CX 4803 CML – Spring 2022

Assignment 6

There are three questions. Please show your work as if you were explaining your solution to another student. Submit your solutions as a single pdf file on Canvas.

1. Consider the following constrained optimization problem:

$$\min_{x_1,x_2} \; 2x_1 + 3x_2 - x_1^3 - 2x_2^2$$

subject to

$$\begin{aligned} x_1 + 3x_2 &\leq 6 \\ 5x_1 + 2x_2 &\leq 10 \\ x_1 &\geq 0 \\ x_2 &\geq 0 \end{aligned}$$

What are all the points that satisfy the Karush–Kuhn–Tucker conditions?

2. Find the solution to the following equality-constrained optimization problem:

$$\min_{x} \; \frac{1}{2} x^T H x + h^T x$$

subject to $Ax = b$, where

$$H = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}, \qquad h = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \qquad A = \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}, \qquad b = 4.$$

3. Consider the problem

$$\min_{x_1,x_2} \; x_1^2 + 2x_1x_2$$

subject to $x_1^2\, x_2 \geq 10$.

   (a) Solve the above problem using the method of Lagrange multipliers. Also state the optimal value of the Lagrange multiplier.

   (b) What is the dual problem?

   (c) Solve the dual problem. Be sure to state the optimal value of the Lagrange multipliers for the dual problem.