14.12 Week 1 Recitation!!

Margaret Zheng (mzheng01@mit.edu)

Fridays 3-4pm, E51-361

OH: Wednesday 3-4pm, E52-548

(unless otherwise announced)

Sign-in

Logistics

- PSETs: 25% grade, lowest dropped, usually due on Fridays
- Midterm: 30% grade
- Final: 45% grade
- **In-class midterm: Thursday, October 26 (4-237).**
- **Final: time and place TBD. Check on finals.mit.edu next week.**
- If you have conflicts or need extensions on PSETs - **contact S^3** (Student Support Services) first!!! We can't give extensions unless you send an email to us where an S^3 dean is cc'ed.
- Also: next Friday (Sep 22) is a student holiday! No classes or recitations <3

Office Hours:

Prof. Sadzik: Tuesday 2-3pm, E52-428

(or by appointment: tsadzik@mit.edu)

Anne: Thursday 2-3pm, E52-516

Win: Thursday 1-2pm, E52-416

Margaret: Wednesday 3-4pm, E52-548

Topics

- Ch.2: Representation of Games
    - Extensive form
        - Information set
        - Strategy
    - Normal form
    - Belief
    - Best Response
    - Mixed Strategy
- Ch.3: Dominance
    - Dominance
    - Dominant-strategy Equilibrium

~~Let’s play games yay~~

Ch.2
Representation of Games
(mostly review)

Representation of a game

Disclaimer: this is not exactly how game theory works. And also, don't do this. (But hopefully it is somewhat memorable.)

Representation of a game

We want to know:

- Who the players are
- What actions they can take
- How much they value each outcome
- What each player knows

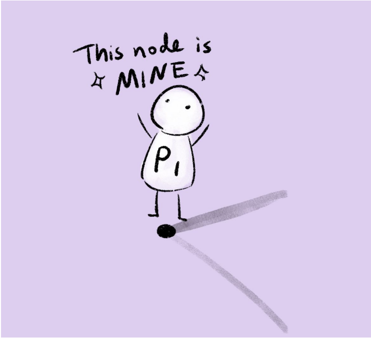
Extensive form: using <u>information sets</u> and <u>game trees</u> to represent a game

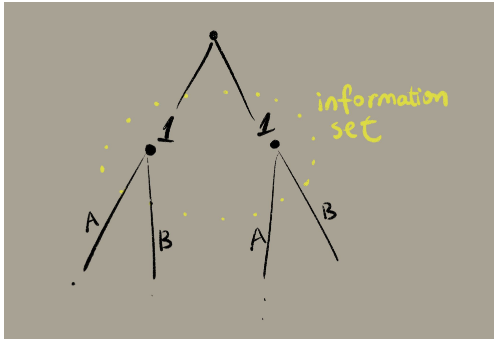
(great if you like visualizations and the game isn't huge)

Normal form: use <u>strategies</u> to summarize the above

(great if the game is really huge, contains many continuous moves, etc)

Extensive Form





Extensive Form

An extensive form game consists of 5 things:

- The set of players
- A tree
- An allocation of non-terminal nodes of the trees to the players (i.e., imagine every node occupied by a player)
- An information partition of the nodes
- Payoffs for each player at each terminal node (i.e., how much does each player get at the end of the game?)

Information Set

Definition 2.4. An *information set* is a collection $I \subseteq H \backslash Z$ of decision nodes such that

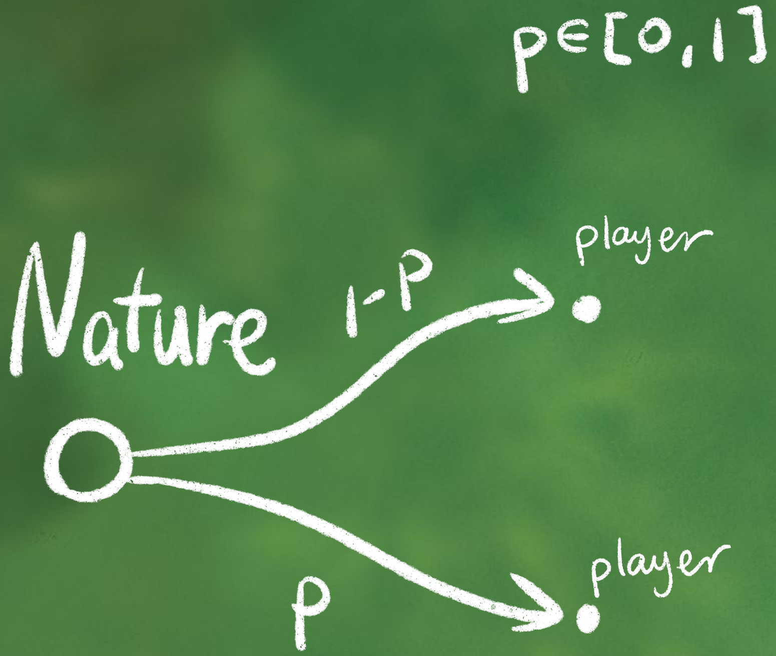
1. the same player i is to move at each of these nodes (i.e., $\iota(h) = \iota(h')$ for all $h, h' \in I$);

2. the same moves are available at each of these nodes (i.e., $A(h) = A(h')$ for all $h, h' \in I$).

Definition 2.5. An *information partition* is an allocation of each non-terminal node of the tree to an information set; the initial node must be "alone".

Nature?

- Nature should be represented by an empty circle
- Nature is NOT a player and therefore does not have strategies
- Instead of moves, what stems out of nature nodes are probabilities; the branches that stem out of each nature node sum to 1)



What exactly is a strategy?

1. In words: A **strategy** of a player is a complete contingency-plan that determines which action to take at each information set.

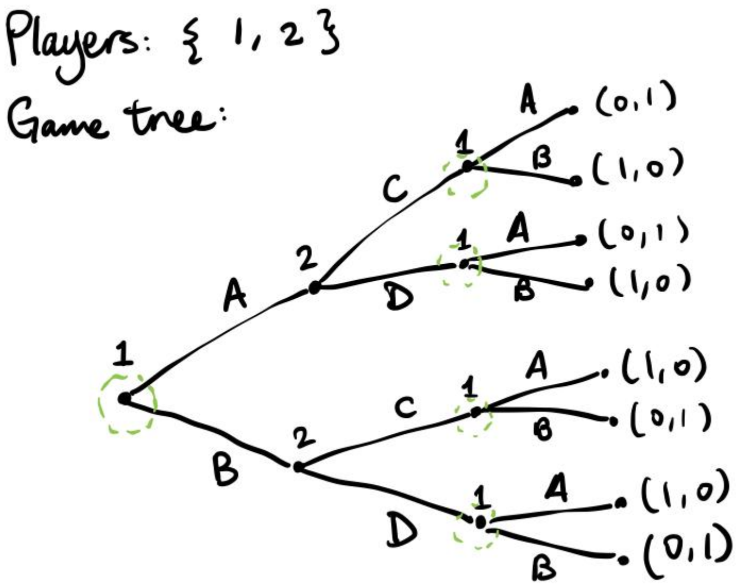
   TLDR, informally: if player P have n information sets, its strategy should be n-letters-long.

2. More mathematically: A **strategy** s_i for player i maps each information set h_i of player i to an action that is available at h_i
3. 3 important notes:
   a. Every information MUST have a move assigned!
   b. The assigned move must be available at the information set.
   c. At all nodes that belong to the same information set, the player plays the same move!

Excel in strategic thinking!!

Example game:

- 2 players, P1 and P2
- 1 can choose moves {A,B} and 2 can choose moves {C,D}
- 1 chooses first, then 2 chooses, then 1 chooses again
- Everyone can observe each move



Excel in strategic thinking!!

Making a strategy is like filling a list of drop-down menus in excel!

Sample strategy	**A**	**B**	**B**	**A**	**A**
	"At the first round, player 1 chooses a move from {A,B}"	"At the second round, knowing that player 1 has chosen A in the first round and player 2 chose C, player 1 chooses a move from {A,B}"	"At the second round, knowing that player 1 has chosen A in the first round and player 2 chose D, player 1 chooses a move from {A,B}"	"At the second round, knowing that player 1 has chosen B in the first round and player 2 chose C, player 1 chooses a move from {A,B}"	"At the second round, knowing that player 1 has chosen B in the first round and player 2 chose D, player 1 chooses a move from {A,B}"
Information sets					

A

B

Example - PSET 1 Q3

One valid strategy for the contestant:

AAAAAAAAAAAAA

because there are 13 information sets and in every information set, the contestant can choose from {A,B,C}

“it’s so long!” [scream] And that’s when normal form comes in a bit more handy…

Normal Form

A normal form of a game is a list

$G = (N, S_1, S_2, \ldots, S_n, u_1, u_2, \ldots, u_n)$

Where

- $N = \{1,\ldots,n\}$ is the set of players
- S_i is the set of all strategies available to i,
- $u_i : S_1 \times S_2 \times \ldots \times S_n \rightarrow \mathbb{R}$ is the utility function of i (given all the strategies for every player, what will i get).

G is common knowledge: every player understands it!

Strategy Profile

- Big "S" and small "s" are slightly different :D
- A **__strategy profile__** **$s = (s_1, s_2, ..., s_n)$** is a list of strategies, one for each of the n players.
  - Here, s_1 is a specific strategy for player 1.
  - Think of this as a very specific situation
- The set of all strategy profiles is

$$\mathbf{S = S_1 \times S_2 \times \ldots \times S_n}$$

Where S_i is the set of all strategies available to player i.

Example Question - Margaret and Wide Tim

In this game, the players are {Margaret, Wide Tim}. Margaret has 3 possible moves: {**E**at, **S**leep, **P**rocrastinate}. Wide Tim has 2 possible moves:{**E**at, **S**leep}.

There is a 0.5 probability that Margaret will have a good day, and when she does, she chooses from {Eat, Sleep}. However, if she does not have a good day, she will choose from {Sleep, Procrastinate}.

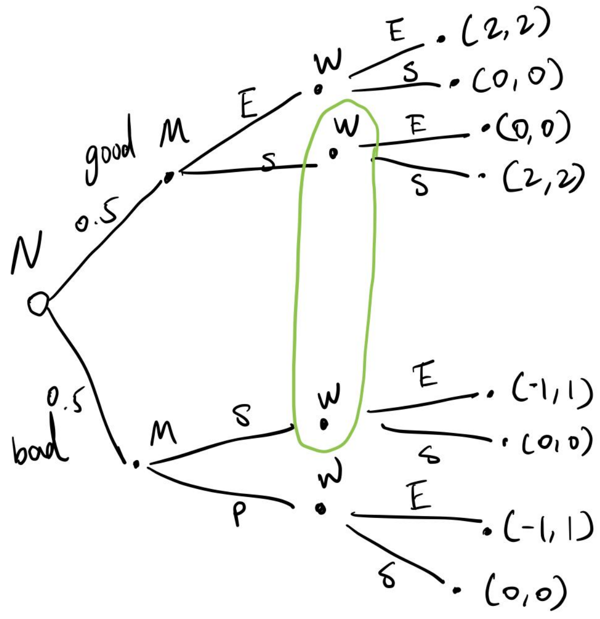
After Margaret executes a move, Wide Tim chooses from {Eat, Sleep}. He observes Margaret's previous moves but does not know if she has had a good day or not.

The payoff is (2,2) if Wide Tim does the same activity as Margaret. The payoff is (-1, 1) if Wide Tim chooses to eat on a bad day. The payoff is (0,0) otherwise.

Write the extensive form of this game, and calculate how many strategies each player has. Then pick your favorite strategy profile and calculate the expected payoff.

Example - Margaret and Wide Tim - Solution

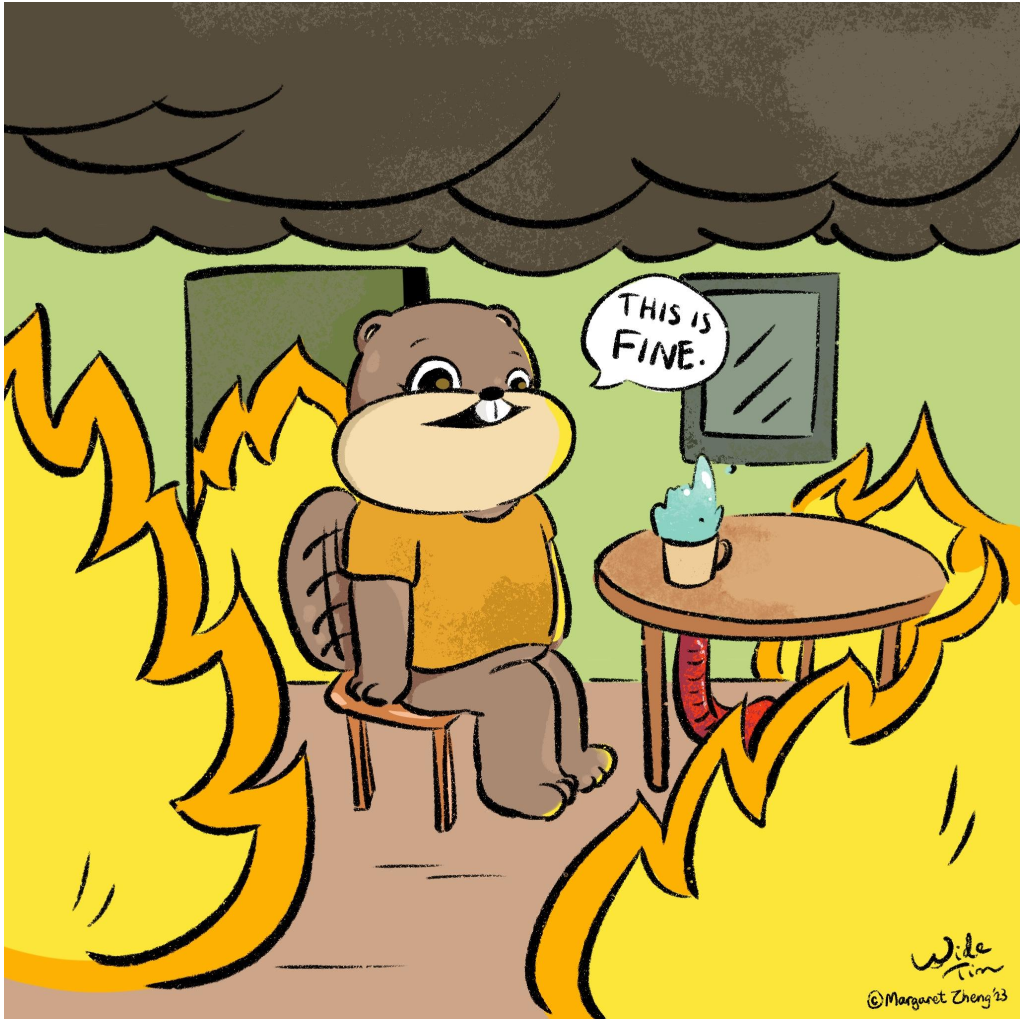
Extensive form:



Margaret has 2*2 = 4 strategies.
Wide Tim has 2*2*2 = 8 strategies.

One strategy would be **SSEE** (i.e., Margaret always sleeps, and Wide Tim always eats.) The payoff is

$u_M = 0.5*0+0.5*(-1) = -0.5$
$u_W = 0.5*0+0.5*1 = 0.5$


THIS IS FINE.
Wide Tim
©Margaret Zheng'23

Some preparatory notations

Notation 2.1. For any player $i \in N$, the notation s_{-i} denotes the list of strategies s_j played by all the players j other than i, i.e.,

$$s_{-i} = (s_1, ...s_{i-1}, s_{i+1}, ...s_n).$$

The set of other players' strategies is denoted by

$$S_{-i} = \prod_{j \neq i} S_j.$$

Finally, the notation (s_i, s_{-i}) denotes the strategy profile in which i plays s_i and other players play according to s_{-i}; i.e., $(s_i, s_{-i}) = (s_1, ...s_{i-1}, s_i, s_{i+1}, ...s_n)$.

Mixed Strategy

"At this specific information set, instead of playing A for 100% of the time, I'm going to play A for 50% of the time and play B 50% of the time"

Definition 2.12. A *mixed strategy* of a player i is a probability distribution over the set S_i of her strategies; the strategies $s_i \in S_i$ are also called *pure* strategies.

Belief

- Informal example: “Player 1 thinks player 2 is going to play move A with probability 70%, and B with probability 30%.”

Definition 2.10. A *belief* of player i (about other players' strategies) is a probability distribution β_{-i} on S_{-i}.

Write $u_i(s_i, \beta_{-i})$ for the expected payoff from playing s_i under belief β_{-i}. When S_{-i} is finite, this is computed as

$$u_i\left(s_i, \beta_{-i}\right) = \sum_{s_{-i} \in S_{-i}} u_i\left(s_i, s_{-i}\right) \beta_{-i}\left(s_{-i}\right).$$

Best Response

Definition 2.11. For any player i, a strategy s_i is said to be a *best response* to s_{-i} if

$$u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i}) \qquad \text{(for all } s_i' \in S_i\text{)}.$$

A strategy s_i is said to be a *best response* to a belief β_{-i} if playing s_i maximizes the expected payoff under β_{-i}, i.e.,

$$u_i\left(s_i, \beta_{-i}\right) \geq u_i\left(s_i', \beta_{-i}\right), \qquad \forall s_i' \in S_i.$$

Best Response

A best response strategy (which could be a mixed strategy) is:

- contingent to a belief (it could no longer be a BR if belief changes)
- not necessarily unique (there could be many best responses)
- NOT strictly dominated by any (pure or mixed) strategy

Theorem 3.1. *Assume that there are finitely many strategies. A strategy s_i is a best response to some belief if and only if s_i is not strictly dominated.*[3]

Example Questions - Belief/BR

Let N = {1,2}, S_1 = {T,B}, S_2={L,R}. The payoff matrix is given as below:

	L	R
T	(1,2)	(3,2)
B	(4,3)	(2,1)

1. What is the payoff of each of player 1's moves if she believes that player 2 is going to play "L" with probability 0.75 and "R" with probability 0.25? What is the best (pure strategy) response in this case?
2. What is the payoff of each of player 2's moves if she believes that player 1 is going to play each strategy with probability 0.5?

Example Solution - Belief/BR

Let $N = \{1,2\}$, $S_1 = \{T,B\}$, $S_2=\{L,R\}$. The payoff matrix is given as below:

	L	R
T	(1,2)	(3,2)
B	(4,3)	(2,1)

1. What is the payoff of each of player 1's moves if she believes that player 2 is going to play "L" with probability 0.75 and "R" with probability 0.25? What is the best (pure strategy) response?

   $u_1(T) = 0.75*1+0.25*3 = 1.5$; $u_1(B) = 0.75*4+0.25*2 = 3.5$; B is the best (pure strategy) response.

2. What is the payoff of each of player 2's moves if she believes that player 1 is going to play each strategy with probability 0.5?

   $u_2(L) = 0.5*2+0.5*3 = 2.5$; $u_2(R) = 0.5*2+0.5*1 = 1.5$.

Example Question - Belief/BR (Continued)

Let $N = \{1,2\}$, $S_1 = \{T,B\}$, $S_2=\{L,R\}$. The payoff matrix is given as below:

	L	R
T	(1,2)	(3,2)
B	(4,3)	(2,1)

3. What is the best response for player 1 if her belief is that player 2 will play L with probability p?

Example Solution - Belief/BR (Continued)

Let $N = \{1,2\}$, $S_1 = \{T,B\}$, $S_2=\{L,R\}$. The payoff matrix is given as below:

	L	R
T	(1,2)	(3,2)
B	(4,3)	(2,1)

3. What is the best response for player 1 if her belief is that player 2 will play L with probability p?

Solution. **$u_1(T) = p*1+(1-p)*3 = 3-2p$; $u_1(B) = p*4+(1-p)*2 = 2+2p$**

If $p>0.25$, then B is the best response. If $p<0.25$, then T is the best response. If $p=0.25$, then any mixed strategy between T and B is a best response.

Ch.3
Dominance

Dominance

Definition 3.1. For any player $i \in N$, a strategy $s_i^* \in S_i$ is said to (*strictly*) *dominate* a strategy $s_i \in S_i$ if

$$u_i\left(s_i^*, s_{-i}\right) > u_i\left(s_i, s_{-i}\right) \qquad \text{(for all } s_{-i} \in S_{-i}\text{)}.$$

Definition 3.4. A strategy s_i^* is said to *weakly dominate* s_i if

$$u_i(s_i^*, s_{-i}) \geq u_i(s_i, s_{-i}) \qquad \text{for all } s_{-i} \in S_{-i},$$

and at least one of these inequalities is strict.

Dominant-Strategy Equilibrium

Definition 3.6. A strategy profile $s^* = (s_1^*, s_2^*, ...s_n^*)$ is said to be a *dominant-strategy equilibrium* if for each player i, s_i^* is a weakly dominant strategy.

Example: Dominant Strategy

What is the dominant strategy equilibrium of the game below?

	L	R
T	(3,3)	(0,1)
M	(1,2)	(1,4)
B	(0,3)	(3,0)

Example: Dominant Strategy

What is the dominant strategy equilibrium of the game below?

	L	R
T	(3,3)	(0,1)
~~M~~	~~(1,2)~~	~~(1,4)~~
B	(0,3)	(3,0)

Example: Dominant Strategy

What is the dominant strategy equilibrium of the game below?

	L	~~R~~
T	(3,3)	~~(0,1)~~
~~M~~	~~(1,2)~~	~~(1,4)~~
B	(0,3)	~~(3,0)~~

Example: Dominant Strategy

What is the dominant strategy equilibrium of the game below?

	L	~~R~~
T	(3,3)	~~(0,1)~~
~~M~~	~~(1,2)~~	~~(1,4)~~
~~B~~	~~(0,3)~~	~~(3,0)~~

Answer: (T, L)

Example Question: Dominant Strategy

Let $N = \{1,2\}$, $S_1 = \{T, B\}$, $S_2 = \{L,R\}$. Make a (separate) normal form game grid and fill the payoffs so that:

1. For player 1, T weakly dominates B, but B is a best response to some belief about player 2's strategy.
2. For each player, none of the strategies weakly dominate the other strategy.

(there are many correct answers.)

Example Solution: Dominant Strategy

Let N = {1,2}, S_1= {T, B}, S_2 = {L,R}. Make a (separate) normal form game grid and fill the payoffs so that:

1. For player 1, T weakly dominates B, but B is a best response to some belief about player 2's strategy.
2. For each player, none of the pure strategies strongly or weakly dominate the other strategy.

Sample solution

1.

	L	R
T	(1, 0)	(1, 0)
B	(1, 0)	(0, 0)

2.

	L	R
T	(1, 0)	(0, 1)
B	(0, 1)	(1, 0)

Explanations:

1. T weakly dominates B for player 1, but for the belief that "player 2 is going to play L with 100% probability", player 1 should be indifferent between T and B.
2. There isn't a pure strategy that dominates the other pure strategy for either player.