CX 4803 CML – Spring 2022

Assignment 5

Please show your work as if you were explaining your solution to another student. In particular, please **show all your code** *used to generate the solutions. Submit your solutions as a single pdf file on Canvas. Include your program listings in your pdf file.*

1. Consider a two-class classification problem with the following distributions:

$$p(x|C_1) = N\left(x \,\middle|\, \begin{bmatrix}0\\2\end{bmatrix}, \begin{bmatrix}2 & 0\\0 & 1\end{bmatrix}\right), \qquad p(x|C_2) = N\left(x \,\middle|\, \begin{bmatrix}2\\0\end{bmatrix}, \begin{bmatrix}1 & 0\\0 & 2\end{bmatrix}\right).$$

   Assume $p(C_1) = p(C_2)$.

   What is the equation of the discriminant(s) that separates the two classes? Be sure to show your algebraic derivation.

2. Consider the following 20 training observations in 3 dimensions. For each observation, the fourth column shows if the point is in class 0 or class 1. You can select the data to copy it into your code.

```
0.8147    0.1576    0.6557    0
0.9058    0.9706    0.0357    0
0.1270    0.9572    0.8491    0
0.9134    0.4854    0.9340    0
0.6324    0.8003    0.6787    0
0.0975    0.1419    0.7577    0
0.2785    0.4218    0.7431    0
0.5469    0.9157    0.3922    0
0.9575    0.7922    0.6555    0
0.9649    0.9595    0.1712    0
1.5060    1.2387    1.0760    1
0.8318    1.1816    1.4797    1
1.0769    1.5655    1.4551    1
0.8462    1.5952    0.9626    1
0.8971    0.9869    0.9190    1
1.6235    1.2898    1.2984    1
1.4948    1.2456    1.7597    1
1.1171    1.4463    1.1404    1
1.7502    1.5094    1.3853    1
0.8344    1.5547    1.0238    1
```

   Given a point $[0.9, 1.0, 0.9]$, what the probability that it is in class 0? What is the probabiility that it is in class 1? Use logistic regression. What is the equation of the discriminant surface?

3. Reproduce Figure 4.12 in Bishop. Generate data similar to the data in the Figure. Choose two Gaussian basis functions, one centered at $[-1, -1]^T$ and the other at the origin. The "standard deviations" of these Gaussian basis functions are not specified by Bishop, so you can choose them however you wish. Your figure will not be exactly the same as Bishop's, due to different data and possibly due to different basis functions.

   To perform the minimization in logistic regression, you can use any function you wish (e.g., `fminunc` in Matlab).

   Plot the linear discriminant for the transformed points.

   You do not need to plot the green crosses or the green contours of your basis function in the original space.

   You also do not need to draw the nonlinear decision boundary in the original space (black curve in the left-hand plot). However, if you explain how to compute this curve and also plot it, you will get bonus points for this question.