Linear transformation of Gaussian random vectors

Let x denote a random vector from the multivariate Gaussian distribution

$$x \sim N(\mu, \Sigma).$$

Let $y = Ax + b$ be a linear transformation of x, where A is any matrix and b is a vector. Then

$$y \sim N(A\mu + b, A\Sigma A^T),$$

that is, y has a (multivariate) Gaussian distribution with mean $A\mu + b$ and covariance matrix $A\Sigma A^T$.

Example. Let z be a sample from a standard multivariate Gaussian distribution,

$$z \sim N(0, I).$$

Let $SS^T = \Sigma$, where Σ is a covariance matrix. There are many possible matrices S that satisfy $SS^T = \Sigma$ for a given Σ. Perhaps the one that can be computed most easily is the lower triangular Cholesky factor of Σ. If $y = Sz$, then by the linear transformation of Gaussian random vectors,

$$y \sim N(0, SS^T) = N(0, \Sigma).$$

Thus if we want to generate a sample y from the distribution $N(0, \Sigma)$, we could use the Matlab code

```
L = chol(Sigma, 'lower');
z = randn(n,1);
y = L*z;
```