Fall 2023
14.12 Game Theory
Tomasz Sadzik

14.12 Midterm Exam

October 26, 2023

You have **80 minutes** to complete the exam.

There are **three questions** worth the following:

- Problem 1: 10 points
- Problem 2: 10 points
- Problem 3: 10 points

This exam is **closed book**. You may not use any electronic devices or written material brought into the exam.

[Intentionally blank]

Problem 1 (10 points)**.** A manufacturer M sells a product to consumers through a retailer R. M chooses a whole-sale price $w \geq 0$ and simultaneously R chooses a surcharge $s \geq 0$. The resulting price to consumers is $p = w+s$. The demand function is $Q(p) = 48-2p$. We ignore all costs of the manufacturer and the retailer. The profit functions are thus given by $\pi_M(w, s) = w(48-2(w+s))$ for the manufacturer and $\pi_R(w, s) = s(48 - 2(w + s))$ for the retailer.

1. What are the best response functions $BR_M(s)$ and $BR_R(w)$?
2. What are the sets of best responses for manufacturer and retailer BR_M^1 and BR_R^1? What are the sets of best responses to best responses BR_M^2 and BR_R^2?
3. What is the lowest n such that price 7 is NOT included in BR_R^n?
4. What is the Nash equilibrium of this game?
5. Manufacturer and retailer consider integrating vertically and charging a single price p to maximize joint profits. Is the optimal price p^I greater or smaller than the total price $w + s$ the consumers face in part 4?

Problem 2 (10 points)**.** Suppose that N people are witnessing a crime. Each witness would not hesitate to contact the police but optimally would like somebody else to contact the police, to spare oneself the trouble. Formally, the players are the N witnesses. Each chooses whether to (C)all the police or (N)ot. The payoff to a witness is $v > 0$ if they do not call, but someone else calls the police. The payoff to a witness is $v - c > 0$ if they call the police (and maybe others too). The payoff is 0 if no one calls the police.

1. There are N pure strategy Nash equilibria. Describe in words (1-2 sentences) what they are.

2. Consider now a mixed strategy Nash equilibrium, in which each player chooses to (N)ot call the police with probability p. If witnesses 2 to N use such strategies, write down the equation that must be satisfied for player 1 to be indifferent between (C)all and (N)ot. *(Hint: Suppose a biased coin falls heads with probability p. If you toss it $N-1$ times, the probability that it falls heads each time is p^{N-1}.)*

3. Think about the probability that in this equilibrium nobody out of the N witnesses reports the crime. When the group size N is very large, is this probability roughly zero, or is it positive?

Problem 3 (10 points)**.** Consider the following game between the seller and the buyer. The seller has a single good to trade. The seller values the good at 0, and the buyer values the good at 1. Each player's utility equals her value from the good (if she gets/keeps it) net any payments she pays/receives. The buyer and seller haggle over the price as follows. Both players discount with discount factor $\delta < 1$. As in class, if a player is indifferent, they accept the offer.

1. Suppose that in period 0, the seller proposes that the good is sold at a price $p_0 \in [0, 1]$. If the buyer accepts, the trade is carried out and the game ends. If the buyer rejects, play proceeds to period 1.

   In period 1, the buyer proposes that the good is sold at a price $p_1 \in [0, 1]$. If the seller accepts, the trade is carried out and the game ends. If the seller rejects, there is no trade and the game ends (i.e., the seller keeps the good).

   In the subgame perfect Nash equilibrium, in which round and at what price is the good sold?

2. Suppose that in period 0, the seller proposes that the good is sold at a price $p_0 \in [0, 1]$. If the buyer accepts, the trade is carried out and the game ends. If the buyer rejects the offer, play proceeds to period 1.

   In period 1, the seller again proposes that the good is sold at a price $p_1 \in [0, 1]$. If the buyer accepts, the trade is carried out and the game ends. If the buyer rejects, play proceeds to period 2.

   In period 2, the buyer proposes that the good is sold at a price $p_2 \in [0, 1]$. If the seller accepts, the trade is carried out and the game ends. If the seller rejects, there is no trade and the game ends (i.e., the seller keeps the good).

   In the subgame perfect Nash equilibrium, in which round and at what price is the good sold?