s_a, s_b) = \frac{1}{2}(2) + \frac{1}{2}\left(p(2) + (1-p)(-1)\right) = 1 + \frac{3p-1}{2} = 1/2 + \frac{3p}{2} \tag{1}$$

since Ann gets renewed if Bob can observe (which happens with probability 1/2) and if Bob cannot observe but project is success (which happens with probability p/2); but in the case of failure, which happens with probability (1 - p)/2, she loses -1 from working.

$$u_B(s_a, s_b) = \frac{1}{2}(p + (1-p)) + \frac{1}{2}\left(p - (1-p)\right) = 1/2 + \frac{2p-1}{2} = p \tag{2}$$

since Bob gets gets 1 if he can observe Ann working and renew the contract (which happens with probability 1/2) and if he cannot observe Ann working but project is success (which happens with probability p/2) while he loses -1 if the project fails (which happens with probability (1-p)/2) and he terminates Ann. The rest of the payoffs can be calculated in this fashion.

As another example of how payoffs are calculated, consider the following strategies: $s_a = S; s_b = \{R, R, R, R, T\}$ - Ann shirks, and Bob will renew unless he sees that Ann is shirking.

Payoffs are:

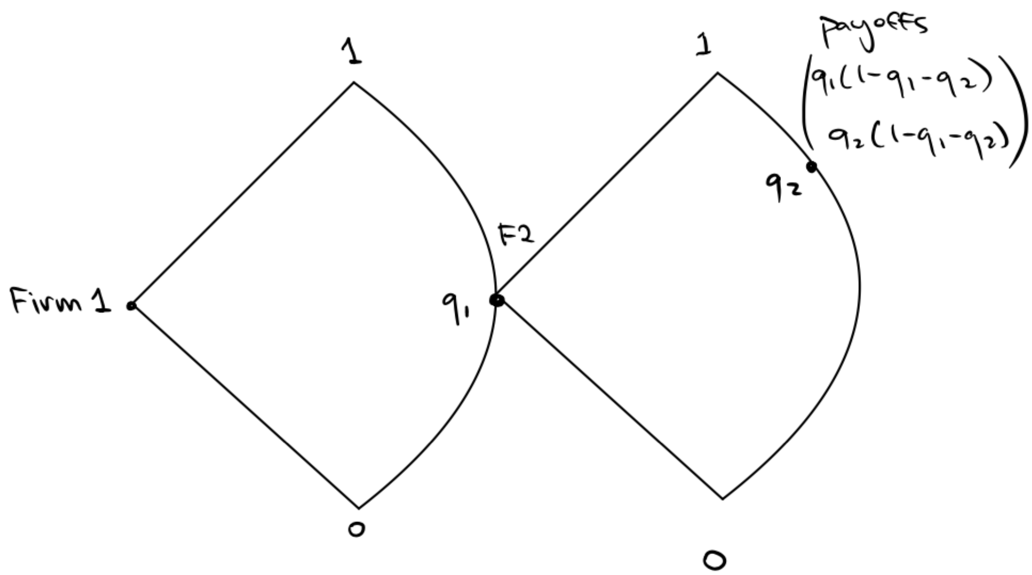
$$u_A(s_a, s_b) = \frac{1}{2}(3) + \frac{1}{2}(0) = \frac{3}{2} \tag{3}$$

$$u_B(s_a, s_b) = \frac{1}{2}(-1) + \frac{1}{2}(1) = 0 \tag{4}$$

Note that for these strategies, the payoffs do not depend on p!

Problem 2

Part 1



Part 2

Players: $N = \{Firm1, Firm2\}$.

Firm 1 has one information set, and an infinite choice of actions available, so $S_1 = \{q_1 \in [0,1]\}$.

Firm 2 can choose anything in response to $q_1 \in [0,1]$ (infinite information sets). One way to write this is $S_2 = \{q_2 \in [0,1]\}^{[0,1]}$. For an example of a potential strategy, Firm 2 could always choose $q_2 = 0.5 \forall q_1$. Then, in this case, payoffs would be (denote $S_1 = q_1$).

$$u_1(S_1, S_2) = q_1(1 - q_1 - 0.5)$$

$$u_2(S_1, S_2) = 0.5(1 - 0.5 - q_1)$$

Or, suppose that S_2 is to choose $q_2 = q_1$. Then, payoffs would be:

$$u_1(S_1, S_2) = q_1(1 - 2q_1)$$

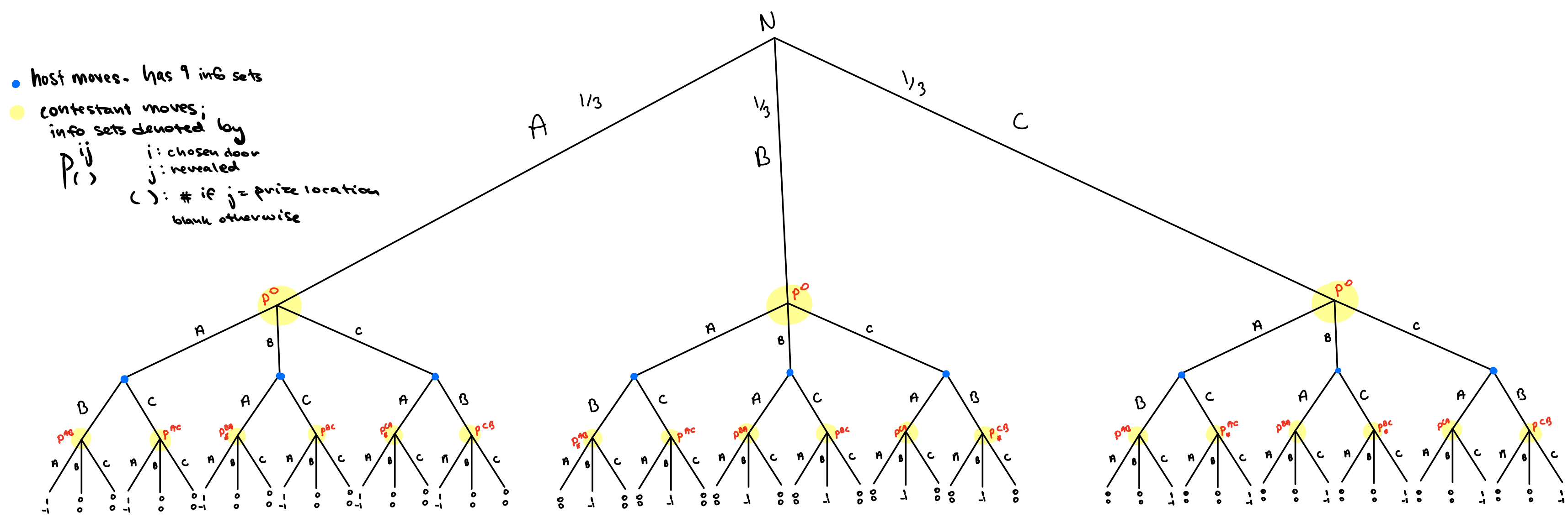
$$u_2(S_1, S_2) = q_1(1 - 2q_1)$$

Generally, for $S_1 = q_1$, and $S_2 = q_2(q_1)$, payoffs are:

$$u_1(S_1, S_2) = q_1(1 - q_1 - q_2(q_1))$$

$$u_2(S_1, S_2) = q_2(q_1)(1 - q_2(q_1) - q_1)$$

2.13



• host moves. has 9 info sets
contestant moves; info sets denoted by $P^{ij}_{(\,)}$
i: chosen door
j: revealed
( ): # if j = prize location
blank otherwise
N
A 1/3
1/3 B
1/3 C

Problem 3

Part 1

(Extensive form above)

The key thing is to observe that the following scenarios belong to the same information set (which has 2 nodes)

- Prize behind A, Contestant chooses A, Host reveals B

- Prize behind C, Contestant chooses A, Host reveals B

whereas the scenario where "Prize behind B, Contestant chooses A, Host reveals B" belongs to an information set with a single node, since in this case the Contestant also observes the prize is behind door B. In the extensive form, we labeled the nodes in the first groups as P^{AB} since door A is chosen and door B is revealed, and there is not prize behind door B, while we labeled the node in the second group as P^{AB}_{*} since door A is chosen and door B is revealed, and there is a prize behind door B.

Part 2

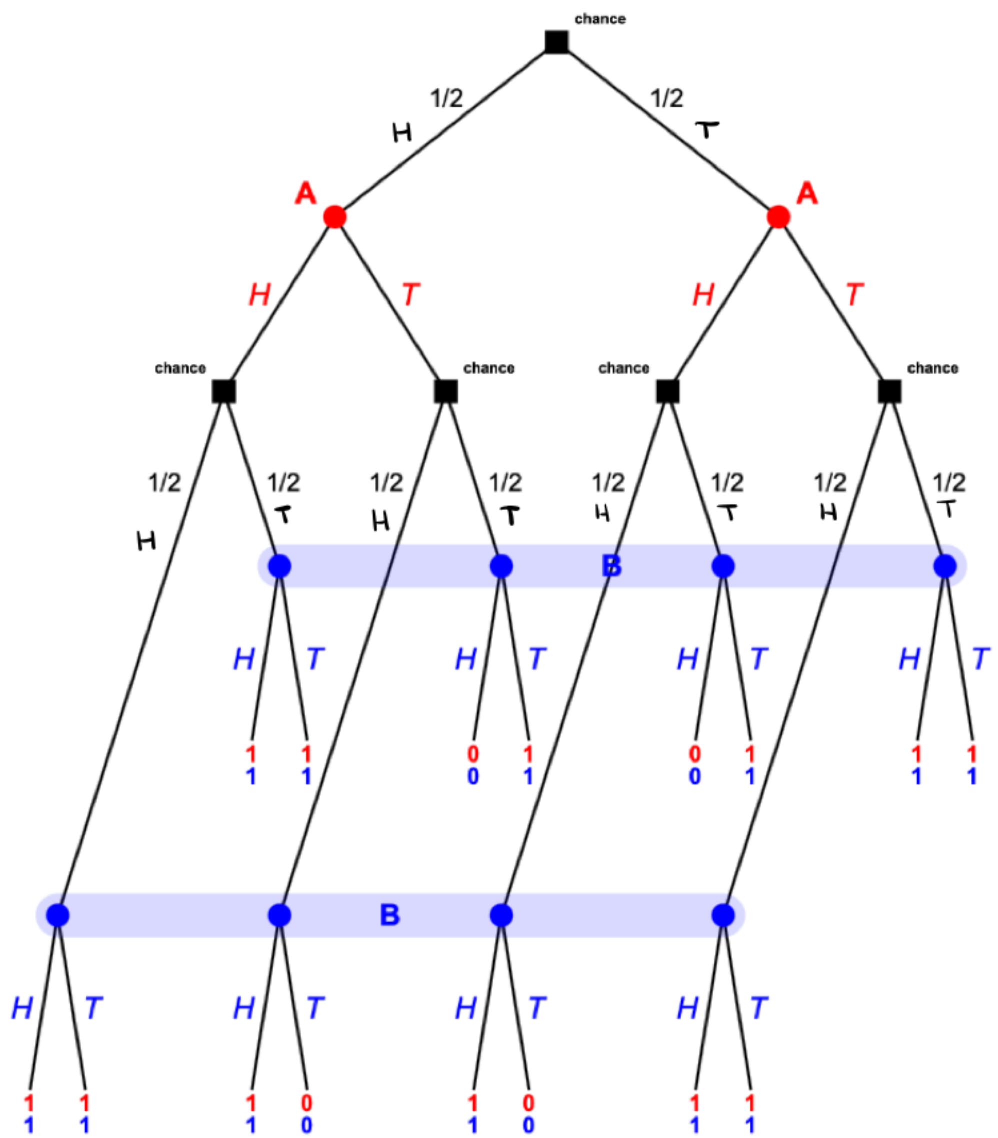
The contestant has 13 different information sets. In all those information sets, they can choose any door, so the set of available actions would be $\{A,B,C\}$. We can represent the strategy space of the Contestant by $S_C = \{A,B,C\}^{13}$. The number of strategies available to the Contestant is 3^{13}.

Host has 9 different information sets. $\{A,B\}$, $\{A,C\}$ and $\{B,C\}$ are the available actions for 3 different information sets. We can represent the strategy space of the Host by $S_H = \{A,B\}^3 \times \{A,C\}^3 \times \{B,C\}^3$. The number of strategies available to the Host is 2^9.

A potential strategy for the contestant is to choose A at all information sets (i.e., choose A to start, and then pick A regardless of the door that the host opens and what it reveals).

Problem 4

Part 1



Here, we note that both Alice and Bob have 2 information sets (they observe one coin flip, and see whether it is Heads or Tails).

In particular, Bob cannot distinguish between the following: (coin flip 1 is Heads, coin flip 2 is Heads, Alice guesses Heads); (coin flip 1 is Heads, coin flip 2 is Heads, Alice guesses Tails); (coin flip 1 is Tails, coin flip 2 is Heads, Alice guesses Heads); (coin flip 1 is Tails, coin flip 2 is Heads, Alice guesses Tails).

Part 2

Here is the normal form, where Ann is the row player and Bob is the column player.

	HH	HT	TH	TT
HH	3/4,3/4	3/4,3/4	3/4,3/4	3/4,3/4
HT	3/4,3/4	1/2,1/2	1,1	3/4,3/4
TH	3/4,3/4	1,1	1/2,1/2	3/4,3/4
TT	3/4,3/4	3/4,3/4	3/4,3/4	3/4,3/4

Part 3

As we can see from the normal form, (perhaps surprisingly) strategies exist where they can be released with probability one: If one of them chooses the outcome they see while the other chooses the reverse, then they get out by probability one. The reason is that when they do this, if Alice guesses what she sees and is wrong, this means that the realization of coin toss is different in both rooms (as otherwise, Alice would have won), and Bob will be choosing he reverse and their release is guaranteed.

If they both guess what they see, if Alice is wrong, again the realization of coin toss is different in both rooms. But this time, Bob will be guessing what he sees, and they will not be released. Thus they are not released unless Alice is right.